Minimal Mixing of Quarks and Leptons in the SU(3) Theory of Flavour

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Abstract

We argue that flavour mixing, both in the quark and lepton sector, follows the minimal mixing pattern, according to which the whole of this mixing is basically determined by the physical mass generation for the first family of fermions. So, in the chiral symmetry limit when the masses of the lightest (u and d) quarks vanish, all the quark mixing angles vanish. This minimal pattern is shown to fit extremely well the already established CKM matrix elements and to give fairly distinctive predictions for the as yet poorly known ones. Remarkably, together with generically small quark mixing, it also leads to large neutrino mixing, provided that neutrino masses appear through the ordinary "see-saw" mechanism. It is natural to think that this minimal flavour mixing pattern presupposes some underlying family symmetry, treating families of quarks and leptons in a special way. Indeed, we have found a local chiral $SU(3)_F$ family symmetry model which leads, through its dominant symmetry breaking vacuum configuration, to a natural realization of the proposed minimal mechanism. It can also naturally generate the quark and lepton mass hierarchies. Furthermore spontaneous CP violation is possible, leading to a maximal CP violating phase $\delta = \frac{\pi}{2}$, in the framework of the MSSM extended by a high-scale $SU(3)_F$ chiral family symmetry.

1 Introduction

The flavour mixing of quarks and leptons is certainly one of the major problems that presently confront particle physics (for a recent review see [1]). Many attempts have been made to interpret the pattern of this mixing in terms of various family symmetries—discrete or continuous, global or local. Among them, the abelian $U(1)$ [2, 3, 4] and/or non-abelian $SU(2)$ [5, 6, 7] and $SU(3)$ [8, 9] family symmetries seem the most promising. They provide some guidance to the expected hierarchy between the elements of the quark-lepton mass matrices and to the presence of texture zeros [10] in them, leading to relationships between the mass and mixing parameters. In the framework of the supersymmetric Standard Model, such a family symmetry should at the same time provide an almost uniform mass spectrum for the superpartners, with a high degree of flavour conservation [11], that makes its existence even more necessary in the SUSY case.

Despite some progress in understanding the flavour mixing problem, one has the uneasy feeling that, in many cases, the problem seems just to be transferred from one place to another. The peculiar quark-lepton mass hierarchy is replaced by a peculiar set of $U(1)$ flavour charges or a peculiar hierarchy of Higgs field VEVs in the non-abelian symmetry case. As a result there are not so many distinctive and testable generic predictions, strictly relating the flavour mixing angles to the quark-lepton masses.

A commonly accepted framework for discussing the flavour problem is based on the picture that, in the absence of flavour mixing, only the particles belonging to the third generation t, b and τ have non-zero masses. All other (physical) masses and the mixing angles then appear as a result of the tree-level mixings of families, related to some underlying family symmetry breaking. They might be proportional to powers of some small parameter, which are determined by the dimensions of the family symmetry allowed operators that generate the effective (diagonal and off-diagonal) Yukawa couplings for the lighter families in the framework of the (ordinary or supersymmetric) Standard Model.

Recently, a new mechanism of flavour mixing, which we call Lightest Family Mass Generation (LFMG), was proposed [12] (see the papers in ref. [13] for further discussion). According to LFMG the whole of flavour mixing for quarks is basically determined by the mechanism responsible for generating the physical masses of the *up* and *down* quarks, m_u and m_d respectively. So,

1

in the chiral symmetry limit when m_u and m_d vanish, all the quark mixing angles vanish. This, in fact minimal, flavour mixing model was found to fit extremely well the already established CKM matrix elements and give fairly distinctive predictions for the yet poorly known ones.

