Calibrating the CreditMetrics™ Correlation Concept for Non-Publicly-Traded Corporations – Empirical Evidence from Germany

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Abstract

This paper deals with the problem of empirically calibrating the CreditMetrics™ correlation concept for a portfolio of loans to non-publicly-traded firms. Using this framework to determine the probabilities of joint default events, requires an estimation of the portion of the obligors' asset return volatility that is firm-specific (idiosyncratic). Whenever the bank's obligors in the credit portfolio under consideration are companies listed on a stock exchange, this can – in principle – be achieved by regressing individual stock returns on the returns of an appropriately composed industry index. The resulting coefficient of determination (R-squared) for each company can be interpreted as an estimate of the portion of return variation that is not firm-specific (systematic), i. e. caused by underlying factors which affect the firm's industry as a whole. But in the case of non-listed obligor firms, which is typical of many medium-sized enterprises in Germany (the so-called "Mittelstand"), this regression model cannot be fitted because of lacking stock price data. We analyze, how the solution to this problem currently offered in CreditManager™, which basically relates the weight of the idiosyncratic component to company size, can be adapted to this case. We show that there is no empirically valid relationship between R-squared and market capitalization (book value of total assets) left, if those German stocks, which are the heavy-weights in their CDAX® industry indices, are excluded from our random sample. Therefore, we suggest that a reasonable calibration of the CreditMetrics™ index model for German non-listed corporate obligors should do without a reference to company size.

JEL-Classification: G11, G21

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I. Introduction

Among the major challenges of credit risk measurement is the issue of modelling the joint default behaviour in a portfolio of fixed-income securities, e. g. corporate bonds or loans. Ignoring the impact of up- or downgrades in either the rating agencies' external or banks' own internal rating systems on the securities' market values and focussing instead on a hold-to-maturity point of view, a proper credit risk measurement has – in principle – to quantify the probabilities of joint default events across all obligors for the relevant risk horizon. Rooted in accounting practice, the time horizon considered in credit risk models is usually set to one year.

Whereas a probability of default (PD) is comparatively easy to estimate for a single obligor firm isolatedly, it is practically impossible to directly estimate probabilities for the joint default events in a loan portfolio comprising several hundreds or thousands of companies.[1] This problem arises mainly because of the sheer number of probabilities needed: For a portfolio with n obligors the bank's internal or other external rating systems should provide n PDs, giving for each firm the probability that just this firm will default on one of its obligations during the year to come, regardless of what will happen to the other n-1 firms. But to get a picture of the entire portfolio loss distribution, one would actually have to estimate the 2^n probabilities of all possible joint default events. In order to provide a workable approximation of the complete loss distribution, several models of credit portfolio risk have been developped in the banking industry.

The beginning of quantitative credit risk modelling at a portfolio level can be traced back to the year 1997, when J. P. Morgan (in conjunction with several co-sponsors) launched CreditMetrics™ and Credit Suisse First Boston released its CreditRisk$^+$.[2] The CreditMetrics™ software implementation CreditManager™ is now marketed by RiskMetricsGroup™, a J. P. Morgan spin-off. Together with KMV's Portfolio Manager™ and McKinsey's CreditPortfolioView™ these two portfolio models still form the cornerstone of current industry practice[3] and their underlying concepts are at the center of an ongoing scientific debate. CreditMetrics™ and the KMV model[4] are both asset value approaches, sometimes also referred to as "structural" – in contrast to "re-

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1 Cf. Schönbucher (2000), p. 4.

2 See the Technical Documents by Gupton/Finger/Bhatia (1997) and Credit Suisse First Boston (1997).

3 See the overview in Basle Committee on Banking Supervision (1999) and in Bluhm/Overbeck/Wagner (2003). Cf. also Saunders/Allen (2002).

4 See Kealhofer/Bohn (2001).

duced-form" – models.[5] Their common notion that default is triggered, if the market value of the firm's total assets falls below some critical level, is due to Merton (1974).

This paper deals with the problem of empirically calibrating the CreditMetrics™ correlation concept for a portfolio of loans to non-publicly-traded German firms.[6] Using the CreditMetrics™ framework to determine the probabilities of joint default events requires for each obligor an estimation of the portion of asset return volatility that is firm-specific (idiosyncratic). From a practitioners point of view, estimating these weights is a very important, probably even the most sensitive calibration problem at all. Setting the percentage portion of idiosyncratic risk has a *tremendous* impact on the resulting loss distribution, especially on its lower tail. A comparatively small variation in these weights can lead to a huge increase in the credit value at risk for a "typical" loan portfolio. This calibration problem seems to be the one that is least understood by CreditMetrics™ users and it involves the hardest guess for them to make. Although the problem lies at the heart of correlation concept, there is – as far as we know – virtually no scientific literature one could resort to.

The paper is organized as follows:

In the sections II.1 and II.2, we briefly review the basics of the CreditMetrics™ asset value model approach and discuss some selected problems arising in its practical application to a bank's corporate loan portfolio. In II.1, we concentrate on the derivation of the loss distribution from companies' asset correlations in a restricted version of the model which is similar to that used by Gordy (2000), p. 124 f. Firstly, we restrict ourselves to the default paradigma, i. e. we distinguish between the states "default" and "non-default" only, but we do not deal with migrations across non-default rating grades. Secondly, we define portfolio loss with respect to the book value losses of the loans contained. Hence, we won't have to deal with tricky calculations of "fair" market values for corporate loans which take their differing collateralization into account.[7] Thirdly, we abstract from the uncertainty which is possibly inherent in the recovery rate

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5 See Basle Committee on Banking Supervision (1999), p. 32.

6 A comparison of the above mentioned credit portfolio models with respect to their application to German middle market loan portfolios is given by Kern/Rudolph (2001).

7 Recently, the CreditMetrics™ valuation framework has been significantly changed. See RiskMetrics™ Group (2001).

realized in the state of a default.[8] In II.2, we introduce a simplified version of the CreditMetrics™ index model that shows decreased data requirements. The aim of paragraph II is rather to sketch the main idea of the correlation concept and to thereby enter into some specific problems of application than to provide a general and comprehensive treatment of the full-fledged CreditMetrics™ model which can be found in the Technical Document.[9]

In paragraph III, we turn to the case of non-listed obligor firms, which is typical of many medium-sized enterprises in Germany (the so-called "Mittelstand") and which is our main concern. In the current version of CreditManager™, users are offered a "general rule" that relates the weight of the idiosyncratic component to company size. We discuss the economic background of this rule and analyze empirically – by means of a two-step regression model – how it can be adapted to fit the stock market data in our sample. After briefly reviewing the RiskMetricsGroup™ approach in III.1, we give a detailed motivation for our own empirical study and introduce our methodology (section III.2). Data and results are presented in section III.3. Section IV concludes.

II. The CreditMetrics™ methodology revisited

1. Deriving the probability distribution of portfolio loss from companies' asset correlations

Consider a bank's loan portfolio with n different corporate obligors. Each obligor firm i is characterized by its (strictly positive) probability of default PD_i. These n default probabilities, which can be regarded as inferred from the bank's (internal) rating system, are assumed to be known with certainty.[10] Let us further assume that each company's asset return $\tilde{r}_i$ obeys a standard normal distribution:

$$\tilde{r}_i \sim N(0;1) \quad \forall\, i = 1..n\,. \tag{1}$$

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8 Cf. Gupton/Finger/Bhatia (1997), especially p. 80, who suggest the use of a beta distribution.

9 Cf. Gupton/Finger/Bhatia (1997), especially p. 85-101. See also the textbook by Saunders/Allen (2002), p. 165-176. A more rigorous treatment is presented in Bluhm/Overbeck/Wagner (2003).

10 See Carey/Hrycay (2001) for the parameterization of credit risk models with rating data.

The asset return, which is in fact a latent variable, can be thought of as describing the yearly percentage change in the market value of the firm's total assets. It is a one-period measure of the company's overall business performance. The standard normal distribution is characterized by its probability density function φ or its cumulative probability density function Φ, respectively:

$$\varphi(r_i) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} r_i^2\right) \text{ resp. } \Phi(r_i) = \int_{-\infty}^{r_i} \varphi(u)\, du \,. \tag{2}$$

Company i defaults, if and only if its realized asset return r_i in the year to come falls below the critical level r_i', the so-called default threshold:

$$PD_i = P\{\tilde{r}_i < r_i'\} = \int_{-\infty}^{r_i'} \varphi(r_i)\, dr_i = \Phi(r_i') \Leftrightarrow r_i' = \Phi^{-1}(PD_i). \tag{3}$$

Obviously, the "cut-off" return r_i' is a given function of the company's PD which may in turn be derived from the firm's rating class. The relationship between asset return distribution, default probability and default threshold is illustrated in Figure 1.

