

Merging probability and possibility for robot localization

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Abstract

A mobile robot localization system based on sensor fusion is described. Data coming from various sensors can require different and often complementary uncertainty models: some are better described by possibility theory, others are intrinsically probabilistic. A logic for belief functions is introduced to axiomatize both semantics as special cases. For each place in a map of the environment, a set of logical rules allows to calculate the belief of the robot's presence, as a function of the partial evidences provided by the individual sensors. Various experimental runs have shown promising results.

1 Introduction

The aim of this work is to apply to robotics a logical framework where different uncertainty models, like belief functions, possibilities (i.e. consonant belief functions) and probabilities (i.e. additive belief functions) can be uniformly axiomatized.

1.1 Uncertainty models

Although probability theory has a leading position in the description of uncertainty, in the last decades other theories have been proposed to characterize different aspects of uncertainty, like possibility theory [Zadeh, 1979] and evidence theory [Dempster, 1967; Shafer, 1976].

So far, as there is no complete agreement on the intended semantics of probability (see [Savage, 1972] for a discussion), even less established are the differences and links between the intended semantics of the three theories.

However, some observations are widely accepted: for example, in evidence theory some properties of probabilities and possibilities can be described. Necessities (the dual measures of possibilities) are consonant belief functions, and probabilities are additive belief functions, or, equivalently, belief functions whose corresponding mass functions assign positive mass only to singletons.

Belief functions describe partially-known probabilities, hence they add the concept of approximation to that of randomness, that characterizes probabilities.

Conversely, if we restrict ourselves to the pure idea of approximation, we find the aspect of uncertainty, sometimes called linguistic uncertainty, described by possibility theory. More precisely, the idea of approximation can be described as a *similarity* relation between an observed event and its prototypical description [Ruspini, 1991]. Various other semantics of possibility theory are illustrated in [Dubois and Prade, 1997; 1998].

Hence probabilities and possibilities describe different and complementary aspects of uncertainty.

1.2 Uncertainty in robot localization

In robot localization, the probabilistic framework is largely predominant. Bayesian methods are proposed in [Burgard *et al.*, 1998; Kortenkamp and Weymouth, 1994; Thrun *et al.*, 1998]. In [Cox and Leonard, 1994; Crowley, 1995; Leonard *et al.*, 1992; Moutarlier and Chatila, 1989; Smith and Cheeseman, 1986] Kalman filter methods are used to update the positions of geometric features of the environment in the presence of uncertainty. In [Burgard *et al.*, 1998; Kaelbling *et al.*, 1996; Nourbakhsh *et al.*, 1995; Simmons and Koenig, 1995] Markov Models are used to describe localization and control strategies in topological or grid-based maps. See [Borenstein *et al.*, 1996; Thrun, 1998] for discussions on various methods for localization and mapping in robotics, based on metric and/or topological maps.

A different perspective is offered by fuzzy/possibilistic approaches (see [Saffiotti, 1997] for a survey). For example, in [Saffiotti and Wesley, 1996] uncertainty in the location of the elements in the environment (doors, walls...) is represented by fuzzy subsets in a global Cartesian map. As the robot navigates, it maintains a local fuzzy map of percepts, and it matches this against the global map, building a number of (fuzzy) hypotheses, one for each percept, of the robot's location. The various hypotheses are then fused using fuzzy intersection. A similar approach is proposed in [Gasós and Martín, 1997], where localization is performed by comparing a partial fuzzy map built during navigation with a pre-existing global map.

In this paper we argue that localization requires a combination of both approaches.

In our experimental work we have often found it useful to interpret sensor data using a degree of reliability, learned from experience and inversely related to the error made by assuming the truth of some event or property; such an error can be seen as the complement of a *similarity degree* between an ideal or prototypical percept and the actual observation. Experiments [Bison *et al.*, 1998; 1999b; Sossai *et al.*, 1999] suggest that the similarity degree has nothing in common with the frequency with which an object is observed; in fact, percepts with a degree of similarity close to 1 are often the most difficult to obtain. This completely rules out the semantics of probabilities (in the frequentist sense).

For example, assume that we have to detect whether we are moving along a wall, using only a sonar sensor: we observe some points, and one way to decide whether they correspond to a wall is to measure how well aligned they are, i.e. to calculate their dispersion around the interpolating line. This dispersion can be assumed as the degree of dissimilarity between the actual observation and the ideal one, where the points are perfectly aligned.

However, it seems reasonable that numbers coming from other sensors may have a completely different semantics. In our experiment we have assumed, following [Thrun *et al.*, 1998; López-Sánchez *et al.*, 1998], that the uncertainty that affects odometry is probabilistic in nature, because odometric errors are more naturally related to pure random events, due to imprecision of the mechanical hardware or unpredictable wheel-floor interaction.

1.3 A logical approach

In order to make it possible to merge the different uncertainty models, we propose the adoption of a logic, called Belief-Functions Logic (BFL) [Sossai, 1999], that can be seen as an extension of possibilistic logic [Boldrin and Sossai, 1997], and has the following properties:

1. BFL contains, for every event L and for every number $\alpha \in (0, 1]$, a formula of the form $(\top_\alpha \rightarrow L)$ with the following meaning: “the belief of L is at least α ”. More precisely, the interpretation of $(\top_\alpha \rightarrow L)$ is the set of all mass functions m s.t. the belief of L computed with m is at least α (shortly: $b_m(L) \geq \alpha$);
2. Dempster’s rule of combination is represented by the connective \otimes ;
3. superadditivity is explicitly representable (see the rule SA in the proof system, section 2.2);
4. probability is axiomatizable: a set of sentences can be defined s.t. the belief functions that satisfy them are only the additive ones (probabilities);
5. possibility is axiomatizable: as in property 4 above, for possibility measures;
6. every proof in BFL determines a function that computes the belief of the conclusion from the beliefs of the premises. This function is directly coded in the robot control algorithm. In this, our approach is

different from AI approaches where the main effort is in implementing the deductive algorithm.

