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Hypersonic flow past slender bodies in dispersive hydrodynamics

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Abstract

The problem of two-dimensional steady hypersonic flow past a slender body is formulated for dispersive media. It is shown that for the hypersonic flow, the original $2+0$ boundary-value problem is asymptotically equivalent to the $1+1$ piston problem for the fully nonlinear flow in the same physical system, which allows one to take advantage of the analytic methods developed for one-dimensional systems. This type of equivalence, well known in ideal Euler gas dynamics, has not been established for dispersive hydrodynamics so far. Two examples pertaining to collisionless plasma dynamics are considered.

Keywords: Nonlinear flow past body; Dispersive shock; Hypersonic flow

1. Introduction

The problem of supersonic flow around objects is one of the central problems in classical hydrodynamics [1,2]. The nonlinear system of Euler equations describing established (time-independent) two-dimensional supersonic flows in ideal gas dynamics is hyperbolic. When the supersonic flow occurs around objects it is accompanied by the onset of singularities of the wave breakdown type. In the vicinity of the breakdown point, the higher order corrections in the equations of ideal gas dynamics should be taken into account. In ordinary hydrodynamics, when these terms are dissipative, this gives rise to an oblique compression jump (an analog of a shock in the one-dimensional nonstationary case).

In dispersive continuous media, where dissipation is small enough or negligible, the resolution of a singularity happens through generation of small-scale nonlinear waves. What forms here instead of an oblique compression jump is a wedge shaped region of space occupied by small-scale nonlinear oscilla-

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tions. The oscillations acquire the shape of solitary waves at the front facing the oncoming flow and the shape of harmonic wave of infinitesimally small amplitude at the opposite front toward the body. This two-dimensional wave structure forming in the supersonic flow around the body is nothing but the stationary two-dimensional dispersive shock.

In this Letter, we consider the problem of two-dimensional steady hypercritical (hypersonic) flow past slender bodies for dispersive dissipationless media.

The problem of the dispersive shock formation in two-dimensional supersonic flow past slender bodies has been studied in [3–5]. It was shown that for moderate Mach numbers of the incident flow $M > 1$ and the parameter of the body slenderness $\alpha = b/l \ll 1$, where b is the characteristic thickness of the body and l is its typical length, the problem is weakly nonlinear and asymptotically reduces to the initial value problem for the KdV equation with the small dispersion term.

$$u_\tau + uu_\xi + \varepsilon^2 u_{\xi\xi\xi} = 0,$$
$$u(\xi, 0) = u_0(\xi), \quad \varepsilon \ll 0. \tag{1}$$

The role of time in the KdV approximation is played by the stretched y-coordinate $\tau = \alpha y$ while the new spatial coordinate ξ is a linear characteristics $\xi = x - \sqrt{M^2 - 1}y$. The KdV initial data $u_0(\xi)$ are determined by the derivative of the body profile. Formation of a dispersive shock in the solutions of the KdV equation has been first studied by Gurevich and Pitaevskii in [6] using the Whitham method of slow modulations [7] (see also [8] for a detailed description of the Whitham method and its applications).

If $M \gg 1$ and $M\alpha \sim 1$ then the KdV approximation is no longer valid and a more general consideration is required in the frame of a fully nonlinear model.

Since for the case of fully nonlinear dynamics one cannot expect the universality degree achievable in a weakly nonlinear case, we perform our study by considering two concrete systems pertaining to collisionless plasma flows. Our first example is the well-known system (2) describing finite-amplitude ion-acoustic waves in strongly nonisothermal plasma with hot electrons and cold ions. The second example is the system (22)–(26) describing magnetoacoustic waves in a cold two-component plasma moving strictly across the magnetic field. We believe that both systems, although