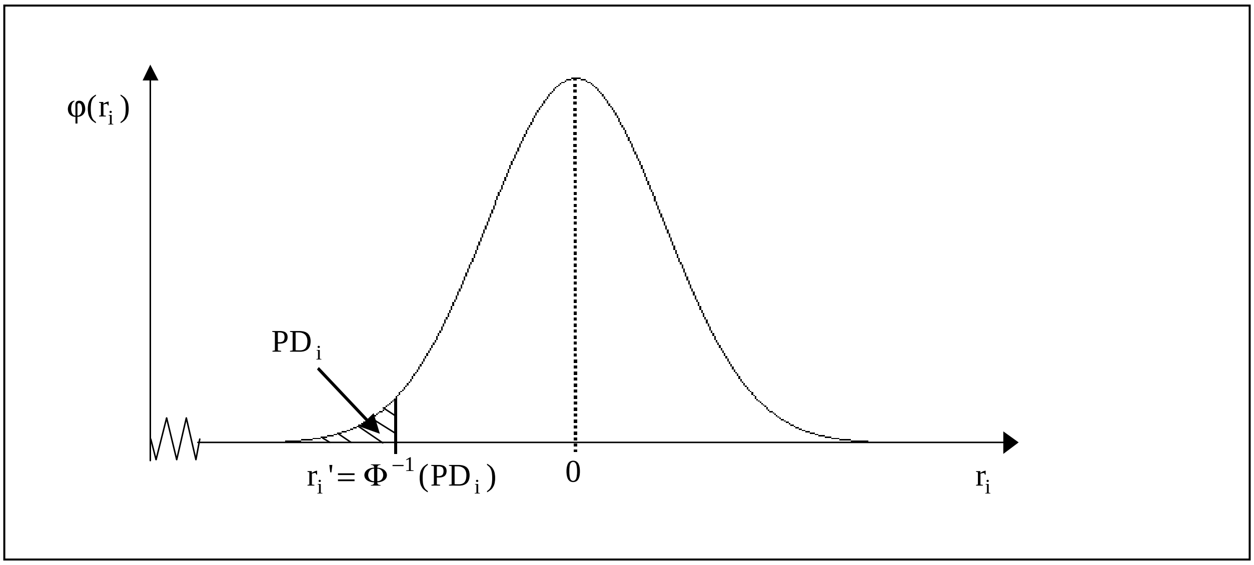

Figure 1: Relationship between asset return distribution, default probability and default threshold.

Note that there is one important difference to the usual option theoretical framework:[11] Here the firms' PDs are given *exogeniously*, e. g. by the bank's rating system, so that there is no need to estimate the volatilities of the n asset returns. Furthermore, assumption (1) of all asset returns being normally distributed with mean zero and standard deviation one is far from being as severe as it may seem at first glance. In fact, a change of assumption (1) would only result in a set of default thresholds differing from that given by equation (3). In this sense, assumption (1) can be made without loss of generality.

We now turn to the case of a simple two-obligor-portfolio for ease of exposition. Both the random asset returns $\tilde{r}_i$ and $\tilde{r}_j$ are not only assumed to obey (1), but moreover they are assumed to be drawn from a *bivariate* standard normal distribution with known correlation coefficient ρ_{ij}:

$$\rho_{ij} = \frac{\mathrm{Cov}\left(\tilde{r}_i;\tilde{r}_j\right)}{\sqrt{\mathrm{Var}(\tilde{r}_i)}\cdot\sqrt{\mathrm{Var}(\tilde{r}_j)}}. \tag{4}$$

The assumption of a bivariate standard normal distribution implies that (1) holds for the two marginal distributions and is much stronger than assumption (1) for i and j separately. The coefficient ρ_{ij} represents the asset return correlation between the companies i and j and is shortly called *asset correlation*. It is a measure of the co-movement of their asset returns which in turn reflect their business success. Starting with Markowitz' (1952) theory of portfolio selection, the notion of correlation itself has not been questioned for a long time as to its aptness for applications in financial risk management. Although correlation only reflects the *linear* dependence between two random variables and there is a growing body of literature dealing with the shortcomings of this measure[12], it is still the predominant paradigm in virtually all practical issues of credit risk management.

The density function of the bivariate standard normal distribution is denoted by φ_2:[13]

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11 Cf. Gupton/Finger/Bhatia (1997), p. 87 f., footnotes 3 and 4. See also Kealhofer/Bohn (2001), p. 2 ff., who summarize the option theoretical approach to default risk.

12 Cf. Embrechts/McNeil/Straumann (1999) and Frey/McNeil/Nyfeler (2001). See also Hahnenstein/Röder (2003), who discuss the limitations of correlation with respect to corporate hedging.

13 See e. g. Crouhy/Galai/Mark (2000), p. 76 f.

$$\varphi_2\left(r_i; r_j; \rho_{ij}\right) = \frac{1}{2\pi\sqrt{1-\rho_{ij}^{\,2}}} \exp\left(-\frac{1}{2(1-\rho_{ij}^{\,2})}(r_i^{\,2} - 2\rho_{ij} r_i r_j + r_j^{\,2})\right). \tag{5}$$

Thus, the shape of the joint asset return density function of two firms is completely described by a single parameter, namely their asset correlation ρ_{ij}.

The probability that both obligors i and j will default jointly is denoted by PD_{ij}. This probability is calculated using the default thresholds $r_i´$ and $r_j´$ that result from PD_i and PD_j via equation (3) together with the bivariate asset return density function (5):[14]

$$PD_{ij} = P\{\tilde{r}_i < r_i' \wedge \tilde{r}_j < r_j'\} = \int_{-\infty}^{r_i'} \int_{-\infty}^{r_j'} \varphi_2\left(r_i; r_j; \rho_{ij}\right) dr_i dr_j = \int_{-\infty}^{\Phi^{-1}(PD_i)} \int_{-\infty}^{\Phi^{-1}(PD_j)} \varphi_2\left(r_i; r_j; \rho_{ij}\right) dr_i dr_j \,. \tag{6a}$$

Alternatively, the joint PD can be written by means of the bivariate standard normal ("Gaussian") distribution function Φ_2, which can be interpretated as the CreditMetrics™ *copula* function:[15]

$$PD_{ij} = \Phi_2\left(\Phi^{-1}(PD_i); \Phi^{-1}(PD_j); \rho_{ij}\right). \tag{6b}$$

Note that only with *uncorrelated* asset returns ($\rho_{ij} = 0$) the probability of a joint default can be derived immediately from the individual PDs:

$$PD_{ij} = \int_{-\infty}^{\Phi^{-1}(PD_i)} \int_{-\infty}^{\Phi^{-1}(PD_j)} \varphi(r_i) \cdot \varphi(r_j)\, dr_i dr_j = \int_{-\infty}^{\Phi^{-1}(PD_i)} \varphi(r_i)\, dr_i \cdot \int_{-\infty}^{\Phi^{-1}(PD_j)} \varphi(r_j)\, dr_j = PD_i \cdot PD_j \,. \tag{7}$$

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14 Cf. Gupton/Finger/Bhatia (1997), p. 89, equation (8.5). See also Overbeck/Stahl (2003) for the relationship between asset and default correlations. Empirical evidence of default correlation is provided by de Servigny/ Renault (2003) for the US and by Hamerle/Rösch (2003) and Rösch (2003) for Germany.

15 See Kealhofer/Bohn (2001), p. 11 f. See also Li (2000), p. 49 f., particularly equations (10) and (11). Bluhm/ Overbeck/Wagner (2003), p. 103 ff. offer an introduction to the use of copulae in credit risk measurement.

A graphical illustration of the relationship between the joint density function of asset returns, the default thresholds and the joint probability of default is given in Figure 2, assuming an asset correlation of $\rho_{ij} = 0.6$.

