A Constant Approximation Algorithm for Link Scheduling in Arbitrary Networks under Physical Interference Model

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ABSTRACT

Link scheduling is crucial in improving the throughput in wireless networks and it has been widely studied under various interference models. In this paper, we study the link scheduling problem under physical interference model where all senders of the links transmit at a given power P and a link can transmit successfully if and only if the Signal-to-Interference-plus-Noise-Ratio (SINR) at the corresponding receiver is at least a certain threshold. The link scheduling problem is to find a maximum "independent set" (MIS) of links, *i.e.*, the maximum number of links that can transmit successfully in one time-slot, given a set of input links. This problem has been shown to be NP-hard [10]. Here we propose the first link scheduling algorithm with a constant approximation ratio for arbitrary background noise $N \geq 0$. When each link l has a weight $w(l) > 0$, we propose a method for weighted MIS with approximation ratio $O(\min(\log \frac{\max_{l \in \mathcal{L}} w(l)}{\min_{l \in \mathcal{L}} w(l)}, \log \frac{\max_{l \in \mathcal{L}} \|l\|}{\min_{l \in \mathcal{L}} \|l\|}))$, where $\|l\|$ is the Euclidean length of a link l.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication, Network topology; G.2.2 [**Graph Theory**]: Graph algorithms, Network problems

General Terms

Algorithms, Design, Theory

Keywords

Link scheduling, independent set, physical interference model, approximation algorithm.

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1. INTRODUCTION

Wireless ad hoc networks have a wide range of applications. A wireless network can be modeled by a communication graph $G = (V, E)$, where V is the set of all wireless terminals (or called nodes sometimes), and E is the set of links (u, v) where nodes u and v can successfully communicate directly under certain interference restrictions. There are a number of interference models to model the wireless interference constraints. Unlike the traditional wired networks, for wireless networks, the wireless interference constraints often depend on the positions of the wireless nodes. An ultimate goal in wireless networks is to increase the network throughput. Assuming there are a number of multihop flow requests in the network, and λ_1, λ_2, $\cdots$, λ_f be the achieved mean data rates of f flows in the network. In other words, the data packets of the flow i arrive randomly with a mean rate λ_i-bps. Maximizing the network throughput (*i.e.*, $\sum_{i=1}^{f} \lambda_i$) with bounded buffer at every node by carefully designing routing schemes and scheduling the set of links to transmit at every time-slot has been widely studied. It was shown in [19, 20] that scheduling links with maximum total weight at every time-slot will maximize the expected throughput using bounded buffer. Here the weight of a link l is defined as $r(l) \cdot q(l)$, where $r(l)$ is the data rate that can be supported by link l and $q(l)$ is the queue size of the link at the current time-slot.

In this paper, we study the following problems related to throughput maximization in wireless networks. Given a set of links where the endpoints of which are distributed in an Euclidean space, we want to compute the maximum number of links that can be scheduled simultaneously in one time-slot. The second question is, given a set of links, what is the minimum number of time-slots needed to schedule all these links successfully. Notice that unlike the traditional wired networks, finding the maximum number of links that can be scheduled simultaneously is NP-hard questions [1, 4, 9, 10] for wireless ad hoc networks.

There are two different lines in studying the capacity of wireless networks in the literature, depending on the topology of the network. The first line of work investigate the asymptotic capacity of networks where nodes are randomly distributed and the flows are randomly selected with random sources and receivers. The problem of determining the capacity of such networks has been extensively studied, starting with the pioneering work of Gupta and Kumar [11]. The second line of work focus on the throughput of a certain given network, *i.e.*, these results mainly focus on designing efficient algorithms to maximize the throughput, by often assuming that the packets arrive at a constant bit rate. The algorithmic challenges of worst-case networks have also received some attention, *e.g.*, [10,14,15]. These studies suggest that there is a significant gap between the capacities of randomly deployed and worst-case networks. For example, when each node transmits at the same power

P and the data rate of each link is a constant W, the total capacity of all random unicast flows is of order $\Theta(\frac{\sqrt{n}}{\sqrt{\log n}})$ [11], while the total capacity in a given network of some specially selected flows could achieve $O(\Theta(1/n))$ in the worst case (*e.g.*, when all flows have to pass the same link). When power control is allowed, this gap is poly-logarithmic in the number of nodes in the network.

An important issue when studying scheduling algorithms for wireless networks is how to model interference. The most commonly used interference models can be roughly classified into graph-based models and fading channel models. In graph-based models, such as the protocol model [11], we typically model the interference of wireless links by a interference graph H, where the vertices are all links in the communication graph G, and two vertices l_1, and l_2 form an edge (l_1, l_2) if these two links l_1 and l_2 cannot be scheduled to transmit at the same time-slot. For any pair of links l_1 and l_2, if it is *not* connected in the interference graph H, then they can transmit simultaneously. Thus a graph-based model considers interference as a binary and a local measure, *i.e.*, they simply ignore interference beyond a certain range. Such models serve as a useful abstraction of wireless networks, often simplifying the protocol design and proof of protocol efficiency. For example, maximizing the throughput is then equivalent to finding the maximum weighted independent set problem in the interference graph H. However, the protocols based on localized graph-based interference models are not guaranteed to work in a real scenario.

Another category of interference models take all interferences experienced by a receiver into account. This is often called *physical interference model*, in which a signal is received successfully if the SINR (the ratio of the received signal strength to the sum of the interference caused by all other nodes sending simultaneously plus noise) is above a threshold depending on hardware and coding method. As opposed to the graph-based model, in this physical interference model, the definition of a successful transmission accounts also interference generated by transmitters located far away. Consequently, we cannot build an interference graph a priori since the SINR of a receiver depends on the set of nodes that are simultaneously transmitting in each time-slot. Observe that, given a set of links, we can decide whether they can transmit simultaneously. In this paper, we call such a set of links *independent set of links* also. Note that here whether links are independent is not a binary relation anymore. Thus, traditional methods for MIS cannot be directly used for finding a maximum weighted independent set of links in physical interference model. This makes the design and analysis of algorithms more challenging than in graph based models [2, 15, 16]. Goussevskaia *et al.* [10] pointed out that we can construct instances that indicate that the relative error between the accuracy of these two different interference models might be as big as $\Theta(n)$, *i.e.*, linear in the number of nodes.

In the paper, given a communication graph $G = (V, E)$, we study the scheduling problem in the physical interference model, where nodes V are arbitrarily distributed in a two-dimensional Euclidean space. We assume that the strength of signal received by a node v, when node u is transmitting at a power $P(u)$, is $P(u) \cdot g(u, v)$, where $g(u, v)$ is the path loss. The majority results in our paper assume that $g(u, v) = 1/\|uv\|^\alpha$ for a constant $\alpha > 2$. We concentrate our attention on finding the maximum number of links that can transmit simultaneously, where all requests are single-hop and all nodes transmit at a fixed power level. This problem was shown to be NP-complete in [9] and several methods were presented in [1, 10, 13, 15, 17] with various approximation guarantees under different conditions.

