Equity Home Bias under Ambiguity

[preliminary and incomplete]

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1 Introduction.

Equity home bias is a well known puzzle in international finance, referring to a wide disparity between the actual portfolio weights and the weights recommended by international equity portfolio theory. Under ideal conditions, the international capital asset market model predicts that investors should hold equities from around the world in proportion to their market capitalization. However, according to the empirical findings of French and Poterba [16] and Tesar and Werner [37], investors hold a substantially larger proportion of their wealth in domestic assets: US investors hold 92.2% of their equity portfolio in domestic stocks; Japanese investors - 95.7%; UK investors - 92%; German investors - 79%; French investors - 89.4%, and Canadian investors - 93.4%. This observed high concentration in domestic equity has become known as "equity home bias".

There have been various attempts to explain this puzzle. The first approach is based on information asymmetries[1], hedging possibilities against domestic risk[2], and barriers to international investment such as restrictions on international capital flows[3], withholding taxes, and transactions costs[4]. Another approach focuses on investors behavioral biases, e.g. optimism about their domestic markets[5] and preference for the familiar[6]. Lewis [31] and Strong and Xin [35] provide an extensive review of proposed explanations. Empirical studies[7] find that home bias is caused by both institutional and behavioral factors.

[1] Gehrig (1993) [17], Kang and Stulz (1997) [24],Ahearne, Griever, Warnock (2004)[1],Jeske (2001) [23]

[2] Baxter and Jermann(1995) [4], Cooper and Kaplanis(1994) [11], Uppal (1992) [39]

[3] Black (1974) [5], Stulz (1981) [36]

[4] Tesar and Werner (1995) [37], Warnock (2002) [41], Obstfeld (2000) [33]

[5] French and Poterba (1991) [16]

[6] Huberman (2001) [22], Coval and Moskowitz (1999) [12]

[7] Bailey, Kumar, and Ng (2005) [3],Karlsson and Norden (2004) [25], Kyrychenko, Shum (2006) [30]

A more recent research direction explains home bias by means of ambiguity aversion. According to the standard expected utility theory, agents are assumed to make decisions under uncertainty as if they have a prior belief about probability distribution over the set of possible states of the world and then maximize the expected utility according to this distribution. However, individuals often fail to accurately assess such probabilities. Knight [29] suggests that there is an important difference between events with objectively (or subjectively) known probabilities, and events where probabilities are unknown. Uncertainty of the first kind is called *risk*, and uncertainty of the second kind is called *ambiguity* or *Knightian uncertainty*. Ellsberg [14] demonstrates the significance of this distinction by showing that individuals may prefer gambles with specified probabilities over gambles with unknown odds. Consider two urns, one contains 50 red balls and 50 black balls, and the other contains 100 red and black balls in unknown proportion. One ball is drawn at random from each urn. In gamble A, the payoff is $100 if a red ball is drawn and $0 if a black ball is drawn. In gamble B, the payoff is $100 if a black ball is drawn and $0 if a red ball is drawn. When surveyed, many people choose to draw from first urn in both gambles. Such behavior contradicts to the standard expected utility paradigm according to which participants form subjective beliefs in the form of a single probability distribution over the composition of balls in the second urn. This experiment has motivated various generalizations of subjective expected utility theory that incorporate ambiguity. One of the most popular approaches is the maxmin multiple prior model of Gilboa and Schmeidler where agents make decisions based on the worst among the many possible probability distributions for any given choice.

This paper presents a simple general equilibrium model that explains how the presence of ambiguity about asset returns affects the equilibrium prices and international portfolio holdings. In particular, I show that if there is a difference in beliefs about perceived uncertainty then it will lead to the bias in portfolio holdings. In the model, the equilibrium portfolio choice depend on the degree of ambiguity aversion as well as parameters that characterize uncertainty. My model builds on the one in Easley and O'Hara where ambiguity averse investors act as if they have a set of distributions on returns and select a portfolio to maximize their utility over this set of distributions. I adopt the smooth model of decision making under ambiguity that has been axiomatized by Klibanoff, Marinacci, and Mukerji [28] .The advantage of using the smooth model is that it allows for intermediate values of ambiguity aversion coefficients rather than the extreme cases of minimal expected utility and standard expected utility maximizing agents. Moreover, it also simplifies the analysis due to smoothness conditions and makes a model analytically tractable.

Within this direction, the two most closely related papers are Epstein and Miao [15] and Uppal and Wang [40]. Epstein and Miao use a recursive multiple-prior model, a multi-period extension of Gilboa and Schmeidler (1989) maxmin model. They consider agents (countries) who are equally ambiguity averse but have different sets of multiple priors, and hence do not agree on which states are ambiguous. However, the notion of maxmin ambiguity aversion can be viewed as an overly pessimistic. In particular, Bossaerts, Guarnaschelli, Ghirardato and Zame [6] have shown that the attitude toward ambiguity varies across individuals. This suggests that modeling investors' decisions by the maxmin rule may significantly overestimate the effects of ambiguity on asset holding and asset prices. Moreover, Condie [10] shows that in an economy where some agents are ambiguity averse (in the maxmin sense), and some are standard expected utility maximizers (in the Bayesian sense), the former are unlikely to survive if there is an aggregate risk. This suggests that agents who exhibit extreme ambiguity aversion may decide not to participate in the market, i.e. not to hold any foreign asset at all. Easley and O'Hara [13] study the non-participation of ambiguity averse individuals and examine its implications for the regulation of financial markets. In my paper, I investigate whether the equity home bias observed in data can be explained by intermediate degrees of ambiguity aversion.

Uppal and Wang [40] study the portfolio choice when an investor accounts for model misspecification. They follow the robust control approach introduced by Hansen, Sargent and Tallarini [20] and Anderson, Hansen, and Sargent [2] where agents use a reference model to differentiate among the priors. This allows for differences in the degree of confidence about the probability distributions for returns of different assets. A preference for robustness is interpreted as aversion to uncertainty. They show that if the confidence about joint stock distribution is low then small differences in the degree of confidence for the marginal return distribution will result in a significant underdiversification relative to the standard mean-variance portfolio. In contrast to the robust control approach, the preference representation by Klibanoff, Marinacci, and Mukerji has an axiomatic foundation and stays within the state independent utility framework. Their model allows to smoothly aggregate the decision maker's information about the subjective relevance of each possible probability measure as the true probability measure. This makes it similar to the Bayesian approach. Unlike in Uppal and Wang, in my model the degree of ambiguity aversion is the same for all assets, but investors perceive uncertainty differently for home and foreign assets. Also, my model provides the effect of the ambiguity on the asset prices and derives the upper bound on the degree of ambiguity aversion for participation in financial markets.

The idea that investors have different beliefs about uncertainty is supported by surveys and empirical studies. Several papers in the home bias literature have identified a systematic bias in investors' return expectations. French and Poterba (1991) show that observed portfolio holdings could be explained by domestic investors having more optimistic expectations about domestic stocks than about foreign stocks. This has been confirmed by empirical studies for Japan (Shiller, Kon-Ya, and Tsutsui [34] and China (Chen, Kim, Nofsinger, Rui [7]), experimental studies for Germany (Kilka and Weber [27]), and surveys of fund-managers (Strong, Xin [35]). Graham, Harvey and Huang [19] also study the link between competence and investor behavior where investor competence is measured through survey responses. They argue that the competence effect contributes to home bias. Tourani-Rad and Kirkby [38] investigate investor overconfidence, socialization and the familiarity effect, using a sample of New Zealand investors. They find support for the investor overconfidence theory, using characteristics such as past success, optimism, confidence in one's abilities, investment experience and investment-related knowledge. Lutje and Menkhoff [32] find that belief in an informational advantage and relative return optimism towards home assets are the driving forces of home bias. They argue that informational advantage often appears to be a perceived advantage, as fund managers with a home preference do not forecast stock indices better than others, and they rely less on fundamental analysis. Christoffersen and Sarkissian [8] relate geographic location and investor behavior by comparing the performance of U.S. equity mutual funds located in and outside of financial centers. They argue that fund managers in financial centers tend to be more overconfident because of their proximity to private information.