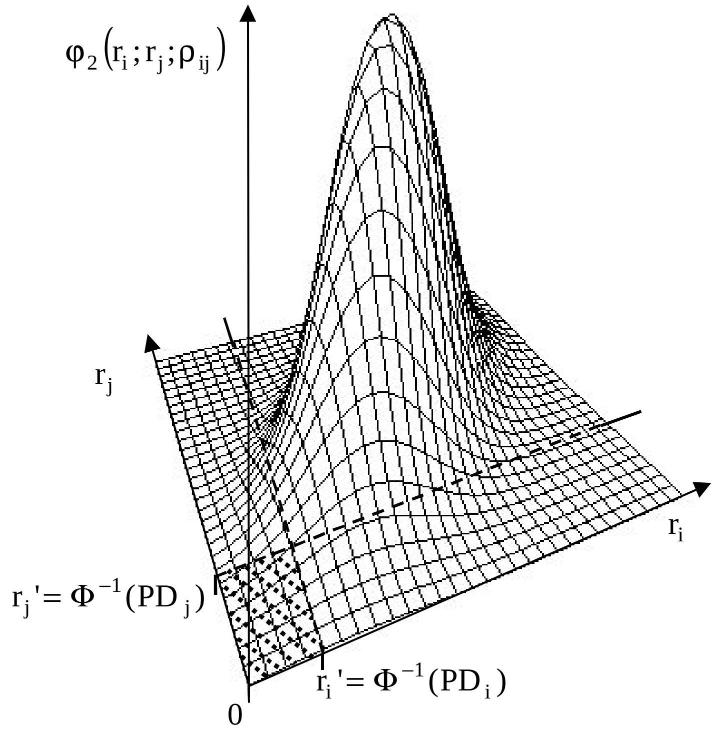

Figure 2: Relationship between the bivariate standard normal density function of asset returns, the default thresholds and the joint probability of default for $\rho_{ij} = 0.6$.

The area that contains those combinations of r_i and r_j that lead to a joint default of both obligors lies in the lower left corner of the base and is hatched. The corresponding probability PD_{ij} is the volume of the solid between the hatched part of the $(r_i; r_j)$-plane and the bivariate density function (5).

Figure 3 contrasts the iso-density ellipses for uncorrelated ($\rho_{ij} = 0$) and positively correlated ($\rho_{ij} = 0.5$) asset correlations. As indicated by the narrower iso-density ellipses for $\rho_{ij} = 0.5$, there is more probability mass lying above the the hatched part of the plane; the probability mass is quasi concentrated in this area by the rise in asset correlation. This is to illustrate, that the probability of a joint default PD_{ij} is a strictly monotically increasing function of the asset correlation between the two obligors. Moreover, the joint PD also strictly rises with increasing isolated PDs, so that the signs of all comparative statics, which are given by Leibniz' rule, are clear-cut:[16]

$$\frac{\partial PD_{ij}}{\partial \rho_{ij}} > 0 \; ; \quad \frac{\partial PD_{ij}}{\partial PD_{i}} > 0 \; ; \quad \frac{\partial PD_{ij}}{\partial PD_{j}} > 0 \, . \qquad (8)$$

16 Note that this result is economically trivial, but can be technically difficult if other copula functions are employed.

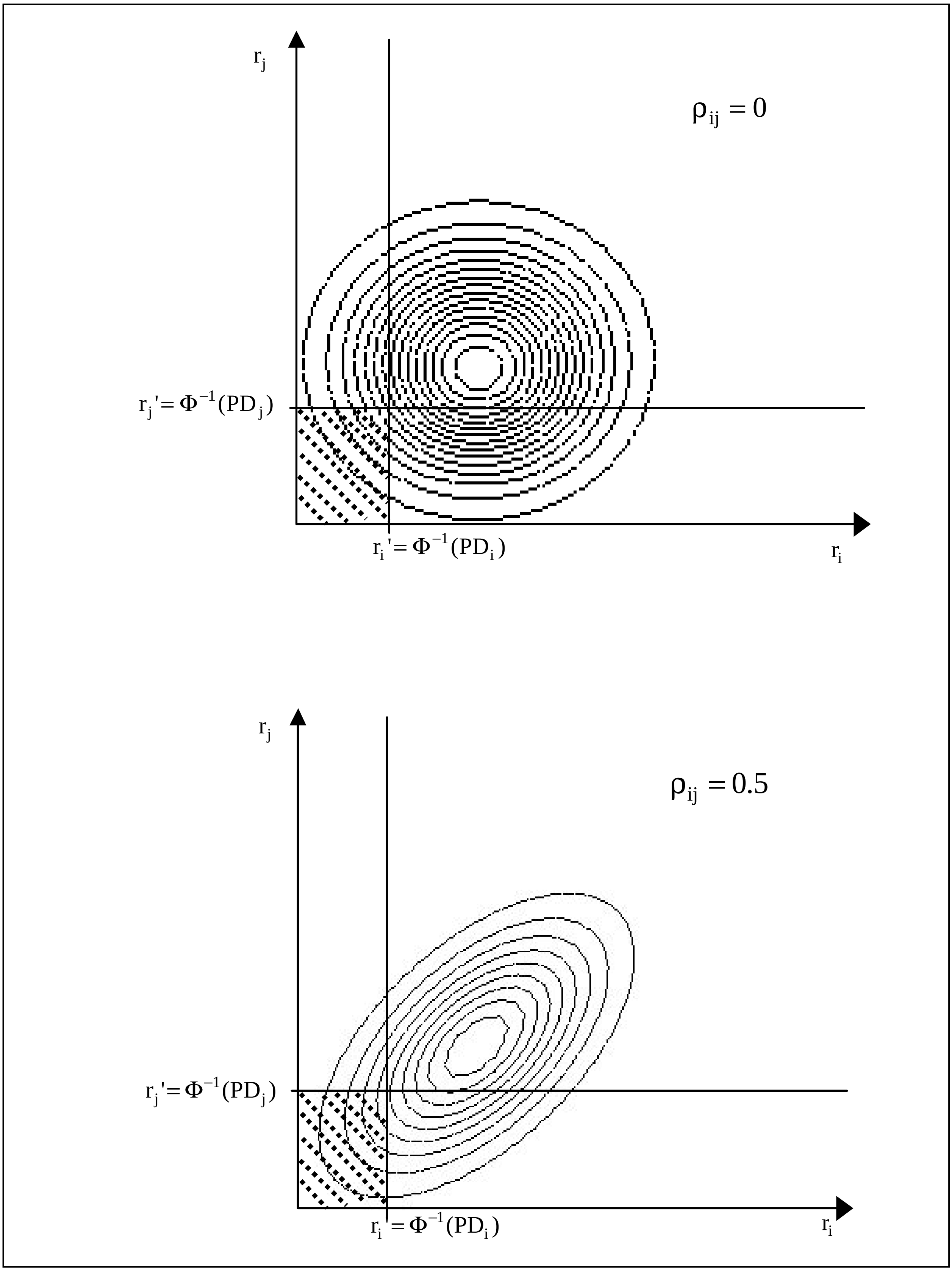

Figure 3: Impact of an increase in asset correlation (from $\rho_{ij} = 0$ to $\rho_{ij} = 0.5$) on the probability of a joint default PD_{ij}.

Having clarified the relationship between asset correlations and joint PDs in the CreditMetrics™ framework, we now turn to the calculation of the portfolio loss distribution for a two-obligor portfolio. Let us assume that the bank has an exposure of E_i dollars to obligor i and of E_j dollars to obligor j. The recovery rates in default are given as a percentage of the respective exposure and denoted by RR_i and RR_j. All of these variables are assumed to be known with certainty. We deliberately exclude the stochastic recovery rate available in CreditMetrics™ as this would needlessly complicate our analysis here. Under these simplifying assumptions, the probability distribution of portfolio loss consists only of the following four states: Either none of the two obligors defaults (state 1), both default (state 2) or just one of them defaults (states 3 and 4). The probabilities p(s) of these four scenarios and the corresponding portfolio losses L(s) are displayed in Figure 4.

obligor i	**obligor j**	**state of the portfolio s**	**state probability p(s)**	**portfolio loss L(s) in state s**
non default	non default	1	p(1)	0
default	default	2	p(2)	$E_i \cdot (1 - RR_i) + E_j \cdot (1 - RR_j)$
non default	default	3	p(3)	$E_j \cdot (1 - RR_j)$
default	non default	4	p(4)	$E_i \cdot (1 - RR_i)$

Figure 4: Probability distribution of portfolio loss.

The calculation of the set of state probabilities which describes the complete loss distribution is straightforward: If the two individual PDs and the asset correlation ρ_{ij} are known, the probability of a joint default is given by (6a) or (6b):

$$p(2) = P\{\tilde{r}_i < r_i' \wedge \tilde{r}_j < r_j'\} = PD_{ij}. \quad (9)$$

We further know from (3) that

$$PD_i = P\{\tilde{r}_i < r_i' \wedge \tilde{r}_j < r_j'\} + P\{\tilde{r}_i < r_i' \wedge \tilde{r}_j \geq r_j'\} = p(2) + p(4) \Leftrightarrow p(4) = PD_i - PD_{ij} \quad (10)$$

and

$$PD_j = P\{\tilde{r}_j < r_j' \wedge \tilde{r}_i < r_i'\} + P\{\tilde{r}_j < r_j' \wedge \tilde{r}_i \geq r_i'\} = p(2) + p(3) \Leftrightarrow p(3) = PD_j - PD_{ij}. \quad (11)$$

The forth state probability is given by

$$p(1) = 1 - p(2) - p(3) - p(4). \qquad (12)$$

Hence, the probability distribution of the potential losses in the loan portfolio is characterized completely by the individual PDs of the obligors together with their asset correlation. This idea – which can be generalized from our simple two obligor illustration to the realistic case of an n obligor loan portfolio in a rather straightforward manner[17] – forms the core of the CreditMetrics^{TM} asset correlation approach to joint default events.

