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Applications of near sets.

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The authors refer to their article as a "bird's-eye view of recent developments in the study of the nearness of sets". They also sketch the long and interesting history of nearness going back to H. Poincaré [Rev. Métaphys. Morale **3** (1895), 631–646; JFM 27.0379.08] and F. Riesz [Atti IV Congresso Internazionale Mat. **2** (1908), 18–24]. The authors briefly mention some applications of nearness to "sets of sensations", "image processing", "color modeling", "ethology", and "visual perception". In modern mathematics, nearness is studied in the category **Near** of nearness spaces and nearness-preserving maps [e.g., see H. Herrlich, General Topology and Appl. **4** (1974), 191–212; MR0350701 (50 #3193)], which contains the category **TOP** of topological spaces and continuous functions, and **Prox**, the category of proximity space and continuous maps [e.g., see A. Di Concilio, in *Beyond topology*, 89–114, Contemp. Math., 486, Amer. Math. Soc., Providence, RI, 2009; MR2521943 (2010j:54044)].

Reviewed by *J. E. Vaughan*

References

1. P. Alexandroff, Diskrete räume, *Mat. Sb. (N.S.)* **2** (1937), 501–518.
2. F. G. Arenas, Alexandroff spaces, *Acta Math. Univ. Comenianae* **1** (1999), 17–25. MR1711071 (2000i:54028)
3. H. L. Bentley, E. Colebunders, and E. Vandermissen, *A Convenient Setting for Completions and Function Spaces*, Contemporary Mathematics (F. Mynard and E. Pearl, eds.), Amer. Math. Soc., Providence, RI, 2009, pp. 37–88. MR2521942 (2010j:54008)
4. N. Bourbaki, *Elements of Mathematics. General Topology, Part 1*, Hermann & Addison-Wesley, Paris & Reading, MA, 1966, i-vii, 437 pp. MR0205210 (34 #5044a)
5. A. Di Concilio, *Proximity: A Powerful Tool in Extension Theory, Function Spaces, Hyperspaces, Boolean Algebras and Point-free Geometry*, Contemporary Mathematics (F. Mynard and E. Pearl, eds.), Amer. Math. Soc., Providence, RI, 2009, pp. 89–114. MR2521943 (2010j:54044)
6. K. Devlin, *Will the real continuous function please stand up?*, Tech. report, 2006, http://www.maa.org/devlin/devlin_11_06.html.
7. R. Z. Domiaty and O. Laback, Semimetric spaces in general relativity (on Hawking–King–McCarthy's path topology), *Russian Math. Surveys* **35** (1980), no. 3, 57–69. MR0580620 (81h:83029)
8. V. A. Efremovič, The geometry of proximity. I, *Mat. Sb.* **31** (1951), 189–200. MR0055659 (14,1106e)

