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Precise Robustness Analysis of Time Petri Nets with Inhibitor Arcs

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Context: Verifying Complex Timed Systems

- Need for early bug detection
 - Bugs discovered when final testing: **expensive**
 - ↪ Need for a thorough **modeling** and verification phase



Motivation: Robustness Analysis

- Timed systems are characterized by a **set of timing constants**
 - “The packet transmission lasts for **50 ms**”
 - “The sensor reads the value every **10 s**”
- Challenge: **Robustness** [Markey, 2011]
 - What happens if **50** is implemented with **49.99**?
 - In which neighbourhood of **50** does the system behave well?

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 - “The packet transmission lasts for **50 ms**”
 - “The sensor reads the value every **10 s**”
- Challenge: **Robustness** [Markey, 2011]
 - What happens if **50** is implemented with **49.99**?
 - In which neighbourhood of **50** does the system behave well?
- **Parametric analysis**
 - Consider that timing constants are **parameters**
 - Find **good values** for the parameters, such that the system still behaves well

Outline

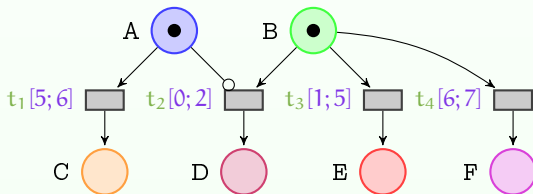
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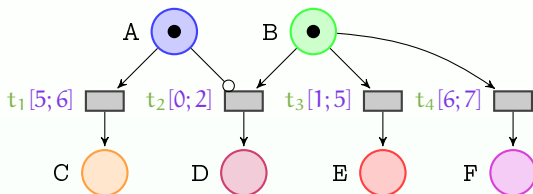
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Time Petri Nets With Inhibitor Arcs (ITPNs)

- Powerful formalism for verifying real-time systems [Merlin, 1974]
- Transition t_1 can fire from 5 to 6 units of time after being enabled
- An enabled transition **must fire** before (or at) its upper bound
- An **inhibitor arc** enables its transition (t_2) when its source place (A) is empty



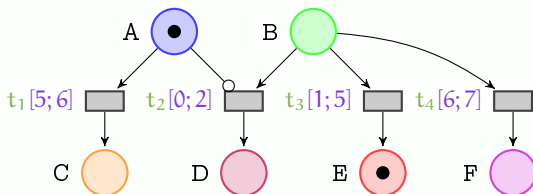
Time Petri Nets With Inhibitor Arcs: Example