The main contributions of this paper are as follows. We first give an algorithm with constant approximation ratio, based on a breakthrough result [10], that maximizes the number of concurrently scheduled links in one time-slot. This means that, given a set of links with an arbitrarily large spread (*i.e.*, the spread of a set of links often is defined as the ratio of the length of longest link over the length of the shortest link), the method returns a subset of links obeying the SINR constraints, of size at least a constant factor of the optimum solution. We then study the problem of finding a subset of "independent" links with maximum weight. When each link l has a weight $w(l) > 0$, we propose a method for weighted MIS with approximation ratio $O(\min(\log \frac{\max_{l\in\mathcal{L}} w(l)}{\min_{l\in\mathcal{L}} w(l)}, \log \frac{\max_{l\in\mathcal{L}} \|l\|}{\min_{l\in\mathcal{L}} \|l\|}))$, where $\|l\|$ is the Euclidean length of a link l.

The rest of the paper is organized as follows. We define the problems to be studied in Section 2 and review the related work in Section 3. Our scheduling algorithms are presented in Section 4. We analyze their performances in Section 5 and conclude the paper in Section 6.

2. PROBLEM FORMULATION

Assume that there is a set V of nodes placed in a 2-dimensional Euclidean space. In the multihop network, there is a set of links $\mathcal{L} = \{l_1, \cdots, l_n\}$, where each link $l_i = (s_i, r_i)$ represents that the sender $s_i \in V$ can directly send a packet to the receiver $r_i \in V$. In this paper, we will study how to maximize the number of links scheduled concurrently in one time-slot.

We assume that each link has a unit-traffic demand, and model the case of non-unit traffic demand by replicating each link multiple times. All nodes are positioned in the Euclidean plane. The distance between two nodes s_i, r_j is denoted by $\|s_i r_j\|$. The length of link l_i is denoted by $\|l_i\|$.

Let $P(u)$ be the transmission power of a node u. We use the notation $P_{ii} = P(s_i) \cdot g(s_i, r_i)$ to denote the power received by receiver r_i from its intended sender s_i, and $I_{ji} = P(s_j) \cdot g(s_j, r_i)$ to denote the interference received by receiver r_i from a concurrent sender s_j. Here $g(u, v)$ is called the path loss from node u to node v. We adopt the physical interference model: a receiver node r_i can successfully receive a packet from a sender s_i if and only if the following condition holds:

$$\frac{P_{ii}}{N + \sum_{l_j \in S_t \setminus \{l_i\}} I_{ji}} \geq \beta \tag{1}$$

Here $\beta > 0$ denotes the minimum SINR required for a packet to be successfully received, N is the ambient noise, and S_t is the set of concurrently scheduled links in time-slot t.

In our studies, as in the majority results in the literature, we assume that $g(s_i, r_i) = \|s_i r_i\|^{-\alpha}$ is the path-loss, where $2 \leq \alpha \leq 5$ is the path-loss exponent. Our results will also hold (with slightly changed proofs) when $g(s_i, r_i) = 1/(1+\|s_i r_i\|^\alpha)$ is the path-loss, to ensure that the received signal strength is at most the transmitted signal strength. In this work we assume that all nodes transmit with the same power level P. Nevertheless, our analysis holds in the case when nodes transmit with different fixed power levels, provided that either the ratio $P_{\max}/P_{\min}$ between the maximum and the minimum power levels is bounded by a constant, or there are only a constant number of possible power levels. Observe that we assume that nodes can be placed arbitrarily in the plane, possibly in a worst-case fashion. The distance between a sender s_i and a receiver r_j could be arbitrarily small or large. The ratio $\max_{l_i\in\mathcal{L}\|l_i\|} / \min_{l_i\in\mathcal{L}} \|l_i\|$ is often called the *spread*, denoted as $\vartheta(\mathcal{L})$, of the links. We generally do not assume that the spread of links is bounded by any constant.

3. RELATED WORK

Link scheduling in randomly deployed wireless networks has been intensely studied from the information theory perspective [8, 11]. However, these results typically do not provide algorithmic tools to find a feasible and efficient link scheduling. In wireless networks, whether a given subset of links can be scheduled to transmit simultaneously depends on the wireless interference model. Typically, in the literature, two different models are used to model the interference. In the simplest models, the interference relations can be described by an interference graph H, in which every link e is a vertex in H and two vertices are connected by an edge if they cannot transmit simultaneously. For example, the widely used protocol interference model, the fixed power protocol interference model are in this category. The unit-disk graph (UDG) model is a special case of protocol interference model in which a receiver v is interfered by another sender w if and only if $\|vw\| \leq 1$. Scheduling problems in this interference graph model can be solved using some graph-based methods [12, 13, 18]. However, this simple interference graph model cannot capture many features of wireless networks. A more complex model is the SINR model in which each sender u transmits at a power $P(u)$ and we assume that the signal strength received by the receiver v is $P(u) \cdot g(u, v)$ where $g(u, v)$ is a distance-dependent path loss. A transmission can only be regraded as successful if the signal-to-interference-plus-noise-ratio (SINR) $\frac{P(u) \cdot g(u,v)}{N + \sum_w P(w) g(w,v)}$ is more than some specified threshold.

The problem of joint scheduling and power control in the SINR model has been well studied previously. For instance, in [5, 6], optimization models and heuristics for this problem are proposed. In [7, 17], topology control with SINR constraints is studied. In [15], a power-assignment algorithm which schedules a strongly connected set of links in poly-logarithmic time is presented. In [3], the combined problem of routing and power control is addressed.

In [9], the scheduling problem without power control in the SINR model, where nodes are arbitrarily distributed in Euclidean space, has been shown to be NP-complete. A greedy scheduling algorithm with approximation ratio of $O(n^{1-2/(\Psi(\alpha)+\epsilon)}(\log n)^2)$, where $\Psi(\alpha)$ is a constant that depends on the path-loss exponent α, is proposed in [2]. Notice that this result can only hold when the nodes are distributed uniformly at random in a square of unit area. In [9], an algorithm with a factor $O(g(L))$ approximation guarantee in arbitrary topologies, where $g(L) = \log \vartheta(L)$ is the diversity of the network, is proposed. In [4], an algorithm with approximation guarantee of $O(\log \Delta)$ was proposed, where Δ is the ratio between the maximum and the minimum distances between nodes. Obviously, it can be arbitrarily larger than $\vartheta(L)$. Most recently, Goussevskaia *et al.* [10] propose an algorithm which has a constant approximation guarantee. Unfortunately, their proofs work only when the background noise is $N = 0$. In contrast to all the above mentioned approaches, our algorithm has a constant approximation guarantee, independent of the network topology, as long as the background nose is a non-negative constant.

There is another line of research assuming that the power of each node is adjustable. Formally, let P_i be the power used by link l_i (it can be 0). We assume that there is a maximum power P_{max} with which any node can transmit. In this model, we try to maximize the number of links scheduled concurrently in one-time slot, *i.e.*, we want to choose the transmission power levels P_i to maximize the total number of links that can transmit simultaneously. This problem has been proven that is NP-Hard in [1] recently. In their paper, they also propose some approximation algorithms: There exists a polynomial time algorithm that always finds a solution that is within a factor $O(\log \frac{\max_{l \in \mathcal{L}} \|l\|}{\min_{l \in \mathcal{L}} \|l\|})$ of optimal.