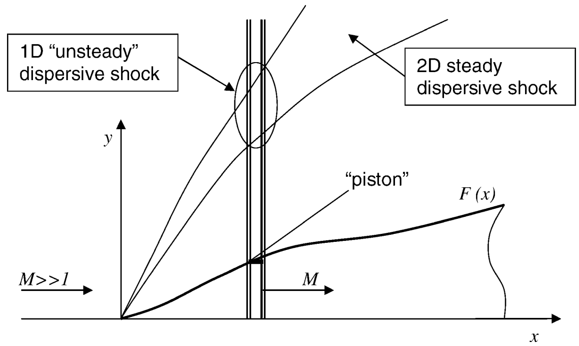

Fig. 1. The piston analogy in the hypersonic flow past a body problem.

extensively studied by physicists for decades (see, for instance, [3,12] and references therein), still retain significant potential as analytically approachable models describing fully nonlinear dispersive wave dynamics.

We show that, in both examples considered, when the Mach number $M \gg 1$, the original stationary 2D problem of the flow past slender body is asymptotically equivalent to the nonstationary one-dimensional piston problem in the *same physical system*. The piston velocity is determined by the derivative of the body profile in the stretched coordinate $T = x/M$. This type of equivalence, although well known in classical hydrodynamics (see, for instance, [2]), is, to the best of our knowledge, not explored for the dispersive hydrodynamics case.

The result of such a "piston motion" (which is actually the vertical motion of the element of the body surface in the reference system associated with the incident flow (see Fig. 1)) is formation of a dispersive shock at a some distance from the body. The problem of generation of fully nonlinear one-dimensional dispersive shock has been recently studied in [9–11], where main parameters of the dispersive shock: its location and the lead solitary wave amplitude have been determined in a general form along with asymptotics of the physical values (mean density, mean velocity, etc.) in the vicinities of the dispersive shock edges.

Thus, the hypersonic reduction considered in this paper provides significant simplification of the system without any sacrifice both in the amplitude and the wavelength range and allows one to understand properties of 2D stationary nonlinear wave behaviour in terms of much better explored 1D nonstationary dynamics. We add that both systems considered are structurally representative for a broad class of analytic models describing fully nonlinear multi-dimensional dispersive waves.

2. Nonlinear ion-acoustic waves

We start our consideration with the system describing fully nonlinear flows in a two temperature ($T_e \gg T_i$) unmagnetized collisionless plasma in dimensionless variables [3,12]:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,$$
$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla \phi = 0,$$
$$\Delta \phi = \exp \phi - \rho. \tag{2}$$

Here ρ and $\mathbf{u}$ are the ion density and velocity normalised by their characteristic values: $\rho/\rho_0 \to \rho$, $\mathbf{u}/c_s \to \mathbf{u}$, $e\phi/T_e \to \phi$ is the electric potential. Here $c_s = (T_e/m_i)^{1/2}$ is the ion sound speed, m_i is the ion mass. The independent variables are normalised as follows: $x/D \to x$, $tc_s/D \to t$, where $D = (T_e/4\pi e^2 \rho_0)^{1/2}$ is the Debye radius determining the scale of dispersive effects, and $e = 1.6 \times 10^{-19}$ C is the electron charge.

The system (2) is structurally representative for a wide class of nonintegrable *dispersion-hydrodynamic* systems. In the dispersionless limit, $\Delta\phi \to 0$ (large-scale smooth flow) it goes over to the Euler polytropic gas dynamics with the index $\gamma = 1$. For the one-dimensional case $\mathbf{u} = u(x,t)\mathbf{e}_x$, $\rho = \rho(x,t)$, $\phi = \phi(x,t)$ the system (2) assumes the form:

$$\partial_t \rho + \partial_x(\rho u) = 0,$$
$$\partial_t u + u\partial_x u + \partial_x \phi = 0,$$
$$\partial^2_{xx}\phi = \exp\phi - \rho. \tag{3}$$

The decay of an initial discontinuity problem for system (3) has been considered in [13] numerically. Analytical study of this problem has been done in [10,11,14] using the Whitham method of slow modulations where the flow in the dispersive shock region is represented in the form of slowly varying travelling wave solution of (3).

