Physical parametric modelling for nonlinear systems

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Indexing terms: Nonlinear models, Genetic algorithms, Parameter estimation, System identification, Nonlinear systems

This letter reports a novel physical model based multivariable nonlinear system identification technique. The parameters of the model are intelligently evolved using a genetic algorithm enhanced by simulated annealing. Implemented results show that step response data can be used directly for system identification and that the developed methodology opens a new way for nonlinear modelling.

Introduction: Existing nonlinear system identification techniques include (a) approximating the practical system with artificial mathematical models; (b) the use of describing functions; and (c) the use of a physical model. In systems and control engineering, a practical system is usually described by a set of differential equations derived from the physical mechanism of the system [1,2]. For a linear system, its artificial mathematical models, such as those based on integral transforms, are equivalent, since their mappings from the system differential equations are bijective. This no longer holds for a nonlinear system, however, and thus the use of a global artificial mathematical model or a set of local models, including the Volterra series, nonlinear auto-regressive moving average, bilinear, nonlinear Wiener, Walsh, fuzzy and neural network models, can only be approximate [1,2]. Further, the information of the physical mechanism is seldom utilised in artificial models. Without this information it is hard to identify, for example, a repeated pole of a system and to identify an unstable system.

On contrast, the describing function method can be based on the physical mechanism in a certain degree. A model based on this is, however, difficult to use in the design of its controller. This letter is thus focused on the development of tractable techniques for direct estimation of parameters of the physical model.

A nonlinear system: A laboratory-scale nonlinear coupled liquid-level regulation system shown in Fig. 1 is used in the pilot study of this work. A physical model of this system is given by:

$$\begin{cases} A\dfrac{dh_1}{dt} = Qv_1 - C_1\sqrt{2g(h_1 - h_2)} \\ A\dfrac{dh_2}{dt} = Qv_2 + C_1\sqrt{2g(h_1 - h_2)} - C_2\sqrt{2g(h_2 - H_3)} \end{cases} \tag{1}$$

where $h_1 > h_2 > H_3$. Here h_1 and h_2 are the liquid levels of Tank 1 and Tank 2, respectively; H_3 the height of the discharging and coupling orifices; $A = 100$ cm^2 the cross-sectional area of both tanks, which can be measured relatively accurately; Q the input liquid flow per actuating volt; C_1 and C_2 discharge constants for the orifice areas of Tank 1 and Tank 2, respectively; and $g = 981\ cm/s^2$ is the gravitational constant. The identification task for this system is thus to accurately estimate the parameter set $[Q, H_3, C_1, C_2]$. These parameters may be slowly time-varying and are usually very difficult to measure accurately in a practical industrial process. Based on physical measurements and the manufacturer's specifications, however, these parameters are initially given by:

$Q = 7.0\ cm^3/s/V$	$H_3 = 3\ cm$
$C_1 = 0.21\ cm^2$	$C_2 = 0.24\ cm^2$

In many control engineering applications, the closed-loop system output is required to follow a step command signal. Data for open-loop step responses are usually available before the closed-loop system is designed. Thus, two identical step inputs, $v_1 = v_2 = 2.5$ V, were applied for 1000 s to the physical model with the above parameters. The outputs of this model, h_1 and h_2, were collected, which are shown in Fig. 2 and are compared with the actual outputs sampled from the real system. For a better validation, a small pseudo random binary sequence (PRBS) is added to both inputs for a further 960 s. The mean-square errors of h_1 and h_2 resulting from this model are 5.2% and 8.7%, respectively.

Widely adopted methods for this task are gradient-guided parameter estimation methods, such as the least mean-squares and instrumental variable methods. These methods can, however, only be applied to a nonlinear system if the parameters to be estimated are linearly separable [1,2]. In eqn. (1), this condition is clearly not satisfied for H_3 and C_2. Further, these conventional methods may fail if the practically constrained objective index is not differentiable or is not "well-behaved". The estimation may also be trapped in a local optimum in the multimodal multidimensional search space. Thus, in this letter, a novel parameter estimation technique is developed using the genetic algorithm based global search technique.

Genetic algorithms with simulated annealing: The genetic algorithm (GA) [3~5] is a global optimisation algorithm based on the mechanism of natural selection and genetics. The algorithm can "intelligently" explore a poorly understood solution space in parallel and find the globally optimised candidate without the need of a differentiable index. This is a non-deterministic polynomial method that has been proven to be very efficient and powerful in solving search and optimisation problems. It has been successfully applied to parameter estimation of nonlinear digital filters [3], linear system identification [4] and improving the performance of maximum likelihood techniques [5].