2. Estimating companies' asset correlations by means of an index model

Determining the loss distribution for a loan portfolio of n obligors in the way described above, requires estimates of the $n \cdot (n-1) / 2$ pairwise asset correlations for the year to come. To distinguish exactly between correlations of random future asset returns ρ_{ij} as defined in (4) and their empirical estimators, we introduce the symbol $\hat{\rho}_{ij}$ for the latter.

In order to reduce data requirements and to simplify the parameter estimation, the CreditMetrics^{TM} methodology deduces estimates of the obligors' individual asset correlations from stock indices by means of a factor model.[18] The problem of using *equity* correlations as a proxy for *asset* correlations, which has already been recognized as a potential drawback by the model's inventors themselves and which has recently been attacked on theoretical and empirical grounds by Zeng/Zhang (2002) of KMV, is *not* the point of our paper.

In the general CreditMetrics^{TM} approach, country as well as industry weights are assigned to each obligor according to its participation therein.[19] For our purposes, we make the following two simplifying assumptions:

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[17] See Bluhm/Overbeck/Wagner (2003), especially p. 71, equations (2.34) and (2.35).

[18] Cf. Gupton/Finger/Bhatia (1997), p. 93. See also Schönbucher (2000) for an introduction to factor models in credit risk measurement.

[19] Cf. Gupton/Finger/Bhatia (1997), p. 98.

- Firstly, we ignore possible calibration problems arising from the issue of cross-country diversification. Since our focus is on a portfolio of German non-listed corporate obligors, all country weights can simply be set to 100 % for Germany, so that all other countries are ignored. Hence, the degree of concentration in such a purely national loan portfolio is mainly driven by the companies' industry composition.

- Secondly, we do not explicitly consider those calibration problems that concern conglomerates. Most banks' internal databases offer just one industry affiliation per customer, e. g. according to the WZ93 code of the Statistisches Bundesamt. A percentage allocation of one customer to more than one industry, which is offered by Reuters or Bloomberg for many publicly-traded companies, is therefore not feasible in practice for most medium-sized corporate obligors. Hence, each obligor must be assigned to its affiliated industry index by 100 %. As a result, all that can be done is mapping each firm i = 1..n to just one industry k(i) = 1..m.

With these simplifications, the CreditMetrics[TM] index model can be written as:

$$\tilde{r}_i = \sqrt{w_i} \cdot \tilde{f}_{k(i)} + \sqrt{1-w_i} \cdot \tilde{\varepsilon}_i \quad \forall\, i = 1..n\,, \tag{13}$$

with

$$w_i \in [0;1], \quad \forall\, i = 1..n\,, \tag{14}$$

$$\tilde{r}_i,\, \tilde{f}_{k(i)},\, \tilde{\varepsilon}_i \sim N(0;1), \quad \forall\, i = 1..n\,, \tag{15}$$

$$\mathrm{Cov}\left(\tilde{\varepsilon}_i;\tilde{\varepsilon}_j\right) = 0, \quad \forall\, i = 1..n\,,\ j = 1..n,\, i \neq j \text{ and} \tag{16a}$$

$$\mathrm{Cov}\left(\tilde{\varepsilon}_i;\tilde{f}_{k(j)}\right) = 0, \quad \forall\, i = 1..n\,,\ j = 1..n\,. \tag{16b}$$

$\tilde{f}_{k(i)}$ denotes the return of the industry index k to which company i is classed. A list of the indices which have been actually used for Germany in the first version of CreditManager[TM] can be found in Appendix I of the Technical Document.[20] The 10 CDAX© indices given there have been amended in the meantime to the 19 CDAX© sub-indices calculated by the Deutsche Börse AG (2001).[21]

The company-specific noise term $\tilde{\varepsilon}_i$ represents the *idiosyncratic* – synonymously called *firm-specific* or *unsystematic* – movements in asset returns. Each obligor's noise term is assumed to be uncorrelated with the noise terms of all other firms and furthermore also uncorrelated with the movements that affect the industry as a whole and which are therefore fully reflected in the respective index return. We use the term idiosyncratic risk in the sense that it describes that component of the total variation in the asset return of a company which cannot be explained by its industry affiliation. Correspondingly, we refer to the industry influence that is embodied in the movements of the respective index as *systematic*, because these movements can be regarded as induced by changes in latent variables which affect many firms (at least more than one firm) in that particular industry.

Note that this definition of systematic and unsystematic return components does *not* necessarily imply that the systematic part is the one that cannot be deleted through proper diversification. Therefore, our definition differs from that used in traditional portfolio theory and in the CAPM. Moreover, the definition used here is neither identical to that of the APT due to Ross (1976, 1977) which defines systematic risk in terms of *unexpected* changes of some factors and does not explicitly identify these factors as industry indices. Finally, the APT risk factors are assumed to be (almost) uncorrelated which is obviously not true for the equity indices used in CreditMetrics™ where their differing correlations represent just a part of the crucial and most valuable input data.

w_i and $1 - w_i$ represent the weights that must be assigned to the industry or alternatively to the firm-specific influence on asset returns. The bigger w_i, the closer tracks the firm its industry performance and the lesser does it move independently from its industry companions. The weights are scaled in such a way that all random variables involved can be modelled as standard normally distributed according to (15). As a result, it is possible to split up the total risk in fluctuating asset returns in two different components, which do not intersect:

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20 Cf. Gupton/Finger/Bhatia (1997), p. 94 – 96 and additionally Appendix I, p. 166.

21 Cf. Deutsche Börse AG (2001), p. 4.

$$\underbrace{\underbrace{\mathrm{Var}(\tilde{r}_i)}_{=1}}_{\text{total risk}} = \underbrace{w_i \cdot \underbrace{\mathrm{Var}(\tilde{f}_{k(i)})}_{=1}}_{\text{systematic risk}} + \underbrace{(1-w_i)\cdot \underbrace{\mathrm{Var}(\tilde{\varepsilon}_i)}_{=1}}_{\text{unsystematic risk}} + \underbrace{2\cdot\sqrt{w_i\cdot(1-w_i)}\cdot \underbrace{\mathrm{Cov}(\tilde{f}_{k(i)};\tilde{\varepsilon}_i)}_{=0}}_{\text{no intersection}} \tag{17}$$

As we are solely interested in an estimation of the pairwise asset correlations ρ_{ij} between the firms i and j, we can assume *standardized* company (asset) returns as well as *standardized* (equity) index returns and idiosyncratic returns according to (15) without loss of generality.

The index model (13) to (16) enables a straightforward calculation of pairwise asset correlations. For two obligors i, j belonging to not necessarily different industries k(i), k(j) (4) yields:

$$\begin{aligned}
\rho_{ij} &= \mathrm{Cov}\left(\tilde{r}_i;\tilde{r}_j\right) = E\left(\tilde{r}_i\cdot\tilde{r}_j\right) - E\left(\tilde{r}_i\right)\cdot E\left(\tilde{r}_j\right) \qquad (18)\\
&= E\left((\sqrt{w_i}\cdot\tilde{f}_{k(i)} + \sqrt{1-w_i}\cdot\tilde{\varepsilon}_i)\cdot(\sqrt{w_j}\cdot\tilde{f}_{k(j)} + \sqrt{1-w_j}\cdot\tilde{\varepsilon}_j)\right) - 0\\
&= \sqrt{w_i\cdot w_j}\cdot E\left(\tilde{f}_{k(i)}\cdot\tilde{f}_{k(j)}\right) + \sqrt{w_i\cdot(1-w_j)}\cdot \underbrace{E\left(\tilde{f}_{k(i)}\cdot\tilde{\varepsilon}_j\right)}_{=\mathrm{Cov}\left(\tilde{f}_{k(i)};\tilde{\varepsilon}_j\right)=0}\\
&\quad + \sqrt{(1-w_i)\cdot w_j}\cdot \underbrace{E\left(\tilde{\varepsilon}_i\cdot\tilde{f}_{k(j)}\right)}_{=\mathrm{Cov}\left(\tilde{\varepsilon}_i;\tilde{f}_{k(j)}\right)=0} + \sqrt{(1-w_i)\cdot(1-w_j)}\cdot \underbrace{E\left(\tilde{\varepsilon}_i\cdot\tilde{\varepsilon}_j\right)}_{=\mathrm{Cov}\left(\tilde{\varepsilon}_i;\tilde{\varepsilon}_j\right)=0}\\
&= \sqrt{w_i\cdot w_j}\cdot \mathrm{Cov}\left(\tilde{f}_{k(i)}\cdot\tilde{f}_{k(j)}\right) = \sqrt{w_i\cdot w_j}\cdot\rho_{k(i)k(j)}\,.
\end{aligned}$$