9. G. T. Fechner, *Elemente der Psychophysik*, 2 vols., E. J. Bonset, Amsterdam, 1860.
10. M. Garat and S. A. Naimpally, Proximity approach to semi-metric and developable spaces, *Pacific J. Math.* **44** (1973), no. 1, 93–105. MR0321015 (47 #9548)
11. M. Garat and S. A. Naimpally, Proximity approach to extension problems, *Fund. Math.* **71** (1971), 63–76. MR0293576 (45 #2653)
12. G. DiMaio and S. A. Naimpally, *D*-proximity spaces, *Czech. Math. J.* **41** (1991), no. 116, 232–248. MR1105438 (92g:54034)
13. S. Gupta and K. Patnaik, Enhancing performance of face recognition systems by using near set approach for selecting facial features, *J. Theoretical and Applied Information Technology* **4** (2008), no. 5, 433–441.
14. A. E. Hassanien, A. Abraham, J. F. Peters, G. Schaefer, and C. Henry, Rough sets and near sets in medical imaging: A review, *IEEE Trans. Info. Tech. in Biomedicine* **13** (2009), no. 6, 955–968, doi:10.1109/TITB.2009.2017017.
15. F. Hausdorff, *Grundzüge der Mengenlehre*, Veit and Company, Leipzig, 1914, viii + 476 pp. MR0031025 (11,88d)
16. F. Hausdorff, *Set Theory*, AMS Chelsea Publishing, Providence, RI, 1914, 352 pp. MR0141601 (25 #4999)
17. C. J. Henry, *Near Sets: Theory and Applications*, Ph.D. dissertation, supervisor: J. F. Peters, Ph.D. thesis, Department of Electrical & Computer Engineering, 2010.
18. C. J. Henry and J. F. Peters, Arthritic handfinger movement similarity measurements: Tolerance near set approach, *Comp. & Math. Methods in Medicine* 2011, Article ID 569898 (2011), 1–14, doi:10.1155/2011/569898. MR2784707
19. C. J. Henry and S. Ramanna, *Parallel Computation with Near Sets*, Rough Sets and Knowledge Technology (RSKT2011), Springer-Verlag, 2011, pp. 523–532.
20. G. T. Herman, On topology as applied to image analysis, *Computer Vision, Graphics, and Image Processing* **52** (1990), 409–415.
21. H. Herrlich, A concept of nearness, *Gen. Top. & Appl.* **4** (1974), 191–212. MR0350701 (50 #3193)
22. J. G. Hocking and S. A. Naimpally, *Nearness—A Better Approach to Continuity and Limits*, Allahabad Mathematical Society Lecture Note Series, 3, The Allahabad Math. Soc., Allahabad, 2009, iv+66 pp., ISBN 978–81-908159–1-8. MR2582143
23. V. M. Ivanova and A. A. Ivanov, Contiguity spaces and bicompact extensions of topological spaces (Russian), *Dokl. Akad. Nauk SSSR* **127** (1959), 20–22. MR0107218 (21 #5943)
24. J. L. Kelley, *General Topology*, Springer-Verlag, Berlin, 1955, xiv + 298 pp. MR0370454 (51 #6681)
25. E. H. Kronheimer, The topology of digital images, *Top. and its Appl.* **46** (1992), 279–303. MR1198735 (94a:54089)
26. H.-P. Künzi, *An introduction to quasi-uniform spaces*, Contemporary Mathematics (F. Mynard and E. Pearl, eds.), Amer. Math. Soc., Providence, RI, 2009, pp. 239–304. MR2521946 (2010j:54046)
27. C. Kuratowski, *Topologie. I*, Panstwowe Wydawnictwo Naukowe, Warsaw, 1958, xiii + 494pp. MR0090795 (19,873d)