By its nature, the LFMG mechanism is not dependent on the number of quark-lepton families nor on any "vertical" symmetry structure, unifying quarks and leptons inside a family as in Grand Unified Theories (GUTs). The basic LFMG leads to mass-matrices whose non-zero diagonal and off-diagonal elements for the N family case are related as follows:

$$M_{22} : M_{33} : ... : M_{NN} = |M_{12}|^2 : |M_{23}|^2 : ... : |M_{N-1\ N}|^2 \qquad (1)$$

This shows clearly that the heavier families (4th, 5th, ...), had they existed, would be more and more decoupled from the lighter ones and from each other. Indeed, this behaviour can to some extent actually be seen in the presently observed CKM matrix elements involving the third family quarks t and b.

The above condition (1) is suggestive of some underlying flavour symmetry, probably non-abelian rather than abelian, treating families in a special way. Indeed, for the observed three-family case, we show that the local family $SU(3)_F$ symmetry [8] treating the quark and lepton families as fundamental chiral triplets, with an appropriate dominant symmetry breaking vacuum configuration, could lead to a natural realization of the LFMG mechanism. Other outstanding problems in flavour physics are also addressed within the framework of the Minimal Supersymmetric Standard Model (MSSM) appropriately extended by a high-scale $SU(3)_F$ chiral family symmetry: the quark and lepton mass hierarchies, neutrino masses and oscillations and the possibility of spontaneous CP violation.

The paper is organized in the following way. In section 2 we give a general exposition of the LFMG mechanism involving two alternative scenarios. In section 3 we present the $SU(3)_F$ theory of flavour, treating the quark and lepton families as fundamental chiral triplets. We show that there is a dominant symmetry breaking vacuum configuration related with the basic horizontal Higgs supermultiplets, triplets and sextets of $SU(3)_F$, which leads to a natural realization of the LFMG mechanism. While we mainly consider the supersymmetric case—just the MSSM—most of our argumentation remains valid for the Standard Model (and ordinary GUTs) as well. Finally, in section 4, we give our summary.

2 Minimal Flavour Mixing: the Model

2.1 Quark Mixing

The proposed flavour mixing mechanism, driven solely by the generation of the lightest family mass, could actually be realized in two generic ways.

The first basic alternative (I) is when the lightest family mass (m_u or m_d) appears as a result of the complex flavour mixing of all three families. It "runs along the main diagonal" of the corresponding 3×3 mass matrix M, from the basic dominant element M_{33} to the element M_{22} (via a rotation in the 2-3 sub-block of M) and then to the primordially texture zero element M_{11} (via a rotation in the 1-2 sub-block). The direct flavour mixing of the first and third families of quarks and leptons is supposed to be absent or negligibly small in M.

The second alternative (II), on the contrary, presupposes direct flavour mixing of just the first and third families. There is no involvement of the second family in the mixing. In this case, the lightest mass appears in the primordially texture zero M_{11} element "walking round the corner" (via a rotation in the 1-3 sub-block of the mass matrix M). Certainly, this second version of the LFMG mechanism cannot be used for both the up and the down quark families simultaneously, since mixing with the second family members is a basic part of the CKM phenomenology (Cabibbo mixing, non-zero V_{cb} element, CP violation). However, the alternative II could work for the up quark family provided that the down quarks follow the alternative I.

So, there are two possible scenarios for the LFMG mechanism to be considered.

2.1.1 Scenario A: "both of the lightest quark masses m_u and m_d running along the diagonal"

We propose that both mass matrices for the Dirac fermions — the up quarks ($U = u$, c, t) and the down quarks ($D = d$, s, b) — in the Standard Model, or supersymmetric Standard Model, are Hermitian with three texture zeros of the following form:

$$M_i = \begin{pmatrix} 0 & a_i & 0 \\ a_i^* & A_i & b_i \\ 0 & b_i^* & B_i \end{pmatrix} \qquad i = U, \ D \tag{2}$$

3

It is, of course, necessary to assume some hierarchy between the elements, which we take to be: $B_i \gg A_i \sim |b_i| \gg |a_i|$. We derive this hierarchical structure from our $SU(3)_F$ family symmetry model in section 3.3. The zeros in the $(M_i)_{11}$ elements correspond to our, and the commonly accepted, conjecture that the lightest family masses appear as a direct result of flavour mixings. The zeros in $(M_i)_{13}$ mean that only minimal "nearest neighbour" interactions occur, giving a tridiagonal matrix structure.