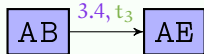
Some possible runs

AB

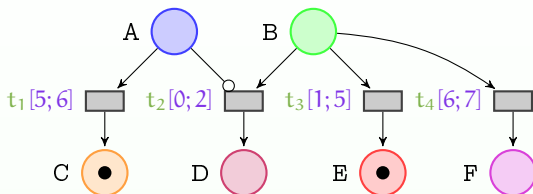
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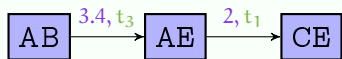
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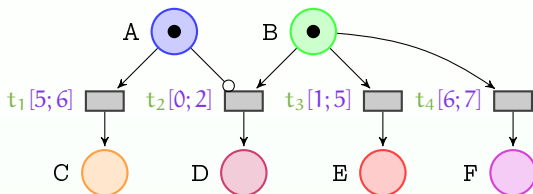
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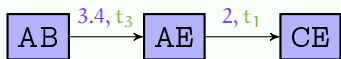
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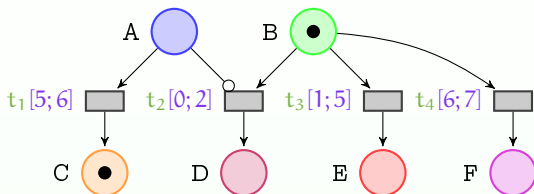
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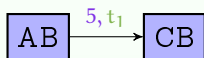
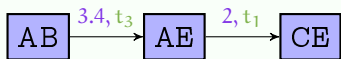
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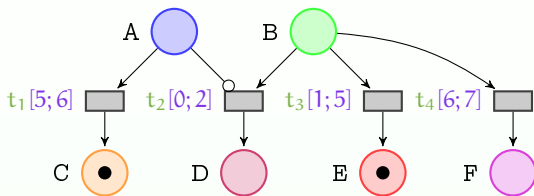
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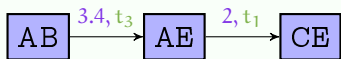
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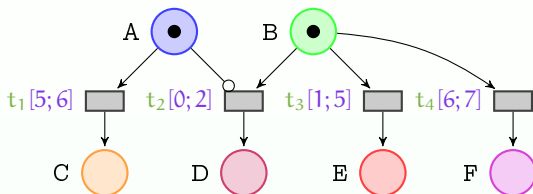
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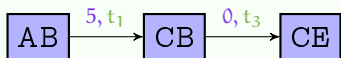
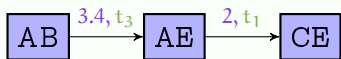
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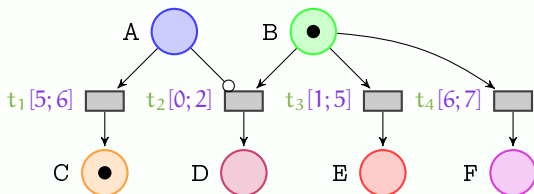
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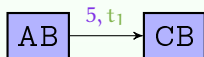
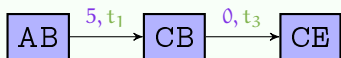
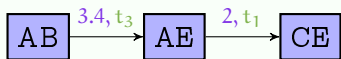
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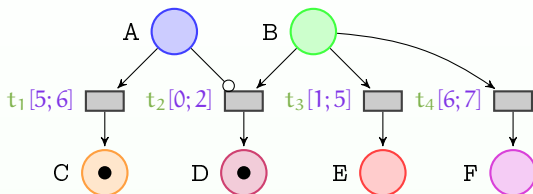
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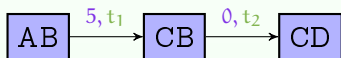
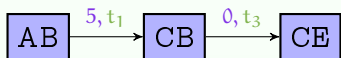
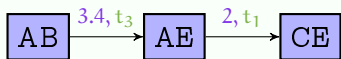
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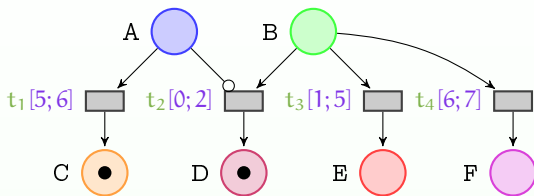
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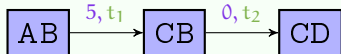
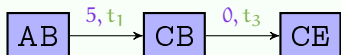
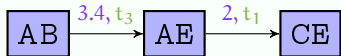
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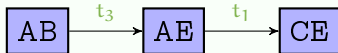
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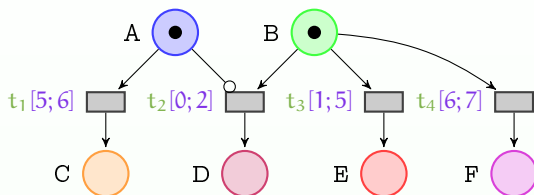


Trace set



Trace: time-abstract behaviour

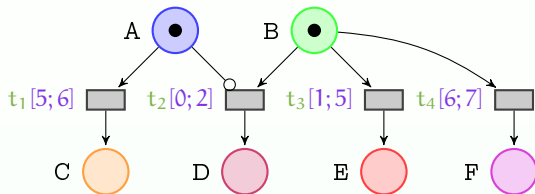
Robustness (1/2)



- What happens if $t_2[0;2]$ is implemented with $t_2[0.01;2]$?

- Trace $AB \xrightarrow{t_1} CB \xrightarrow{t_2} CD$ cannot happen anymore

Robustness (1/2)



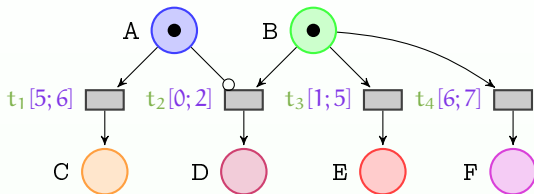
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- What happens if $t_3[1; 5]$ is implemented with $t_3[1; 4.99]$?