28. C. Kuratowski, *Introduction to Calculus*, Pergamon Press, Oxford, UK, 1961, 316pp.
29. C. Kuratowski, *Introduction to Set Theory and Topology*, 2nd ed., Pergamon Press, Oxford, UK, 1962, 1972, 349pp. MR0346724 (49 #11449)
30. S. Leader, On clusters in proximity spaces, *Fund. Math.* **47** (1959), 205–213. MR0112120 (22 #2978)
31. R. Lowen, *Approach Spaces: The Missing Link in the Topology-Uniformity-Metric Triad*, Oxford Mathematical Monographs, Oxford University Press, Oxford, UK, 1997, viii + 253pp. MR1472024 (98f:54002)
32. R. Lowen and C. V. Olmen, *Approach Theory*, Contemporary Mathematics (F. Mynard and E. Pearl, eds.), Amer. Math. Soc., Providence, RI, 2009, pp. 305–332. MR2521947 (2011a:54002)
33. R. Lowen, D. Vaughan, and M. Sioen, Completing quasi metric spaces: An alternative approach, *Houston J. Math.* **29** (2003), no. 1, 113–136. MR1952498 (2004b:54038)
34. C. J. Mozzochi, M. S. Gagrat, and S. A. Naimpally, *Symmetric Generalized Topological Structures*, Exposition Press, Hicksville, N.Y., 1976, xii+74 pp., ISBN 978–3-486–58917-7. MR0482674 (58 #2732)
35. C. J. Mozzochi and S. A. Naimpally, *Uniformity and Proximity*, Allahabad Mathematical Society Lecture Note Series, 2, The Allahabad Math. Soc., Allahabad, 2009, xii+153 pp., ISBN 978–81-908159–1-8. MR2526304 (2010h:54051)
36. S. A. Naimpally, Reflective functors via nearness, *Fund. Math.* **85** (1974), 245–255. MR0353244 (50 #5728)
37. S. A. Naimpally, Near and far. A centennial tribute to Frigyes Riesz, *Siberian Electronic Mathematical Reports* **2** (2005), 144–153.
38. S. A. Naimpally, *Proximity Approach to Problems in Topology and Analysis*, Oldenburg Verlag, München, 2009, xiv + 204pp, ISBN 978–3-486–58917-7. MR2531321 (2010h:54052)
39. S. A. Naimpally and J. F. Peters, *Topology with Applications. Topological Spaces via Near and Far*, World Scientific, Singapore, 2012, *to appear.*
40. S. A. Naimpally and B. D. Warrack, *Proximity Spaces*, Cambridge Tract in Mathematics No. 59, Cambridge University Press, Cambridge, UK, 1970, x+128 pp., paperback (2008). MR0278261 (43 #3992)
41. S. K. Pal and J. F. Peters, *Rough Fuzzy Image Analysis. Foundations and Methodologies*, CRC Press, Taylor & Francis Group, Sept., 2010, ISBN 13: 9781439803295 ISBN 10: 1439803293.
42. Z. Pawlak and J. F. Peters, Jak blisko, *Systemy Wspomagania Decyzji* **I** (2007), 57.
43. J. F. Peters, Tolerance near sets and image correspondence, *International Journal of Bio-Inspired Computation* **1** (2009), no. 4, 239–245.
44. J. F. Peters and P. Wasilewski, Foundations of near sets, *Info. Sciences* **179** (2009), no. 18, 3091–3109. MR2588809 (2010m:68160)
45. J. F. Peters, Near sets. General theory about nearness of objects, *Applied Mathematical Sciences* **1** (2007), no. 53, 2609–2029. MR2380165
46. J. F. Peters, Near sets. Special theory about nearness of objects, *Fund. Inform.* **75** (2007), no. 1–4, 407–433. MR2293708 (2008a:03046)
47. J. F. Peters, Corrigenda and addenda: Tolerance near sets and image correspondence, *Int. J. Bio-Inspired Computation* **2** (2010), no. 5, 310–319.

48. J. F. Peters, *How Near are Zdzisław Pawlak's Paintings? Merotopic Distance Between Regions of Interest,* Intelligent Systems Reference Library, volume dedicated to Prof. Zdzisław Pawlak (A. Skowron and S. Suraj, eds.), Springer, Berlin, 2011, pp. 1–19.
49. J. F. Peters, *Sufficiently Near Sets of Neighbourhoods,* Lecture Notes in Artificial Intelligence 6954 (J. T. Yao, S. Ramanna, G. Wang, and Z. Suraj, eds.), Springer, Berlin, 2011, pp. 17–24.
50. J. F. Peters and M. Borkowski, *ε-Near Collections,* Lecture Notes in Artificial Intelligence 6954 (J. T. Yao, S. Ramanna, G. Wang, and Z. Suraj, eds.), Springer, Berlin, 2011, pp. 533–542.
51. J. F. Peters and S. A. Naimpally, Approach spaces for near families, *Gen. Math. Notes* **2** (2011), no. 1, 159–164.
52. J. F. Peters and L. Puzio, Image analysis with anisotropic wavelet-based nearness measures, *International Journal of Computational Intelligence Systems* **2** (2009), no. 3, 168–183, doi:10.1016/j.ins.2009.04.018.
53. J. F. Peters, S. Shahfar, S. Ramanna, and T. Szturm, *Biologically-Inspired Adaptive Learning: A Near Set Approach,* Frontiers in the Convergence of Bioscience and Information Technologies (Korea), 2007.
54. J. F. Peters, A. Skowron, and J. Stepaniuk, *Nearness in Approximation Spaces,* Proc. Concurrency, Specification and Programming (CS&P 2006) (Humboldt Universität), 2006, pp. 435–445.
55. J. F. Peters, A. Skowron, and J. Stepaniuk, Nearness of objects: Extension of approximation space model, *Fund. Inform.* **79** (2007), no. 3–4, 497–512. MR2346263 (2008i:68120)
56. J. F. Peters and S. Tiwari, Approach merotopies and near filters, *Gen. Math. Notes* **3** (2011), no. 1, 1–15.
57. J. Picado, Weil nearness spaces, *Portugaliae Math.* **55,** no. 2. MR1629638 (99e:54008)
58. J. H. Poincaré, L'espace et la géomètrie, *Revue de métaphysique et de morale* **3** (1895), 631–646.
59. J. H. Poincaré, Sur certaines surfaces algébriques; troisième complément a l'analysis situs, *Bulletin de la Société de France* **30** (1902), 49–70. MR1504408
60. J. H. Poincaré, *Dernières Pensées,* trans. by J. W. Bolduc as mathematics and science: Last essays, Flammarion & Kessinger Pub., Paris & NY, 1913 & 2009. MR0175729 (31 #6)
61. J. H. Poincaré, Sur la nature du raisonnement mathématique, *Revue de métaphysique et de morale* **2** (juil. 1894), 371–384.
62. P. Pták and W. G. Kropatsch, Nearness in digital images and proximity spaces, *LNCS* **1953** (2000), 69–77, In Proc. 9th International Conference on Discrete Geometry for Computer Imagery.
63. S. Ramanna and A. H. Meghdadi, Measuring resemblances between swarm behaviours: A perceptual tolerance near set approach, *Fund. Inform.* **95** (2009), no. 4, 533–552, doi: 10.3233/FI-2009–163. MR2582188
64. F. Riesz, Stetigkeitsbegriff und abstrakte mengenlehre, *Atti del IV Congresso Internazionale dei Matematici* **II** (1908), 18–24.
65. Ju. A. Shreider, *Equality, Resemblance, and Order,* Mir Publishers, Russia, 1975, 279 pp. MR0389600 (52 #10431)
66. J. M. Smirnov, On proximity spaces, *Mat. Sb.* **31** (1952), no. 73, 543–574. MR0055661