Now our main hypothesis, that the second and third family diagonal mass matrix elements are practically the same in the gauge and physical quark-lepton bases, means that :

$$B_i = (m_t, \ m_b) + \delta_i \qquad A_i = (m_c, \ m_s) + \delta_i' \tag{3}$$

The components δ_i and δ_i' are supposed to be much less than the masses of the particles in the next lightest family, meaning the second and first families respectively:

$$|\delta_i| \ll (m_c, \ m_s) \qquad |\delta_i'| \ll (m_u, \ m_d) \tag{4}$$

Since the trace and determinant of the Hermitian matrix M_i gives the sum and product of its eigenvalues, it follows that

$$\delta_i \simeq -(m_u, \ m_d) \tag{5}$$

while the δ_i' are vanishingly small and can be neglected in further considerations.

It may easily be shown that our hypothesis and related equations (3 - 5) are entirely equivalent to the condition that the diagonal elements (A_i, B_i), of the mass matrices M_i, are proportional to the modulus square of the off-diagonal elements (a_i, b_i):

$$\frac{A_i}{B_i} = \left| \frac{a_i}{b_i} \right|^2 \qquad i = U, \ D \tag{6}$$

For the moment we leave aside the question of deriving this proportionality condition, Eq. (6), from some underlying theory beyond the Standard Model (see section 3) and proceed to calculate expressions for all the elements of the matrices M_i and the corresponding CKM quark mixing matrix, in terms of the physical masses.

Using the conservation of the trace, determinant and sum of principal minors of the Hermitian matrices M_i under unitary transformations, we are

led to a complete determination of the moduli of all their elements. The results can be expressed to high accuracy as follows:

$$A_i = (m_c, \ m_s), \qquad B_i = (m_t - m_u, \ m_b - m_d), \tag{7}$$

$$|a_i| = (\sqrt{m_u m_c}, \ \sqrt{m_d m_s}) \tag{8}$$

$$|b_i| = (\sqrt{m_u m_t}, \ \sqrt{m_d m_b}) \tag{9}$$

As to the CKM matrix V, we must first choose a parameterisation appropriate to our picture of flavour mixing. Among many possible ones, the original Euler parameterisation recently advocated [1] is most convenient:

$$
\begin{aligned}
V &= \begin{pmatrix} c_U & s_U & 0 \\ -s_U & c_U & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_D & -s_D & 0 \\ s_D & c_D & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} s_U s_D c + c_U c_D e^{-i\phi} & s_U c_D c - c_U s_D e^{-i\phi} & s_U s \\ c_U s_D c - s_U c_D e^{-i\phi} & c_U c_D c + s_U s_D e^{-i\phi} & c_U s \\ -s_D s & -c_D s & c \end{pmatrix}
\end{aligned}
\tag{10}
$$

$$\tag{11}$$

Here $s_{U, \ D} \equiv \sin\theta_{U, \ D}$ and $c_{U, \ D} \equiv \cos\theta_{U, \ D}$ parameterise simple rotations $R_{12}^{U, \ D}$ between the first and second families for the up and down quarks respectively, while $s \equiv \sin\theta$ and $c \equiv \cos\theta$ parameterise a rotation between the second and third families. This representation of V takes into account the observed hierarchical structure of the quark masses and mixing angles. The CP violating phase is connected directly to the first and second families alone.