4. ONE-SLOT SCHEDULING ALGORITHM

Here we provide algorithms to achieve the first objectives defined in Section 2: One-Slot scheduling which is to maximize the number of links scheduled concurrently in one time-slot. Our method is built upon a recent breakthrough result presented in [10]. To describe our algorithms, we start with some definitions, also used in [10]. The *relative interference* (RI) of a link l_j on link l_i is defined as,

$$RI_j(l_i) = I_{ji}/P_{ii}$$

The *affectedness* on link l_i, denoted as $a_S(l_i)$, caused by a set S of links, is the sum of the relative interferences of the links in S on l_i, as well as the effect of noise, scaled by β. Specifically, we have

$$a_S(l_i) = \beta(\frac{N}{P_{ii}} + \sum_{l_j \in S} RI_j(l_i)) = \beta \cdot \frac{N + \sum_{l_j \in S} I_{ji}}{P_{ii}}$$

From Equation 1, we know that a solution S (*i.e.*, a subset of links) is *valid* if and only if the affectedness (by the other nodes in S) on each link in S is at most 1. Thus for any link l_i in S,

$$1 \geq a_S(l_i) \geq \frac{\beta N}{P_{ii}} \geq \frac{\beta N \|l_i\|^\alpha}{P} \Longrightarrow \|l_i\| \leq \sqrt[\alpha]{\frac{P}{\beta N}}$$

Assume $\delta = \sqrt[\alpha]{\frac{P}{3\beta N}}$, then $\|l_i\| \leq \sqrt[\alpha]{3}\delta$. Thus any link with length larger than $\sqrt[\alpha]{3}\delta$ can not transmit and can be discarded. Now all remaining links in $\mathcal{L}$ has length no longer than $\sqrt[\alpha]{3}\delta$.

The basic idea of our scheduling method is as follows: We partition the links into two disjoint groups $\mathcal{L}_1$ and $\mathcal{L}_2$: one group contains all links with length at most δ, and the other group contains all links with length larger than δ. For the first group of links, we apply Algorithm 2 which was first presented in [10], to find a solution $\mathcal{S}_1$. For the second group of links, we apply a new method (Algorithm 3) to find a solution $\mathcal{S}_2$. Algorithm 2 and 3 will be described in Section 4.1 and 4.2 respectively. Then the better of these two solutions will be returned as the final solution.

Algorithm 1 One-Slot Link Scheduling

Input: Set of links $\mathcal{L} = \{l_1, l_2, \cdots, l_n\}$, and a constant $\delta = \sqrt[\alpha]{\frac{P}{3\beta N}}$ where P is the uniform transmission power, N is the background noise;
Output: One slot schedule $\mathcal{S}$;

for $i = 1, \cdots, n$ **do**
  if $\|l_i\| \leq \delta$ **then**
    add l_i to $\mathcal{L}_1$
  else
    add l_i to $\mathcal{L}_2$
apply First-Fit Scheduling (Algorithm 2) to $\mathcal{L}_1$. Assume the result is $\mathcal{S}_1$.
apply Partition Scheduling (Algorithm 3) to $\mathcal{L}_2$. Assume the result is $\mathcal{S}_2$.
if $|\mathcal{S}_1| \geq |\mathcal{S}_2|$ **then**
  $\mathcal{S} = \mathcal{S}_1$
else
  $\mathcal{S} = \mathcal{S}_2$
return $\mathcal{S}$

Algorithm 1 describes our method for link scheduling in one time-slot. Here $|S|$ is the cardinality of a set S. We will analyze its approximation ratio in next section.

4.1 Link scheduling for $\mathcal{L}_1$

Algorithm 2 greedily schedules links in an increasing order of lengths. After a link l_i is selected to the solution S, its validness

Algorithm 2 First-Fit Scheduling (Goussevskaia *et al.* [10])

Input: Set of links $\mathcal{L}_1 = \{l_1, l_2, \cdots, l_n\}$ sorted in the increasing order of length;
Output: Set of links $\mathcal{S}$;

```
1: Set c_1 according to Equation (2);
2: repeat
3:     Add the shortest link l_i ∈ L_1 to S;
4:     Delete all l_j ∈ L_1, where ||s_j r_i|| ≤ c_1 · ||s_i r_i||;
5:     Delete all l_j ∈ L_1, where a_S(l_j) ≥ 2/3;
6: until L_1 = ∅;
7: return S;
```

is guaranteed by two deleting rules. First (line 4), all links l_j (remaining in $\mathcal{L}_1$) within the radius $c_1 \cdot \|s_i r_i\|$ of the receiver r_i are deleted. (c_1 is a constant always bigger than 2). Second (line 5), all links l_j, with the affectedness $a_S(l_j)$ at least a threshold of $2/3$, are deleted. The process is repeated until all links in $\mathcal{L}_1$ have been either scheduled or deleted. It has been shown in [10] that steps 1 and 2 ensures that the greedily constructed solution does not lose its feasibility after adding new links. Later we prove that the obtained schedule for $\mathcal{L}_1$ is both correct and competitive. Notice that here in Algorithm 2, the constant $2/3$ could be any constant in $(0, 1)$.

4.2 Link scheduling for $\mathcal{L}_2$

For any link $l_i \in \mathcal{L}_2$, we know $\delta \leq \|l_i\| \leq \sqrt[\alpha]{3}\delta < 2\delta$. Using this property, we can schedule links in $\mathcal{L}_2$ based on a partition scheme. We subdivide the plane into grids by using a set of vertical lines $a_v : x = v \cdot \delta$ ($v \in \mathbb{Z}$) and horizontal lines $b_h : y = h \cdot \delta$ ($h \in \mathbb{Z}$). Hereafter $v(h)$ is called the index of the vertical (horizontal) line a_v (b_h). We call the square formed by a pair of neighboring vertical lines a_v, a_{v+1} and neighboring horizontal lines b_h, b_{h+1} as $g_{v,h}$. Then our partition scheduling method for $\mathcal{L}_2$ within constant approximation is shown in Algorithm 3.

Algorithm 3 Partition Scheduling

Input: Set of links $\mathcal{L}_2 = \{l_1, l_2, \cdots, l_n\}$;
Output: Set of links $\mathcal{S}$;

```
Set c_2 according to Equation (3);
for r = 0, ..., c_2 and s = 0, ..., c_2 do
    for i, j ∈ Z do
        if i mod (c_2 + 1) = r and j mod (c_2 + 1) = s then
            select one link whose sender located within g_{i,j};
    all the selected links form a set S_{r,s};
Let S be the S_{r,s} with the largest size;
return S;
```

5. PERFORMANCE ANALYSIS

5.1 Correctness

Since the returning result for Algorithm 1 comes from either Algorithm 2 or 3, thus we only need to prove the correctness of Algorithm 2 and 3.

