A NEW VOID-AND-CLUSTER METHOD FOR IMPROVED HALFTONE UNIFORMITY

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ABSTRACT

Dithering quality of the void and cluster algorithm suffers due to fixed filter width and absence of a well-defined criterion for selecting among equally likely candidates during the computation of the locations of the tightest clusters and largest voids. Various researchers have addressed the issue of fixed filter width by adaptively changing the width with experimentally determined values. This paper addresses both aforementioned issues by using a Voronoi tessellation and three criteria to select among equally likely candidates. The algorithm uses vertices of the Voronoi tessellation, and the areas of the Voronoi regions to determine the locations of the largest voids and the tightest clusters. During void and cluster operations there may be multiple equally likely candidates for the locations of the largest voids and the tightest clusters. The selection among equally-likely candidates is important when the number of candidates is larger than the number of dots for a given quantization level, or if there are candidates within the local neighborhood of one of the candidate points, or if a candidate’s Voronoi region shares one or more vertices with another candidate’s Voronoi region. Use of these methods leads to more uniform dot patterns for light and dark tones. The improved algorithm is compared with other dithering methods based on power spectrum characteristics and visual evaluation.

Keywords: void and cluster, dithering, stochastic screening, halftone, Voronoi tessellation, power spectrum analysis
1. INTRODUCTION

Continuous tone images can be displayed or printed on bi-level or multi-level devices by a halftoning process. Dispersed dithering is one of the halftoning methods that have visually pleasing characteristics. Error diffusion is a well-known and widely applied dispersed dithering algorithm that is applied to monochrome and color images. This method was first proposed by Floyd and Steinberg\(^1\). The method diffuses the quantization error to the neighboring pixels to perform adaptive quantization. Error diffusion produces sharp and visually pleasing halftones. The algorithm inherently enhances high spatial frequencies, therefore increases the sharpness of the edges in the images\(^2\). The Floyd and Steinberg error diffusion algorithm can be improved in terms of smoothness by adaptive selection of thresholds or diffusion kernels\(^3, 4\). Ulichney showed that the visually pleasing characteristic of error diffusion were due in part to blue noise characteristics\(^4\).

Model based halftoning methods can be used to improve error diffusion. The model based halftoning methods utilize a model of the human visual system\(^5, 6\) and/or a printer model\(^7\). Analoui and Allebach\(^5\) proposed a method using direct binary search (DBS) to generate halftone images that are visually optimized for the display device. The algorithm searched for a binary array of pixel values that minimizes the difference between the perceived displayed continuous tone image and the perceived displayed halftoned image. This was a computationally expensive method. Recently, Lieberman and Allebach\(^6\) reported speed and quality improvement over DBS. Pappas and Neuhoff\(^7\) modeled printer distortions to increase both spatial and grayscale resolutions of the printed image. They modified the error diffusion algorithm based on the printer model and achieved substantial improvement over conventional halftoning techniques.
Stochastic screening or frequency modulated (FM) screening is an efficient dispersed dithering algorithm that does not require the processing of neighboring pixels and is faster than the techniques described earlier. Some of FM techniques are blue noise mask\textsuperscript{8, 9}, void and cluster\textsuperscript{10}, DBS screen\textsuperscript{11}. The blue noise mask method constructs the dither array with given frequency domain characteristics, whereas the void and cluster method constructs the dither array in the spatial domain. The DBS screen constructs a dithering array by utilizing optimization-based direct binary search algorithm that searches for the closest visual match between a continuous tone image and its halftone. A review of blue-noise matrix generation techniques was published by Spaulding et al.\textsuperscript{12} They also described methods to extend basic blue noise dither techniques for multilevel and color output devices.

FM screening methods construct a dither array that consists of thresholds. The dither array is tiled periodically to cover the image to be halftoned. Each pixel in the image is compared with the corresponding threshold value from the dither array to decide whether a dot will be placed at that location. This method places same size dots in various locations depending on the grayscale. The problem with the method is that since the dither array is fixed, if a dot is present at a particular location in a light tone, it will be present at the same location in all of the darker tones. For this reason, visually unpleasing placement of dots in light tones leads to artifacts in darker tones. On the other hand, FM screening has high frequency characteristics in the frequency domain and it is visually pleasing to the human eye due to the lowpass characteristics of the human visual system.