- Trace $AB \xrightarrow{t_1} CB \xrightarrow{t_3} CE$ cannot happen anymore

Robustness (1/2)



- What happens if $t_2[0; 2]$ is implemented with $t_2[0.01; 2]$?

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- What happens if $t_3[1; 5]$ is implemented with $t_3[1; 4.99]$?

- Trace $AB \xrightarrow{t_1} CB \xrightarrow{t_3} CE$ cannot happen anymore

↪ This system is not **robust**, in the sense that infinitesimal variations of the bounds lead to a different discrete behaviour (trace set).

Robustness (2/2)

Definition (LT-robustness)

An ITPN N is **LT-robust** if there exists $\gamma > 0$ such that N_γ and N have **the same trace sets**.

(where N_γ is any ITPN similar to N where each timing bound c can be replaced with any value within $[c - \gamma, c + \gamma]$)

Challenges:

- Is an ITPN robust?
- If not, why is it non-robust?
- Is it possible to render robust a non-robust ITPN? If so, how?

Outline

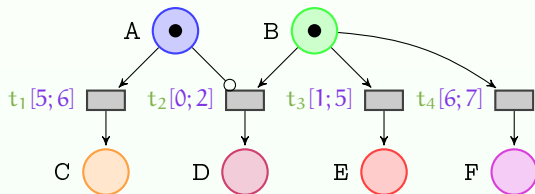
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Parametric Time Petri Nets

Idea: Reason parametrically (using **unknown constants**)

Parametric Time Petri Nets with Inhibitor Arcs (PITPNs)

- Constants in firing intervals replaced with **parameters**
[Traonouez et al., 2009]

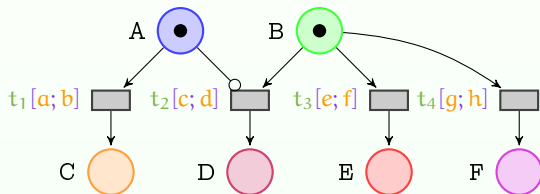


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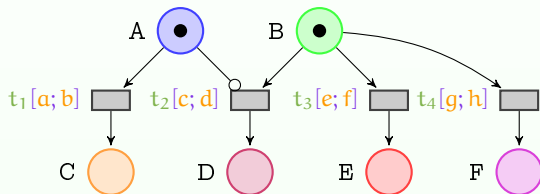


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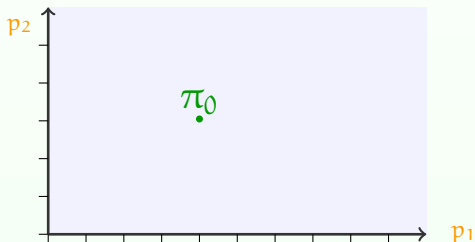


- Notation: given a PITPN \mathcal{N} and a valuation π of the parameters, we denote by $[[\mathcal{N}]]_{\pi}$ the ITPN obtained from \mathcal{N} by replacing all parameters with their valuation in π

The Inverse Method (*IM*)

■ Input

- A PITPN \mathcal{N}
- A reference valuation π_0 of all the parameters of \mathcal{N}



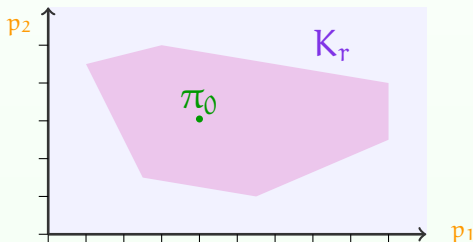
The Inverse Method (IM)

Input

- A PITPN \mathcal{N}
- A reference valuation π_0 of all the parameters of \mathcal{N}

Output: K_r

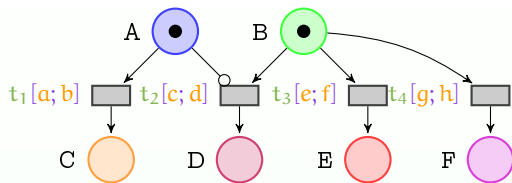
- Convex constraint on the parameters such that
 - $\pi_0 \models K_r$
 - For all points $\pi \models K_r$, $[[\mathcal{N}]]_\pi$ and $[[\mathcal{N}]]_{\pi_0}$ have the same trace sets