Here we are going to take advantage of the multi-dimensional system (2) to consider the problem of the 2D steady flow past a wedge-shaped body. In fact, we will show that for steady plane hypersonic flows, $M \gg 1$, where $M = u_0/c_s$ is the Mach number of the incident flow, the system (2) is asymptotically equivalent to (3) with variables $u \equiv u_x$, x and t in (3) replaced by $v \equiv u_y$, y and x/M correspondingly.

For simplicity we assume that the body is placed in the upper half-plane with the pointed part directed to the left (Fig. 1). Let the incident flow be uniform, come from the left and have the density $\rho = 1$ and the velocity $\mathbf{u} = M\mathbf{e}_x$. Then far from the body surface:

$$\mathbf{u} \to M\mathbf{e}_x, \qquad \rho \to 1, \qquad \phi \to 0$$
$$\text{as} \quad (x - x_b)^2 + (y - y_b)^2 \to \infty, \tag{4}$$

where (x_b, y_b) is any point on the body surface.

On the body surface we require the standard impenetrability condition

$$(\mathbf{u} \cdot \mathbf{n})_{\text{body}} = 0, \tag{5}$$

where $\mathbf{n}$ is the outer normal vector. Also we require the body to be electrically insulating plus absence of free charges on the body surface

$$(\nabla\phi \cdot \mathbf{n})_{\text{body}} = 0, \qquad (\phi)_{\text{body}} = (\ln\rho)_{\text{body}}. \tag{6}$$

Conditions (6) provide smoothness of the flow near the body surface and guarantee absence of macroscopic boundary layer effects, which could complicate our consideration. We note that the system (2) admits the steady plane flow reduction where $u_z \equiv 0$ and the remaining variables depend on x, y only:

$$\partial_t \equiv 0, \qquad \mathbf{u} = \big(u(x,y), v(x,y), 0\big),$$
$$\rho = \rho(x,y), \qquad \phi = \phi(x,y). \tag{7}$$

The system (2) then assumes the form:

$$\partial_x(\rho u) + \partial_y(\rho v) = 0,$$
$$u\partial_x u + v\partial_y u + \partial_x\phi = 0,$$
$$u\partial_x v + v\partial_y v + \partial_y\phi = 0,$$
$$\big(\partial^2_{xx} + \partial^2_{yy}\big)\phi = \exp\phi - \rho. \tag{8}$$

Let the body shape be given by the function

$$y = F(x), \qquad y \geqslant 0, \qquad F(0) = 0, \qquad F' > 0. \tag{9}$$

The boundary conditions (4)–(6) then take the form:

$$\rho \to 1, \qquad \phi \to 0, \qquad u \to M, \qquad v \to 0$$
$$\text{as} \quad (x - x_b)^2 + \big(y - F(x_b)\big)^2 \to \infty. \tag{10}$$

$$v = uF'(x) \quad \text{on} \quad y = F(x), \tag{11}$$

$$\partial_y \phi = \partial_x \phi F'(x), \quad \phi = \ln\rho \quad \text{on} \quad y = F(x). \tag{12}$$

Now we consider the hypersonic flow, i.e., we put $M \gg 1$ and introduce the asymptotic decomposition for the x-component of the flow velocity

$$u = M + u_1 + O(1/M). \tag{13}$$

Simultaneously, we introduce new independent variables

$$T = x/M, \qquad X = y. \tag{14}$$

Substituting (13), (14) into (8) we readily obtain to the leading order in M:

$$\partial_T \rho + \partial_X(\rho v) = 0,$$
$$\partial_T v + v\partial_X v + \partial_X \phi = 0,$$
$$\partial^2_{XX}\phi = e^\phi - \rho, \tag{15}$$

which coincides with 1D nonstationary system (3) but with v, X, T instead of u, x, t. The correction u_1 to the x-component of the flow velocity is connected with v by the linear equation

$$\partial_T u_1 + v\partial_X u_1 = 0, \tag{16}$$

which can be easily integrated by characteristics.

