Recognising Randomness

David Green
Lougliborough University, England.

◆INTRODUCTION◆

THE concept of randomness is something we all think we understand, but to provide an acceptable definition is by no means straightforward. To us it is more of an intuition and we all know that intuition can let us down! In a recent interesting article Chatterjee (1996) discussed the difficulties encountered when considering randomness, which has its echoes in a project undertaken by the author some time ago (see Green, 1991 for a fuller report).

The project involved 1600 English primary school pupils, aged 7 to 11, and 300 secondary school pupils aged 13-14, each of whom was given a half-hour paper-based test on concepts of randomness. One test item involved selection from a uniform distribution. A class demonstration was first undertaken with pupils selecting, with replacement, numbered counters using the scenario depicted in Fig. 1.

> Paul plays a game using 16 counters numbered 1, 2, 3, 4,...16.
> Paul puts the counters in a tin. He shakes the tin a lot. Rachel shuts her eyes and picks out a counter.
>
> It is number 7.
> Paul puts a cross in box 7. The 7 is put back in the tin and someone else picks a counter.

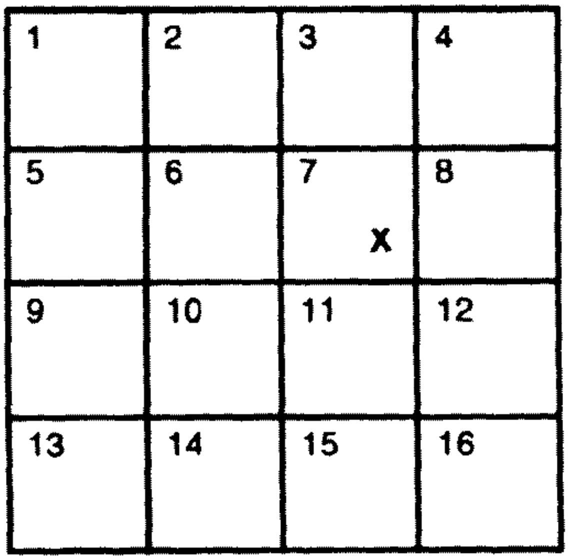

Figure 1 Randomness test: counter selection demonstration

The instructions were read out to the class and an experiment was then conducted with six different pupils each selecting one counter and then replacing it. All pupils in the class recorded each number selected by putting a cross in the appropriate box in their answer booklets. If no duplication had occurred naturally after six trials, the researcher secretly made it happen in one extra (seventh) trial without the children being aware of the intervention. (This was rarely necessary.) There followed three questions in which the pupils had to illustrate what they thought would happen if this experiment were continued for twelve, sixteen or thirty selections. A follow-up question explored this in a multiple choice format (Fig. 2).

◆RESULTS ◆

First of all, the reader may like to study the questions in Figure 2 and decide which patterns show evidence of ‘cheating’ (non-random) and which do not (random). Then the reader may consider how children aged 7 to 14 of varying abilities are likely to answer. Which items are very easy and which are difficult? What order of difficulty would you place them in? What approximate percentages correct would you expect at age 7 and age 14? The results obtained with English children are presented in Table 1 which shows the percentages correct (facilities) for each of the eight pictures.

Age is clearly a highly significant factor for the first five items. Gender is significant for none.

Alice: Alice’s pattern (see Fig. 2) looks so unlikely that one would expect virtually every pupil to declare that Alice cheated. However, one third of pupils aged 7 to 8 do not, and, even at age 13 to 14, 16% seem to consider the pattern reasonable.

Rebecca, Emma and Nicola: Most pupils reject as unnatural the extremely regular pattern. The lower facility for Emma compared to Rebecca shows that the picture itself misleads about *25%*, for the distributions are logically identical in the context of putting crosses in boxes. An encouragingly high proportion recognise that Nicola has an acceptable (random) pattern.

Janet: Compared with Nicola, the slightly reduced clustering in Janet produces an appreciably higher facility showing the tendency of pupils to underestimate how uneven a randomly produced distribution may be.

Sally, Tracey and Claire: The previous five items are relatively straightforward and amenable to mature logic.

Some children are told to play the counters game by themselves using 16 real counters. Did some cheat and make it up? Did they all cheat? Did none cheat? **You must decide!**