The Inverse Method: General Idea

- Initially defined for timed automata [A., Chatain, Encrenaz, Fribourg, 2009]
- Extended to PITPNs [A., Pellegrino, Petrucci, 2013]
- The idea
 - Exploration of the parametric state space
 - Instead of negating bad states (as in “CEGAR” approaches), remove π_0 -incompatible states
 - Return the intersection of all constraints on the parameters

Application to an Example

 π_0

a = 5 b = 6

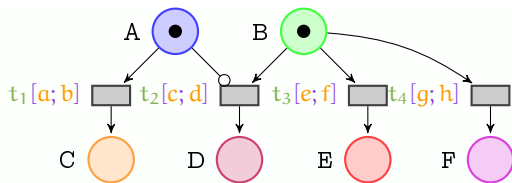
c = 0 d = 2

e = 1 f = 5

g = 6 h = 7

Forward analysis

Application to an Example

 π_0 $a = 5$ $b = 6$ $c = 0$ $d = 2$ $e = 1$ $f = 5$ $g = 6$ $h = 7$

Forward analysis

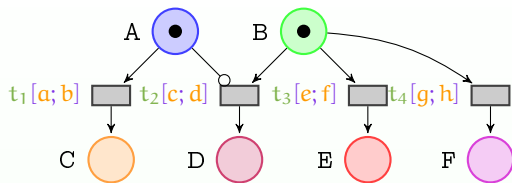
K:

true

AB

 $a \leq b$ $c \leq d$ $e \leq f$ $g \leq h$

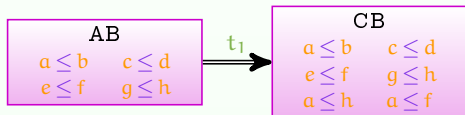
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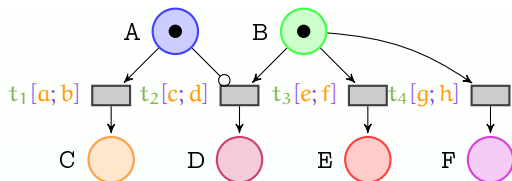
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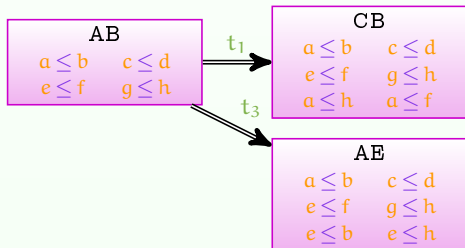
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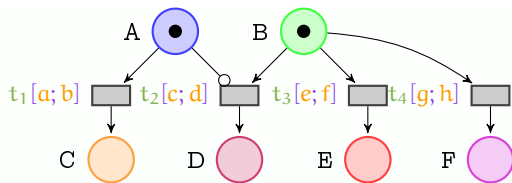
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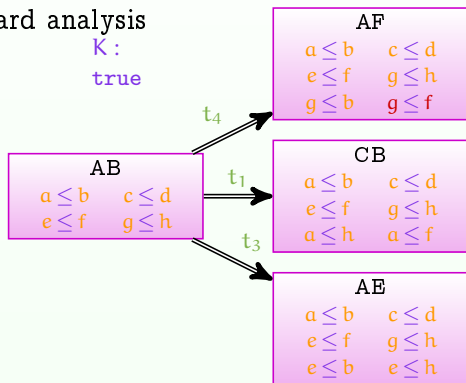
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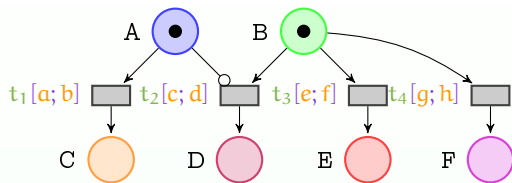
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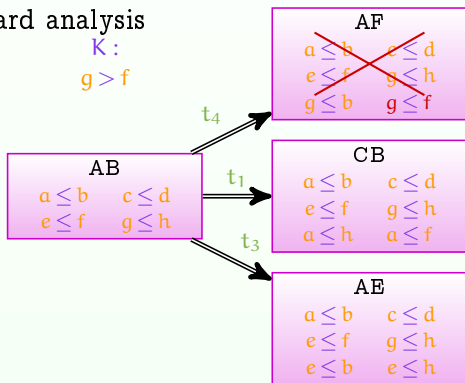
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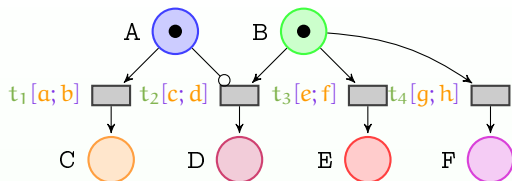
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K:

g > f



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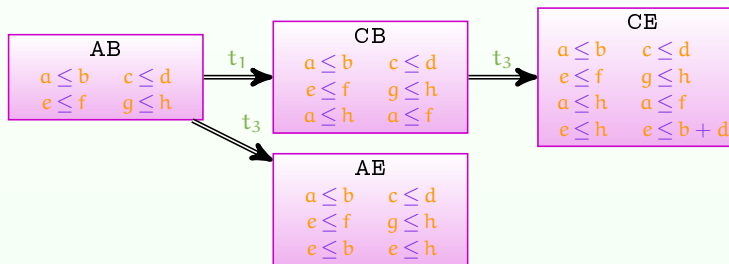
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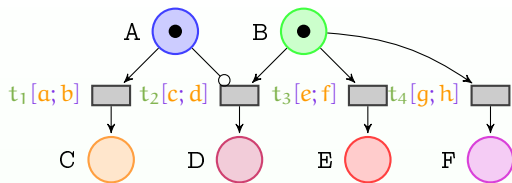
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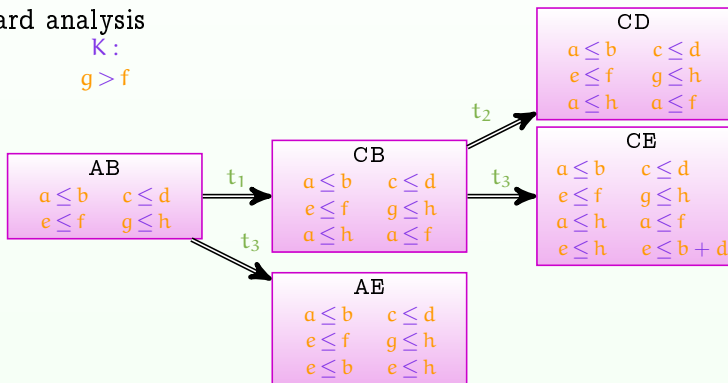
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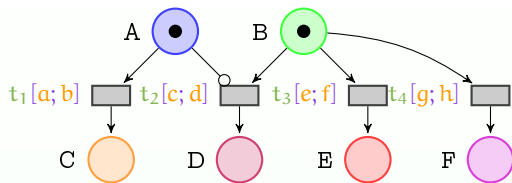
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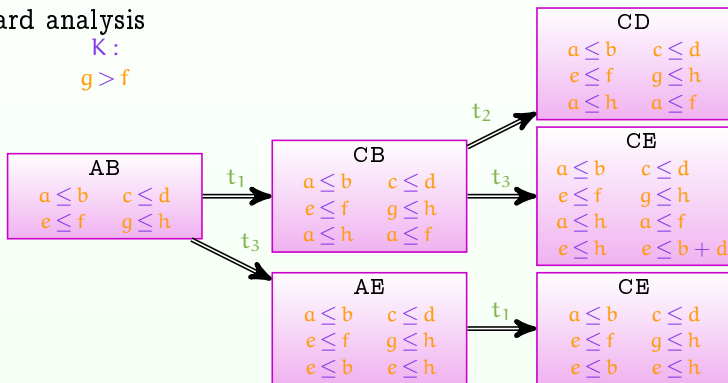
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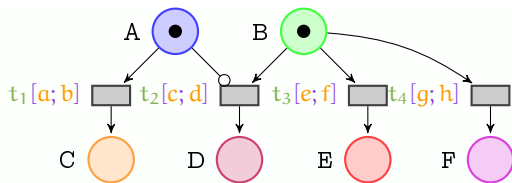
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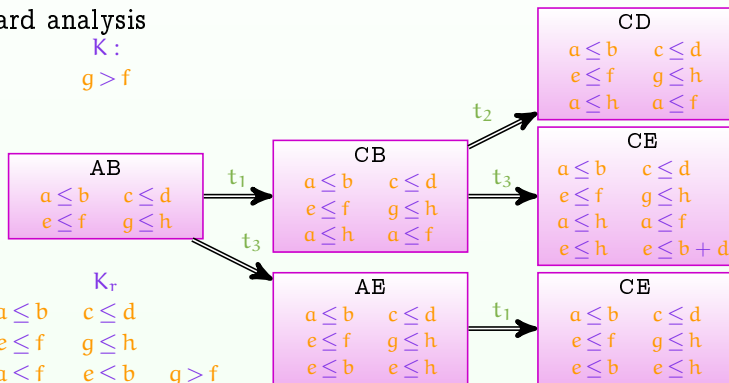
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Forward analysis