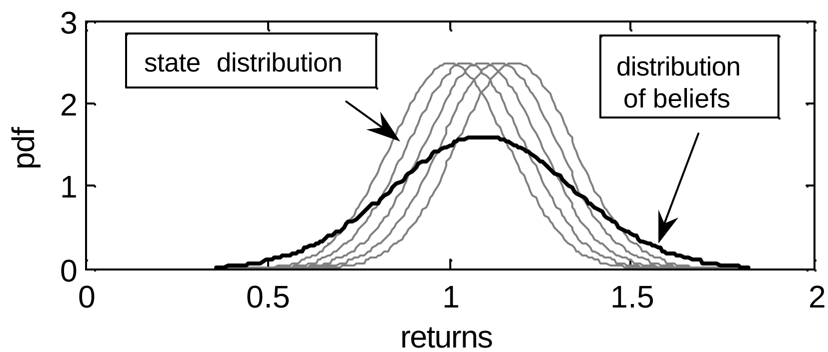
To check whether equity home bias can be explained by a less extreme degree of ambiguity aversion, I calibrate the model to data. I find that even if the degree of ambiguity aversion is relatively small, it is possible that the difference in beliefs about perceived uncertainty can generate a home bias in portfolio holdings that is close to the data.

The remainder of the paper is organized as follows: the model environment is described next. In section 3, I consider a model with a constant variance across assets where (1) there is no uncertainty about the home asset, (2) investors are overconfident about the home asset, and (3) investors are more optimistic about the home asset. Section 4 provides the equilibrium characterization. Section 5 presents the results of the calibration exercise, and Section 6 concludes.

2 Model environment.

I consider a model with two countries: A and B. One can think about country A as one particular country (for example, the US), and about B as the rest of the world. The total number of investors in both countries is I, with λI living in country A and $(1-\lambda)I$ living in country B, where λ is between zero and one. There are three assets in the economy: one risk-free asset m (money) and two risky assets. The risk-free asset is the same in both countries and its price and return are normalized to one. In addition, a is a risky asset in country A, and b is a risky asset in country B. All assets are traded on the international market so investors from each country could trade them. The price of risky asset k is p_k and the return is r_k where $k = a, b$

There are infinitely many possible states; in each state s, asset return r_k is normally distributed with mean $\overline{r}_k(s)$ and variance $\sigma_k^2(s)$. I assume there is no uncertainty about the asset's variances: for each s, $\sigma_k(s) = \sigma_k$ and the returns and variances of both assets are independent. Investors do not know which state will be realized, so they form beliefs about a set of possible realizations of mean returns for each asset.



– possible distribution of returns, ***— distribution of beliefs over state space***

All investors are identical within the country they live in, and the endowment of each investor is $e = (\overline{m}, \overline{x}_k)$. The total endowment in the economy is $(I\overline{m}, \lambda I\overline{x}_a, (1-\lambda)I\overline{x}_b)$, so the wealth of each investor is equal to $w = (r_a - p_a)x_a + (r_b - p_b)x_b + \overline{m} + p_k\overline{x}_k$. Investors choose their optimal portfolio (x_a, x_b) to maximize their utility function.

The utility function I use is adapted from the smooth model of decision making under ambiguity by Klibanoff, Marinacci, and Mukerji (2005). The individual preferences are represented by:

$$U(w) = E_{\mu}\left[\phi(E_{\pi_s}[u(w)|s_n])\right]$$

where $u(\cdot)$ is a von Neumann-Morgenstern utility function, π_s is a known probability distribution in each state s, and μ is subjective probability distribution over the possible probabilities π_s. The subjective prior μ weights the importance of each distribution π_s reflecting an investor's beliefs about how likely each state is going to occur. The increasing function ϕ characterizes the attitudes towards ambiguity. The degree of ambiguity aversion is defined as $\alpha(y) = -\phi''(y)/\phi'(y)$. If function ϕ is concave then it characterizes ambiguity aversion, which is defined as an aversion to mean preserving spreads in μ. If function ϕ is linear then the reduction of compound lotteries can be applied and it becomes equivalent to the standard subjective expected utility. The model of maxmin expected utility: $U(\cdot) = \min_{\pi_s} E_{\pi_s}[u(\cdot)]$) may be seen as an extreme case of my model with infinite degree of ambiguity aversion.

The smooth model allows the separation between ambiguity (a decision maker's subjective beliefs μ) and ambiguity attitude (a characteristic of the decision maker's preferences α). It smoothly aggregates the decision maker's information about plausibility of each possible probability distributions, consequently, the indifference curves are smooth rather then kinked. Note that in maxmin models, the decision maker only looks at the the worst value.

I assume $u(w) = -e^{-\gamma w}$ is a CARA utility function where γ is the degree of risk aversion. If investors are ambiguity neutral then ϕ is linear: $\phi(y) = y$; if investors are ambiguity averse then $\phi(y) = -e^{-\alpha y}$ where α is the degree of ambiguity aversion. These strong assumptions on investors preferences and returns distribution allow to derive results for prices and asset holdings in closed-form.

3 Overconfidence and Optimism.

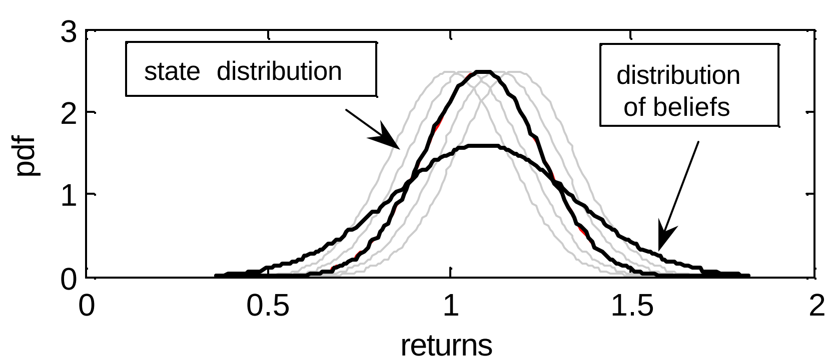
Investors believe that possible mean returns $(\overline{r}_a(s), \overline{r}_b(s))$ are jointly normally distributed with mean $(\overline{r}_a, \overline{r}_b)$ and variance $\Delta = \begin{bmatrix} \delta_a^2 & \delta_{ab} \\ \delta_{ab} & \delta_b^2 \end{bmatrix}$, where δ_k^2 characterizes the dispersion of possible distributions for each asset and δ_{ab} characterizes the correlation between states. For example, investors may believe that if the return on the home asset turn out to be high then the return on the foreign asset is more likely to be high. Even though in every state returns are independent, the state realizations for the two assets may be correlated. The correlation is based on investors' expectations rather than fundamentals, allowing to capture the possible contagion effect between two countries (Goldstein and Pauzner [18] and Keister[26]).

The beliefs about the dispersion of returns distributions depend on whether the asset is domestic or foreign. Investors believe there is less uncertainty about the home asset than about the foreign asset, and they are more optimistic about the returns on the home asset. These assumptions are supported by findings of Kilka and Weber [27]. They conduct a cross country study in Germany and the U.S. to investigate whether people's subjective probability distributions on average exhibit systematic differences in location and in dispersion. Their results show that people consider themselves to be on average more competent in forecasting domestic stock prices than in forecasting foreign stocks prices. Subjective probability distributions of stock returns are significantly less dispersed for domestic stocks (associated with high confidence levels) than for foreign stocks (associated with low confidence levels). Furthermore, domestic stocks are judged significantly more optimistically than foreign stocks. These observed patterns are consistent with biases in individual judgment documented by psychological research (Heath and Tversky [21]).