Asymptotical equivalence of the mathematical description of two-dimensional hypersonic steady flows to that of nonstationary one-dimensional flows is well known in classical gas dynamics (see [2] for instance). For the dispersive hydrodynamics this property, to the best of our knowledge, has not been establihed so far.

We look now at what happens to the boundary conditions (10)–(12) under the asymptotic transformations (13), (14). First, we note that the condition (11) is consistent with the hypersonic decompositions only if the body is slender enough, namely

$$M^{-2} \ll \alpha \ll 1, \quad \alpha = \max F'(x). \tag{17}$$

The most interesting asymptotic arises when

$$\alpha M \sim 1. \tag{18}$$

Let $F(x) = f(x/M)$. Then, as $M \to \infty$, we have the following transformations of the boundary conditions. Condition (11) transforms into

$$v = \frac{dX}{dT} \geqslant 0 \quad \text{on} \quad X = f(T). \tag{19}$$

Condition (12) becomes

$$\phi_X = 0, \qquad \phi = \ln\rho \quad \text{on} \quad X = f(T). \tag{20}$$

And, instead (10) we have

$$\rho \to 1, \qquad \phi \to 0, \qquad v \to 0 \quad \text{as} \quad X \to \infty. \tag{21}$$

Conditions (19)–(21) represent an analogue of the piston problem conditions in classical gas dynamics [1]. Indeed, the condition (19) expresses the equality of the gas velocity at the piston surface to the velocity of the piston itself. Actually, in the hypersonic flow past body problem, the role of a piston is played by the body surface. This correspondence is illustrated in Fig. 1. Conditions (21), as usual, determine the uniform flow at infinity. And conditions (20) ensure absence of flow singularities near the piston.

Thus, one can see that even for slender bodies with $\alpha \ll 1$ the problem of the hypersonic flow past the body is described by a fully nonlinear system (15).

3. Nonlinear magnetoacoustic waves

Now we consider the fundamental system describing the dynamics of two-component cold collisionless plasma in an external magnetic field [3].

$$m_i \frac{d\mathbf{v}_i}{dt} = e\big[\mathbf{E} + (\mathbf{v}_i \times \mathbf{B})\big], \tag{22}$$

$$m_e \frac{d\mathbf{v}_e}{dt} = -e\big[\mathbf{E} + (\mathbf{v}_i \times \mathbf{B})\big], \tag{23}$$

$$\frac{\partial n_{i,e}}{\partial t} + \nabla \cdot (n_{i,e}\mathbf{v}_{i,e}) = 0, \tag{24}$$

$$\operatorname{curl}\mathbf{B} = e\mu_0(n_i\mathbf{v}_i - n_e\mathbf{v}_e), \tag{25}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\operatorname{curl}\mathbf{E}, \quad \nabla \cdot \mathbf{B} = 0. \tag{26}$$

Here $m_{i,e}$, $\mathbf{v}_{i,e}$, $n_{i,e}$ are masses, velocities and concentrations of ions and electrons respectively, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, and $\mu_0 = 4\pi \times 10^{-7}$ m/A is the magnetic constant. We note that Eqs. (22) and (23) contain different material derivative operators: $d/dt = \partial/\partial t + (\mathbf{v}_i \cdot \nabla)$ for ions and $d/dt = \partial/\partial t + (\mathbf{v}_e \cdot \nabla)$ for electrons. The physical assumptions behind the system (22)–(26) are the standard ones:

- $c_A/c \ll 1$, where $c_A = B_0/(\mu_0 m_i n_i)^{1/2}$ is the Alfvén speed, c is the speed of light, and B_0 is the characteristic magnitude of the magnetc field.
- $\beta = 2\mu_0 k T_{i,e}/B_0^2 \ll 1$, where k is the Boltzmann constant, $T_{i,e}$ is the temperature of the particles (cold plasma).
- $d_{i,e}/l_f \ll 1$, where $d_{i,e} = c/\omega_{i,e}$ are two characteristic dispersion lengths in the system (22)–(26), and l_f is the free path. Here $\omega_{i,e} = 4\pi n_{i,e} e^2/m_{i,e}$ are the ion and electron plasma frequencies.