(14,1107b)
67. A. B. Sossinsky, Tolerance space theory and some applications, *Acta Applicandae Mathematicae: An International Survey Journal on Applying Mathematics and Mathematical Applications* **5** (1986), no. 2, 137–167. MR0823824 (87d:92070)
68. A. D. Taimanov, On the extension of continuous mappings of topological spaces, *Mat. Sb.* **31** (1952), 451–463. MR0050871 (14,395a)
69. W. J. Thron, Proximity structures and grills, *Math. Ann.* **206** (1973), 35–62. MR0336710 (49 #1483)
70. S. Tiwari, *Some aspects of general topology and applications. Approach merotopic structures and applications,* supervisor: M. Khare, Ph.D. thesis, Department of Mathematics, Allahabad (U.P.), India, Jan. 2010.
71. J. W. Tukey, *Convergence and Uniformity in Topology,* Princeton Univ. Press, Annals of Math. Studies AM-2, Princeton, NJ, 1940, 90pp. MR0002515 (2,67a)
72. E. Čech, *Topological Spaces,* revised ed. by Z. Frolik and M. Katătov, John Wiley & Sons, NY, 1966. MR0211373 (35 #2254)
73. H. Wallman, Lattices and topological spaces, *Ann. of Math.* **39** (1938), no. 1, 112–126. MR1503392
74. P. Wasilewski, J. F. Peters, and S. Ramanna, Perceptual tolerance intersection, *Trans. on Rough Sets* **XIII** (2011), 159–174.
75. A. Weil, *Sur les Espaces à Structure Uniforme et sur la Topologie Générale,* Harmann & cie, Actualités scientifique et industrielles, Paris, 1938.
76. M. Wolski, Perception and classification. A note on near sets and rough sets, *Fund. Inform.* **101** (2010), 143–155. MR2732874 (2012a:68218)
77. M. Wolski, *Gauges, Pregauges and Completions: Some Theoretical Aspects of Near and Rough Set Approaches to Data,* Lecture Notes in Artificial Intelligence 6954 (J. T. Yao, S. Ramanna, G. Wang, and Z. Suraj, eds.), Springer, Berlin, 2011, pp. 559–568.
78. E. C. Zeeman, *The Topology of the Brain and Visual Perception,* University of Georgia Institute Conference Proceedings (1962), Published in M. K. Fort, Jr. (ed.), Topology of 3-Manifolds and Related Topics, Prentice-Hall, Inc., 1962, 240–256. MR0140374 (25 #3796)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.