BFL is described more formally in section 2.

1.4 Using the logic

In the style of current research [Kortenkamp and Weymouth, 1994; Kuipers, 1996; Nourbakhsh *et al.*, 1995], we perform localization with respect to a topological map, consisting of places connected by paths, with the addition of local metric information, as done for example in [Gasós and Saffiotti, 1999]. In our work, each place is symbolically described by BFL sentences (more precisely *sequents*, in the form $\Gamma \vdash C$, where Γ is a multiset of formulae and C a formula of the logical language), which relate an abstract description of the place to the perceptual picture provided by the sensors. The logic adopted allows one to code uncertainty by attaching to each sentence a number in $[0,1]$, that represents a degree of confidence (a *belief value* in the terminology of evidence theory). In addition, the formalism allows one to fuse all the pieces of available information and to return, at each time instant t , a resulting degree for each place P_i in the map (see Fig. 1), expressed as the belief value of a proposition like $at_place_{\langle P_i, t \rangle}$. Localization in the map then consists in choosing the place with the highest belief (or in acknowledging disorientation).

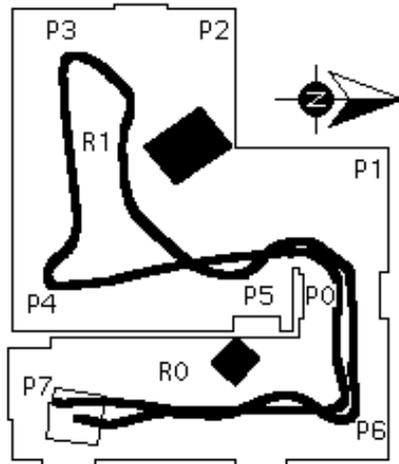


Figure 1: The rooms and the places

2 Belief-Functions Logic (BFL)

2.1 Semantics

As a starting point, we assume that the frame of discernment is described by (the Lindembaum Boolean algebra of) a suitable logical (propositional) language \mathcal{L}_C defined as:

$$\mathcal{L}_C ::= P | \top_C | \perp_C | R \wedge S | R \vee S | \neg R | R \Rightarrow S$$

where \top_C , \perp_C are the symbols for “true” and “false”, \wedge , \vee , \neg , \Rightarrow are the symbols of the usual connectives, \leq

the order relation of the Boolean algebra, and R, S, T denote propositions of \mathcal{L}_C . Let \mathcal{L}_C/\vdash be the Lindembaum (Boolean) algebra of \mathcal{L}_C , i.e. $\mathcal{L}_C/\vdash = \{[R] : R \in \mathcal{L}_C\}$, where $[R] = \{S : S \in \mathcal{L}_C, \vdash R \leftrightarrow S\}$.

Let \mathcal{RM} be the class of functions defined as:

Def. 1 *The set \mathcal{RM} is the set of functions $m : \mathcal{L}_C/\vdash \rightarrow [0, 1]$ such that: $\sum_{R \in \mathcal{L}_C} m(R) = 1$*

In the sequel, to make the notation simpler, we will drop the square brackets, identifying $[R]$ with R . Note that, following the ideas of P. Smets [Smets, 1988], we allow \perp_C to have positive mass. As usual, we will denote with m_1 the vacuous mass function ($m_1(\top_C) = 1$, 0 otherwise), moreover we will consider for every $\alpha \in (0, 1]$ the mass assignments $\mu_\alpha = \frac{1}{\alpha} \cdot m_1$, and $m_{\perp_C}(R) = m_1(\neg R)$.

We assume that $m_{\perp_C} \in \mathcal{RM}$; note also that we are dealing with particular variants of the vacuous mass assignment. Dempster's rule of combination is defined as follows:

Def. 2 *For every $m, n \in \mathcal{RM}$: $m \times n(R) = \sum_{S \wedge T = R} m(S) \cdot n(T)$.*

Our complete set of mass functions \mathcal{M} is the closure of $\mathcal{RM} \cup \{\mu_\alpha : \alpha \in (0, 1]\}$ with respect to \times . Hence $\mathcal{M} = \bigcup_{\alpha \in (0, 1]} \{\frac{1}{\alpha} \cdot m : m \in \mathcal{RM}\}$. The extension of definition 2 to \mathcal{M} is trivial.