For the cold, collisionless plasma under consideration the formal hydrodynamic description (22)–(26) is well established using the consistent kinetic approach (see [15] for instance).

Since the Debye radius $D = c_T/\omega_e$, where $c_T = \sqrt{kT_e/m_e}$ is the heat motion speed of electrons, then $D \ll d_{i,e}$ and we can regard plasma as quasineutral:

$$n_e \approx n_i = n.$$

Now we introduce the mass density and velocity of the plasma by

$$\rho = (m_i + m_e)n, \qquad \mathbf{u} = \frac{m_i \mathbf{v}_i + m_e \mathbf{v}_e}{m_i + m_e}. \tag{27}$$

Then the basic system (22)–(26), upon eliminating the electric field $\mathbf{E}$ and neglecting small corrections $\sim m_e/m_i \ll 1$, assumes the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{28}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \frac{1}{\rho}(\mathbf{B} \times \operatorname{curl} \mathbf{B}) - \left(\frac{\operatorname{curl} \mathbf{B}}{\rho} \cdot \nabla\right)\frac{\operatorname{curl} \mathbf{B}}{\rho} = 0, \tag{29}$$

$$\frac{\partial}{\partial t}\left\{\mathbf{B} + \frac{m_i}{m_e}\operatorname{curl} \mathbf{u} + \operatorname{curl}\left(\frac{\operatorname{curl} \mathbf{B}}{\rho}\right)\right\} = \operatorname{curl}(\mathbf{u} \times \mathbf{B}) - \frac{m_i}{m_e}\operatorname{curl}(\mathbf{u} \cdot \nabla)\mathbf{u} - \operatorname{curl}\left[(\mathbf{u} \cdot \nabla)\frac{\operatorname{curl} \mathbf{B}}{\rho} + \left(\frac{\operatorname{curl} \mathbf{B}}{\rho} \cdot \nabla\right)\mathbf{u}\right], \tag{30}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{31}$$

All variables in (28)–(31) are dimensionless: $\rho/\rho_0 \to \rho$, $\mathbf{u}/c_A \to \mathbf{u}$, $\mathbf{B}/B_0 \to \mathbf{B}$, $x/d_e \to x$, $y/d_e \to y$, $tc_A/d_e \to t$. Although Eq. (30) contains terms proportional to $m_i/m_e \gg 1$, we will show that for the plasma propagating strictly across the magnetic field, these terms will not survive in the final equations.

For one-dimensional propagation across the magnetic field $\mathbf{u} = u(x,t)\mathbf{e}_x$, $\mathbf{B} = B(x,t)\mathbf{e}_z$ equations (28)–(30) assume the form

$$\partial_t \rho + \partial_x(\rho u) = 0, \tag{32}$$

$$\partial_t u + u\partial_x u + \frac{B}{\rho}\partial_x B = 0, \tag{33}$$

$$\partial_t\left[B - \partial_x\left(\frac{1}{\rho}\partial_x B\right)\right] + \partial_x\left[u\left(B - \partial_x\left(\frac{1}{\rho}\partial_x B\right)\right)\right] = 0. \tag{34}$$

The system (32)–(34) has been first derived in [16] (see also [7]) in a slightly different form.

Comparing (34) with (32) we readily find the nonlocal relationship between B and ρ [7]

$$B - \partial_x\left(\frac{1}{\rho}\partial_x B\right) = C\rho,$$

where C is a constant. Setting $B = \rho = 1$ at infinity we obtain $C = 1$. Then the system becomes

$$\partial_t \rho + \partial_x(\rho u) = 0,$$
$$\partial_t u + u\partial_x u + \frac{B}{\rho}\partial_x B = 0,$$
$$\partial_x\left(\frac{1}{\rho}\partial_x B\right) = B - \rho. \tag{35}$$

Our purpose, again, is to show that the system (28)–(30), considered for steady two-dimensional hypersonic flows, is asymptotically equivalent to (35) (although with different x, t, u).