First, I will consider the economy with the extreme version of overconfidence when investors completely ignore uncertainty about the home asset, and consequently, they behave as standard expected utility maximizers with respect to home asset. In the next case, investors believe there is less uncertainty about the home asset, i.e. the dispersion of possible distributions is smaller for the home asset than for the foreign asset. Third, I consider the model where investors face the same uncertainty about home and foreign assets but they are more optimistic about returns on the home asset.

3.1 Case 1. No uncertainty about the home asset.

Suppose investors ignore uncertainty about the home asset but not about the foreign asset. Investors form a single prior about returns on home asset, i.e. instead of considering all possible distributions they put a mass point weight on one average distribution with mean $\overline{r}_k$ and variance σ_k . In this case investors can exhibit any degree of ambiguity aversion α with respect to the home asset but it is irrelevant since their beliefs about asset returns consist of a single prior. Consequently, results are equivalent to having a linear ϕ- function with respect to the asset returns, and therefore, it is equivalent to ambiguity neutrality with respect to that asset. For foreign asset, investors take into consideration all possible distributions and, therefore, the degree of ambiguity aversion α matters. Effectively, investors are ambiguity neutral with respect to domestic asset and ambiguity averse with respect to the foreign asset. The asset returns are normally distributed with some mean $\overline{r}_k(s)$ and variance $\sigma_k^2 : r_k \sim N(\overline{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that the possible mean returns are equal to the average of mean returns if it is a home asset, or normally distributed with mean $\overline{r}_k$ and variance δ_k^2, if it is a foreign asset; i.e. $\overline{r}_k(s) \begin{cases} = \overline{r}_k & if \quad k \text{ is home asset} \\ \sim N(\overline{r}_k, \delta_k^2) & if \quad k \text{ is foreign asset} \end{cases}$



In the competitive equilibrium, investors choose portfolio holdings to maximize their expected utility, and prices are determined such that markets clear. Since asset returns are assumed to be distributed normally and investors have a CARA utility function, maximization problem can be expressed in terms of mean and variance.

Investors solve the following optimization problem:

$$\max_{x_a, x_b} \left\{ \gamma\alpha(\overline{r}_h - p_h)x_h + \gamma\alpha(\overline{r}_f - p_f)x_f + \gamma\alpha e - \frac{1}{2}\alpha\gamma^2(\sigma_h^2 x_h^2 + \sigma_f^2 x_f^2) - \frac{1}{2}\gamma^2\alpha^2\delta_f^2 x_f^2 \right\}$$

For country A investors: $i = 1, .., \lambda I$, home asset is a, so subscript $\cdot_h$ refers to asset a and subscript $\cdot_f$ to asset b. Symmetrically for investors from country B: $j = (\lambda I + 1), .., I$, subscript $\cdot_h$ refers to asset b and subscript $\cdot_f$ to asset a.

The optimal demands for each asset are given by:

$$\text{for country A investors:} \quad .x_{ai}^* = \frac{\overline{r}_a - p_a}{\gamma\sigma_a^2} \qquad x_{bi}^* = \frac{\overline{r}_b - p_b}{\gamma(\sigma_b^2 + \alpha\delta_b^2)}$$

$$\text{for country B investors:} \quad x_{aj}^* = \frac{\overline{r}_a - p_a}{\gamma(\sigma_a^2 + \alpha\delta_a^2)} \qquad x_{bj}^* = \frac{\overline{r}_b - p_b}{\gamma\sigma_b^2}$$

In equilibrium, the demand for optimal asset holdings should sum up to the total endowment:

$$\begin{aligned} \lambda x_{ai}^* + (1-\lambda)x_{aj}^* &= \lambda\overline{x}_a \\ \lambda x_{bi}^* + (1-\lambda)x_{bj}^* &= (1-\lambda)\overline{x}_b \end{aligned}$$

Solving for the equilibrium prices:

$$\begin{aligned} p_a^* &= \overline{r}_a - \lambda\overline{x}_a\gamma\sigma_a^2 \frac{\sigma_a^2 + \alpha\delta_a^2}{\sigma_a^2 + \alpha\lambda\delta_a} \\ p_b^* &= \overline{r}_b - (1-\lambda)\overline{x}_b\gamma\sigma_b^2 \frac{\sigma_b^2 + \alpha\delta_b^2}{\sigma_b^2 + (1-\lambda)\alpha\delta_b^2} \end{aligned}$$

Substituting prices into utility maximizing asset holdings, we can get the equilibrium portfolio holdings:

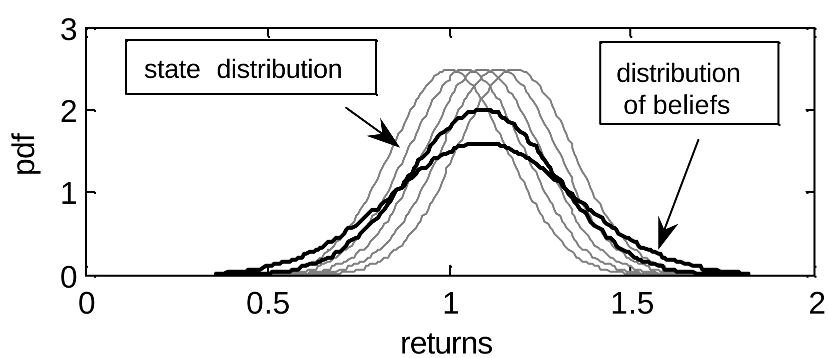
$$\begin{aligned} x_{ai}^* &= \lambda\overline{x}_a \frac{\sigma_a^2 + \alpha\delta_a^2}{\sigma_a^2 + \alpha\lambda\delta_a^2} \\ x_{bi}^* &= (1-\lambda)\overline{x}_b \frac{\sigma_b^2}{\sigma_b^2 + (1-\lambda)\alpha\delta_b^2} \end{aligned}$$

$$x^*_{aj} = \lambda \overline{x}_a \frac{\sigma_a^2}{\sigma_a^2 + \lambda\alpha\delta_a^2}$$
$$x^*_{bj} = (1-\lambda)\overline{x}_b \frac{\sigma_b^2 + \alpha\delta_b^2}{\sigma_b^2 + \alpha(1-\lambda)\delta_b^2}$$

If there is no ambiguity than the asset holding x^*_k should be equal to the per capita supply of that asset $\lambda_k \overline{x}_k$. However, if there is a difference in perceived uncertainty about a home and a foreign asset then portfolio holdings will be biased towards the home asset. Note that the bias in asset holdings does not directly depend on the degree of risk aversion.

3.2 Case 2. Overconfidence about the home asset.

It is a strong assumption that investors completely ignore uncertainty about the home asset, i.e. they behave as if they know the true distribution. Now I will relax this assumption: investors are now effectively ambiguity averse with respect to both assets, home and foreign, but they believe there is less uncertainty about the home asset. In their beliefs they put more weight on distributions that are close to the average "correct" distribution so the dispersion of possible distributions is smaller for the home asset than for the foreign asset. This is equivalent to a smaller ambiguity aversion with respect to home asset. The asset returns are normally distributed with some mean $\overline{r}_k(s)$ and variance $\sigma_k^2 : r_k \sim N(\overline{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that possible mean returns are normally distributed with mean $\overline{r}_k$ and variance $\delta^2_{k.}$, i.e. $\overline{r}_k(s) \sim \begin{cases} N(\overline{r}_k, \delta^2_{kh}) & \text{if } k \text{ is a home asset} \\ N(\overline{r}_k, \delta^2_{kf}) & \text{if } k \text{ is a foreign asset} \end{cases}$ where $\delta_{kh} < \delta_{kf}$.