From now on we will use the following convention: $l, m, n \in \mathcal{RM}$, $\lambda, \mu, \nu \in \mathcal{M}$. Beliefs are defined as usual with the proviso that the mass of \perp_C must be added. Following P. Smets, we will call such functions *b-functions*:

Def. 3 *For every $\mu \in \mathcal{M}$ and $R \in \mathcal{L}_C$: $b_\mu(R) = \sum_{S \leq R} \mu(S)$*

Let us define the following order relation over the set \mathcal{M} of mass-functions:

Def. 4 *For every $\mu, \nu \in \mathcal{M}$: $\mu \leq \nu$ if and only if $(\forall R)(b_\mu(R) \leq b_\nu(R))$.*

We construct the entire logic in two steps. First, we define the language $\mathcal{L}_I ::= P \mid \perp \mid \top_\alpha \mid \phi \otimes \psi \mid \phi \rightarrow \psi$, where P is an atomic sentence of \mathcal{L}_C and \top_α , with $\alpha \in (0, 1]$, is the proposition with the following meaning: "all the available mass $\geq \frac{1}{\alpha}$ is given to \top_C ". Moreover, we call $\mathcal{L}_1 ::= P \mid \perp \mid L \otimes M \mid L \rightarrow \perp$, and we also assume that $L, M, N \in \mathcal{L}_1$.

The following is the central definition of Local Truth, i.e. the forcing relation $\Vdash \subseteq \mathcal{M} \times \mathcal{L}_I$:

Def. 5 *For every $\lambda, \mu, \nu \in \mathcal{M}$, $\frac{1}{\alpha}$:*

$$\begin{aligned} \lambda \Vdash \perp & \text{ iff } b_\lambda(\perp_C) \geq 1 \\ \lambda \Vdash \top_\alpha & \text{ iff } b_\lambda(\top_C) \geq \frac{1}{\alpha} \\ \lambda \Vdash P & \text{ iff } b_\lambda(P) \geq 1 \\ \lambda \Vdash \phi \otimes \psi & \text{ iff } (\exists \mu \Vdash \phi)(\exists \nu \Vdash \psi)(\lambda \geq \mu \times \nu) \\ \lambda \Vdash \phi \rightarrow \psi & \text{ iff } (\forall \mu \Vdash \phi)(\lambda \times \mu \Vdash \psi) \end{aligned}$$

We use the following abbreviation: $\star\phi = \{\mu : \mu \Vdash \phi\}$.

Let τ be the function from \mathcal{L}_1 to \mathcal{L}_C defined by induction on the length of the formulae as follows: $\tau(\perp) = \perp_C$, $\tau(P) = P$, $\tau(L \otimes M) = \tau(L) \wedge \tau(M)$, $\tau(L \rightarrow \perp) = \neg \tau(L)$.

It can be proved that the logic \mathcal{L}_I contains a copy (\mathcal{L}_1) of the classical (or Boolean) logic \mathcal{L}_C . To this aim, let us define the states of complete information: $m_L(X) = 1$ if $X = \tau(L)$, 0 otherwise, for every $L \in \mathcal{L}_1$.

We use $\uparrow \mu$ to indicate the set $\{\nu : \nu \geq \mu\}$. The following property 1 shows that τ gives an isomorphism between \mathcal{L}_1 and \mathcal{L}_C , while property 2 allows to represent the main fact, i.e. $b(L) \geq \alpha$, in the language:

Property 1 $\star N = \uparrow m_{\tau(N)}$.

Property 2 $\star(\top_\alpha \rightarrow L) = \{\mu : b_\mu(\tau(L)) \geq \alpha\}$.

Note that the two equalities $\star(L \rightarrow \perp) = \uparrow m_{\neg \tau(L)}$ and $\star(L \otimes M) = \uparrow m_{\tau(L) \wedge \tau(M)}$ are consequences of the above property 1, and that using these two equalities we can define all Boolean connectives. In the sequel, when the context is clear, we will identify L with $\tau(L)$. Moreover, \mathcal{L}_I gives a symbolic description of separable support functions. In fact, it is easy to see that $\star(\top_\alpha \rightarrow L) = \uparrow m_L^\alpha$, where m_L^α is the simple support function that assigns α to L and $1 - \alpha$ to \top_C . Separable support functions form a proper subset of b-functions; to have a complete description of all b-functions, we need a language (\mathcal{L}) that is strong enough to describe every b-function. This means that for every mass function m there is a formula s.t. its truth value is $\uparrow m$. To this aim, the extension \mathcal{L} of \mathcal{L}_I is introduced.

We can define new operations between the sets of mass functions of the form $\star\psi$ as follows: for every formula ϕ, ψ : $\star\phi \times \star\psi =_{def} \star(\phi \otimes \psi)$ and $\star\phi \rightarrow \star\psi =_{def} \star(\phi \rightarrow \psi)$. Using these definitions, the following holds: the structure $S = \langle \{\star\phi : \phi \in \mathcal{L}_I\}, \times, \star 1 \rangle$ is a commutative monoid. Moreover, the entailment (\rightarrow) satisfies the adjoint condition: $\star\phi \times \star\psi \subseteq \star\xi$ if and only if $\star\phi \subseteq \star\psi \rightarrow \star\xi$. Thus we have a commutative monoid, S , with \times as product, \rightarrow as its adjoint operator, and \subseteq as order relation, hence using Dedekind-MacNeille theorem [Sambin, 1994] the above structure can be embedded into a Quantale Q .

To construct Q we need the following closure operation: if $X \subseteq S$, then the closure of X is defined as $\overline{X} = \{\star\phi : (\forall \star\psi)(X \subseteq \star\psi \rightarrow \star\phi \subseteq \star\psi)\}$. Using the above closure operator, we have that $Q = \{X \subseteq S : X = \overline{X}\}$. It can be endowed with a lattice structure: $\bigvee_{Q, i \in I} A_i = \overline{\bigcup_{i \in I} A_i}$, $\bigwedge_{Q, i \in I} A_i = \bigcap_{i \in I} A_i$. Moreover, it inherits also the monoidal operation and the entailment: $A \times_Q B = \overline{A \times B}$, where $A \times B = \{\star\phi \times \star\psi : \star\phi \in A, \star\psi \in B\}$, and $A \rightarrow_Q B = \{\star\phi : (\forall \star\psi \in A)(\star\phi \times \star\psi \in B)\}$. It can be shown that $\langle Q, \times_Q, \subseteq, \star 1 \rangle$ is a commutative unital Quantale and $S \subseteq Q$.