The method presented in this paper is derived from the void and cluster algorithm\textsuperscript{10} for FM screening. The traditional void and cluster algorithm\textsuperscript{10} creates a dither array to produce patterns of 1s and 0s for bitonal dithering. The goal of the algorithm is to produce patterns that are homogeneously distributed,
and free from disturbing artifacts. Each element in the dither array is assigned a rank corresponding to the order in which that location in an input image is to be assigned a 1. Thus, the elements of the array are uniquely ranked by integers from 0 to one less than the number of elements in the dither array. Successive ranks are assigned by using fixed-width filter responses to identify candidate locations in the dither array. However, the traditional algorithm does not have a strategy for selecting among candidates that have the same filter response. The lack of such a strategy leads to the selection of either the first candidate or random selection among the candidates. Both choices may lead to non-uniform dot patterns that are especially disturbing in light and mid tones. Problems related to fixed filter width were addressed in Ref. 13 by experimentally determining an optimum filter width for different gray levels.

Methods presented in this paper improve the dot placement in light and dark tones. The dither array generation begins with binary pattern of 0s and 1s representing the dot pattern for an initial gray level. The set of candidates for the next gray level is determined by partitioning the set of dots in the current gray level into Voronoi regions. The Voronoi vertices are used to determine the location of the largest void while the Voronoi regions are used to determine the location of the tightest cluster. In case of equally likely candidates, three different criteria are used to determine the best candidate. Gotsman and Allebach suggested using a Voronoi diagram to add dots, which are positioned at the point in the unit square most distant from all other previous dots. Our method uses areas of Voronoi regions for removing dots from the initial binary pattern, and Voronoi vertices for adding dots to the binary pattern. The maximal distance procedure adopted by Gotsman and Allebach may give rise to multiple equivalent candidates, especially in mid tones. In this case, they choose a candidate at random among these candidates. In our method, additional selection criteria based on global dot-distribution characteristics are used when equivalent candidates arise.
The remainder of this paper is organized as follows. Section 2 discusses initial pattern generator. Section 3 describes dither array generation. Section 4 compares the improved method with other dithering methods in terms of spatial dot distribution and radial frequency responses. Finally, conclusions are given in Section 5.

2. INITIAL PATTERN GENERATOR

Homogeneity of the initial dot pattern affects the quality of the dithering arrays generated by the FM screening methods. In this paper, error diffusion is used to generate an initial binary pattern from a light uniform patch. Since the initial pattern has blue noise characteristics it performs better than the patterns generated from white noise. Error diffusion is implemented on a torus shape patch and error is diffused in a wrapped-around fashion. If the number of dots generated by the error diffusion algorithm is less than the required nearest gray level, additional dots are added to the binary pattern. These dots are inserted in the largest voids. The initial pattern is then homogenized using a method similar to the one given in Ref. 10. Throughout this paper, 1s and 0s in the binary pattern represent dots and voids, respectively.

The initial pattern is homogenized as follows. Initially, the tightest cluster location is determined and the dot at this location is removed. After this step, the largest void location is determined and a dot is placed at this location. Successive computations of tightest clusters and largest voids continue until the tightest cluster location is the same as the location of the largest void. The location of the tightest cluster is determined by computing the area of each Voronoi region. The smallest of these regions denotes the location of the tightest cluster. The location of the largest void is determined by utilization of Voronoi
vertices. Gaussian filters that are centered on these vertices are applied to the binary pattern, and the vertex with the smallest filter response is selected as the location of the largest void. Since the dither array tiles the Euclidean plane infinitely along two orthogonal axes, the Voronoi tessellation is computed on a torus shaped binary pattern.

Carefully designed selection criteria are used to choose among equally likely candidates. This selection minimizes low frequency noise in the dot distribution. The homogenized pattern is used to compute the dither array, which is given in the next section. Figure 1 shows the initial pattern that is used to generate the dither array in Section 3.

Figure 1: An initial pattern that is used to generate a $256 \times 256$ dither array. The initial pattern was generated by error diffusing a patch with gray value 240 and was homogenized by the method described in Section 2.
3. DITHER ARRAY GENERATION

Dither array generation consists of two operations, viz., computing the locations of the tightest clusters and the locations of the largest voids. Within each step, the best candidate among equally likely candidates is selected using one of the three criteria. The cluster and void operations and the candidate selection criteria are described below.