 K :

$g > f$



Properties

■ Correctness

- $\pi_0 \models K_r$ and
- $\forall \pi \models K_r, \llbracket \mathcal{N} \rrbracket_\pi$ and $\llbracket \mathcal{N} \rrbracket_{\pi_0}$ have the same trace set.

■ IM is non-confluent

- Several executions with the same input may lead to different outputs

■ IM is non-complete

- K_r may not be the maximum set of parameter valuations with the same trace set as $\llbracket \mathcal{N} \rrbracket_{\pi_0}$

■ Termination of IM is not guaranteed in general

- Parameter synthesis for PITPNs **undecidable**
[Traonouez et al., 2009]

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Robustness Using the Inverse Method

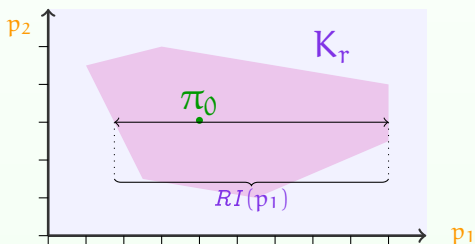
Let \mathbf{N} be an ITPN.

General idea

- 1 Construct the parametric version \mathcal{N} of \mathbf{N} , and π_0 the reference valuation such that $[[\mathcal{N}]]_{\pi_0} = \mathbf{N}$
- 2 Call $IM(\mathcal{N}, \pi_0)$ and assume K_r is the resulting constraint
- 3 Measure the system robustness
- 4 If the system is non-robust, render it robust (if possible)

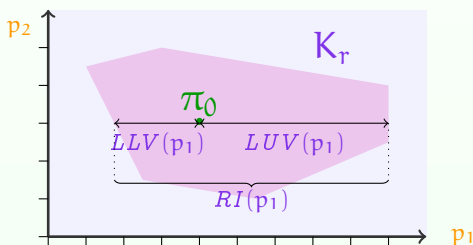
Metrics for Measuring Local Robustness

- **Ranging interval** of a parameter $RI(p)$
 - Minimum and maximum admissible values within K_r



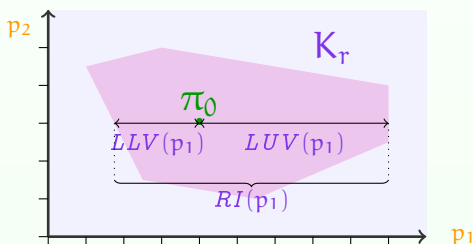
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- **Local lower/upper variability** of a parameter
 - Distance between $\pi_0(p)$ and the lower/upper bound of $RI(p)$
 - Given $RI(p) = (a, b)$, then $LLV(p) = \pi_0(p) - a$ and $LUV(p) = b - \pi_0(p)$



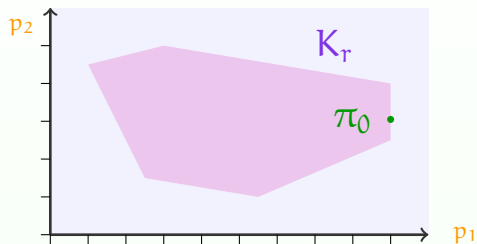
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 - Distance between $\pi_0(p)$ and the lower/upper bound of $RI(p)$
 - Given $RI(p) = (a, b)$, then $LLV(p) = \pi_0(p) - a$ and $LUV(p) = b - \pi_0(p)$
- **Local robustness**: distance between $\pi_0(p)$ and the closest border within K_r
 - $LR(p) = \min(LLV(p), LUV(p))$



Critical Timing Bounds

- Critical timing bounds are those with a null local robustness



Remark

If any of the timing bounds is critical, classical (“ Δ -based”) approaches will just classify the system as non-robust.