The quark mass matrices M_U and M_D are diagonalised by unitary transformations which can be written in the form:

$$V_U = R_{12}^U R_{23}^U \Phi^U \qquad V_D = R_{12}^D R_{23}^D \Phi^D \tag{12}$$

where $\Phi^{U, \ D}$ are phase matrices, depending on the phases of the off-diagonal elements $a_i = |a_i| \, e^{i\alpha_i}$ and $b_i = |b_i| \, e^{i\beta_i}$:

$$\Phi^U = \begin{pmatrix} e^{i\alpha_U} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\beta_U} \end{pmatrix} \qquad \Phi^D = \begin{pmatrix} e^{i\alpha_D} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\beta_D} \end{pmatrix} \tag{13}$$

The CKM matrix is defined by

$$V = V_U V_D^\dagger = R_{12}^U R_{23}^U \Phi^U \left(\Phi^D\right)^* \left(R_{23}^D\right)^{-1} \left(R_{12}^D\right)^{-1} \tag{14}$$

5

and, after a suitable re-phasing of the quark fields, we can use the representation

$$R_{23}^U \Phi^U \left(\Phi^D \right)^* \left(R_{23}^D \right)^{-1} = \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \tag{15}$$

The rotation matrices $R_{23}^{U,\ D}$ and $R_{12}^{U,\ D}$ for our mass matrices are readily calculated and the CKM matrix expressed in terms of quark mass ratios

$$s_U = \sqrt{\frac{m_u}{m_c}} \qquad s_D = \sqrt{\frac{m_d}{m_s}} \tag{16}$$

$$s = \left| \sqrt{\frac{m_d}{m_b}} - e^{i\gamma} \sqrt{\frac{m_u}{m_t}} \right| \tag{17}$$

and two phases $\phi = \alpha_D - \alpha_U$ and $\gamma = \beta_D - \beta_U$.

It follows that the Cabibbo mixing is given by the well-known Fritzsch formula [14]

$$|V_{us}| \simeq \left| s_U - s_D e^{-i\phi} \right| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \tag{18}$$

which fits the experimental value well, provided that the CP violating phase ϕ is required to be close to $\frac{\pi}{2}$. So, in the following, we shall assume maximal CP violation in the form $\phi = \frac{\pi}{2}$, as is suggested by spontaneous CP violation in the framework of $SU(3)_F$ family symmetry (see section 3.3). The other phase γ appearing in V_{cb} and V_{ub}

$$|V_{cb}| \simeq s \qquad |V_{ub}| = s_U s \tag{19}$$

can be rather arbitrary, since the contribution $\sqrt{\frac{m_u}{m_t}}$ to s is relatively small, even compared with the uncertainties coming from the light quark masses themselves. This leads to our most interesting prediction (with the mass ratios calculated at the electroweak scale [15]):

$$|V_{cb}| \simeq \sqrt{\frac{m_d}{m_b}} = 0.038 \pm 0.007 \tag{20}$$

in good agreement with the current data $|V_{cb}| = 0.039 \pm 0.003$ [16]. For definiteness we shall assume the phase γ in s, see Eq. (17), to be aligned with the CP violating phase ϕ, again as suggested by $SU(3)_F$ family symmetry,

and take $\gamma = \frac{\pi}{2}$. This has the effect of reducing the uncertainty in our prediction Eq. (20) from 0.007 to 0.004. Another prediction for the ratio:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\frac{m_u}{m_c}} \tag{21}$$

is quite general for models with "nearest-neighbour" mixing [1].

2.1.2 Scenario B: "up quark mass m_u walking around the corner, while down quark mass m_d runs along the diagonal"

Now the mass matrices for the down quarks M_D and charged leptons M_E are supposed to have the same form as in Eq. (2), while the Hermitian mass matrix for the up quarks is taken to be:

$$M_U = \begin{pmatrix} 0 & 0 & c_U \\ 0 & A_U & 0 \\ c_U^* & 0 & B_U \end{pmatrix} \tag{22}$$

All the elements of M_U can again be readily determined in terms of the physical masses as:

$$A_U = m_c \qquad B_U = m_t - m_u \qquad |c_U| = \sqrt{m_u m_t} \tag{23}$$

The quark mass matrices are diagonalised again by unitary transformations as in Eq. (12), provided that the matrix V_U is changed to

$$V_U = R_{13}^U \Phi^U \tag{24}$$

where the 1-3 plane rotation of the u and t quarks and the phase matrix Φ^U (depending on the phase of the element $c_U = |c_U| e^{i\alpha_U}$) are parameterised in the following way:

$$R_{13}^U = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \qquad \Phi^U = \begin{pmatrix} e^{i\alpha_U} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{25}$$

Here $s_{13} \equiv \sin\theta_{13}$ and $c_{13} \equiv \cos\theta_{13}$.