The simplification offered by (18) is as follows: If one can get an (independent) estimate of the percentage portion of total asset variance that is firm-specific for each company i = 1..n, then all n · (n – 1) / 2 asset correlations can be calculated immediately from the m · (m – 1) / 2 correlations between the k = 1..m industry indices via (18). Empirical estimates of the latter – $\hat{\rho}_{k(i)k(j)}$ – are calculated within CreditManager™ from historical weekly index returns by means of the standard formulae[22] traditionally used in value at risk concepts for market risk. These formulae provide unbiased estimators for variances and covariances, if the time series observations are interpreted as independent drawings from the unchanged "true" distributions (i.i.d. property). In CreditManager™ version 2.5 these estimators have been calculated from the last 52 weekly observations before the effective date of the analysis.[23]

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[22] Cf. Gupton/Finger/Bhatia (1997), p. 97, equations [8.7] to [8.9].

[23] Note that this sample size differs significantly from the 190 observations mentioned in the Technical Document, p. 97. Furthermore, in CreditManager™ version 2.5 the index values were transformed into discrete, not

We do not want to touch here on the ubiquitous problem of choosing the proper sample size to grant the best forecast for the year to come, nor deal with more sophisticated estimators that can be found in most econometrics textbooks. As explained in the introduction, the point we want to make regards the calibration of the weights of the systematic components w_i.

Whenever all the bank's obligors in the credit portfolio under consideration are companies listed on a stock exchange, individual estimates of the weights w_i ($\hat{w}_i$) can be derived from the coefficient of determination R_i^2 in a standard time-series regression model:[24]

$$\hat{w}_i = R_i^2 \quad , \forall\, i = 1..n . \tag{19}$$

To see this, recall that the coefficient of determination (R-squared) in a univariate, linear ordinary-least-square (OLS) regression model of the form[25]

$$(\hat{r}_{i,t} - r_{i,t})^2 = (a_i + b_i \cdot f_{k(i),t} - r_{i,t})^2 \rightarrow \min_{a,b} \tag{20}$$

is defined as:

$$R_i^2 = \frac{\sum_{t=1}^{T} (\hat{r}_{i,t} - \bar{r}_i)^2}{\sum_{t=1}^{T} (r_{i,t} - \bar{r}_i)^2} \quad \text{with} \quad \bar{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t} \ . \tag{21}$$

We denote the observed discrete return on stock i during the period [t – 1; t] as $r_{i,t}$ and its estimate – as given by the fitted coefficients a, b in the regression equation $\hat{r}_{i,t} = a_i + b_i \cdot f_{k(i),t}$ – as $\hat{r}_{i,t}$. Thus, (21) states that the coefficient of determination R_i^2 is the quotient of that part of

log returns. With the introduction of the CreditManager™ version 3.1, the estimation of asset correlations cannot be duplicated in detail by users any more. The CDAX© index set has been replaced by one from MSCI and the index time series actually used do not lay open, because of missing redistribution rights.

24 But note the criticism uttered by Kealhofer/Bohn (2001), p. 12.

25 See e. g. Copeland/Weston (1992), Appendix C, p. 877-893, for a short brush-up in regression analysis.

variation of the dependent variable which is explained by the regression equation (numerator) and its total variation around its mean (denominator).

To clarify the relationship between the estimated slope term in the regression equation b_i, which is a sensitivity measure that resembles the beta coefficients known from CAPM and APT, the coefficient of determination R_i^2 and the coefficient of correlation between the stock return and its industry index $\rho_{i,k(i)}$, we transform (21) in the following way:[26]

$$R_i^2 = \frac{\frac{1}{T-1} \cdot \sum_{t=1}^{T} (a_i + b_i \cdot f_{k(i),t} - \bar{r}_i)^2}{\frac{1}{T-1} \cdot \sum_{t=1}^{T} (r_{i,t} - \bar{r}_i)^2} = \frac{\hat{Var}\left(a_i + b_i \cdot \tilde{f}_{k(i)}\right)}{\hat{Var}\left(\tilde{r}_i\right)} = \frac{b_i^2 \cdot \hat{Var}\left(\tilde{f}_{k(i)}\right)}{\hat{Var}\left(\tilde{r}_i\right)}. \tag{22}$$

We further know that the sensitivity coefficient b_i can be written as:[27]

$$b_i = \frac{\hat{Cov}\left(\tilde{r}_i ; \tilde{f}_{k(i)}\right)}{\hat{Var}\left(\tilde{f}_{k(i)}\right)} = \frac{\hat{\rho}_{i,k(i)} \cdot \sqrt{\hat{Var}\left(\tilde{r}_i\right)}}{\sqrt{\hat{Var}\left(\tilde{f}_{k(i)}\right)}}. \tag{23}$$

Putting (22) and (23) together, we see the well-known fact that the coefficient of determination is simply the square of the estimated coefficient of correlation between stock and index returns:

$$R_i^2 = \hat{\rho}_{i,k(i)}^2 . \tag{24}$$

With (18), (19) and (24), the estimator of the asset (= equity) correlation between two companies can be written as:

$$\hat{\rho}_{i,j} = \sqrt{\hat{w}_i \cdot \hat{w}_j} \cdot \rho_{k(i),k(j)} = \hat{\rho}_{i,k(i)} \cdot \hat{\rho}_{j,k(j)} \cdot \hat{\rho}_{k(i),k(j)} . \tag{25}$$

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[26] Cf. Bluhm/Overbeck/Wagner (2003), p. 45, equation (1.19) for a similar result in the context of the KMV model.

[27] Cf. Copeland/Weston (1992), p. 879, equation (C.3) and p. 881, footnote 5.

Thus, the pairwise asset correlation between two companies is decomposed into the product of the equity correlations between the two firms and their respective industry index and the correlation between these two indices.

In the case of non-publicly traded obligor firms, which is typical for many medium-sized enterprises in Germany (the so-called "Mittelstand"), the regression model described above cannot be fitted because of lacking stock price data. In the next section, we analyze, if the approach currently implemented in CreditManager™ offers a reasonable solution to this dilemma situation.

III. The relationship between systematic risk and company size

1. The approach currently implemented in CreditManager™

Although not explicitly formulated in the CreditMetrics™ Technical Document, the idea that company size is an important driver of systematic risk is already anchored there: "Generally, prices for companies with large market capitalization will track the indices closely, and the idiosyncratic portion of the risk to these companies is small; on the other hand, prices for companies with less market capitalization will move more independently of the indices, and the idiosyncratic risk will be greater."[28] The Interface File Specification to CreditManager™ version 2.5 is more precise: "In general, obligor-specific risk can be considered to be a function of company size. Larger companies have relatively small firm-specific risk because their behavior tends to be like that of the overall market (often they are components of market benchmarks). Smaller companies can have larger firm-specific risk since they are more likely to behave independently of broad market trends and are less likely to be index components."[29] This is the main economic argument behind the solution to the estimation problem (19) offered in CreditManager™:[30]

$$\hat{w}_i^{CM} = \hat{R}_i^2(M_i) = 1 - \frac{1}{1 + M_i^{\gamma} \cdot \exp(\lambda)} \quad , \forall\, i = 1..n . \tag{26}$$

Equation (26) provides a "general rule" that gives an estimate of the overall weight of the sys-

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28 Gupton/Finger/Bhatia (1997), p. 98.

29 RiskMetrics™ Group (1999), p. 54.

30 Cf. RiskMetrics™ Group (2000), p. 20, especially the rearrangement of equation (C.3). See also again RiskMetrics™ Group (1999), p. 54 and RiskMetrics™ Group (2002), p. 3.

tematic component in asset returns for each obligor i. The only input data required by users for the calculation of these weights are the market values of the firms' total assets (in USD), which we denote by M_i. The two parameters in the logistic function (26), which produces R-squared estimates between zero and one, are actually set to $\gamma = 0.550$ and $\lambda = -12.600$ in the CreditManager™ version 3.1. A plot of the function for these parameter values is given in Figure 6.