By analogy with Section 2, we consider the 2D problem of the steady flow past a wedge-shaped body. Now the boundary conditions are (cf. (4)–(6)):

$$\mathbf{u} \to M\mathbf{e}_x, \qquad \rho \to 1, \qquad \mathbf{B} \to \mathbf{e}_z \quad \text{as} \quad (x - x_b)^2 + (y - y_b)^2 \to \infty, \tag{36}$$

where $M = u_0/c_A$ is the Alfvén Mach number.

$$(\mathbf{u} \cdot \mathbf{n})_{\text{body}} = 0, \tag{37}$$

$$(\operatorname{curl} \mathbf{B} \cdot \mathbf{n})_{\text{body}} = 0, \qquad (B)_{\text{body}} = (\rho)_{\text{body}}. \tag{38}$$

Again, basic equations (28)–(30) admit the steady plane flow reduction, i.e.,

$$\partial_t \equiv 0, \qquad \mathbf{u} = \big(u(x,y), v(x,y), 0\big),$$
$$\rho = \rho(x,y), \qquad \mathbf{B} = \big(0, 0, B(x,y)\big). \tag{39}$$

Correponding scalar equations for ρ, u, v, B are then easily obtained from (28)–(30). They are quite cumbersome though so we do not write them here. For the body with the countour shape given by the function $y = F(x)$ (see (9)) boundary conditions (36)–(38) assume the form

$$\rho \to 1, \qquad B \to 1, \qquad u \to M, \qquad v \to 0$$
$$\text{as} \quad (x - x_b)^2 + \big(y - F(x_b)\big)^2 \to \infty, \tag{40}$$

$$v = uF'(x) \quad \text{on} \quad y = F(x), \tag{41}$$
$$\partial_y B = \partial_x B F'(x), \quad B = \rho \quad \text{on} \quad y = F(x). \tag{42}$$

Introducing the asymptotic decompositions (13), (14) we obtain that the steady 2D reduction of the system (28)–(30) assumes to the leading order in M the form:

$$\partial_T \rho + \partial_X(\rho v) = 0, \tag{43}$$
$$\partial_T v + v\partial_X v + \frac{B}{\rho}\partial_X B = 0, \tag{44}$$
$$\partial_T u_1 + v\partial_X u_1 = 0, \tag{45}$$
$$\partial_T B + \partial_x(vB) - \frac{m_i}{m_e}\partial_X(\partial_T u_1 + v\partial_X u_1)$$
$$- \left[\partial_X\left(\partial_T\left(\frac{1}{\rho}\partial_X B\right) + v\partial_X\left(\frac{1}{\rho}\partial_X B\right)\right)\right] = 0, \tag{46}$$

provided $M^{-1}m_i/m_e < O(1)$. Owing to (45) the third term in (46) disappears and the equation becomes

$$\partial_T\left[B - \partial_X\left(\frac{1}{\rho}\partial_X B\right)\right]$$
$$+ \partial_X\left[\left(B - \partial_X\left(\frac{1}{\rho}\partial_X B\right)\right)v\right] = 0. \tag{47}$$

Comparing (47) with (43) we immediately obtain

$$B - \partial_X\left(\frac{1}{\rho}\partial_X B\right) = C\rho, \tag{48}$$

where C is a constant. Using the condition at infinity (40) we see that $C = 1$. Therefore, for B, ρ, v we obtain a closed system

$$\partial_T \rho + \partial_X(\rho v) = 0,$$
$$\partial_T v + u\partial_X v + \frac{B}{\rho}\partial_X B = 0,$$
$$\partial_X\left(\frac{1}{\rho}\partial_X B\right) = B - \rho, \tag{49}$$

which, as expected (cf. (15)), coincides with one-dimensional nonstationary system (35) but with v, X, T instead u, x, t. The correction u_1 to the x-component of the flow velocity is connected with v by the linear equation (45).