Now we are ready to introduce the complete language of our logic:

$$\mathcal{L} = P \mid \top_\alpha \mid A \& B \mid A \otimes B \mid A \oplus B \mid A \rightarrow B \mid 0 \mid \top \mid \perp$$

and the corresponding interpretation function $\| \cdot \| : \mathcal{L} \rightarrow Q$.

Def. 6

$$\begin{aligned}
\|\top\| &= S \\
\|0\| &= \emptyset \\
\|\perp\| &= \{\star\psi : \star\psi \subseteq \star\perp\} \\
\|\top_\alpha\| &= \{\star\psi : \star\psi \subseteq \star\top_\alpha\} \\
\|P\| &= \{\star\psi : \star\psi \subseteq \star P\} \\
\|A \otimes B\| &= \|A\| \times_Q \|B\| \\
\|A \oplus B\| &= \|A\| \vee_Q \|B\| \\
\|A \& B\| &= \|A\| \wedge_Q \|B\| \\
\|A \rightarrow B\| &= \|A\| \rightarrow_Q \|B\|
\end{aligned}$$

From now on we will say $\mu \in \|A\|$ meaning that there is $\star\phi \in \|A\|$ s.t. $\mu \in \star\phi$. Moreover, when the context is clear, we will drop the subscript Q from the connectives.

2.2 Proof system of BFL

1. *b-rules:*

(a) *Structural rules:*

$$\begin{aligned}
\text{id)} & \frac{A \vdash A}{A \vdash A} \\
\text{cut)} & \frac{\Gamma \vdash B \quad \Delta, B \vdash C}{\Delta, \Gamma \vdash C} \\
\text{ex)} & \frac{\Gamma, B, A, \Delta \vdash C}{\Gamma, A, B, \Delta \vdash C} \\
\text{we)} & \frac{\Gamma \vdash C}{\Gamma, L \vdash C}
\end{aligned}$$

(b) *Logical rules:*

$$\begin{aligned}
\&) & \frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C} \\
& \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B} \\
\otimes) & \frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C} \\
& \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B} \\
\oplus) & \frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, B \oplus A \vdash C} \\
& \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B} \\
\rightarrow) & \frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \rightarrow B \vdash C} \\
& \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \\
\top) & \frac{}{\Gamma \vdash \top} \\
0) & \frac{}{\Gamma, 0 \vdash A}
\end{aligned}$$

(c) *Classical rules ($L \in \mathcal{L}_1$):*

$$\begin{aligned}
\text{dn)} & \frac{}{\neg\neg L \vdash L} \\
\text{cont)} & \frac{\Gamma, L, L \vdash C}{\Gamma, L \vdash C}
\end{aligned}$$

(d) *Numerical rules:*

$$\begin{aligned}
\text{PR)} & \top_\alpha \otimes \top_\beta \dashv\vdash \top_{\alpha \cdot \beta} \\
\text{OR)} & \top_\beta \vdash \top_\alpha \text{ for any } \beta \leq \alpha \\
\text{SA)} & \&_{I \subseteq \{1, \dots, n\}} (\top_{\alpha_I} \rightarrow \&_{i \in I} L_i) \\
& \vdash \top_{\sum_{I \subseteq \{1, \dots, n\}} (-1)^{|I|+1} \alpha_I} \rightarrow \bigvee_{i \leq n} L_i
\end{aligned}$$

2. *Additive b-functions:*

$$\text{AD)} \quad \Gamma, \top_\alpha \rightarrow L \vdash \top_{1-\alpha} \rightarrow \neg L$$

3. *Consonant b-functions:*

$$\text{CO)} \quad \Gamma, (\top_\alpha \rightarrow L) \& (\top_\beta \rightarrow M) \vdash \top_{\min\{\alpha, \beta\}} \rightarrow L \& M$$

Theorem 1 *The b-rules are valid.*

Theorem 2 *For every m classical (i.e. $m(\perp_C) = 0$) the following are equivalent:*

1. b_m is additive;
2. for all L there is α s.t. if $m \in \|\top_\alpha \rightarrow L\|$ then $m \in \|\top_{1-\alpha} \rightarrow \neg L\|$.

In a similar way, consonant b-functions can be described in BFL.

Moreover, the following sequents can be proved and will be used in the sequel:

$$\begin{aligned}
\text{IND)} & (\top_\alpha \rightarrow L) \otimes (\top_\beta \rightarrow M) \vdash \top_{\alpha \cdot \beta} \rightarrow L \& M \\
\text{GMP)} & (\top_\alpha \rightarrow L), (\top_\beta \rightarrow (L \rightarrow M)) \vdash \top_{\alpha \cdot \beta} \rightarrow M
\end{aligned}$$

The second sequent describes Generalized Modus Ponens for b-functions.