A basic GA involves three types of operations, namely, *reproduction*, *crossover* and *mutation*. All quantised candidate solutions are coded into a string termed "chromosome". A "population" of a fixed number of chromosomes can be randomly generated initially. Theoretical values of the parameter set can also be included in this initial population, which will usually result in a faster convergence. Reproduction is then used to evolve for a new generation of better candidates, according to their individual performance, which is termed the “fitness”. Crossover is used in the production of off-spring that inherit portion of their parental genetic material, while mutation restores lost, or generates new, genes. In this letter, mutation is realised by multi-point simulated annealing (SA) to achieve a faster and more accurate convergence [4].

Evolutionary parameter estimation: In the evolutionary identification process, the same step input of 1000 s are used to excite candidate models described by eqn. (1). The discrepancies between the outputs of a candidate model and the actual system data are used to measure the performance of the parameter sets. The fitness that guides the evolution is evaluated by:

$$f = \exp\left\{-a\sqrt{\sum_{i=1}^{N}\left[e_1^2 + e_2^2\right]}\right\} \qquad (2)$$

where e_1 and e_2 are errors between the actual and modelled liquid levels in Tank 1 and Tank 2, respectively; i the time index in simulation; N = 1000, the window size; and a = 0.01, the scaling factor dependent upon N.

The SA enhanced GA is programmed in Turbo Pascal and was run for 50 generations with a candidate population size of 20. Convergence traces of the fitness and estimated parameters for several runs are very consistent and a set of these are shown in Fig. 3 and Fig 4, respectively. It can be seen that both the fitness and the parameters converge rapidly. The optimally estimated parameters are given by:

$Q = 6.133\ cm^3/s/V$ $\qquad$ $H_3 = 2.50\ cm$

$C_1 = 0.2087\ cm^2$ $\qquad$ $C_2 = 0.2002\ cm^2$

The outputs of the physical model with identified parameters are shown in Fig. 5, indicating that the estimation method with step inputs yields accurate results. For further validation, the same PRBS is added. The mean-square errors for this model are 4.3% and 4.8%, which are significantly smaller than those obtained from measured parameters.

Conclusions and further work: This letter has developed a simulated annealing enhanced genetic algorithm technique for estimation of physical parameters of nonlinear systems. It is shown that, using this technique, simple step response data available in an engineering process are enough for accurate and global parameter estimation tasks. This methodology can be applied to both continuous and discrete-time systems. Further work on extending this method to uncertainty bound estimation and to real-time adaptive control is being carried out. Identification and refinement of the structure of a nonlinear system by genetic programming, in the same process of parameter estimation, are also investigated at Glasgow.

Acknowledgements: Mr. K.C. Tan is grateful to the University of Glasgow and CVCP for their financial support. The authors would like to thank colleagues in their evolutionary computing and control group for useful discussions.

9 June 1995

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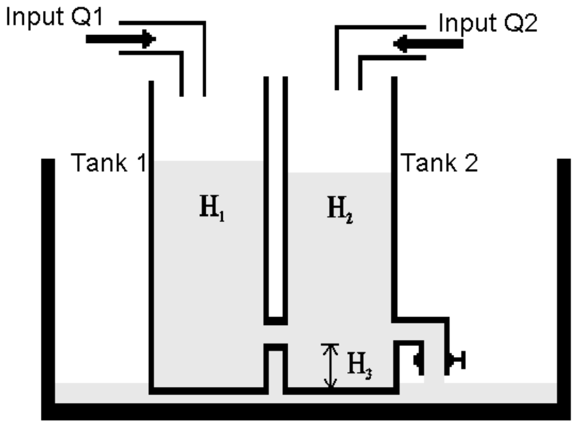

Fig. 1 A nonlinear coupled liquid-level regulation system.

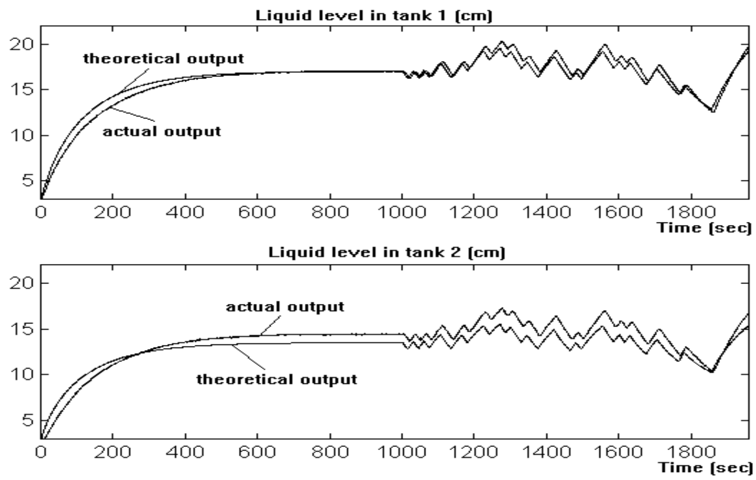

Fig. 2 Theoretical outputs of the model and actual outputs of the physical system.