In this case I assume that the interstate correlation between assets is zero. The results hold for non-zero correlation and model has the closed-form solution but the expression for equilibrium prices and asset holdings are extremely cumbersome and, therefore, less tractable.

In equilibrium the prices are given by:

$$p_a^{**} = \overline{r}_a - \lambda \overline{x}_a \gamma \frac{\left(\sigma_a^2 + \alpha \delta_{ah}^2\right)\left(\sigma_a^2 + \alpha \delta_{af}^2\right)}{\sigma_a^2 + \alpha\left(\lambda \delta_{af}^2 + (1-\lambda)\delta_{ah}^2\right)}$$

$$p_b^{**} = \overline{r}_b - (1-\lambda)\overline{x}_b \gamma \frac{\left(\sigma_b^2 + \alpha \delta_{bh}^2\right)\left(\sigma_b^2 + \alpha \delta_{bf}^2\right)}{\sigma_b^2 + \alpha\left(\lambda \delta_{bh}^2 + (1-\lambda)\delta_{bf}^2\right)}$$

The equilibrium portfolio holdings for investors from country A

$$x_{ai}^{**} = \lambda \overline{x}_a \frac{\sigma_a^2 + \alpha \delta_{af}^2}{\sigma_a^2 + \alpha\left(\lambda \delta_{af}^2 + (1-\lambda)\delta_{ah}^2\right)}$$

$$x_{bi}^{**} = (1-\lambda)\overline{x}_b \frac{\sigma_b + \alpha \delta_{bh}^2}{\sigma_b^2 + \alpha\left(\lambda \delta_{bh}^2 + (1-\lambda)\delta_{bf}^2\right)}$$

The equilibrium portfolio holdings for investors from country B

$$x_{aj}^{*} = \lambda \overline{x}_a \frac{\sigma_a^2 + \alpha \delta_{ah}^2}{\sigma_a^2 + \alpha\left(\lambda \delta_{af}^2 + (1-\lambda)\delta_{ah}^2\right)}$$

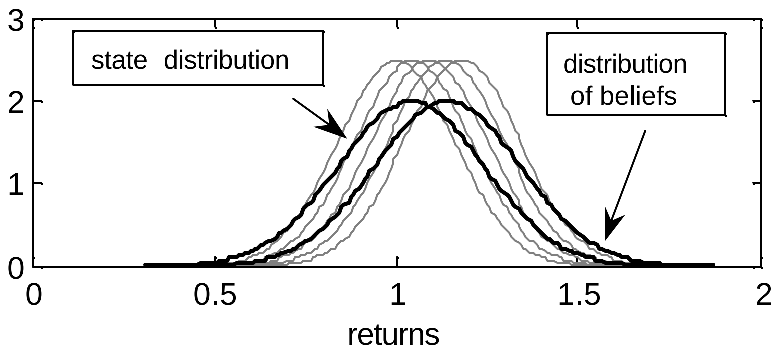
$$x_{bj}^{*} = (1-\lambda)\overline{x}_b \frac{\sigma_b^2 + \alpha \delta_{bf}^2}{\sigma_b^2 + \alpha\left(\lambda \delta_{bh}^2 + (1-\lambda)\delta_{bf}^2\right)}$$

Clearly, the portfolio holdings are biased towards the home asset.

3.3 Case 3. Optimism about the home asset.

Suppose now investors face the same uncertainty about a home and a foreign asset but they are more optimistic about returns on the home asset. Investors have 'wrong' beliefs about the second-order distributions over states but with respect to the home asset they are optimistic about the realization of 'good' states with returns realization above average, and with respect to the foreign asset they think that the states with returns realization below average are more likely to occur.

The asset returns are normally distributed with some mean $\overline{r}_k(s)$ and variance $\sigma_k^2 : r_k \sim N(\overline{r}_k(s), \sigma_k^2),\ k = a, b$. Investors believe that possible mean returns are normally distributed with mean $\overline{r}_{k.}$ and variance δ_k^2, i.e. $\overline{r}_k(s) \sim \begin{cases} N(\overline{r}_{kh}, \delta_k^2) & \text{if } k \text{ is a home asset} \\ N(\overline{r}_{kf}, \delta_k^2) & \text{if } k \text{ is a foreign asset} \end{cases}$ where $\overline{r}_{kh} > \overline{r}_{kf}$.



In this economy the equilibrium prices are given by

$$p_a^{***} = [\lambda\overline{r}_{ah} + (1-\lambda)\overline{r}_{af}] - \lambda\overline{x}_a\gamma(\sigma_a^2 + \alpha\delta_a^2) - (1-\lambda)\overline{x}_b\gamma\alpha\delta_{ab}$$
$$p_b^{***} = [\lambda\overline{r}_{bf} + (1-\lambda)\overline{r}_{bh}] - (1-\lambda)\overline{x}_b\gamma(\sigma_b + \alpha\delta_b) - \lambda\overline{x}_a\gamma\alpha\delta_{ab}$$

The equilibrium asset holding of investors in country A are given by

$$x_{ai}^* = \lambda\overline{x}_a\left[1 + \frac{(1-\lambda)(\overline{r}_{ah} - \overline{r}_{af})(\sigma_b^2 + \alpha\delta_b^2) + (1-\lambda)\left(\overline{r}_{bh} - \overline{r}_{bf}\right)\alpha\delta_{ab}}{\gamma\left[\left(\sigma_a^2 + \alpha\delta_a^2\right)\left(\sigma_b^2 + \alpha\delta_b^2\right) - \left(\alpha\delta_{ab}\right)^2\right]}\right]$$
$$x_{bi}^* = (1-\lambda)\overline{x}_b\left[1 - \frac{(1-\lambda)\left(\overline{r}_{bh} - \overline{r}_{bf}\right)\left(\sigma_a^2 + \alpha\delta_a^2\right) + (1-\lambda)(\overline{r}_{ah} - \overline{r}_{af})\alpha\delta_{ab}}{\gamma\left[\left(\sigma_a^2 + \alpha\delta_a^2\right)\left(\sigma_b^2 + \alpha\delta_b^2\right) - \left(\alpha\delta_{ab}\right)^2\right]}\right]$$

The equilibrium asset holding of investors in country B are given by

$$x_{aj}^* = \lambda\overline{x}_a\left[1 - \frac{\lambda(\overline{r}_{ah} - \overline{r}_{af})(\sigma_b^2 + \alpha\delta_b^2)) + \lambda\left(\overline{r}_{bh} - \overline{r}_{bf}\right)\alpha\delta_{ab}}{\gamma\left[\left(\sigma_a^2 + \alpha\delta_a^2\right)\left(\sigma_b^2 + \alpha\delta_b^2\right) - \left(\alpha\delta_{ab}\right)^2\right]}\right]$$
$$x_{bj}^* = (1-\lambda)\overline{x}_b\left[1 + \frac{\lambda\left(\overline{r}_{bh} - \overline{r}_{bf}\right)\left(\sigma_a^2 + \alpha\delta_a^2\right) + \lambda(\overline{r}_{ah} - \overline{r}_{af})\alpha\delta_{ab}}{\gamma\left[\left(\sigma_a^2 + \alpha\delta_a^2\right)\left(\sigma_b^2 + \alpha\delta_b^2\right) - \left(\alpha\delta_{ab}\right)^2\right]}\right]$$

As expected, there is a bias towards the home asset in portfolio holdings.

The Proposition 1 summarizes and generalizes results considered in the above cases.