LEMMA 1. *Algorithm 2 produces a valid solution.*

PROOF. When a link l_i is selected, since it was not deleted by the second deleting rule in previous steps, the interference on l_i by selected links shorter than l_i (noted as $\mathcal{S}_i^-$) plus the ambient noise N satisfies:

$$a_{\mathcal{S}_i^-}(l_i) < 2/3$$

It remains to show that the set of selected links longer than l_i (noted as $\mathcal{S}_i^+$) affects l_i by at most $1/3$.

For any two links l_j and l_k in $\mathcal{S}_i^+$ (assume $\|l_j\| \leq \|l_k\|$), by the first deleting rule, we know $\|s_k r_j\| \geq c_1 \cdot \|l_j\|$, otherwise l_k will be deleted when l_j is selected. From triangular inequality, $\|s_k s_j\| \geq \|s_k r_j\| - \|l_j\| \geq c_1 \cdot \|l_j\| - \|l_j\| = (c_1 - 1) \cdot \|l_j\| \geq (c_1 - 1) \cdot \|l_i\|$. Therefore, disks D_i of radius $\frac{(c_1-1)\cdot\|l_i\|}{2}$ around senders in $\mathcal{S}_i^+$ do not intersect.

Then the sender set in $\mathcal{S}_i^+$ is partitioned into concentric rings $Ring_k$ ($k = 0, 1, \cdots, +\infty$) of width $c_1 \cdot \|l_i\|$ around the receiver r_i. Each ring $Ring_k$ contains all senders $s_j \in \mathcal{S}_i^+$, for which $k \cdot (c_1 \cdot \|l_i\|) < \|s_j r_i\| \leq (k+1) \cdot (c_1 \cdot \|l_i\|)$. The first ring $Ring_0$ does not contain any sender. Consider all senders $s_j \in Ring_k$ for some integer $k > 0$, all discs of radius $\|l_i\|(c_1 - 1)/2$ around each sender s_j must be located entirely in an extended ring $Ring_k$ of area

$$\begin{aligned} A(Ring_k) &= [((k+1) \cdot c_1 \cdot \|l_i\| + \frac{(c_1-1)\cdot\|l_i\|}{2})^2 - \\ &\quad (k \cdot c_1 \cdot \|l_i\| - \frac{(c_1-1)\cdot\|l_i\|}{2})^2] \cdot \pi \\ &= (2k+1) \cdot 2c_1(c_1-1) \cdot \|l_i\|^2 \cdot \pi \\ &< (2k+1) \cdot 2c_1^2 \cdot \|l_i\|^2 \cdot \pi \end{aligned}$$

Since disks D_i of area $A(D_i) = (\frac{(c_1-1)\cdot\|l_i\|}{2})^2 \cdot \pi$ around senders in S_i^+ do not intersect, we can use an area argument to bound the number of senders inside each ring. The total interference by senders located in $Ring_k$ ($k \geq 1$) is bounded by

$$\begin{aligned} I_{Ring_k}(l_i) &\leq \sum_{s_j \in Ring_k} I_{s_j}(l_i) \leq \frac{A(Ring_k)}{A(D_i)} \cdot \frac{P}{(kc_1\|l_i\|)^\alpha} \\ &\leq \frac{(2k+1)}{k^\alpha} \cdot \frac{P}{\|l_i\|^\alpha} \cdot \frac{2^3 c_1^2}{c_1^\alpha (c_1-1)^2} \\ &\leq \frac{1}{k^{\alpha-1}} \cdot \frac{P}{\|l_i\|^\alpha} \cdot \frac{2^5 3}{c_1^\alpha} \end{aligned}$$

where the last inequality holds since $k \geq 1 \Rightarrow 2k+1 \leq 3k$ and $c_1 \geq 2 \Rightarrow c_1 - 1 \geq c_1/2$. Summing up the interferences over all rings yields

$$\begin{aligned} I_{\mathcal{S}_i^+}(l_i) &< \sum_{k=1}^{\infty} I_{Ring_k}(l_i) \leq \sum_{k=1}^{\infty} \frac{1}{k^{\alpha-1}} \cdot \frac{P}{\|l_i\|^\alpha} \cdot \frac{2^5 3}{c_1^\alpha} \\ &< \frac{\alpha-1}{\alpha-2} \cdot \frac{P}{\|l_i\|^\alpha} \cdot \frac{2^5 3}{c_1^\alpha} \end{aligned}$$

where the last inequality holds since $\alpha > 2$. This results in affectedness

$$a_{\mathcal{S}_i^+}(l_i) = \frac{\beta I_{\mathcal{S}_i^+}(l_i)}{P_{ii}} < \frac{\alpha-1}{\alpha-2} \cdot \frac{2^5 3\beta}{c_1^\alpha} \leq 1/3$$

$$\text{where } c_1 = \max\left(2, \left(2^5 3^2 \beta \frac{\alpha-1}{\alpha-2}\right)^{\frac{1}{\alpha}}\right) \tag{2}$$

We have shown that $\forall l_i \in \mathcal{S}$, $a_{\mathcal{S}}(l_i) \leq 2/3 + 1/3 = 1$, which means that $SINR(l_i) \geq \beta$ for every scheduled link. This concludes the proof. □

LEMMA 2. *Algorithm 3 produces a valid solution.*

PROOF. For any two links l_i and l_j in $\mathcal{S}$, since they are in different grids, $\|s_i s_j\| \geq c_2 \cdot \delta$. Thus, disks D of radius $\frac{c_2\cdot\delta}{2}$ around senders in $\mathcal{S}$ do not intersect.

For a link $l_i \in \mathcal{S}$, we partition the sender set in $\mathcal{S}$ into concentric rings $Ring_k$ ($k = 0, 1, \cdots, +\infty$) of width $d = (c_2 - \sqrt[\alpha]{3}) \cdot$

δ around the receiver r_i. Each ring $Ring_k$ contains all senders $s_j \in \mathcal{S}$, for which $k \cdot d < \|s_j r_i\| \leq (k+1) \cdot d$.

$$\text{Since } \|s_j s_i\| \geq c_2 \cdot \delta \text{ and } \|l_i\| \leq \sqrt[\alpha]{\frac{P}{\beta N}} = \sqrt[\alpha]{3}\delta \Rightarrow$$
$$\|s_j r_j\| \geq c_2 \cdot \delta - \|l_i\| \geq c_2 \cdot \delta - \sqrt[\alpha]{3}\delta = (c_2 - \sqrt[\alpha]{3}) \cdot \delta$$

The first ring $Ring_0$ does not contain any sender.