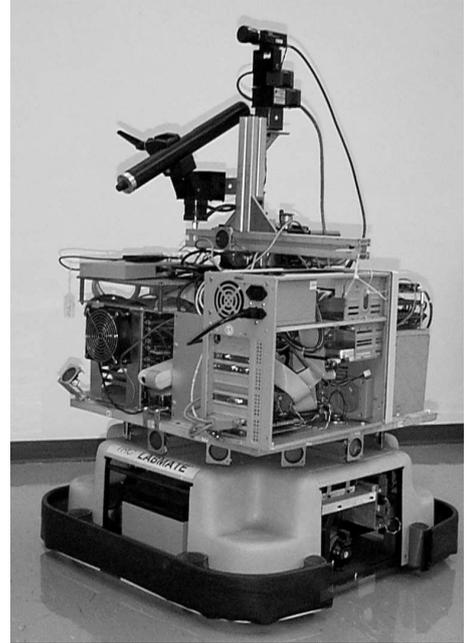


Figure 2: The robot

3 The robot and the environment

The robot (Fig. 2) consists of a commercial base, the TRC Labmate, equipped with a number of additional hardware modules. These include a belt of twelve static Polaroid sonars and a structured-light vision system. This consists of a laser-beam projector generating a horizontal line, and a camera which monitors the changes in the line shape.

The scene used in the experiments is an indoor environment, shown in Fig. 1, consisting of two rooms: a hall (R_1), roughly 6 by 4 meters, and a corridor (R_0). Eight places, denoted in the figure as P_0, \dots, P_7 , have

been defined. For each place, its relative position with respect to the room entrance is known.

The sensors that are used in this experiment are the sonar belt, the structured-light system and the robot odometer.

For odometry, each place P_i is represented by a square S_i . Let τ be the “odometric path”, that is the trajectory denoted by the initial and final readings of the odometer, and by the distance traveled. To each trajectory τ , a probability distribution p_τ is associated, uniform over a square centered on the ending point, and with side proportional to the distance. Let α_2 be the probability that the ending point of τ belongs to S_i . Then the information coming from the odometry is coded in the system using sequent 11 in section 4.3.

Studies on the robot dynamics are in progress, to define a less trivial model, as the triangular distribution of translational and rotational errors used in [Thrun *et al.*, 1998].

For the information coming from the other two sensors we have chosen a possibilistic interpretation.

The sonar belt is used to avoid obstacles during navigation and to detect walls for localization. Wall detection is done using the sonars mounted on the lateral sides of the robot: for each sonar, the last 8 readings, taken at significantly different robot positions, are passed to a least-squares line-fitting algorithm. This returns the best-fitting line for each side, with the associated dispersion value Δ , which is then normalized, to obtain δ_w , the degree of dissimilarity between the observed points and the collinear points representing an ideal wall:

$$\delta_w = \min\left(\frac{\Delta}{\Delta_{max}}, 1\right) \quad (1)$$

The value Δ_{max} is a threshold for dispersion values, beyond which the fitted lines are considered completely unreliable. When δ_w is close to 0, the fitted line is considered a very good approximation of a real wall. The information coming from the sonar belt is coded at the symbolic level using formulae of the form $\vdash \top_{(1-\delta_w)} \rightarrow \text{sonar_wall}_{\langle d,t \rangle}$.

The structured-light system uses the binary image of the projected line as a visual cue for establishing whether the robot is facing a wall or a corner. It provides the degrees of dissimilarity δ_f and δ_c between the visual pattern and, respectively, the prototypical images of a front wall or of a corner.

4 Using BFL for localization

4.1 The map

The operating environment is described by a map consisting of a set of places connected by paths. Each place is represented by:

- a symbolic description of the features (corners, walls) that characterize it,
- metric positional information,

- the architectural element (room, corridor, door) containing the place.

Place representation is given in logical form using BFL sequents.

4.2 Simplified description of a feature

We give below a simplified description of a corner feature (sequent 2) and of the symbolic encoding of sensor information (sequents 3–7):

$$\begin{aligned} \vdash & \left(((\text{sonar_wall}_{\langle North,t \rangle} \ \& \ \text{laser_wall}_{\langle West,t \rangle}) \oplus \right. \\ & \left. (\text{sonar_wall}_{\langle West,t \rangle} \ \& \ \text{laser_wall}_{\langle North,t \rangle}) \oplus \right. \\ & \left. \text{laser_corner}_{\langle North,West,t \rangle} \right) \\ & \rightarrow \text{corner}_{\langle North,West,t \rangle} \end{aligned} \quad (2)$$

$$\vdash \top_{1-\delta_{w_N}} \rightarrow \text{sonar_wall}_{\langle North,t \rangle} \quad (3)$$

$$\vdash \top_{1-\delta_{w_W}} \rightarrow \text{sonar_wall}_{\langle West,t \rangle} \quad (4)$$

$$\vdash \top_{1-\delta_{f_N}} \rightarrow \text{laser_wall}_{\langle North,t \rangle} \quad (5)$$

$$\vdash \top_{1-\delta_{f_W}} \rightarrow \text{laser_wall}_{\langle West,t \rangle} \quad (6)$$

$$\vdash \top_{1-\delta_c} \rightarrow \text{laser_corner}_{\langle North,West,t \rangle} \quad (7)$$

Note that, as we have previously described, $1-\delta_{w_N}$, $1-\delta_{w_W}$, $1-\delta_{f_N}$, $1-\delta_{f_W}$, $1-\delta_c$, computed directly from the sensor data, are interpreted as similarity degrees, thus all the above atomic propositions behave according to a possibilistic semantics. Due to this fact, we are allowed to use the proof system consisting of the b-rules with the addition of CO, the rules that characterize consonant b-functions. Such a proof system, with sequents 2–7, gives a proof of:

$$\vdash \top_\alpha \rightarrow \text{corner}_{\langle North,West,t \rangle} \quad (8)$$

where $\alpha = \max(\min(1-\delta_{w_N}, 1-\delta_{f_W}), \min(1-\delta_{w_W}, 1-\delta_{f_N}), 1-\delta_c)$.