Proposition 1 *If investors are ambiguity averse with respect to both assets and believe that the dispersion of possible distributions of asset returns means are smaller for the home asset than for the* *foreign asset:* $\delta_{kh} < \delta_{kf}$*,* $k = a, b$*, and they are optimistic about the home asset relative to foreign:* $\overline{r}_{kh} > \overline{r}_{kf}$*,* $k = a, b$ *then investors will choose their portfolio so that the proportion of the home asset is larger than its market share and the proportion of the foreign asset is smaller than its market share:*

$$\frac{x_a^*}{x_a^* + x_b^*} > \frac{\lambda \overline{x}_a}{\lambda \overline{x}_a + (1-\lambda)\overline{x}_b} \quad \textit{if } a \textit{ is a home asset}$$
$$\frac{x_a^*}{x_a^* + x_b^*} < \frac{\lambda \overline{x}_a}{\lambda \overline{x}_a + (1-\lambda)\overline{x}_b} \quad \textit{if } a \textit{ is a foreign asset.}$$

4 Asset Holdings and Ambiguity.

4.1 Asset Prices.

Consider investors from country A. In Case 1, they believe that the correct price for home asset should be $p_a^o = \overline{r}_a - \lambda \overline{x}_a \gamma \sigma_a^2$,([8]) which is lower than the actual equilibrium price

$$p_a^* = \overline{r}_a - \lambda \overline{x}_a \gamma \sigma_a^2 \frac{\sigma_a^2 + \alpha \delta_a^2}{\sigma_a^2 + \alpha \lambda \delta_a^2}$$

. Therefore, country A investors believe that asset a is overpriced, so they have incentive to hold more of home asset. Similarly, they believe that the price for asset b should be $\widetilde{p}_b = \overline{r}_b - (1-\lambda)\overline{x}_b \gamma(\sigma_b + \alpha \delta_b^2)$, ([9]) which is higher than the actual equilibrium price.

$$p_b^* = \overline{r}_b - (1-\lambda)\overline{x}_b \gamma \sigma_b^2 \frac{\sigma_b^2 + \alpha \delta_b^2}{\sigma_b^2 + (1-\lambda)\alpha \delta_b^2}$$

.So country A investors believe that asset b is underpriced and, hence, they will hold less of the foreign asset. The same conclusions hold for country B investors due to the symmetry.

The same logic holds for the case 2 and 3, the equilibrium price for asset a is given by

$$\text{Case 2} \quad : \quad p_a^{**} = \overline{r}_a - \lambda \overline{x}_a \gamma \frac{\left(\sigma_a^2 + \alpha \delta_{ah}^2\right)\left(\sigma_a^2 + \alpha \delta_{af}^2\right)}{\sigma_a^2 + \alpha\left(\lambda \delta_{af}^2 + (1-\lambda)\delta_{ah}^2\right)}$$
$$\text{Case 3} \quad : \quad p_a^{***} = [\lambda \overline{r}_{ah} + (1-\lambda)\overline{r}_{af}] - \lambda \overline{x}_a \gamma (\sigma_a^2 + \alpha \delta_a^2)$$

If asset a is a home asset investors believe that the correct price should be lower than the actual equilibrium price.

$$\widetilde{p}_{ah} = \overline{r}_{ah} - \lambda \overline{x}_a \gamma \left(\sigma_a^2 + \alpha \delta_{ah}^2\right)$$

[8] This is an equilibrium price when all investors ignore uncertainty about asset a. It is equivalent to the case where all investors behave as standard expected utility maximizers, almost identical to the one considered in Easley, O'Hara [2005]

[9] This is an equilibrium price when all investors consider uncertainty about asset b (see appendix for derivations).

If asset a is a foreign asset investors believe that the correct price should be higher than the actual equilibrium price

$$\widetilde{p}_{af} = \overline{r}_{af} - \lambda \overline{x}_a \gamma \left(\sigma_a^2 + \alpha \delta_{af}^2\right)$$

Hence, investors believe that asset a is overpriced if it is home asset, and underpriced if it is a foreign asset. The same conclusions hold for asset b investors by symmetry.

Therefore, the difference in beliefs about uncertainty of the asset returns is generating a home bias. Unlike models with asymmetric information, here prices are not informative. When an investor observes prices different from what she/he thinks they should be, she/he has no incentive to adjust her/his beliefs.

4.2 Non-participation.

Another interesting feature to note is that there is an upper bound on the degree of ambiguity aversion that comes from the requirement of the asset price to be non-negative. Investors will choose to participate in the market only if they believe that the price for the foreign asset is positive: $\widetilde{p}_f > 0$. This means that if investors have a degree of ambiguity aversion α such that

$$\alpha > \frac{\overline{r}_f - \lambda_f \overline{x}_f \gamma \sigma_f^2}{\lambda_f \overline{x}_f \gamma \delta_{kf}^2}$$

they will not hold any of the foreign asset. This condition rules out the participation of agents who have maxmin type of preferences.

4.3 Equity Premium

Define the equity premium as $EP \equiv (E[r_k] - 1)/p$. If all investors are ambiguity neutral then

$$EP_{EU} = \frac{\overline{r}_k - 1}{\overline{r}_k - \lambda_k \overline{x}_k \gamma \sigma_k^2}$$

.If all investors are ambiguity averse then

$$EP_{AA} = \frac{\overline{r}_k - 1}{\overline{r}_k - \lambda_k \overline{x}_k \gamma (\sigma_k^2 + \alpha \delta_k^2)}$$

Equity premium is higher under ambiguity, and as degree of ambiguity aversion and/or the dispersion of possible distribution increases, the premium will become larger. The positive effect of ambiguity on equity premium have been adressed by several papers on decision theory under uncertainty.

4.4 Comparative Statics

Proposition 2

- degree of ambiguity aversion α: *If the degree of ambiguity aversion increases then the prices of both assets go down, the holding of the home asset may increase or decrease and the foreign asset holding decreases. Overall, the equity home bias becomes larger.*

- difference in the perceived dispersions of mean asset returns $\triangle\delta_k = \delta_{kf} - \delta_{kh}$: *If for a given asset the difference in the perceived dispersions increases then its equilibrium price goes down. The holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.*

- difference in the perceived mean returns $\triangle\overline{r}_k = \overline{r}_{kh} - \overline{r}_{kf}$: *If for a given asset the difference in the perceived mean returns increases then its equilibrium price goes up. The holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.*

- correlation of asset returns $\rho_{ab} = \frac{\delta_{ab}}{\delta_a \delta_b}$: *If the interstate correlation between assets is positive (negative) than the equity home bias becomes larger (smaller).*

- population fraction λ : *If population in country A decreases relative to country B then the equilibrium price for asset a goes up and the equilibrium price for asset b goes down. In country A, the home asset holding increases and the foreign asset holding decreases; vice versa for country B. Therefore, the equity home bias becomes larger.*

In the following table all effects of possible changes in parameters of asset a are summarized:

$\backslash$	$\alpha\uparrow$	$\triangle\delta_a\uparrow$	$\triangle\overline{r}_a\uparrow$	$\rho_{ab}\uparrow$	$\lambda\uparrow$
p_a	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
x_{ah}	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
x_{af}	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$

5 Calibration Exercise:investors are overconfident and optimistic about the home asset

In this section I will investigate the quantative joint effect[10] of optimism and overconfidence on the asset holding. The asset returns are normally distributed with some mean $\overline{r}_k(s)$ and variance

[10]The theoretical results are presented in the Appendix in the proof of Proposition 1

σ_k^2 : $r_k \sim N(\overline{r}_k(s), \sigma_k^2)$, $k = a, b$. Investors believe that the possible mean returns are normally distributed with mean $\overline{r}_{k.}$ and variance $\delta_{k.}^2$, i.e. $\overline{r}_k(s) \sim \begin{cases} N(\overline{r}_{kh}, \delta_{kh}^2) & \text{if } k \text{ is a home asset} \\ N(\overline{r}_{kf}, \delta_{kf}^2) & \text{if } k \text{ is a foreign asset} \end{cases}$ where $\delta_{kh} < \delta_{kf}$ and $\overline{r}_{kh} > \overline{r}_{kf}$.