Consider all senders $s_j \in Ring_k$ for some integer $k > 0$. All discs of radius $\frac{c_2 \cdot \delta}{2}$ around each s_j must be located entirely in an extended ring $Ring_k$ of area

$$A(Ring_k) = [((k+1)d + \frac{c_2\delta}{2})^2 - (kd - \frac{c_2\delta}{2})^2] \cdot \pi$$
$$= (2k+1) \cdot (d^2 + c_2\delta \cdot d) \cdot \pi$$

Since disks D of area $A(D) = (\frac{c_2 \cdot \delta}{2})^2\pi$ around senders in $\mathcal{S}$ do not intersect, we can use an area argument to bound the number of senders inside each ring. The total interference by senders located in $Ring_k$ ($k \geq 1$) is bounded by

$$I_{Ring_k}(l_i) \leq \sum_{s_j \in Ring_k} I_{s_j}(l_i) \leq \frac{A(Ring_k)}{A(D)} \cdot \frac{P}{(k \cdot d)^\alpha}$$
$$\leq \frac{1}{k^{\alpha-1}} \cdot \frac{P}{d^\alpha} \cdot \frac{12 \cdot (d^2 + c_2 \cdot \delta d)}{c_2^2 \cdot \delta^2}$$
$$= \frac{1}{k^{\alpha-1}} \cdot \frac{12 \cdot P}{((c_2 - \sqrt[\alpha]{3}) \cdot \delta)^\alpha} \cdot \frac{((c_2 - \sqrt[\alpha]{3}) \cdot \delta)^2 + c_2 \cdot \delta(c_2 - \sqrt[\alpha]{3}) \cdot \delta}{c_2^2 \cdot \delta^2}$$
$$= \frac{1}{k^{\alpha-1}} \cdot \frac{12 \cdot P}{\delta^\alpha} \cdot \frac{(c_2 - \sqrt[\alpha]{3})^2 + c_2 \cdot (c_2 - \sqrt[\alpha]{3})}{(c_2 - \sqrt[\alpha]{3})^\alpha \cdot c_2^2}$$
$$= \frac{1}{k^{\alpha-1}} \cdot \frac{P}{\delta^\alpha} \cdot \frac{12 \cdot (2c_2 - \sqrt[\alpha]{3})}{c_2^2 \cdot (c_2 - \sqrt[\alpha]{3})^{\alpha-1}}$$

where the last inequality holds since $k \geq 1 \Rightarrow 2k+1 \leq 3k$ and $c \geq 2 \Rightarrow c-1 \geq c/2$. Summing up the interferences over all rings yields

$$I_{\mathcal{S}}(l_i) < \sum_{k=1}^{\infty} I_{Ring_k}(l_i)$$
$$\leq \sum_{k=1}^{\infty} \frac{1}{k^{\alpha-1}} \cdot \frac{P}{\delta^\alpha} \cdot \frac{12 \cdot (2c_2 - \sqrt[\alpha]{3})}{c_2^2 \cdot (c_2 - \sqrt[\alpha]{3})^{\alpha-1}}$$
$$< \frac{\alpha-1}{\alpha-2} \cdot \frac{P}{\delta^\alpha} \cdot \frac{12 \cdot (2c_2 - \sqrt[\alpha]{3})}{c_2^2 \cdot (c_2 - \sqrt[\alpha]{3})^{\alpha-1}}$$

where the last inequality holds since $\alpha > 2$. Since $P_{ii} = \frac{P}{\|l_i\|^\alpha} \geq \frac{P}{(\sqrt[\alpha]{3}\delta)^\alpha} \geq \frac{1}{3} \cdot \frac{P}{\delta^\alpha}$ and $\frac{\beta N}{P_{ii}} = \frac{\beta N}{P} \cdot \|l_i\|^\alpha$, this results in affectedness

$$a_{\mathcal{S}}(l_i) = \beta \cdot \frac{I_{\mathcal{S}}(l_i) + N}{P_{ii}} = \frac{\beta N}{P_{ii}} + \frac{\beta \cdot I_{\mathcal{S}}(l_i)}{P_{ii}}$$
$$< \frac{\beta N}{P_{ii}} + \frac{\alpha-1}{\alpha-2} \cdot 3 \cdot \frac{12 \cdot (2c_2 - \sqrt[\alpha]{3})\beta}{c_2^2 \cdot (c_2 - \sqrt[\alpha]{3})^{\alpha-1}}$$
$$< \frac{\beta N}{P_{ii}} + \frac{\alpha-1}{\alpha-2} \cdot \frac{54 \cdot 2^{\alpha-1} \cdot \beta}{c_2^\alpha}$$

$$\text{where } c_2 = \max\left(2, \left(\frac{\alpha-1}{\alpha-2} \cdot 54 \cdot 2^{\alpha-1} \cdot \beta\right)^{\frac{1}{\alpha}}\right) \quad (3)$$

We have shown that $\forall l_i \in \mathcal{S}$, $a_{\mathcal{S}}(l_i) \leq 1$, which means that $SINR(l_i) \geq \beta$ for every scheduled link. This concludes the proof. □

From Lemma 1 and 2, we can derive Theorem 3 immediately.

THEOREM 3. *Algorithm 1 is a valid solution.*

5.2 Approximation Ratio

Similarly, to analyze approximation ratio of Algorithm 1, we need the approximation ratio analysis of Algorithm 2 and 3. We define our solution for $\mathcal{L}_1$ produced by Algorithm 2 is $ALG(\mathcal{L}_1)$ and the optimal solution is $OPT(\mathcal{L}_1)$ (similarly we define $ALG(\mathcal{L}_2)$ and $OPT(\mathcal{L}_2)$). We overload these terms to refer also to the sizes of these sets.

First we analyze the relationship between $ALG(\mathcal{L}_1)$ and $OPT(\mathcal{L}_1)$). We define $OPT'(\mathcal{L}_1) = OPT(\mathcal{L}_1) \backslash ALG(\mathcal{L}_1)$ and divide $OPT'(\mathcal{L}_1)$ into $OPT_a(\mathcal{L}_1)$ and $OPT_b(\mathcal{L}_1)$. $OPT_a(\mathcal{L}_1)$ consists of links in $OPT'(\mathcal{L}_1)$ deleted by the first deleting rule of Algorithm 2(line 4), and $OPT_b(\mathcal{L}_1)$ consists of links deleted by the second rule (line 5). If we can bound both ratios of $OPT_a(\mathcal{L}_1)$ and $OPT_b(\mathcal{L}_1)$ to $ALG(\mathcal{L}_1)$, then our proof is done.

LEMMA 4. *$OPT_a(\mathcal{L}_1) \leq \rho_a \cdot ALG(\mathcal{L}_1)$, where $\rho_a = (2c_1+1)^\alpha$ and c_1 is defined in Equation 2.*

PROOF. Consider the set X_i from $OPT_a(\mathcal{L}_1)$ deleted by the first rule of Algorithm 2 when link l_i was selected. Each link in X_i is of length at least $\|l_i\|$. Consider the longest link l_j, the affectness on l_j by X_i. Assume $\|X_i\| \geq (2c+1)^\alpha + 1$, then

$$a_{X_i}(l_j) = \beta \cdot \frac{\sum_{s_k \in X_i} \frac{P}{\|s_k r_j\|^\alpha} + N}{\frac{P}{\|l_j\|^\alpha}} \geq \frac{\sum_{s_k \in X_i} \frac{P}{\|s_k r_j\|^\alpha}}{\frac{P}{\|l_j\|^\alpha}}$$
$$\geq \frac{\sum_{s_k \in X_i} (\|s_k s_j\| + \|l_j\|)^{-\alpha}}{\|l_j\|^{-\alpha}}$$
$$\geq \frac{\sum_{s_k \in X_i} (2c \cdot \|l_i\| + \|l_j\|)^{-\alpha}}{\|l_j\|^{-\alpha}}$$
$$\geq \frac{\sum_{s_k \in X_i} (2c \cdot \|l_j\| + \|l_j\|)^{-\alpha}}{\|l_j\|^{-\alpha}}$$
$$\geq \frac{\rho_a \cdot (2c \cdot \|l_j\| + \|l_j\|)^{-\alpha}}{\|l_j\|^{-\alpha}}$$
$$\geq \frac{\rho_a}{(2c+1)^\alpha} \geq 1$$

which leads to a contradiction. Thus $\|X_i\| \leq (2c+1)^\alpha = \rho_a$. Therefore $OPT_a(\mathcal{L}_1) \leq \rho_a \cdot ALG(\mathcal{L}_1)$. □

To bound the ratio of $OPT_b(\mathcal{L}_1)$ to $ALG(\mathcal{L}_1)$, we need the following two definitions and Lemma 5.