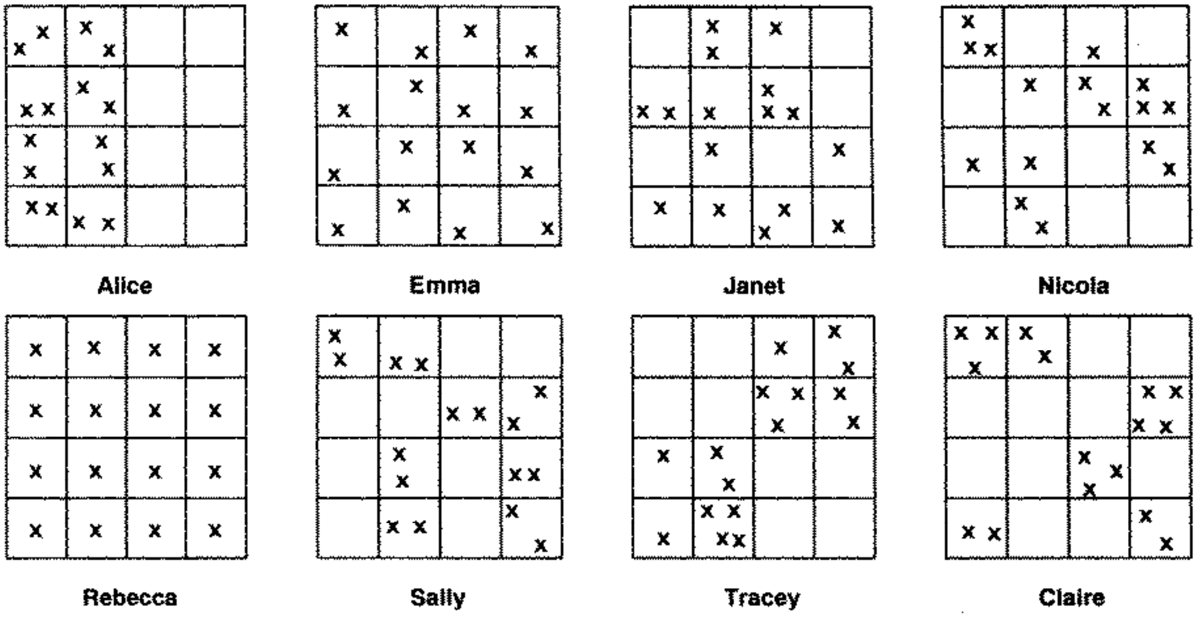

Figure 2 Randomness test: counter selection question

Year	Age	Alice Yes	Emma Yes	Janet No	Nicola No	Rebecca Yes	Sally Yes	Tracey Yes	Claire Yes
Primary									
4	7-8	67	45	70	66	70	40	53	47
5	8-9	74	52	79	67	82	40	49	48
6	9-10	84	67	86	76	90	38	53	42
7	10-11	84	67	88	81	90	43	50	48
Secondary									
9	13-14	84	74	95	90	95	40	50	41

Table 1 Randomness test: counter selection question facilities

Thus their facilities correlate strongly with age. By contrast, these last three items are more demanding. This leads to a lower facility (as expected), but also to a surprising lack of improvement with age. Both Sally and Claire have manifestly improbable box frequencies: no occurrence of a single cross, always two or more together. The low facilities of around 40% might be explained by the fact that many pupils concentrate on the overall visual pattern (which is quite random) and ignore the detailed box frequencies. However, in contrast, Tracey has a very non-random overall visual appearance and variable (random?) box frequencies but, despite this, achieves a facility of around *50%* which seems to contradict the previous assertion. The explanation may be that there are two groups of children who give different weights to the two aspects. For some, randomness is seen in global terms whereas for others a more local focus is preferred. Randomness is evidently not a simple concept and it has different facets, so perhaps we should not be surprised to get such anomalous results!

◆IMPLICATIONS ◆

The increase in facility with age on some items and the lack of any such increase for other apparently very similar items shows the very subtle intellectual demands which the concept of randomness makes. What to the teacher may appear trivial and obvious may, in fact, be deep and subtle and so should not be hastily dismissed or glossed over. In studying random events, aspects of local and global frequency distribution and visual appearance arise together. These need to be considered separately and focused upon. To use the blanket term random' without exploring its different manifestations and interpretations can create rather than avoid difficulties.

Neither natural experience nor maturation nor the existing educational curriculum would appear adequate. If improved understanding of these deeper aspects is to be achieved by many pupils, then a definite intervention by the teacher with a planned curriculum seems essential. Guided experiences for the pupils to explore variability, which is the essence of statistical thought, may help to bring about an awareness that is missing from a traditional mathematical approach to stochastics. With this in mind the author has prepared a game which might be helpful in developing understanding of two dimensional random distributions. This is now described and readers are encouraged to try it out and assess whether it has an appreciable effect on students' (apparent) understanding.

References

Chatterjee, S. (1996) Statistics and Intuition for the Classroom. *Teaching Statistics*, 18(2), 34-38. [see also the editorial for *18(3)*]

Green D. (1991) Chapter 2 in *Studies in Mathematics Education* Volume 7: The teaching of statistics, UNESCO.

SUBMARINE

Number of players

1 . solo; 2 or more - competitive

Apparatus needed per player

- Nine discs numbered 1 to 9
- A bag or shaker (eg Yoghurt carton) for the discs
- A pencil.
- A set of five game boards (example below)

1	2	3
4	5	6
7	8	9

A Recording Table

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Submarine prediction game

There is one submarine in each of the nine squares. The skill is in trying to predict how many of the nine detected after playing a round of Sonar described below.

Sonar

Place the nine numbered discs in the container and shake thoroughly to mix them well, then pick one out without looking and note the number. That number shows which submarine has been detected by Sonar. Draw a cross in the square (even if there is one there already). Replace the numbered disc in the container and mix the discs well, ready to play again. Repeat until you have done this exactly nine times (you can check b) counting the crosses drawn).

Scoring

Predicting the number of submarines detected by Sonar

exactly - 4 points
out by 1 - 2 points
out by 2 - 1 point

Playing Submarine

(1) Make your PREDICTION of how many submarines will be detected when playing a round of Sonar. Enter your prediction in the Recording Table. (Each player should do this separately.)
(2) Play a round of Sonar and count up how main submarines were actually detected. Record the number of DETECTIONS in the Table. (Players can take turns or play simultaneously.
(3) Find the DIFFERENCE between your prediction and the actual result, and calculate your POINTS
(4) Repeat until you have played five rounds of Sonar
(5) Total up your score.
(6) The winner of Submarine is the player with mos points after the five rounds.

Questions

Are the results surprising?
Do you improve with practice?
What is the best strategy?
How often will all nine submarines be detected?

Submarine Sonar Boards *and* Recording Tables

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

ROUND	PREDICTION	DETECTIONS	Difference	POINTS
1				
2				
3				
4				
5				
			TOTAL	

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
7	8	9

1	2	3
4	5	6
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