I use the following stylized facts[11]: expected asset return r_t^m is 9%, asset standard deviation $\sigma(r_t^m)$ is 16%, coefficient of risk aversion is equal to 2. According to Ahearne, Griever, Warnock (2004), the US market capitalization is about 48.3% ($\lambda \simeq 0.5$) and the estimated home asset holding about 89.9%.

Table 1 presents the home asset holdings for several values of the difference in perceived mean returns $\triangle\overline{r}_k$ and perceived dispersions $\triangle\delta_k$, for different degrees of ambiguity aversion. The perceived mean returns for the home asset is $\overline{r}_{kh} = 1.09 + \triangle\overline{r}_k/2$, and for the foreign asset it is $\overline{r}_{kf} = 1.09 - \triangle\overline{r}_k/2$. The exact values of dispersions are chosen to match the 8% of the equity premium; these values are presented in Table 1b.

As the degree of ambiguity aversion increases, the bias toward the home asset becomes larger for any given difference in perceived mean returns $\triangle\overline{r}_k$ and perceived dispersions $\triangle\delta_k$. The difference in mean returns $\triangle\overline{r}_k$ contributes more to the bias then the difference in perceived dispersions $\triangle\delta_k$. The significant portion of the bias can be explained with relatively small degree of ambiguity aversion and differences in beliefs within 5%. It is possible to matchcexactly the US domestic asset holding observed in data but it requires large (but still finite) degree of ambiguity aversion or large differences in beliefs. The dispersion levels required to match the equity premium is smaller for a higher degree of ambiguity aversion.

[11] Cochrane [9]

Table 1a. Optimal home asset holdings $\lambda = 0.5$

$\alpha = 1$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	50	53.10	56.19	65.48
1%	51.30	54.39	57.47	66.45
2%	52.60	55.67	58.73	67.78
5%	56.45	59.45	62.41	71.03
10%	62.56	65.37	68.10	75.83

$\alpha = 2$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	50	53.10	56.19	65.48
1%	52.05	55.14	58.20	67.31
2%	54.10	57.15	60.18	69.07
5%	60.06	62.96	65.79	73.91
10%	68.93	71.42	73.81	80.43

$\alpha = 5$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	50	53.10	56.19	65.48
1%	53.24	56.31	59.35	68.34
2%	56.45	59.45	62.41	71.03
5%	65.41	68.09	70.68	77.93
10%	77.02	78.97	80.81	85.81

$\alpha = 10$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	50	53.10	56.19	65.48
1%	54.58	57.62	64.84	69.48
2%	59.03	61.97	75.43	73.11
5%	70.77	73.15	75.43	81.70
10%	83.44	84.87	86.22	89.84

Table 1b. Dispersion of the foreign asset $\lambda = 0.5$

$\alpha = 1$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	23.48	23.48	23.48	23.48
1%	23.99	24.02	24.05	24.08
2%	24.52	24.58	24.64	24.70
5%	26.21	26.36	26.51	26.66
10%	29.38	29.67	29.96	30.24

$\alpha = 2$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	16.6	16.6	16.6	16.6
1%	17.12	17.15	17.18	17.27
2%	17.66	17.72	17.78	17.96
5%	19.43	19.58	19.72	20.15
10%	22.84	23.12	23.39	24.17

$\alpha = 5$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	10.5	10.5	10.5	10.5
1%	11.02	11.17	11.08	11.17
2%	11.58	11.88	11.70	11.88
5%	13.50	14.19	13.79	14.19
10%	17.32	18.49	17.82	18.49

$\alpha = 10$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	7.43	7.43	7.43	7.43
1%	7.95	7.99	8.02	8.11
2%	8.54	8.60	8.66	8.83
5%	10.61	10.74	10.88	11.25
10%	14.77	14.99	15.20	15.77

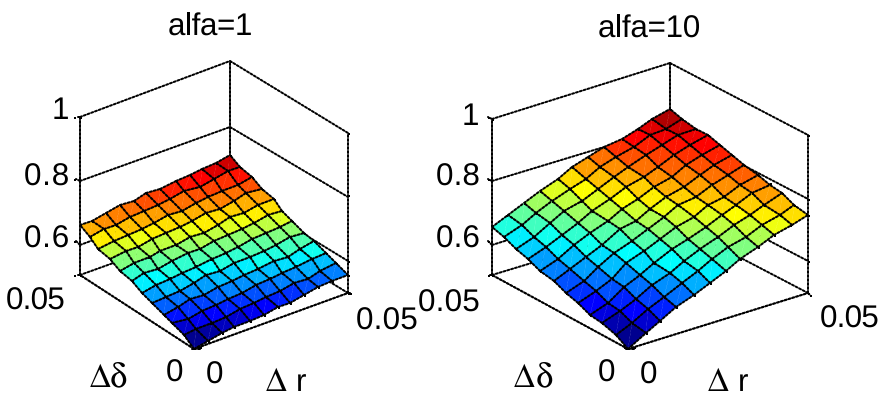


Figure 1:

Figure 1 presents the home asset holdings as a function of the difference in perceived mean returns $\triangle\overline{r}_k$ and dispersion $\triangle\delta_k$ when $\alpha = 1$ (ambiguity neutrality) and $\alpha = 10$. The difference in perceived mean returns $\triangle\overline{r}_k$ and perceived dispersions $\triangle\delta_k$ ranges from 0% to 5%.

As proportion λ of country A investors increases, the equity home bias becomes smaller. The intuition is the following: if investors from one country dominate the market then they have a large impact on equilibrium asset prices. So the deviation between equilibrium prices and the 'correct' prices (that investors believe should be) is smaller, hence, the portfolio holdings are closer to the market capitalization weights. Similarly, if the proportion of investors from one country is relatively small than their asset holding will be strongly biased towards the home asset. Table 2 presents the home asset holdings when market capitalization is 10%.

Table 2a. Optimal home asset holdings $\lambda = 0.1$

$\alpha = 1$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	10	15.81	21.56	38.52
1%	10.99	17.02	22.94	40.18
2%	12.05	18.29	24.38	41.86
5%	15.68	22.48	28.98	46.92
10%	23.12	30.52	37.36	55.14

$\alpha = 2$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	10	15.81	21.56	38.52
1%	11.42	17.54	23.53	40.88
2%	12.99	19.39	25.61	43.26
5%	18.56	25.67	32.36	50.38
10%	30.34	37.84	44.60	61.44

$\alpha = 5$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	10	15.81	21.56	38.52
1%	12.31	18.60	24.73	42.26
2%	14.99	21.70	28.14	46.04
5%	25.10	32.57	39.41	56.99
10%	45.77	52.46	58.28	72.00

$\alpha = 10$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	10	15.81	21.56	38.52
1%	13.38	19.85	26.11	43.83
2%	17.49	24.49	31.13	49.14
5%	33.42	40.86	47.50	63.81
10%	61.55	66.54	70.80	80.64

Table 2b. Dispersions of the foreign asset $\lambda = 0.1$

$\alpha = 1$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	22.67	22.77	22.85	23.09
1%	23.51	23.61	23.71	23.97
2%	24.35	24.46	24.57	24.86
5%	26.90	27.04	27.18	27.55
10%	31.20	31.39	31.57	32.07

$\alpha = 2$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	16.03	16.10	16.16	16.33
1%	16.87	16.95	17.02	17.21
2%	17.71	17.80	17.88	18.10
5%	20.28	20.39	20.50	20.80
10%	24.61	24.77	24.92	25.35

$\alpha = 5$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	10.14	10.18	10.22	10.32
1%	10.98	11.03	11.08	11.21
2%	11.83	11.89	11.95	12.11
5%	14.50	14.50	14.58	14.82
10%	18.92	18.92	19.05	19.41

$\alpha = 10$				
$\triangle\delta_k \backslash \triangle\overline{r}_k$	0%	1%	2%	5%
0%	7.17	7.20	7.23	7.30
1%	8.01	8.05	8.09	8.19
2%	8.86	8.91	8.96	9.09
5%	11.47	11.54	11.62	11.82
10%	15.89	16.01	16.12	16.45

6 Conclusion.

My paper provides a simple theoretical framework which illustrates how the difference in beliefs can generate the equity home bias. In my model, all investors possess the same information about set of possible states and the corresponding returns distribution in each state but they have different beliefs about the likelyhood of these states. This heterogeneity of beliefs generates asymmetry of choices. This asymmetry is fundamentally different from information asymmetry in the sense that prices are not informative. The idea that investors have biased beliefs about uncertainty of asset returns is supported by several papers in the literature on home bias.