3.1 Cluster Operation

Denote the number of dots in the initial binary pattern by \( N \). The cluster operation assigns rank values from \( N-1 \) to 0, to all dots in the initial binary pattern. The assignment of ranks proceeds as follows. First, a Voronoi tessellation is computed on the binary pattern. The area of each Voronoi region is computed and the smallest one is determined. The dot within the smallest region denotes the location of the tightest cluster. A rank value of \( N-1 \) is assigned to this dot in the dither array, and the dot is removed from the binary pattern. If there are two or more dots with the same smallest region size, one of the three criteria explained in Section 3.3 is used to determine the dot to be removed. Successive ranks are assigned iteratively by recomputing Voronoi regions for the binary pattern (after the dot has been removed), and repeating the procedure for tightest cluster location and dot removal, until all dots have been ranked.

Dots that are removed by the cluster operation during the gray level transition from 244 to 245 are shown in Figure 2. These dots are removed from the densest clusters. New dots are added to the binary pattern using the void operation as described in the next section.
Figure 2: Illustrates cluster operation on a set of dots that have been cropped from a binary pattern with dimensions $256 \times 256$. During the transition from gray level 244 to 245 two dots have been removed by the cluster operation. (a) Dots at gray level 244. (b) Voronoi tessellation for the binary pattern shown in (a). Shaded Voronoi regions have smaller sizes than the other regions. (c) Dots at gray level 245. Dots in shaded regions are removed in this figure.

3.2 Void Operation

The void operation assigns the remaining ranks, viz., $N$ to one less than the number of elements in the dither array, to the void locations in the initial binary pattern. Starting from the initial binary pattern, the void locations are ranked as follows. While the current rank to be assigned is less than a predetermined value, $R_1$, a Voronoi tessellation is computed on the dots of the binary pattern. The vertices of the tessellation represent candidate locations for inserting the next dot. At these vertices Gaussian kernels are applied on the binary pattern. Gaussian kernels with dimensions 11 and 9 were used in this paper during void operation. The vertex with the smallest filter response is selected as the location of the largest void. A rank value is assigned for this location in the dither array and a 1 is inserted into the corresponding location in the binary pattern. This procedure is repeated until the rank value, $R_1$, has been assigned. Dots generated in this manner are shown in Figure 3.
At the rank value of $R_1$, dots begin to dominate the binary pattern. After this point there is no advantage of using the Voronoi tessellation, a few Gaussian based filters are sufficient to determine the rank values of the dots up to a certain rank, $R_2$, where $R_2 > R_1$. In this paper, we used rank values 12800 and 61696 for $R_1$ and $R_2$, respectively to generate a $256 \times 256$ dither array from an initial gray level 240.

At $R_2$, 0s become very sparse. At this level Gaussian filters have very weak discrimination leading to large number of equally likely candidates. This problem can be resolved by interchanging the roles of 0s and 1s. With this change the problem of finding largest void in the original domain becomes equivalent to computing the tightest cluster in the new domain. The cluster operation explained in the previous section can be applied on the new binary pattern. This process continues until all entries in the dither array have valid rank values. Ranks in the dither array are converted to thresholds to perform bitonal or multilevel dithering as described in Ref 10. All the ranks that are assigned to the same threshold value are said to belong to the same quantization level. All quantization levels may not have the same number of dots (ranks).

The determination of the largest void using filter responses may result in multiple equally likely candidates. In this case, candidate selection proceeds by using one of the three criteria explained in the following section.
Figure 3: Illustrates void operation on a set of dots that have been cropped from a binary pattern with dimensions $256 \times 256$. During the transition from gray level 235 to 234 three dots have been added by the void operation. (a) Dots at gray level 235. (b) Voronoi tessellation for the binary pattern shown in (a). Crosses in this figure denote positions of the dots that will be inserted in gray level 234. (c) Dots at gray level 234.

3.3 Candidate Selection Criteria

Selection among equally-likely candidates is important when the number of the candidates is larger than the number of dots within a quantization level, or two or more dots are within an area covered by a filter kernel which is centered on one of the dots, or if a candidate’s Voronoi region shares one or more vertices with another candidate’s Voronoi region. In these situations, the selected dot can change the distribution of dots for the current and future quantization levels.

3.3.1 Selection Among Equally Likely Candidates by Spatial Separation Criterion

The spatial separation criterion helps to resolve degeneracy between equally likely candidates by selecting the candidate that leads to the most uniform dot distribution. This criterion is defined as follows.