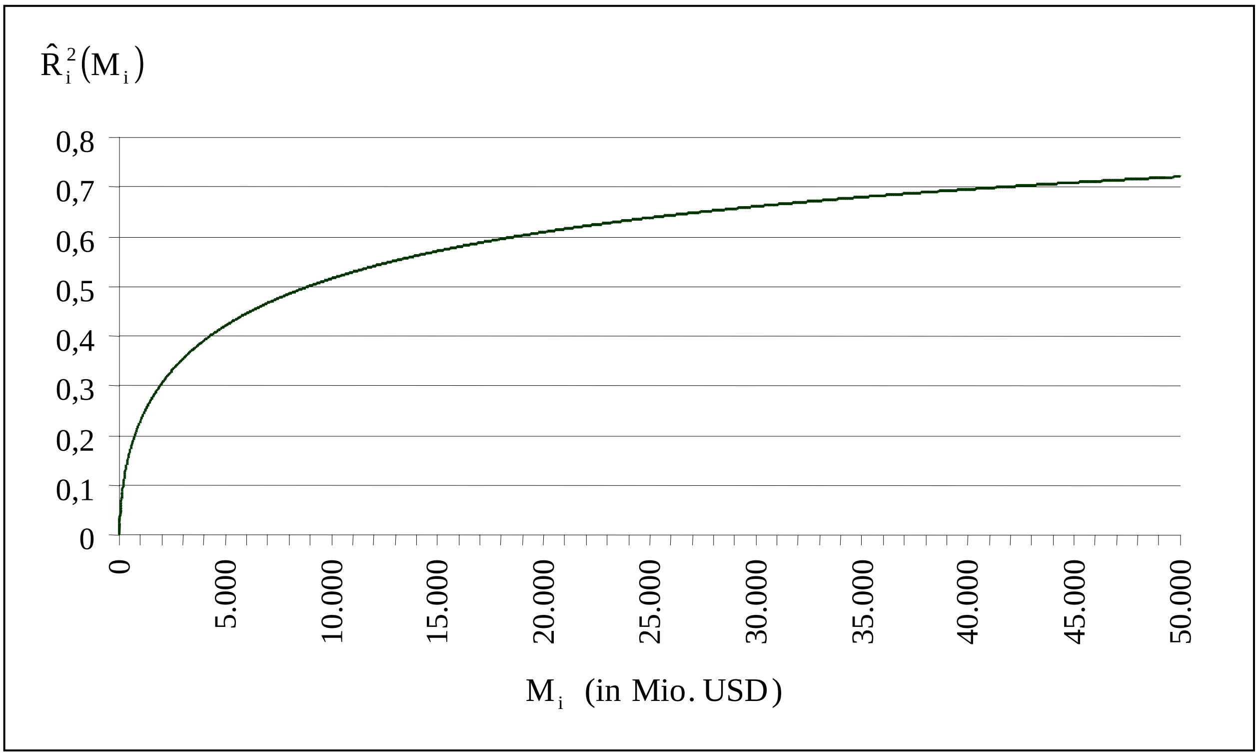

Figure 6: Relationship between the market value of the firm's total assets (in Mio. USD) and the portion of its systematic risk.

To calibrate the logistic function (26), RiskMetrics™ Group has developed its own econometric methodology, which is basically a refined quadratic programming approach.[31] The data used in their latest study dealing with this calibration problem consist of 200 stocks representing 14 nations and covering the period from 1993 to 1997.

Our main objection against using the result of this study, on which the calibration of (26) presented in Figure 6 is based, was simply that it could hardly be regarded as representative, when considering a bank with a focus on the German mid-cap loan market.

31 See RiskMetrics™ Group (2000) for further details.

2. Empirical estimation of a two-step regression model with a random sample of German stocks

2.1 Motivation and statistical design

In detail, our reasons to do an empirical study of our own were as follows:

- The study carried out by RiskMetrics™ Group was an international one, with the focus – at least implicitly – on a globally diversified portfolio. The sample contained only 10 German stocks (BASF AG, Bayer AG, Commerzbank AG, Daimler-Benz AG, Deutsche Bank AG, Dresdner Bank AG, DSL Holding AG, Lufthansa AG, Siemens AG and Volkswagen AG) at all. These seemed just too few to get a precise picture as to the calibration of the credit value at risk for a loan portfolio concentrated in Germany.
- Virtually all of the stocks chosen by RiskMetrics™ Group for the German sub-sample were big "global players", with correspondingly high market capitalizations. From our point of view, this could hardly lead to a "good fit" in the parameters when applied to a portfolio of medium-sized businesses, a typical German "Mittelstandsportfolio". These obligors all lie at the extreme left margin of the x-axis in Figure 6, where the curve is very steep.
- The stocks in the German sub-sample were clustered in just 5 industries, covering not even the half of the 11 CDAX© indices named in the study.
- We favoured the simple, but robust econometric approach of a two-step (OLS-) regression model, which was introduced into the field of capital market research by Fama/MacBeth (1973) and which was – following Chen/Roll/Ross (1986) – used in the bulk of empirical studies dealing with the derivation of risk premia from beta coefficients in the context of the APT.
- As the parameter values of γ and λ were fitted in the RiskMetrics™ Group study with respect to market capitalizations denominated in USD, fluctuating USD/EUR exchange rates would influence company sizes measured in USD and thereby exert an unjustified and quite arbitrary influence on the obligors' portions of idiosyncratic risk.
- As there are no market capitalizations available for the large group of non-listed mid-cap firms in Germany, we decided to use the book value of total assets as a second, alternative measure of company size. These book values should already be available in banks' internal databases and, as the regression parameters are calibrated with respect to this variable, there is no need for further adjustments involving e. g. EBITDA-multiples to generate arti-

ficial “market values”.

- We wanted our calibration of the weights of the systematic components to fit in with our simplified factor model according to (14) to (16). Therefore, since we could not map an obligor to more than one industry, we needed an estimate of the extent to which each obligor’s return was driven by that particular industry. As a consequence, we had to restrict ourselves to a set of *univariate* linear time-series regressions in the first step (to calculate estimates of the coefficients of determination $\hat{R}_i^2$), which were followed in the second step by a cross-sectional regression. Moreover, as the industry correlations were estimated in CreditManager™ version 2.5 from the last 52 weekly observations, we decided to use roughly one year of weekly returns for the estimation of the R-squared as well.
- Last but not least, the necessary tranfer of the economic reasoning behind (26) to the case of non-listed obligor firms seemed a bit dubious to us. As a matter of fact, the part of variation explained in the time-series regression of a stock on an index will be greater – all else equal – if the stock has a higher weight in the index. This effect is *tautological*, insofar as the stock returns are to a larger extent explained by themselves. The problem is that this relationship is just an artificial reflex of the index composition and does not contain any worthwhile economic meaning for our application. Therefore, to arrive at a reasonable rule relating company size and the portion of idiosyncratic risk, which can be applied to non-publicly-traded companies as well, the influence of the index weights has to be filtered out. This can either be achieved by constructing a new set of index time series with equal weights for all stocks in that industry or – as a kind of “quick and dirty” solution – by excluding those stocks with particularly high index weights from the sample.

2.2 Data and results

We drew a random sample of 44 companies from the total 790 companies listed on the Frankfurt stock exchange on the 30.01.2002. For these 44 ISIN codes we received the weekly Friday fixing prices from Bloomberg for the year 2001, exactly from the 05.01.2001 to the 25.01.2002. Because of missing entries, we had to eliminate 7 stocks from our sample. Additionally, we removed two more titels, one bank and one insurance company, because our aim was to get a calibration for a portfolio of corporates, not pure financial intermediaries with rather untypical balance sheet characteristics.

Selected data on the remaining 35 stocks (company names, ISIN codes, industry affiliations, weights in the respective CDAX© industry index)[32] are given in Table 1.