My model allows to quantify the effect of ambiguity and ambiguity attitude on the portfolio holdings and asset prices. It shows that the difference in perceived uncertainty can significantly contribute to the bias towards domestic assets. The extent to which the observed bias can be explained by the differences in beliefs and ambiguity aversion depends on the what parameter values one is willing to accept as reasonable. Even though the ambiguity does significantly contribute to the explanation of equity home bias, it is unlikely that the observed lack of diversification is entirely due to ambiguity aversion. This leaves room for the explanations based on the institutional factors and information assymetries. My results do not contradict to the equity home bias explanations based on institutional factors and asymmetric information but rather complements them. This is consistent with empirical findings that equity home bias is caused by both institutional and behavioral factors.

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7 Appendix.

7.1 A1.1 Investors are ambiguity neutral with respect to both assets.

$$max\{\gamma(\overline{r}_a - p_a)x_a + \gamma(\overline{r}_b - p_b)x_b + \gamma e - \frac{\gamma^2}{2}(\sigma_a x_a^2 + \sigma_b x_b^2)\}$$

optimal demand:

$$x_k^\circ = \frac{\overline{r}_k - p_k}{\gamma\sigma_k^2}$$

equilibrium prices:

$$p_k^o = \overline{r}_k - \overline{x}_k\gamma\sigma_k^2$$

$$x_k^o = \overline{x}_k$$

7.2 A1.2 Ambiguity aversion w.r.t both assets.

asset k

$$\begin{aligned} r_k &\sim N(\overline{r}_{ks}, \sigma_k^2) \\ \overline{r}_{ks} &\sim N(\overline{r}_k, \delta_k^2) \end{aligned}$$

$\max \alpha E\left[E[w|s] - \frac{1}{2}var[w|s]\right] - \frac{1}{2}\alpha^2 Var\left[E[w|s] - \frac{1}{2}var[w|s]\right]$

(i) $Cov(\overline{r}_a, \overline{r}_b) = \delta_{ab} \neq 0$

$\max\left\{\alpha\gamma(\overline{r}_a - p_a)x_a + \alpha\gamma(\overline{r}_b - p_b)x_b + \alpha\gamma e - \frac{\alpha\gamma^2}{2}(\sigma_a^2 x_a^2 + \sigma_b^2 x_b^2) - \frac{\alpha^2\gamma^2}{2}\left(\delta_a^2 x_a^2 + \delta_b^2 x_b^2 + 2\delta_{ab}x_a x_b\right)\right\}$

Optimal demand for assets:

$$\begin{aligned} x_a &= \frac{(\overline{r}_a - p_a)(\sigma_b^2 + \alpha\delta_b^2) - (\overline{r}_b - p_b)\,\alpha\delta_{ab}^2}{\gamma\left[(\sigma_a^2\sigma_b^2 + \alpha\sigma_a^2\delta_b^2 + \alpha\sigma_b^2\delta_a^2\right]} \\ x_b &= \frac{(\overline{r}_b - p_b)\left(\sigma_a^2 + \alpha\delta_a^2\right) - (\overline{r}_a - p_a)\,\alpha\delta_{ab}}{\gamma\left[(\sigma_a^2\sigma_b^2 + \alpha\sigma_a^2\delta_b^2 + \alpha\sigma_b^2\delta_a^2\right]} \end{aligned}$$

Equilibrium prices:

$$\begin{aligned} \widetilde{p}_a &= \overline{r}_a - \overline{x}_a\gamma\left(\sigma_a^2 + \alpha\delta_a^2\right) - \overline{x}_b\gamma\alpha\delta_{ab} \\ \widetilde{p}_b &= \overline{r}_b - \overline{x}_b\gamma(\sigma_b^2 + \alpha\delta_b^2) - \overline{x}_a\gamma\alpha\delta_{ab} \end{aligned}$$

(ii) returns on assets are independent across states than $Cov(\overline{r}_a, \overline{r}_b) = \delta_{ab} = 0$

Optimal demand for assets:

$$\begin{aligned} x_a &= \frac{(\overline{r}_a - p_a)(\sigma_b^2 + \alpha\delta_b^2)}{\gamma\left[(\sigma_a^2\sigma_b^2 + \alpha\sigma_a^2\delta_b^2 + \alpha\sigma_b^2\delta_a^2\right]} \\ x_b &= \frac{(\overline{r}_b - p_b)\left(\sigma_a^2 + \alpha\delta_a^2\right)}{\gamma\left[(\sigma_a^2\sigma_b^2 + \alpha\sigma_a^2\delta_b^2 + \alpha\sigma_b^2\delta_a^2\right]} \end{aligned}$$

Equilibrium prices:

$$\begin{aligned} \widetilde{p}_a &= \overline{r}_a - \overline{x}_a\gamma\left(\sigma_a^2 + \alpha\delta_a^2\right) \\ \widetilde{p}_b &= \overline{r}_b - \overline{x}_b\gamma(\sigma_b^2 + \alpha\delta_b^2) \end{aligned}$$

Proof of Proposition 1:

7.3 $\overline{r}_{ah} > \overline{r}_{af}, \delta_{ah} < \delta_{af}$

maximization problem of investors from country A:

$\max \alpha E\left[E[w|s_n] - \frac{1}{2}var[w|s_n]\right] - \frac{1}{2}\alpha^2 Var\left[E[w|s_n] - \frac{1}{2}var[w|s_n]\right]$

$\max\left\{\alpha\gamma(\overline{r}_{ah} - p_a)x_a + \alpha\gamma(\overline{r}_{bf} - p_b)x_b + \alpha\gamma e - \frac{\alpha\gamma^2}{2}(\sigma_a x_a^2 + \sigma_b x_b^2) - \frac{\alpha^2\gamma^2}{2}\left(\delta_{ah}\gamma^2 x_a^2 + \delta_{bf}\gamma^2 x_b^2\right)\right\}$

FOC: $\overline{r}_{ah} - p_a - \gamma\sigma_a x_a - \alpha\gamma\delta_{ah}x_a = 0$

$\overline{r}_{bf} - p_b - \gamma\sigma_b x_b - \alpha\gamma\delta_{bf}x_b = 0$

$$x_{ai}^* = \frac{(\overline{r}_{ah} - p_a)}{\gamma\left(\sigma_a + \alpha\delta_{ah}\right)} \quad x_{bi}^* = \frac{(\overline{r}_{bf} - p_b)}{\gamma(\sigma_b + \alpha\delta_{bf})}$$

$$x_{aj}^* = \frac{(\overline{r}_{af} - p_a)}{\gamma\left(\sigma_a + \alpha\delta_{af}\right)} \quad x_{bj}^* = \frac{(\overline{r}_{bh} - p_b)}{\gamma(\sigma_b + \alpha\delta_{bh})}$$

Equilibrium:

$$\begin{aligned} \lambda x_{ai}^* + (1-\lambda)x_{aj}^* &= \lambda\overline{x}_a \\ \lambda x_{bi}^* + (1-\lambda)x_{bj}^* &= (1-\lambda)\overline{x}_b \end{aligned}$$