From now on, we can use the above conclusion only with b- and/or CO-rules, but we are not allowed to use the rule AD if the main formula L contains the proposition *corner*.

4.3 Proofs and functions

As anticipated at point 6 of section 1.3, in this section we want to determine the function f_{P_1} , that computes the belief of being at P_1 using the description of the above feature.

For the sake of simplicity, we consider a single-room environment. See [Bison *et al.*, 1998] for an extended description of the possibilistic part, and [Bison *et al.*, 1999a] for a complete axiomatization. The sequents used in this example are:

$$\vdash \top_\alpha \rightarrow \text{corner}_{\langle North,West,t \rangle} \quad (9)$$

$$\vdash \top_{\alpha_1} \rightarrow \text{at_place}_{\langle P_0,t' \rangle} \quad (10)$$

$$\vdash \top_{\alpha_2} \rightarrow \text{path}_{\langle \tau, P_0, P_1, t', t \rangle} \quad (11)$$

$$\begin{aligned} \vdash \top_{k_1} \rightarrow & ((\text{at_place}_{\langle P_0,t' \rangle} \ \& \ \text{path}_{\langle \tau, P_0, P_1, t', t \rangle}) \\ & \rightarrow \text{reached_place}_{\langle P_1,t \rangle}) \end{aligned} \quad (12)$$

$$\vdash \top_{k_2} \rightarrow (\text{corner}_{\langle North,West,t \rangle})$$

$$\begin{aligned}
& \rightarrow \text{sensed_place}_{\langle P_1, t \rangle} & (13) \\
\vdash & (\text{reached_place}_{\langle P_1, t \rangle} \oplus \text{sensed_place}_{\langle P_1, t \rangle}) \\
& \rightarrow \text{at_place}_{\langle P_1, t \rangle} & (14)
\end{aligned}$$

The above sequents have the following readings:

- (9) this is sequent 8 in section 4.2.
- (10) α_1 is the belief that the robot was at P_0 at time t' .
- (11) α_2 is the probability that the robot is at P_1 , knowing that it has certainly been at P_0 and it has followed trajectory τ . It is computed using the probabilistic model of the odometer, described in section 3.
- (12) the value k_1 is the belief that, if the robot was at place P_0 at time t' , and moved along path τ , it reached place P_1 at time t . k_1 is a tuning parameter which accounts for the fact that odometry alone is never sufficient to determine a place.
- (13) this sequent links the feature description to place P_1 . k_2 is a tuning parameter like k_1 .
- (14) the robot knows that it is at P_1 if the odometer or the other sensors believe so.

Using the following notation:

$$\begin{aligned}
L &= \text{at_place}_{\langle P_0, t' \rangle} \\
M &= \text{path}_{\langle \tau, P_0, P_1, t', t \rangle} \\
N &= \text{reached_place}_{\langle P_1, t \rangle} \\
H &= \text{sensed_place}_{\langle P_1, t \rangle} \\
J &= \text{at_place}_{\langle P_1, t \rangle} \\
K &= \text{corner}_{\langle \text{North}, \text{West}, t \rangle}
\end{aligned}$$

sequents 9–14 become:

$$\begin{aligned}
\vdash & \top_{\alpha} \rightarrow K \\
\vdash & \top_{\alpha_1} \rightarrow L \\
\vdash & \top_{\alpha_2} \rightarrow M \\
\vdash & \top_{k_1} \rightarrow (L \& M \rightarrow N) \\
\vdash & \top_{k_2} \rightarrow (K \rightarrow H) \\
\vdash & (N \oplus H \rightarrow J)
\end{aligned}$$

The following sequents are provable. From IND:

$$(\top_{\alpha_1} \rightarrow L) \otimes (\top_{\alpha_2} \rightarrow M) \vdash \top_{\alpha_1 \cdot \alpha_2} \rightarrow (L \& M)$$

From GMP:

$$\begin{aligned}
& \top_{\alpha_1 \cdot \alpha_2} \rightarrow (L \& M), \top_{k_1} \rightarrow ((L \& M) \rightarrow N) \\
& \vdash \top_{k_1 \cdot \alpha_1 \cdot \alpha_2} \rightarrow N \\
& \top_{\alpha} \rightarrow K, \top_{k_2} \rightarrow (K \rightarrow H) \\
& \vdash \top_{k_2 \cdot \alpha} \rightarrow H
\end{aligned}$$

N and H come from independent sensors, hence we can combine them using the \otimes connective, therefore, using IND in the form:

$$\begin{aligned}
& \top_{k_1 \cdot \alpha_1 \cdot \alpha_2} \rightarrow N \otimes \top_{k_2 \cdot \alpha} \rightarrow H \\
& \vdash \top_{k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha} \rightarrow (N \& H)
\end{aligned}$$

we can prove:

$$\vdash \top_{k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha} \rightarrow (N \& H)$$