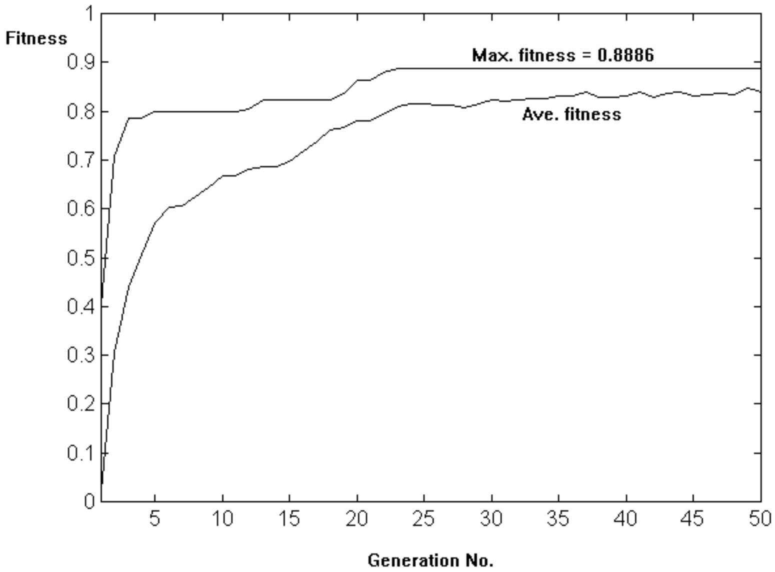

Fig. 3 A convergence trace of fitness in the evolution.

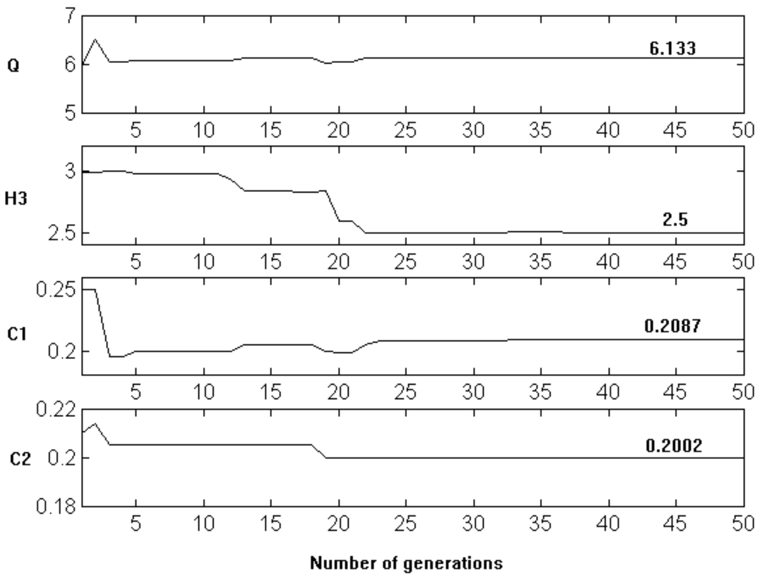

Fig. 4 Convergence trace of the parameter set with the highest fitness.

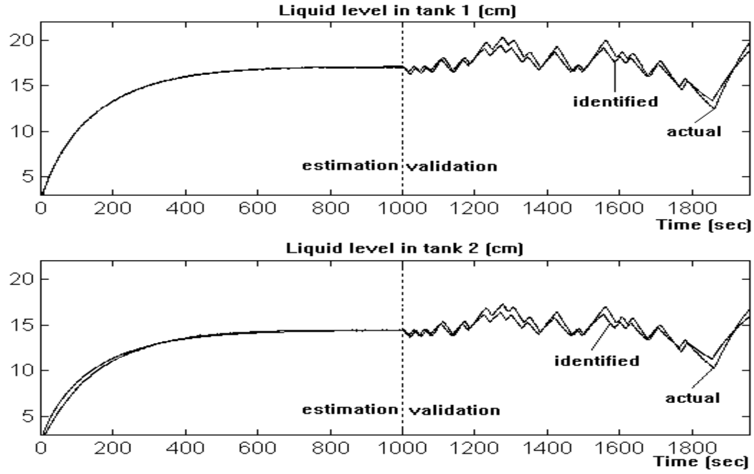

Fig. 5 Outputs of the identified model and actual outputs of the physical system.