Equilibrium prices:

$$
\begin{aligned}
p_a^* &= \frac{\lambda\overline{r}_{ah}\left(\sigma_a+\alpha\delta_{af}\right)+(1-\lambda)\overline{r}_{af}\left(\sigma_a+\alpha\delta_{ah}\right)}{\left[\sigma_a+\alpha\left(\lambda\delta_{af}+(1-\lambda)\delta_{ah}\right)\right]}-\lambda\overline{x}_a\gamma\frac{\left(\sigma_a+\alpha\delta_{ah}\right)\left(\sigma_a+\alpha\delta_{af}\right)}{\left[\sigma_a+\alpha\left(\lambda\delta_{af}+(1-\lambda)\delta_{ah}\right)\right]} \\
p_b^* &= \frac{(1-\lambda)\overline{r}_{bh}\left(\sigma_b+\alpha\delta_{bf}\right)+\lambda\overline{r}_{bf}\left(\sigma_b+\alpha\delta_{bh}\right)}{\left[\sigma_b+\alpha\left(\lambda\delta_{bh}+(1-\lambda)\delta_{bf}\right)\right]}-(1-\lambda)\overline{x}_b\gamma\frac{\left(\sigma_b+\alpha\delta_{bh}\right)\left(\sigma_b+\alpha\delta_{bf}\right)}{\left[\sigma_b+\alpha\left(\lambda\delta_{bh}+(1-\lambda)\delta_{bf}\right)\right]}
\end{aligned}
$$

Equilibrium portfolio holdings for investors from country A:

$$
\begin{aligned}
x_{ai}^* &= \lambda\overline{x}_a\frac{\left(\sigma_a+\alpha\delta_{af}\right)}{\left[\sigma_a+\alpha\left(\lambda\delta_{af}+(1-\lambda)\delta_{ah}\right)\right]}+\frac{(1-\lambda)\left(\overline{r}_{ah}-\overline{r}_{af}\right)}{\gamma\left[\sigma_a+\alpha\left(\lambda\delta_{af}+(1-\lambda)\delta_{ah}\right)\right]}>\lambda\overline{x}_a \\
x_{bi}^* &= (1-\lambda)\overline{x}_b\frac{\left(\sigma_b+\alpha\delta_{bh}\right)}{\left[\sigma_b+\alpha\left(\lambda\delta_{bh}+(1-\lambda)\delta_{bf}\right)\right]}-\frac{\lambda\left(\overline{r}_{bh}-\overline{r}_{bf}\right)}{\gamma\left[\sigma_b+\alpha\left(\lambda\delta_{bh}+(1-\lambda)\delta_{bf}\right)\right]}<(1-\lambda)\overline{x}_b
\end{aligned}
$$

Asset Prices

$$
\begin{aligned}
\widetilde{p}_{ah} &= \overline{r}_{ah}-\overline{x}_a\gamma\left(\sigma_a+\alpha\delta_{ah}\right) \\
\widetilde{p}_{af} &= \overline{r}_{af}-\overline{x}_a\gamma\left(\sigma_a+\alpha\delta_{af}\right) \\
p_a^* &= \frac{\overline{r}_{ah}\lambda\left(\sigma_a+\alpha\delta_{af}\right)+\overline{r}_{af}(1-\lambda)\left(\sigma_a+\alpha\delta_{ah}\right)}{\left[\sigma_a+\alpha\left(\lambda\delta_{af}+(1-\lambda)\delta_{ah}\right)\right]}-\overline{x}_a\gamma\frac{\left(\sigma_a+\alpha\delta_{ah}\right)\left(\sigma_a+\alpha\delta_{af}\right)}{\left[\sigma_a+\alpha\left(\lambda\delta_{af}+(1-\lambda)\delta_{ah}\right)\right]}
\end{aligned}
$$

$$
\widetilde{p}_{ah}>p_a^*>\widetilde{p}_{af}
$$

Proof of Proposition 2:

$$
\begin{aligned}
x_{ai}^* &= \lambda\overline{x}_a\frac{\sigma_a+\alpha\left(\triangle\delta_a+\delta_{ah}\right)}{\sigma_a+\alpha\left(\lambda\triangle\delta_a+\delta_{ah}\right)}+\frac{(1-\lambda)\triangle\overline{r}_a}{\gamma\left[\sigma_a+\alpha\left(\lambda\triangle\delta_a+\delta_{ah}\right)\right]} \\
x_{bi}^* &= (1-\lambda)\overline{x}_b\frac{\left(\sigma_b+\alpha(\triangle\delta_b+\delta_{bh}\right)}{\left[\sigma_b+\alpha\left(\lambda\triangle\delta_b+\delta_{bh}\right)\right]}-\frac{\lambda\triangle\overline{r}_b}{\gamma\left[\sigma_b+\alpha\left(\lambda\triangle\delta_b+\delta_{bh}\right)\right]}
\end{aligned}
$$

- degree of ambiguity aversion α: If the degree of ambiguity aversion increases then the prices of both assets go down, the holding of the home asset increases and the foreign asset holding decreases. Hence, the equity home bias becomes larger.

$$
\begin{aligned}
\frac{\partial x_{ai}^*}{\partial\alpha} &= \frac{-\lambda\overline{x}_a\gamma^2(1-\lambda)\triangle\delta_a+\lambda\triangle\delta_a+\delta_{ah}}{\gamma^2(\sigma_a+\alpha\left(\lambda\triangle\delta_a+\delta_{ah}\right))^2} \\
\frac{\partial x_{bi}^*}{\partial\alpha} &= \frac{-(1-\lambda)\overline{x}_b\gamma^2\lambda\triangle\delta_b-\lambda\triangle\delta_b-\delta_{bh}}{\gamma^2\left[\sigma_b+\alpha\left(\lambda\triangle\delta_b+\delta_{bh}\right)\right]^2}<0
\end{aligned}
$$

If the degree of ambiguity aversion increases then the home asset holdings may increase or decrease and the foreign asset holding decreases. Overall, the equity home bias becomes larger.

- difference in perceived dispersion of mean asset returns $\triangle\delta_k = \delta_{kf} - \delta_{kh}$:

$$\frac{\partial x^*_{ai}}{\partial \triangle\delta_a} = \frac{\alpha\lambda(1-\lambda)\triangle\overline{r}_a}{\gamma^2\left[\sigma_a + \alpha\left(\lambda\triangle\delta_a + \delta_{ah}\right)\right]^2} > 0$$

$$\frac{\partial x^*_{bi}}{\partial \triangle\delta_b} = \frac{-\alpha(1-\lambda)\lambda\triangle\overline{r}_b}{\gamma^2(\left[\sigma_b + \alpha\left(\lambda\delta_{bh} + (1-\lambda)\delta_{bf}\right)\right]^2} < 0$$

If for a given asset difference in perceived dispersion of mean asset returns $\triangle\delta_k$ increases then the holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.

- difference in perceived mean returns of mean asset returns $\triangle\overline{r}_k = \overline{r}_{kh} - \overline{r}_{kf}$:

$$\frac{\partial x^*_{ai}}{\partial \triangle\overline{r}_a} = \frac{(1-\lambda)}{\gamma\left[\sigma_a + \alpha\left(\lambda\triangle\delta_a + \delta_{ah}\right)\right]} > 0$$

$$\frac{\partial x^*_{bi}}{\partial \triangle\overline{r}_b} = \frac{-\lambda}{\gamma(\left[\sigma_b + \alpha\left(\lambda\delta_{bh} + (1-\lambda)\delta_{bf}\right)\right]} < 0$$

If for a given asset difference in perceived mean returns $\triangle\overline{r}_k$ increases then the holding of this asset decreases if it is a foreign asset, and increases if it is a home asset. Therefore, the equity home bias becomes larger.