ISIN-Code	Company Name	CDAX-Subindex	Index weight (Deutsche Börse AG, 30.01.2002)	Market capitalization (in Mio. EUR, Bloomberg, 13.03.2002)	Book Value of Total Assets (in Mio. EUR, Bloomberg, 13.03.2002)	R-squared
DE0005067300	SZ TESTSYSTEME AG	Technology	0,044%	39,811	84,520	0,180
DE0005085906	AWD HOLDING AG	Financial services	3,967%	1.103,268	749,780	0,106
DE0005165906	BARMAG AG	Machinery	0,850%	199,920	314,790	0,000
DE0005183701	EICHBORN AG	Media	0,391%	15,000	23,400	0,023
DE0005220909	NEXUS AG	Software	0,043%	27,788	42,760	0,267
DE0005285704	B.U.S BERZEL.UM.ST.	Industrial	1,216%	130,243	291,260	0,070
DE0005341408	NEUE SENTIM.FILM	Media	0,616%	18,095	43,340	0,091
DE0005422703	BIODATA INFOR.TECHN.AG	Technology	0,004%	4,208	99,430	0,072
DE0005501456	SYSKOPLAN AG	Software	0,132%	86,919	41,830	0,262
DE0005508105	DT.BETEILIG.AG	Industrial	3,903%	289,520	292,420	0,238
DE0005558696	PARAGON AG	Technology	0,018%	14,000	24,210	0,202
DE0005751002	MAN ROLAND DRUCKM.AG	Machinery	1,855%	418,080	986,000	0,000
DE0006005200	HAITEC AG	Software	0,009%	6,269	74,780	0,017
DE0006083439	HORNBACH HOLD.VZO	Retail	0,704%	472,000	1.122,600	0,015
DE0006205701	IVG HOLDING AG	Financial services	5,218%	1.356,040	2.584,930	0,255
DE0006223803	IN-MOTION AG	Media	2,493%	43,320	130,930	0,332
DE0006228604	I-D MEDIA AG	Software	0,024%	13,860	48,060	0,073
DE0006350002	KALI-CHEMIE AG	Chemicals	1,398%	858,000	152,190	0,067
DE0006496003	CUSTODIA HOLDING AG	Food&Beverages	11,935%	390,710	314,950	0,008
DE0006595101	MEDICLIN AG	Pharma & Health	0,215%	92,925	375,560	0,108
DE0006602006	MG TECHNOLOGIES AG	Industrial	27,195%	2.428,481	6.910,440	0,821
DE0006622905	MONACHIA AG AKTIEN	Financial services	1,611%	419,020	54,170	0,000
DE0006656101	MWB WERTPAPIERH.AG	Financial services	0,041%	8,969	43,410	0,085
DE0006904709	PEGASUS BETEILIG.AG	Financial services	0,031%	4,461	27,720	0,016
DE0006943608	PROUT AG	Software	0,004%	2,051	13,240	0,118
DE0007057705	ROHWEDDER AG	Machinery	0,343%	65,565	76,520	0,141
DE0007072001	RUETGERS AG	Chemicals	2,052%	1.370,800	2.640,990	0,003
DE0007164030	JIL SANDER AG VZO	Consumer-cyclical	0,601%	106,250	103,160	0,055
DE0007224404	SENATOR ENTERTMT. AG	Media	2,477%	75,820	289,470	0,296
DE0007249104	SOFTM SOFTW.U.BER.	Software	0,047%	33,117	33,790	0,101
DE0007283004	STRABAG AG	Construction	0,879%	78,787	1.099,020	0,032
DE0007507501	WASHTEC AG	Machinery	0,073%	17,176	225,450	0,029
DE0007608101	UMWELTKONT.RENEW.	Technology	0,132%	109,750	65,920	0,192
DE0007614406	E.ON AG	Utilities	49,560%	39.358,180	106.215,000	0,855
DE0007852139	ZANDERS FEINPAP.	Basic Resources	0,552%	218,880	478,800	0,024

Table 1: ISIN codes, company names, industry affiliations, CDAX© weights, market capitalizations, book values of total assets and coefficients of determination (R-squared) calculated according to (21).

The firms' market capitalizations and book values of total assets per 13.03.2002, which we got from Bloomberg, are included in Table 1 as well. The market capitalizations (book values of total assets) in our sample vary between 2.051 (13.240) and 39.358 (106.215) Mio. EUR with an average of 1.425 (3.602) Mio. EUR. The index weights have a minimum of 0.004 % and a maximum of 49.560 %, 3.477 % on average.

The stocks in our sample cover 14 of the total 19 CDAX© industry indices. The time series of the 14 indices for the 56 Fridays were also taken from Bloomberg. As the index values are calculated by the Deutsche Börse AG on the basis of closing prices (at about 20.15 h), there is a time

32 These data were provided by Deutsche Börse AG, ip.hotline@deutsche-boerse.com.

lag of approximately 8 hours to the corresponding stock prices which are fixed at 12.30 h. This lag has certainly led to an overall downward bias in our OLS-estimates of the 37 coefficients of determination, which are disclosed in Table 1, too. They were calculated in the way described in section II.2 from the 55 discrete weekly returns by regressing the stock returns of firm i on its corresponding industry index k(i).[33] But, as long as this bias can assumed to be of nearly equal size for all stocks, the error will probably cancel out in the cross-sectional regressions. The coefficients of determination which are the result of the first-step-regressions vary between 0.000 and 0.855 with an average of 0.147 (see Table 1). The scatter plots in the Figures 7 and 8 reveal, in what way they are related to our two measures of company size.

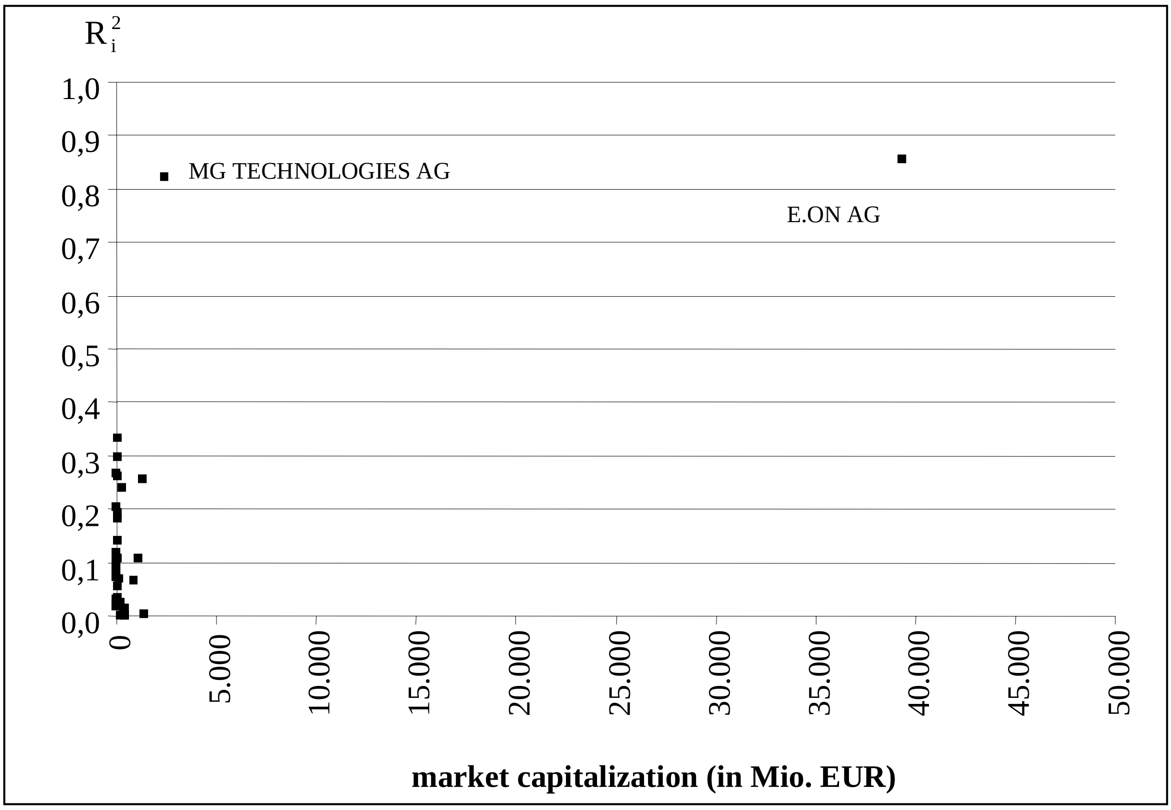

Figure 7: Scatter plot of coefficients of determination (R-squared) against market capitalizations.

[33] Moreover, the returns have been standardized to have an average of zero and a sample standard deviation of one. Note that this linear transformation is irrelevant for the resulting coefficients of determination. All regressions were calculated with SPSS® version 10.0 under Microsoft® Windows NT.