From this, by superadditivity (rule SA) in the form:

$$\begin{aligned}
& \top_{k_1 \cdot \alpha_1 \cdot \alpha_2} \rightarrow N \& \top_{k_2 \cdot \alpha} \rightarrow H \& \\
& \top_{k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha} \rightarrow (N \& H) \\
& \vdash \top_{k_1 \cdot \alpha_1 \cdot \alpha_2 + k_2 \cdot \alpha - k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha} \rightarrow (N \oplus H)
\end{aligned}$$

we obtain:

$$\vdash \top_{k_1 \cdot \alpha_1 \cdot \alpha_2 + k_2 \cdot \alpha - k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha} \rightarrow (N \oplus H)$$

By GMP, we have that:

$$\vdash \top_{k_1 \cdot \alpha_1 \cdot \alpha_2 + k_2 \cdot \alpha - k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha} \rightarrow \text{at_place}_{\langle P_1, t \rangle}$$

thus we can express the belief of $\text{at_place}_{\langle P_1, t \rangle}$ as the function:

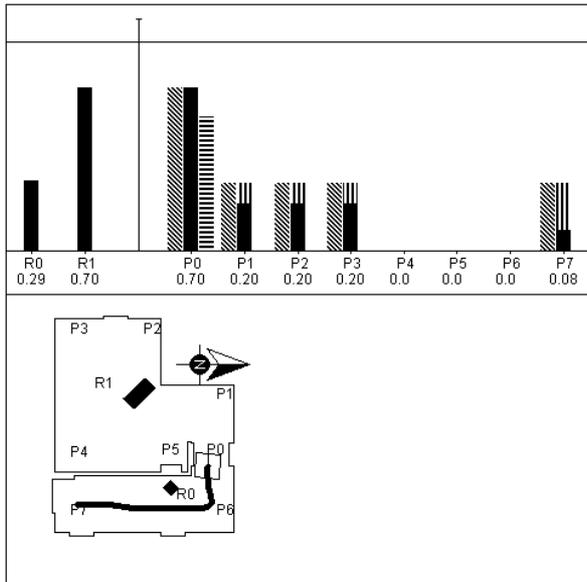
$$\begin{aligned}
f_{P_1}(k_1, \alpha_1, \alpha_2, k_2, \alpha) &= \\
& k_1 \cdot \alpha_1 \cdot \alpha_2 + k_2 \cdot \alpha - k_1 \cdot \alpha_1 \cdot \alpha_2 \cdot k_2 \cdot \alpha
\end{aligned}$$

5 Experimental results

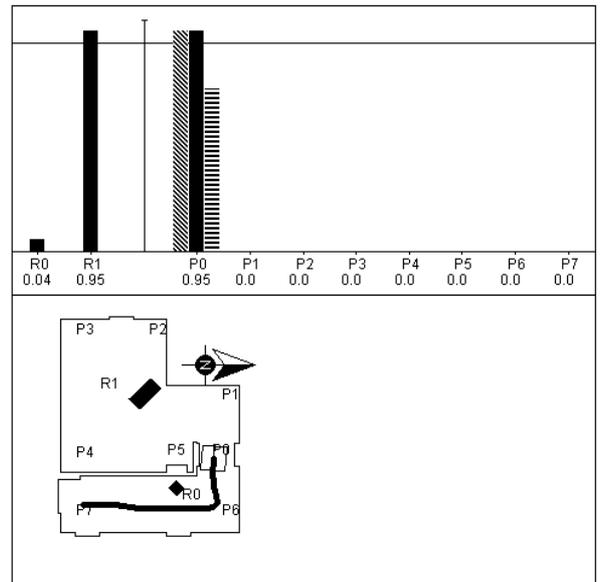
The localization system has been tested in several experimental runs, either by teleoperating the robot, or by using a behavior-based control system. In figure 1 one of these runs is shown, plotted using the position values returned by the odometer.

For each point in the path, the beliefs corresponding to places and rooms were computed. Each figure (3–5) shows a particular position of the robot in the path and the corresponding beliefs through diagrams which plot the belief values for each place and room: the greater the belief, the more likely is the robot to be at the corresponding place. For rooms, the solid bar shows the belief of being in the corresponding room: in the example of figure 3-b, the robot has just entered R_1 and identified it. For places, the solid bar corresponds to the resulting global belief, the horizontally-striped bar on its right to the belief due to the odometry, and the diagonally-striped bar on its left to the belief due to sonars and structured-light system. The vertically-striped bar gives the belief relative to the place, computed without taking into account the information about the room. We say that the robot is at a place P_i when the corresponding belief reaches at least 0.9. If more places had beliefs greater than 0.9, the largest one would be selected. This never happened in the experiments.

The results of the runs can be usefully compared to those of previous experiments [Bison *et al.*, 1998; 1999b; Sossai *et al.*, 1999] with the same robot in the same environment, using only the possibilistic model and no odometry, but no unaccounted obstacles. In the new tests the system performs at least equally well in terms of sheer place-recognition rate, in spite of the presence of obstacles. In addition, the localization appears more robust and stable during navigation, because the degree of a place grows more smoothly and with no oscillations as the robot approaches it.