DEFINITION 1. *Let $\mathcal{R}$ and $\mathcal{B}$ be two disjoint sets of points in a metric space $(\mathcal{V}, d)$. We call them red and blue points, respectively. For q a positive integer, a point $b \in \mathcal{B}$ is q-blue-dominant if every ball $B_\delta(b)$ around b, comprised by points w such that $\|w, b\| \leq \delta$, contains $q \in Z^+$ times more blue points than red points. Formally, $\forall \delta \in R_0^+ : \|B_\delta(b) \cap \mathcal{B}\| > q \cdot \|B_\delta(b) \cap \mathcal{R}\|$.*

DEFINITION 2. *Let $\mathcal{R}$ and $\mathcal{B}$ be defined as above. Let $r \in \mathcal{R}$ be a red point and $G \subset \mathcal{B}$ be a set of blue points. We say that G is guarding r if for all $b \in \mathcal{B} \setminus G$, we have that $B_{\|b,r\|}(b) \cap G \neq \emptyset$.*

LEMMA 5. *[10] (Blue-dominant centers lemma in $2D$) Let R and B be two disjoint sets of red and blue points in a 2-dimensional Euclidean space, and q be a positive integer. If $\|B\| > 5q \cdot \|R\|$ then there exists at least one q-blue-dominant point in B.*

LEMMA 6. *$OPT_b(\mathcal{L}_1)$ can be partitioned into $\omega = \lceil 3^\alpha \rceil \cdot \beta^{-1} + 1$ disjoint subsets $S_i (i = 1, \cdots, \omega)$ such that $\cup_{i=1}^{\omega} S_i = OPT_b(\mathcal{L}_1)$ and for any pair of links l_i, l_j belong to the same subset, the distance between their corresponding senders are greater than $\|l_i\| + \|l_j\|$.*

PROOF. We construct a graph G by denoting the corresponding sender of each link in $OPT_b(\mathcal{L}_1)$ as a node, and drawing an edge between any pair of senders s_i, s_j if and only if their mutual distance is no greater than $\|l_i\| + \|l_j\|$. Then we sort the nodes in G in the non-decreasing order of the lengths of their corresponding links. We prove by contradiction that each node has at most $\omega - 1$ neighbors appearing before it in this ordering.

Otherwise, let s_i be the node(which is a sender) who has ω neighboring nodes(which are all senders) appearing before it. We denote the set of all these senders as S_ω, The distance between s_i and any sender $s_j \in S_\omega$ satisfies

$$\|s_j s_i\| \le \|l_j\| + \|l_i\| \le 2\|l_i\|$$

The distance between the corresponding receiver (which is r_i) of s_i and any sender $s_j \in S_\omega$ satisfies

$$\|s_j r_i\| \le \|s_j s_i\| + \|l_i\| \le \|l_i\| + 2\|l_i\| = 3\|l_i\|$$

Thus the total affectedness on link l_i from the senders in S_ω is

$$a_{S_\omega}(l_i) = \beta \cdot \frac{\sum_{s_j \in S_\omega} \frac{P}{\|s_j r_i\|^\alpha} + N}{\frac{P}{\|l_i\|^\alpha}} \ge \beta \cdot \frac{(\omega - 1)\frac{P}{(3\|l_i\|)^\alpha} + N}{\frac{P}{\|l_i\|^\alpha}}$$
$$\ge \beta \cdot \beta^{-1} = 1$$

This contradicts the fact that $OPT_b(\mathcal{L}_1)$ is a valid scheduling.

Thus in this ordering each node has at most $\omega - 1$ neighbors appearing before it. Therefore using first-fit coloring in this order, at most ω colors are needed to ensure that each neighboring nodes are assigned distinct colors. Thus $OPT_b(\mathcal{L}_1)$ can be partitioned into ω subsets with each color as a subset. □

LEMMA 7. *$OPT_b(\mathcal{L}_1) \le \rho_b \cdot ALG(\mathcal{L}_1)$, where $\rho_b = 5 \cdot 2^{\alpha+1} \cdot \omega$.*

PROOF. We prove by contradiction. By Lemma 6, we can partition $OPT_b(\mathcal{L}_1)$ into ω disjoint subsets $S_k (k = 1, 2, \cdots, \omega)$ such that for any pair of links $l_i, l_j \in S_k$, the distance between their corresponding senders are greater than $\|l_i\| + \|l_j\|$. Assume $OPT_b(\mathcal{L}_1) > \rho_b \cdot ALG(\mathcal{L}_1)$, consider the subset with the largest size as S, by pigeonhole principle, $\|S\| \ge 5 \cdot 2^{\alpha+1}$.

Consider the set of receivers from S and senders from $ALG(\mathcal{L}_1)$. Label those from S as blue and those from $ALG(\mathcal{L}_1)$ as red. By Lemma 5, since $|S| > (5 \cdot 2^{\alpha+1}) \cdot ALG(\mathcal{L}_1)$, there is a $2^{\alpha+1}$-blue-dominant point (receiver) r_b in S. We claim that link l_b would have been picked by our algorithm, leading to a contradiction. Since r_b is $2^{\alpha+1}$-blue-dominant, the total interference it receives from blue receivers (those in $S \setminus s_b$) is at least $2^{\alpha+1}$ times as high as the interference it would receive from the red points (the senders in $ALG(\mathcal{L}_1)$).