Relaxing Timing Bounds

Definition (Potential robustness)

An ITPN N is **potentially robust** if, for all timing bounds p_i , $LLV(p_i) > 0$ or $LUV(p_i) > 0$.

Intuitively: A system is potentially robust if each parameter can vary within K_r .

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Theorem (Rendering a system robust)

If \mathcal{N} is potentially robust, then there exists π_R such that $\llbracket \mathcal{N} \rrbracket_{\pi_R}$ is LT-robust, and has the same trace set as \mathcal{N} .

Construction: choose $\frac{LLV(p)+LUV(p)}{2}$ for each parameter p .

Relaxing Timing Bounds: Remarks

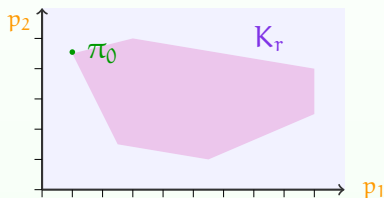
The potential robustness is a **non-necessary** condition to render a system robust

- 1 The potential robustness is based on the local robustness, that comes from K_r , that may be non-complete

Relaxing Timing Bounds: Remarks

The potential robustness is a **non-necessary** condition to render a system robust

- 1 The potential robustness is based on the local robustness, that comes from K_r , that may be non-complete
- 2 The potential robustness considers the variability of each timing bound in an independent manner



- In that case, the system is not potentially robust (since $LLV(p_2) = LUV(p_2) = 0$), but could still be made robust (by choosing a point in the middle of K_r)

Comparison with Related Work (1/2)

- Robustness studied for timed automata and time Petri nets (see [Markey, 2011] for a survey)
- “ Δ -based” approaches
 - Robustness studied with respect to a single **enlargement** Δ for all bounds
 - **or** to a single **shrinking** Δ for all bounds
 - Extension to a (constant) **vector**
 - Extension to independent variations Δ for each parameter, but for shrinking only [Sankur, 2013]

Comparison with Related Work (2/2)

■ Recent approaches

- Parameterized robust reachability in timed automata is decidable [Bouyer et al., 2012]
- Computing the greatest acceptable variation Δ is decidable for flat timed automata with progressive clocks [Jaubert and Reynier, 2011]
- CEGAR-based approach using parametric techniques to evaluate the greatest acceptable variation Δ for parametric timed automata (not decidable in general) [Traonouez, 2012]

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■ In contrast to most approaches, we consider a **local robustness measure** for each delay

- ☺ For linear-time properties
- ☺ More flexible: Bounds can be both enlarged and shrunk
- ☺ More precise: Exhibits the critical timing bounds
- ☹ May not terminate

Outline

- 1 Time Petri Nets with Inhibitor Arcs
- 2 The Inverse Method for ITPNs
- 3 Precise Robustness Analysis
- 4 Conclusion and Perspectives**

Conclusion

- Local robustness analysis of timed systems
 - For linear-time properties
 - Using the inverse method
 - Quantifies the robustness of **each** timing bound
 - ↪ Identifies **critical bounds**
- Sufficient condition for rendering a non-robust system robust
- Comparison with related approaches
 - 😊 More precise than most existing approaches
 - 😞 May not terminate

Conclusion

- Local robustness analysis of timed systems
 - For linear-time properties
 - Using the inverse method
 - Quantifies the robustness of **each** timing bound
 - ↪ Identifies **critical bounds**
- Sufficient condition for rendering a non-robust system robust
- Comparison with related approaches
 - 😊 More precise than most existing approaches
 - ☹ May not terminate
- Linear-time properties, hence **untimed**
 - But timed properties can be considered using **observers**

Perspectives

- **Implementation**
 - Work in progress
 - Comparison with other tools such as **Shrinktech** [Sankur, 2013]
- Improve conditions for rendering non-robust systems robust
- **Variation of the clocks speed** (“ ϵ ”)
 - Addition of two parameters for the admissible decrease and increase of the clock rate
 - Extension of the inverse method to **non-linear (hybrid) systems**

Bibliography

References I



André, É., Chatain, Th., Encrenaz, E., and Fribourg, L. (2009).
An inverse method for parametric timed automata.
IJFCS, 20(5):819–836.