Let the distance between any two points be defined as the minimum geodesic distance on a torus and denote the distance by $D(\cdot, \cdot)$. Denote the $N$ candidate locations that are found to be equally likely under a
local neighborhood criterion by the set $X$, $X = \{x(j); j = 0, \cdots, N-1\}$. This neighborhood can be
determined by Voronoi tessellation or by the application of a conventional kernel. The set of points that
have already been ranked in the current and the previous quantization levels are denoted by
$P = \{p(j); j = 0, \cdots, M-1\}$. The index $J$, given by the following expression,
\[ J = \arg \max_j \left\{ \min_i D(x_j, p_i) \right\} \quad (1) \]
is the index of the point in the set $X$ that will be ranked next during the void operation. Equation (1)
maximizes the minimum geodesic distance to identify the largest void location.

Similarly, the index $J$ for the cluster operation is computed by,
\[ J = \arg \min_j \left\{ \min_i D(x_j, p_i) \right\} . \quad (2) \]
The minimum geodesic distance is minimized in Equation (2) because a dot is removed from the tightest
cluster during the cluster operation. If Equations (1) and (2) generate more than one candidate, then the
criterion described in the following section is used to determine the next candidate.

### 3.3.2 Selection Among Equally Likely Candidates by Assuring Uniform Distribution of Dots

Within Blocks

The criterion described in this section is used to select a candidate based on the number of dots in a
spatial region. The binary pattern is partitioned into blocks as shown in Figure 4. As shown in this figure,
blocks $B$, $C$, and $D$ are portions of neighboring blocks of the dither array on a torus. Within each block
the number of dots is counted and a new dot location is chosen to achieve a uniformly distributed binary
pattern. During the clustering operation the candidate is chosen from the block that contains the largest number of points, whereas during the void operation the candidate is chosen from the block with the smallest number of points. This criterion enforces a uniform distribution of dots within a quantization level.

In some cases, application of both criteria may not generate a unique candidate. In this case, a candidate is selected randomly from the available set.

![Partitioning Scheme](image)

**Figure 4:** Shows a partitioning scheme that partitions a binary pattern into four equal size blocks. Blocks B, C, and D are located along the boundaries and are wrapped around as they are illustrated in the figure.

### 4. RESULTS

In this section the method presented in this paper is compared to the standard method given in Ref. 10 and Floyd and Steinberg’s error diffusion in terms of spatial dot distribution and radial spectrum characteristics. Dither arrays for both void and cluster methods have been generated from the same initial pattern. Having the same initial pattern for both methods enables us to do a fair comparison. The initial pattern has been generated by error diffusing a patch with gray value 240.
Halftone patterns generated from the gray level 253 are shown in Figure 5. This figure illustrates the improvement in light tone over the standard method and Floyd and Steinberg error diffusion algorithm. The improved method exhibits a dot distribution that is more visually pleasing than the standard method.

According to Ulichney a well-formed dither array should have the following characteristics\(^4\). It should have low-frequency cutoff at principal frequency, sharp transition region, and flat high frequency blue noise region. These characteristics are illustrated by computing radially averaged power spectrum of \(256 \times 256\) halftone patterns that are dithered by the same size dither arrays. The radially averaged power spectrum has been computed on 181 concentric annuli. Spectrum estimation has been computed by averaging 16 periodograms. Radial spectra for gray level 253 has been shown in Figure 6. The improved method has smaller amplitude in the low frequency regions than the standard method. The transition region is sharper and high frequency region is flatter for the new method.

Similar desired frequency characteristics are also satisfied on dark tones. The magnitudes of the Fourier transforms of the halftone patterns of patches with gray value 3 are shown in Figure 7. As can be observed the lower frequency regions for the improved method have lower amplitudes. Similar conclusions can be drawn from the radially averaged power spectrum of these patterns in Figure 8. According to our observations radially averaged power spectrum characteristics for both methods were comparable in mid tones, but both were slightly worse than the error diffusion.