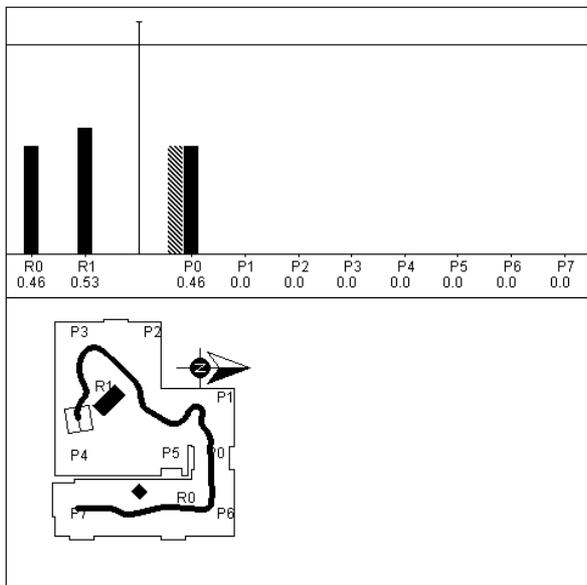


a

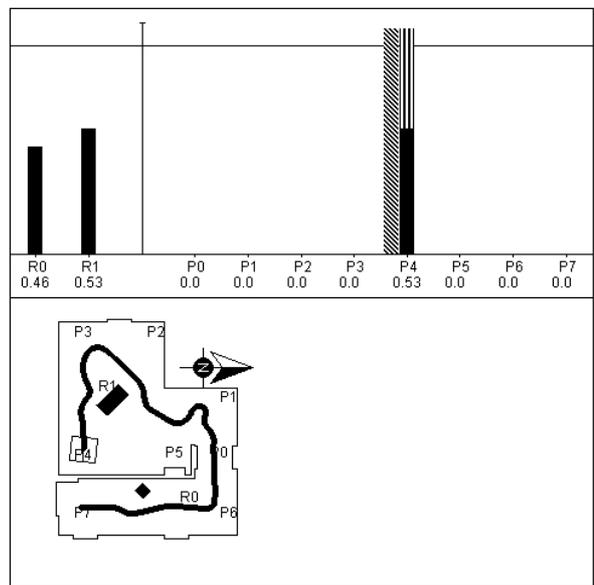


b

Figure 3: Recognition of P_0 and R_1 heading West



a



b

Figure 4: Misjudgment of a place

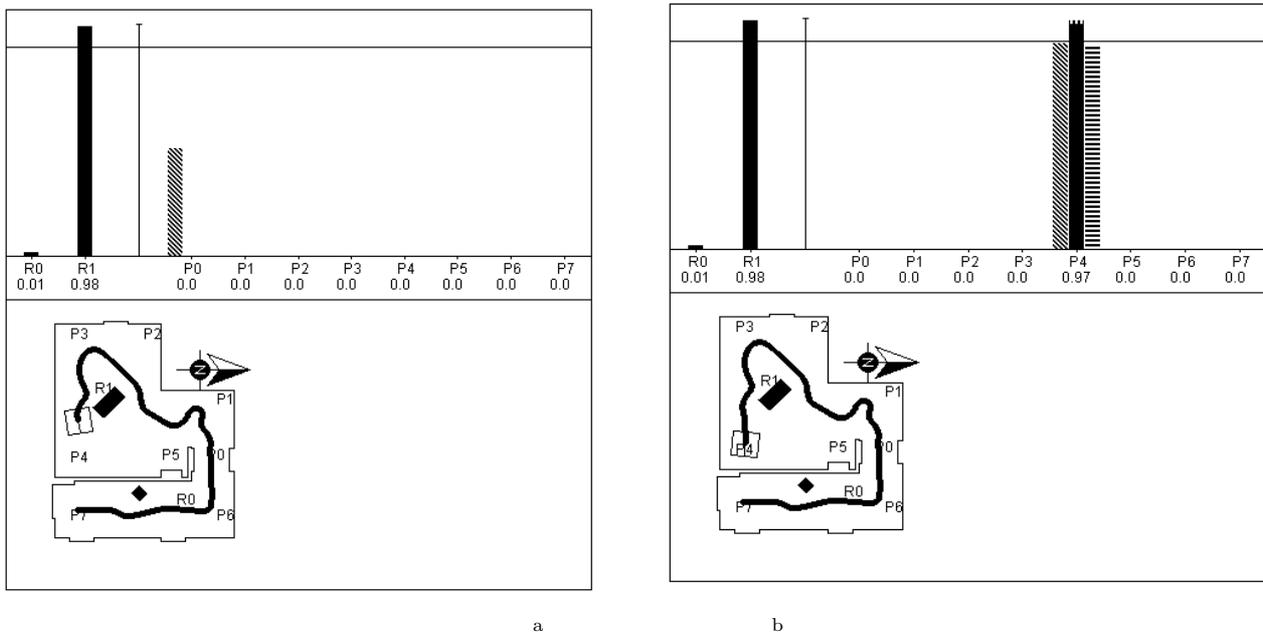


Figure 5: The correct estimate

At some places, recognition is impossible without information fusion, because false positives are produced. For such places it was then assumed that no single percept could provide enough evidence. This was obtained by introducing a scaling degree, like k_1 in sequent 12, which limits the belief of odometry to k_1 . We have chosen $k_1 = 0.89$, just below the recognition threshold.

To give an example (fig. 4-a), with the previous system the robot would recognize as P_0 (i.e. the door between the two rooms) the area between the South-wall and the obstacle. From then on, the robot loses the notion of which room it is in, and therefore fails to recognize P_4 (fig. 4-b) and any other place. With the new approach, the possibilistic part of the system alone is not allowed to provide sufficient evidence to produce the above misjudgment. Only by adding the odometric evidence (fig. 5-a), the robot avoids the pitfall and then correctly spots place P_4 (fig. 5-b).

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