André, É., Petrucci, L., and Pellegrino, G. (2013).
Precise robustness analysis of time Petri nets with inhibitor arcs.
In *FORMATS*, volume 8053 of *Lecture Notes in Computer Science*, pages 1–15.
Springer.



Bouyer, P., Markey, N., and Sankur, O. (2012).
Robust reachability in timed automata: A game-based approach.
In *ICALP*, volume 7392 of *Lecture Notes in Computer Science*, pages 128–140.
Springer.



Jaubert, R. and Reynier, P.-A. (2011).
Quantitative robustness analysis of flat timed automata.
In *FoSSaCS*, volume 6604 of *Lecture Notes in Computer Science*, pages 229–244.
Springer-Verlag.



Markey, N. (2011).
Robustness in real-time systems.
In *SIES*, pages 28–34. IEEE Computer Society Press.

References II



Merlin, P. M. (1974).

A study of the recoverability of computing systems.

PhD thesis, University of California, Irvine, CA, USA.



Sankur, O. (2013).

Shrinktech: A tool for the robustness analysis of timed automata.

In *CAV'13*, volume 8044 of *Lecture Notes in Computer Science*, pages 1006–1012.

Springer.



Traonouez, L.-M. (2012).

A parametric counterexample refinement approach for robust timed specifications.

In *FIT*, volume 87 of *EPTCS*, pages 17–33.



Traonouez, L.-M., Lime, D., and Roux, O. H. (2009).

Parametric model-checking of stopwatch Petri nets.

Journal of Universal Computer Science, 15(17):3273–3304.

Additional explanation

The Algorithm

Algorithm 1: $IM(\mathcal{N}, \pi)$

input : PITPN \mathcal{N} of initial class c_0 and initial constraint K_0 , valuation π

output: Constraint K_r

```

1  $i \leftarrow 0$ ;  $K_c \leftarrow K_0$ ;  $C \leftarrow \{c_0\}$ 
2 while true do
3   while  $\exists \pi$ -incompatible classes in  $C$  do
4     Select a  $\pi$ -incompatible class  $(M, D)$  of  $C$ 
5     Select a  $\pi$ -incompatible  $J$  in  $D \downarrow_P$ 
6      $K_c \leftarrow K_c \wedge \neg J$ ;  $C \leftarrow \bigcup_{j=0}^i Post_{\mathcal{N}(K_c)}^j(\{c_0\})$ 
7   if  $Post_{\mathcal{N}(K_c)}(C) \subseteq C$  then
8     return  $K_r \leftarrow \bigcap_{(M,D) \in C} D \downarrow_P$ 
9    $i \leftarrow i + 1$ ;  $C \leftarrow C \cup Post_{\mathcal{N}(K_c)}(C)$ 

```

Explanation for the 4 pictures in the beginning



Allusion to the Northeast blackout (USA, 2003)

Computer bug

Consequences: 11 fatalities, huge cost

(Picture actually from the Sandy Hurricane, 2012)



Allusion to any plane crash

(Picture actually from the happy-ending US Airways Flight 1549, 2009)



Allusion to the sinking of the Sleipner A offshore platform (Norway, 1991)

No fatalities

Computer bug: inaccurate finite element analysis modeling

(Picture actually from the Deepwater Horizon Offshore Drilling Platform)



Allusion to the MIM-104 Patriot Missile Failure (Iraq, 1991)

28 fatalities, hundreds of injured

Computer bug: software error (clock drift)

(Picture of an actual MIM-104 Patriot Missile, though not the one of 1991)

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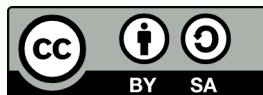
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