The boundary conditions (40)–(42), as well as in the previous section, transform into the piston conditions as $M \to \infty$ provided $\alpha M \sim 1$ where $\alpha = \max F'(x)$:

$$v = \frac{dX}{dT} \quad \text{on} \quad X = f(T), \tag{50}$$
$$\partial_X B = 0, \quad B = \rho \quad \text{on} \quad X = f(T), \tag{51}$$
$$\rho \to 1, \qquad B \to 1, \qquad v \to 0 \quad \text{as} \quad X \to \infty. \tag{52}$$

Thus, we have established the sought equivalence for nonlinear magnetoacoustic waves in a cold plasma propagating across the magnetic field.

4. Conclusion

We have shown that the problem of steady hypercritical plane flow around a slender body is asymptotically equivalent to a one-dimensional piston problem in the same physical system with the piston speed determined by the derivative of the body profile in stretched coordinate. An important feature of the obtained approximation is that it leads to a fully nonlinear system even for thin bodies with the slenderness parameter $\alpha \sim M^{-1} \ll 1$ in contrast to the weakly nonlinear case with $M > 1$, $\alpha \ll 1$, which is asymptotically described by the KdV equation.

We would like to emphasize that, although using examples from plasma physics, this Letter did not aim the study of specific plasma phenomena. Rather, our purpose was to demonstrate some general (possibly universal) transformations and relationships occuring in physically relevant multidimensional nonlinear wave equations under the hypersonic limiting transition and put them in the context with already obtained results in one-dimensional nonstationary theory.

The full description of the hypersonic flow past thin wedge in dispersive hydrodynamics will be published elsewhere.

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References

[1] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, Pergamon, Oxford, 1987.
[2] L.V. Ovsyannikov, 1981 Lectures on the Foundations of Gas Dynamics, Nauka, Moscow, 1981 (in Russian).
[3] V.I. Karpman, Nonlinear Waves in Dispersive Media, Pergamon, Oxford, 1975.
[4] A.V. Gurevich, A.L. Krylov, V.V. Khodorovskii, G.A. El, J. Exp. Theor. Phys. 81 (1995) 87.
[5] A.V. Gurevich, A.L. Krylov, V.V. Khodorovskii, G.A. El, J. Exp. Theor. Phys. 82 (1996) 709.
[6] A.V. Gurevich, L.P. Pitaevsky, Sov. Phys. JETP 38 (1974) 291.
[7] G.B. Whitham, Proc. R. Soc. London, Ser. A 283 (1965) 238.
[8] A.M. Kamchatnov, Nonlinear Periodic Waves and Their Modulations: An Introductory Course, World Scientific, Singapore, 2000.
[9] A.V. Gurevich, A.L. Krylov, G.A. El, Sov. Phys. JETP 71 (1990) 899.
[10] A.V. Tyurina, G.A. El, J. Exp. Theor. Phys 88 (1999) 615.
[11] G.A. El, V.V. Khodorovskii, A.V. Tyurina, Phys. Lett. A 318 (2003) 526.
[12] R.Z. Sagdeev, in: M.A. Leontovich (Ed.), Reviews of Plasma Physics, vol. 4, Consultants Bureau, New York, 1968, p. 20.
[13] A.V. Gurevich, A.P. Meshcherkin, Sov. Phys. JETP 60 (1984) 732.
[14] A.V. Gurevich, A.L. Krylov, G.A. El, Sov. J. Plasma Phys. 16 (1990) 139.
[15] A.I. Akhiezer, Plasma Electrodynamics, Pergamon, Oxford, 1975.
[16] J.H. Adlam, J.E. Allen, Philos. Mag. 3 (1958) 448.