For any link l_i in S, we have

$$\|s_i r_b\| \le \|s_i r_i\| + \|r_i r_b\| \le 2\|r_i r_b\| \Longrightarrow \|r_i r_b\| \ge \frac{1}{2} \cdot \|s_i r_b\|.$$

Thus,

$$\frac{\frac{P}{\|r_i r_b\|^\alpha}}{\frac{P}{\|l_b\|^\alpha}} \le \frac{\frac{P}{(\frac{1}{2} \cdot \|s_i r_b\|)^\alpha}}{\frac{P}{\|l_b\|^\alpha}} = 2^\alpha \cdot \frac{\frac{P}{\|s_i r_b\|^\alpha}}{\frac{P}{\|l_b\|^\alpha}} = 2^\alpha \cdot \frac{I_{ib}}{P_{bb}} = 2^\alpha \cdot RI_i(l_b).$$

Since $l_b \in \mathcal{L}_1 \Rightarrow \|l_b\| < \sqrt[n]{\frac{P}{3\beta N}}$, we have

$$\frac{\beta N}{2P_{bb}} = \frac{\beta N \|l_b\|^\alpha}{2P} \le \frac{1}{6}.$$

Therefore,

$$a_{ALG(\mathcal{L}_1)}(l_b) = \beta \cdot \left(\frac{N}{P_{bb}} + \sum_{l_i \in ALG} RI_i(l_b) \right)$$
$$= \beta \cdot \left(\frac{N}{P_{bb}} + \sum_{l_i \in ALG} \frac{\frac{P}{\|s_i r_b\|^\alpha}}{\frac{P}{\|l_b\|^\alpha}} \right)$$
$$\le \beta \cdot \left(\frac{N}{P_{bb}} + \frac{1}{2^{\alpha+1}} \cdot \sum_{l_i \in S} \frac{\frac{P}{\|r_i r_b\|^\alpha}}{\frac{P}{\|l_b\|^\alpha}} \right)$$
$$\le \beta \cdot \left(\frac{N}{P_{bb}} + \frac{1}{2^{\alpha+1}} \cdot 2^\alpha \cdot \sum_{l_i \in S} RI_i(l_b) \right)$$
$$= \beta \cdot \left(\frac{N}{P_{bb}} + \frac{1}{2} \cdot \sum_{l_i \in S} RI_i(l_b) \right)$$
$$\le \frac{\beta N}{2P_{bb}} + \frac{1}{2} \cdot a_S(l_i) < \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$$

However this receiver was deleted from $ALG(\mathcal{L}_1)$ because it had been affected by at least $2/3$ by $S^-_{r_k}$ (assume l_b was deleted when l_k was added), and thus at least that amount by $ALG(\mathcal{L}_1)$, which establishes the contradiction. □

THEOREM 8. *$ALG(\mathcal{L}_1) \le \rho_1 \cdot OPT(\mathcal{L}_1)$, where $ALG(\mathcal{L}_1)$ is the solution obtained by Algorithm 2, $OPT(\mathcal{L}_1)$ is the corresponding optimum solution and $\rho_1 = (\rho_a + \rho_b + 1)^{-1}$.*

PROOF. The result follows by adding the bounds of Lemma 4 and Lemma 7: $OPT(\mathcal{L}_1) \le OPT'(\mathcal{L}_1) + ALG(\mathcal{L}_1) \le (\rho_a + \rho_b + 1) \cdot ALG(\mathcal{L}_1)$. Thus, $ALG(\mathcal{L}_1) \ge \rho_1 \cdot OPT(\mathcal{L}_1)$. □

Next, we show that the approximation ratio of Algorithm 3 is constant.

LEMMA 9. *In Algorithm 3, each grid contains at most $m = (\sqrt{2}\sqrt[\alpha]{3} + 1)^\alpha$ scheduled links.*

PROOF. Consider a grid, assume all the links it contains form a set X. For a link $l_i \in X$, $\|l_i\| \ge \sqrt[\alpha]{\frac{P}{3\beta N}} = \delta/2$, thus $\delta \le \sqrt[\alpha]{3}\|l_i\|$. Assume $|X| > (\sqrt{2}\sqrt[\alpha]{3} + 1)^\alpha$.

$$a_X(l_i) = \beta \cdot \frac{\sum_{s_j \in X} \frac{P}{\|s_j r_i\|^\alpha} + N}{\frac{P}{\|l_i\|^\alpha}} \ge \frac{\sum_{s_j \in X} \frac{P}{\|s_j r_i\|^\alpha}}{\frac{P}{\|l_i\|^\alpha}}$$
$$\ge \sum_{s_j \in X} \frac{\|l_i\|^\alpha}{(\|s_j s_i\| + \|l_i\|)^\alpha} \ge \sum_{s_j \in X} \frac{\|l_i\|^\alpha}{(\sqrt{2}\delta + \|l_i\|)^\alpha}$$
$$\ge \sum_{s_j \in X} \frac{\|l_i\|^\alpha}{(\sqrt{2}\sqrt[\alpha]{3}\|l_i\| + \|l_i\|)^\alpha} = \sum_{s_j \in X} \frac{1}{(\sqrt{2}\sqrt[\alpha]{3} + 1)^\alpha}$$
$$= \frac{|X|}{(\sqrt{2}\sqrt[\alpha]{3} + 1)^\alpha} \ge 1$$

which leads to a contradiction. □

THEOREM 10. *$ALG(\mathcal{L}_2) \le \rho_2 \cdot OPT(\mathcal{L}_2)$, where $ALG(\mathcal{L}_2)$ is the solution obtained by Algorithm 3, $OPT(\mathcal{L}_2)$ is the corresponding optimum solution and $\rho_2 = (m(c_2 + 1)^2)^{-1}$.*

PROOF. From Algorithm 3, $\bigcup_{r,s=0,1,\cdots,c_2} \mathcal{S}_{r,s}$ select one link whose sender located within a grid for each grid (if there exists one sender in the grid).

According to Lemma 9, $OPT(\mathcal{L}_2)$ can select at most m links for each grid, thus,

$$OPT(\mathcal{L}_2) \le m \cdot \left| \bigcup_{r,s=0,1,\cdots,c_2} \mathcal{S}_{r,s} \right|$$

By pigeonhole principle, our scheduled obtained by Algorithm 3:

$$ALG(\mathcal{L}_2) \geq \frac{1}{(c_2+1)^2} \cdot \left| \bigcup_{r,s=0,1,\cdots,c_2} \mathcal{S}_{r,s} \right|$$

Thus, $OPT(\mathcal{L}_2) \leq m(c_2+1)^2 \cdot ALG(\mathcal{L}_2)$. □

THEOREM 11. *Algorithm 1 can find a constant approximation solution for One-Slot link scheduling problem.*

PROOF. By Theorem 8 and Theorem 10,

$$ALG(\mathcal{L}_1) \geq \rho_1 \cdot OPT(\mathcal{L}_1), ALG(\mathcal{L}_2) \geq \rho_2 \cdot OPT(\mathcal{L}_2)$$

Therefore,

$$\begin{aligned} & \max(ALG(\mathcal{L}_1), ALG(\mathcal{L}_2)) \\ \geq\ & \frac{1}{2} \cdot (\rho_1 \cdot OPT(\mathcal{L}_1) + \rho_2 \cdot OPT(\mathcal{L}_2)) \\ \geq\ & \frac{1}{2} \cdot \max(\rho_1, \rho_2) \cdot (OPT(\mathcal{L}_1) + OPT(\mathcal{L}_2)) \\ \geq\ & \frac{1}{2} \cdot \max(\rho_1, \rho_2) \cdot OPT(\mathcal{L}) \end{aligned}$$

Therefore $\max(ALG(\mathcal{L}_1), ALG(\mathcal{L}_2))$ is within constant approximation ratio of the optimum solution for One-Slot scheduling. Since Algorithm 1 chooses it as the returning result, our proof is done. □

5.3 One-Slot weighted link scheduling

In this section, we assume each link l has a weight $w(l)$, and the goal is to maximize the weighted sum of scheduled links rather than just the number of such links. We will propose two different methods for this problem with two different approximation ratios. We then return the better of these two solutions as a solution. Here a better solution means the solution has a larger weighted sum of all links in it.