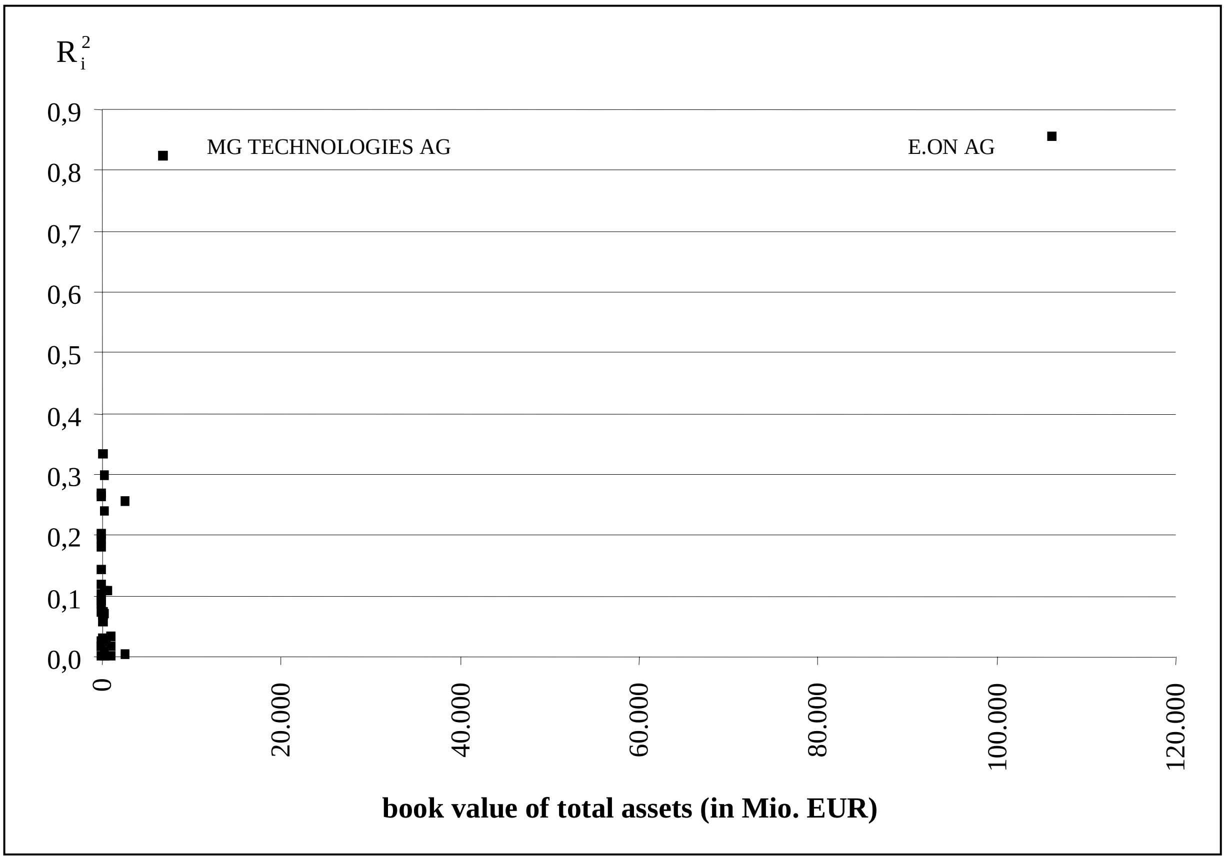

Figure 8: Scatter plot of coefficients of determination (R-squared) against book value of total assets.

From a first visual inspection, it is quite obvious that a logistic function of the type given in (26) could well be used to fit the data, if we were willing to accept that this shape is induced solely by the two outliers E.ON AG and MG Technologies AG, which are characterized by extremely high weights of 49.560 % respectively 27.195 % in the CDAX© sub-indices for Utilities and Industrials.

Regressing all the 35 R-Squared values on the two measures of company size we employ leads to the following estimates of the parameter values:

Parameter / Independent variable	γ estimate	γ 95%-Lower Bound	γ 95%-Upper Bound	λ estimate	λ 95%-Lower Bound	λ 95%-Upper Bound	Percentage of variation explained in the cross-sectional regression
Market capitalization	***0.634***	0.293	0.972	***-14.272***	-21.323	-7.221	38.742%
Book value of total assets	***0.623***	0.325	0.920	***-14.392***	-20.722	-8.061	45.528%

Table 2: Nonlinear cross-sectional regression summary statistics, complete sample of 35 stocks.

Astonishingly, our estimates are quite similar to those obtained in the above mentioned RiskMetrics™ Group study despite the many differences in methodology and data. Moreover, the difference caused by the change in the independent variable is rather low and the signs of the coefficients are significant (at least) at the 5 % level.

The use of book values even raises the percentage of variation in the dependent variable which is explained in the cross-sectional regression from 38.742% to 45.528%. On the whole, one could be tempted to say, that – based on these results – the calibration of (26) currently implemented in CreditManager™ will work rather well for our purposes, even if EUR-book values of total assets are entered instead of USD-market capitalizations.

This alleged robustness of calibration results is thoroughly shakened, if we remove the two heavy-weighted outliers from the sample, because of the tautological character inherent in their coefficients of determination. For the reduced sample, the average index weight is only 1.330% with a maximum of still 11.935 %. The summary statistics for the adjusted sample are given in Table 3:

Parameter / Independent variable	γ estimate	γ 95%-Lower Bound	γ 95%-Upper Bound	λ estimate	λ 95%-Lower Bound	λ 95%-Upper Bound	Percentage of variation explained in the cross-sectional regression
Market capitalization	***-0.022***	-0.237	0.194	***-1.750***	-5.635	2.136	0.002%
Book value of total assets	***-0.108***	-0.393	0.176	***-0.112***	-5.388	5.165	0.021%

Table 3: Nonlinear cross-sectional regression summary statistics, reduced sample of 33 stocks.

The exclusion of the two outliers has removed nearly all of the explanatory power from the cross-sectional regression. Moreover, the parameter estimates have changed dramatically and their signs are no longer significant at the 5 % level. These results call the whole idea of transferring a valid empirical relationship between company size and the portion of idiosyncratic risk to the case of non-listed obligors massively into question.

To support our argument that the positive relationship between a company's size and its systematic risk is not valid beyond traded stocks from which the indices are built, we have done another four *linear* cross-sectional OLS-regressions. This is to test, if the sign of the beta coefficient does significantly differ from zero. Our results are displayed in Tables 4 and 5:

Parameter	a (constant)			b (slope)			Percentage of variation explained in the cross-sectional regression
Independent variable	***estimate***	t-test statistic	signifi-cance	***estimate***	t-test statistic	signifi-cance	
Market capitalization	***0.119***	4.555	0.000	***0.00001949***	4.963	0.000	42.7 %
Book value of total assets	***0.121***	4.655	0.000	***0.00000726***	5.027	0.000	43.4 %

Table 4: Linear cross-sectional regression summary statistics, complete sample of 35 stocks.

Using the full sample, the sign of the slope b (as well as the sign of the constant a) is significantly positive, even at a confidence level of 99 % for both proxies of company size. The independent variables seem to explain more than 40 % of the variation in systematic risk. This apparently positive relationship between size and the extent of systematic risk does completely disappear again, if we exclude the two outliers:

Parameter	a (constant)			b (slope)			Percentage of variation explained in the cross-sectional regression
Independent variable	***estimate***	t-test statistic	signifi-cance	***estimate***	t-test statistic	signifi-cance	
Market capitalization	***0.111***	5.327	0.000	***-0.00002161***	-0.467	0.644	8.4 %
Book value of total assets	***0.111***	5.419	0.000	***-0.00001318***	-0.486	0.631	8.7 %

Table 5: Linear cross-sectional regression summary statistics, reduced sample of 33 stocks.

Now, the estimated slope terms bear the "wrong" sign and do not significantly differ from zero. Moreover, the percentage of variation in the 33 time-series-regression R-squared values, which is explained in the cross-sectional regressions, is below 10 %.

IV. Conclusions

This paper presents new empirical evidence on the problem of calibrating the CreditMetrics™ asset correlation concept for a German mid-cap loan portfolio. Its aim is to go a step beyond the standard solution for the determination of r-squared which is currently offered in CreditManager™.

Putting it all together, our results seem to indicate quite clearly that there is no valid empirical relationship between company size and systematic risk present in our data, if the tautological influence via index weights is faded out. Therefore, a calibration of the weights of the idiosyncratic components in our restricted version of the CreditMetrics™ index model that is tailor-made for non-publicly traded German companies cannot be built on this relationship.

As a consequence for our practical implementation, we decided to do without the "general rule" offered in CreditManager™ and to use the same average R-squared of the "reduced" sample across all obligors instead:

$$\hat{w}_i = 0.1054 \quad , \forall\, i = 1..n \,. \tag{27}$$

This value of about 10 % remains quite stable when sorting out further companies from our sample in the order of their index weights. It seems to be a reasonable "best guess" starting point for credit value at risk calculations with a portfolio of German non-listed corporate obligors in CreditManager™.

Future research is needed in order to confirm these findings with respect to a larger database (more obligors in the sample, not just one year of observations etc). More refined estimation methods should be based on the construction of "new", primarily credit-risk-orientated industry indices with *relative* or *equal* weights for the companies included.

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