Homogeneity of the dot distribution of halftoned patterns is compared in terms of their local means. The local mean is computed by multiplying a dithered pattern with a window function. Lets denote \(N \times N\) dithered pattern with \(P\), then windowed pattern \(W\) is computed as,
where, $w_j$ denotes the Welch window function and is defined as,

$$w_j = 1 - \left( \frac{2j - N}{N} \right)^2.$$  \hspace{1cm} (4)

A number of dithered arrays can be generated from the same dithered dither array by circularly shifting the pixel locations along the rows and columns. For each shifted pattern corresponding windowed pattern is computed and the mean value of the pattern is calculated. If the original dithered pattern is truly homogeneous in terms of dot distribution, all mean values should be approximately equal. Figure 9 shows local mean values of 16 shifted patterns that are generated from the halftone patterns that are presented in Figures 5a and 5b. The plot for the improved method is flatter than the standard method. This implies that the dot pattern is more homogeneous than the standard method in light tones. Thus, for uniform gray surfaces, repeating halftone patterns will be less noticeable to the viewer. The new method exhibits flat local mean for dark tones, but in mid tones the mean values of both methods were comparable.

Finally, halftoned gray ramps are shown in Figures 10 and 11. Figure 10 shows the gray levels from 0 to 255. Perceptual quality of the error diffusion was better than both screening methods in mid tones. The improved method generated better halftone in light tones and dark tones. The perceptual quality of the light tones is illustrated on a halftoned ramp that contains gray values 226-255 in Figure 11.
Figure 5: Halftone patterns at gray value 253. (a) Standard method. (b) Improved method that has better spatial distribution. (c) Floyd and Steinberg’s error diffusion by serpentine scanning. Pattern uniformity is worse than the other two.

Figure 6: Radially averaged power spectra of halftone patterns shown in Figures 5a and 5b. The cutoff frequency was 23. The improved method exhibits better frequency characteristics in both low and high frequency regions, and has sharper transition at the cutoff frequency.
Figure 7: Surface plots of magnitude of Fourier transforms of halftoned patches with gray value 3. (a) Standard method. (b) Improved method that has more desirable transform characteristics.

![Surface plots of magnitude of Fourier transforms of halftoned patches with gray value 3.](image)

Figure 8: Radially averaged power spectra of the Fourier transforms shown Figure 7. The cutoff frequency was 28. The new method exhibits better frequency characteristics in both low and high frequency regions, and has sharper transition at the cutoff frequency.

![Radially averaged power spectra of the Fourier transforms](image)
Figure 9: Local mean values for 16 dithering arrays that are generated by circularly shifting the halftone patterns shown in Figures 5a and 5b along x and y axes.

Figure 10: Illustrates the improvement in dot uniformity on gray ramps. (a) Error diffusion with Floyd and Steinberg’s method. Random noise has been added to the fixed threshold value. (b) Dithering with the standard void-and-cluster algorithm that uses random selection for equally likely candidates and uses only one Gaussian kernel. (c) Dithering with the method described in this paper.
Figure 11: Illustrates the improvement in dot uniformity on light tones. The ramps contain gray values 226-255. (a) Error diffusion with Floyd and Steinberg’s method. (b) The standard void-and-cluster. (c) Dithering with the method described in this paper. This ramp has more homogeneous dot distribution.
5. CONCLUSIONS

We presented improvements over the standard void and cluster algorithm by using Voronoi tessellation and selection criteria to determine the best candidate among equally likely candidates that may occur during void and cluster operations. Voronoi tessellation reduces the number of locations that need to be searched for determining the best dither array location to remove or add a dot. The clustering process that is implemented both in light and dark tones, does not rely on kernels but relies on Voronoi regions. The selection among equally-likely candidates is important when the number of candidates is larger than the number of dots for a given quantization level, or if there are candidates within the local neighborhood of one of the candidate points, or if a candidate’s Voronoi region shares one or more vertices with another candidate’s Voronoi region. Application of the selection criteria improved the dot homogeneity in light and dark tones.

Improvements over the standard method have been shown by computing the radially averaged power spectrum, and comparing the characteristics of the spectrum for both methods. Our plots show that the new method results in lower amplitudes in low frequency regions, flatter high frequency components, and sharper transition regions for both light and dark tones. Mid tones are comparable for both methods. Homogeneity of the dot distributions is illustrated by plotting local mean values for the dithered patterns. These plots also show that the new method has better uniformity in light and dark tones. The new method generates noisier halftones for mid tones when it is compared with Floyd and Steinberg’s error diffusion algorithm, but performs better in very light tones. We thus demonstrated methods that improve halftone quality for void and clustering in light and dark tones. The selection criteria presented in this paper can be used for other bitonal or multilevel screening methods that need to choose a candidate among many
candidates. The improved quality of the dither array has significant impact on the smoothness of color halftones as presented in Ref. 16.

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