The first method is to partition links into $g(\mathcal{L})$ different groups such that the spread of links in each group is constant, here $g(\mathcal{L}) = \log \vartheta(\mathcal{L})$ is the length diversity. Thus, we can design a method for weighted link scheduling with a constant approximation ratio for links in a group. The second method is to partition links into another different groups such that the weights of links in any group are within a constant factor of each other. Again, we can design a method for weighted link scheduling with a constant approximation ratio for links in a group.

Algorithm 4 Weighted Partition Scheduling

Input: Set of links $\mathcal{L} = \{l_1, l_2, \cdots, l_n\}$;
Output: One slot schedule $\mathcal{S}$;

1: Set c_2 according to Equation 3;
2: **for** $r = 0, \cdots, c_2$ and $s = 0, \cdots, c_2$ **do**
3:     **for** $i, j \in \mathbb{Z}$ **do**
4:         **if** $i \mod (c_2+1) = r$ and $j \mod (c_2+1) = s$ **then**
5:             select one link whose sender located within $g_{i,j}$;
6:     all the selected links form a set $\mathcal{S}_{r,s}$;
7: Let $\mathcal{S}$ be the $\mathcal{S}_{r,s}$ with the largest weight;
8: **return** $\mathcal{S}$;

LEMMA 12. *There exists a polynomial time algorithm that finds a feasible solution within a factor $O(\log \frac{d_{\max}}{d_{\min}})$ where $d_{\max} = \max_{l \in \mathcal{L}} \|l\|$ and $d_{\min} = \min_{l \in \mathcal{L}} \|l\|$).*

PROOF. We divide all links in $\mathcal{L}$ into groups based on their lengths. Group $\mathcal{L}_j$ contains all links l satisfying $d_{\max}/2^{j-1} \geq \|l\| \geq d_{\max}/2^j$. By pigeonhole principle, there exists a j such that the intersection of $\mathcal{L}_j$ and the optimal solution for $\mathcal{L}$ (noted as OPT) contains links with weighted sum within a factor $\log \frac{d_{\max}}{d_{\min}}$ of OPT. Thus if we can find a constant approximation solution to schedule links in $\mathcal{L}_j$ for each j ($0 \leq j \leq \log \frac{d_{\max}}{d_{\min}}$), then we can find a solution within a factor $O(\frac{d_{\max}}{d_{\min}})$ of OPT by selecting the best one of all the $\log \frac{d_{\max}}{d_{\min}}$ solutions. Note we only focus on a j for which $d_{\max}/2^j \geq d_{\min}$.

Since all links in $\mathcal{L}_j$ has length in $d_{\max}/2^{j-1} \geq \|l_i\| \geq d_{\max}/2^j$. Assume $\delta = d_{\max}/2^j$, then for any link $l_i \in \mathcal{L}_j$, we know $\delta \leq \|l_i\| \leq 2\delta$. Using the property, we subdivide the plane into grids by using a set of vertical lines $a_v : x = v \cdot \delta$ ($v \in \mathbb{Z}$) and horizontal lines $b_h : y = h \cdot \delta$ ($h \in \mathbb{Z}$). Hereafter $v(h)$ is called the index of the vertical (horizontal) line a_v (b_h). We call the square formed by a pair of neighboring vertical lines a_v, a_{v+1} and neighboring horizontal lines b_h, b_{h+1} as $g_{v,h}$. Then our constant factor scheduling for $\mathcal{L}_2$ is shown in Algorithm 4.

Similar to Lemma 9, we can prove the scheduled links are both feasible and within constant approximation of the optimum solution for links in $\mathcal{L}_j$. Thus our proof is done. □

LEMMA 13. *There exists a polynomial time algorithm that finds a feasible solution within a factor $O(\log \frac{w_{\max}}{w_{\min}})$ where $w_{\max} = \max_{l \in \mathcal{L}} w(l)$ and $w_{\min} = \min_{l \in \mathcal{L}} w(l)$.*

PROOF. We divide all links in $\mathcal{L}$ into groups based on their weights. Group $\mathcal{L}_j$ contains all links l satisfying $w_{\max}/2^{j-1} \geq w(l) \geq w_{\max}/2^j$. Note that there exists a j such that the intersection of $\mathcal{L}_j$ and the optimal solution for $\mathcal{L}$ (noted as OPT) contains links with weighted sum within a factor $O(\log \frac{w_{\max}}{w_{\min}})$ of OPT. Then we consider each j in turn and obtain a constant approximation for the links in $\mathcal{L}_j$ only. We focus on a j for which $w_{\max}/2^j \geq w_{\min}$.

By Theorem 11, we can obtain a solution of One-Slot link scheduling for $\mathcal{L}_j (j \in N)$ within constant approximation in terms of the number of links. Since the weights of all links in $\mathcal{L}_j$ are within a constant factor of each other, the constant approximation ratio also works in terms of weighted sum. Thus our proof is done. □

From Lemma 12 and 13, we can derive Theorem 14 immediately.

THEOREM 14. *There exists a polynomial time algorithm that finds a feasible solution for One-Slot weighted link scheduling within a factor $O\left(\min(\log \frac{\max_{l \in \mathcal{L}} w(l)}{\min_{l \in \mathcal{L}} w(l)}, \log \frac{\max_{l \in \mathcal{L}} \|l\|}{\min_{l \in \mathcal{L}} \|l\|})\right)$.*

6. CONCLUSION

In this paper we study the link scheduling problem under the physical interference model in an arbitrary wireless networks. We proposed a scheduling method for maximizing the number of concurrently transmitting links with a constant approximation ratio guarantee. Furthermore, when each link is associated with a weight, we prove that we can achieve $O\left(\min(\log \frac{\max_{l \in \mathcal{L}} w(l)}{\min_{l \in \mathcal{L}} w(l)}, \log \frac{\max_{l \in \mathcal{L}} \|l\|}{\min_{l \in \mathcal{L}} \|l\|})\right)$ approximation solution. Applying the One-Slot algorithm to multi-hop case, we can achieve an approximation of $O(\min(\log n, \log \vartheta(\mathcal{L})))$ where n and $\vartheta(\mathcal{L})$ are the size and spread of the input network respectively. We hope that it is a big step towards understanding the problem for link scheduling in wireless networks of arbitrary topologies.

There are still a number of interesting problems that we would like to address. The first problem is whether there is a constant approximation method for finding a subset of independent links with maximum weight under physical interference model when each node has a fixed transmitting power. The second problem is whether a method with a constant approximation-ratio exists for one-slot maximum (weighted) link scheduling when nodes can adjust the transmitting power in a range $[P_{\min}, P_{\max}]$, where $P_{\max}$ and $P_{\min}$ are not necessarily constants. The third problem is what is the best approximation-ratio we can achieve for the problem of minimizing the number of time-slots needed to schedule all links.

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