The structure of the CKM matrix now differs from that of Eq. (14) as it contains the direct 1-3 plane rotation for the up quarks:

$$V = V_U V_D^\dagger = R_{13}^U \Phi^U \left(\Phi^D\right)^* \left(R_{23}^D\right)^{-1} \left(R_{12}^D\right)^{-1} \tag{26}$$

7

although the phases and rotations associated with the down quarks are left the same as before. This natural parameterisation is now quite close to the standard one [16]. The proper mixing angles and CP violating phase (after a suitable re-phasing of the c quark, $c \to c e^{-i\beta_D}$) are given by the simple and compact formulae:

$$|V_{us}| \simeq s_{12} = \sqrt{\frac{m_d}{m_s}} \qquad |V_{cb}| \simeq s_{23} = \sqrt{\frac{m_d}{m_b}} \qquad |V_{ub}| \simeq s_{13} = \sqrt{\frac{m_u}{m_t}} \quad (27)$$

and

$$\delta = \alpha_D + \beta_D - \alpha_U \tag{28}$$

While the values of $|V_{us}|$ and $|V_{cb}|$ are practically the same as in scenario A and in good agreement with experiment, a new prediction for $|V_{ub}|$ (not depending on the value of the CP violating phase) should allow experiment to differentiate between the two scenarios in the near future.

2.1.3 The CKM Matrix

Our numerical results for both versions of our model, with a maximal CP violating phase (see discussion in section 3.3.6), are summarized in the following CKM matrix:

$$V_{CKM} = \begin{pmatrix} 0.975(1) & 0.222(4) & 0.0023(5) \ A \\ & & 0.0036(6) \ B \\ 0.222(4) & 0.975(1) & 0.038(4) \\ 0.009(2) & 0.038(4) & 0.999(1) \end{pmatrix} \tag{29}$$

The uncertainties in brackets are largely given by the uncertainties in the quark masses. There is clearly a real and testable difference between scenarios A and B given by the value of the V_{ub} element. Agreement with the experimental values of the already known CKM matrix elements [16] looks quite impressive. The distinctive predictions for the presently relatively poorly known V_{ub} and V_{td} elements[1] should be tested in the near future, by experimental data from the B-factories.

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[1]Present data from CLEO [17] and LEP [18] favour scenario B.

2.2 Lepton Sector

The lepton mixing matrix is defined analogously to the CKM matrix:

$$U = U_\nu U_E{}^\dagger \tag{30}$$

where the indices ν and E stand for $\nu = (\nu_e, \nu_\mu, \nu_\tau)$ and $E = (e, \mu, \tau)$. Our model predicts the charged lepton mixing angles in the matrix U_E with high accuracy to be:

$$\sin\theta_{e\mu} = \sqrt{\frac{m_e}{m_\mu}} \qquad \sin\theta_{\mu\tau} = \sqrt{\frac{m_e}{m_\tau}} \qquad \sin\theta_{e\tau} \simeq 0 \tag{31}$$

provided that the charged lepton masses follow alternative I, along with the down quarks ones, or

$$\sin\theta_{e\mu} = 0 \qquad \sin\theta_{\mu\tau} = 0 \qquad \sin\theta_{e\tau} = \sqrt{\frac{m_e}{m_\tau}} \tag{32}$$

if they follow alternative II, like the up quarks. We consider only alternative I for the charged leptons in our $SU(3)_F$ family symmetry model, since the alternative II for them would lead in general to the same hierarchy between their masses as appears for the up quarks, which is experimentally excluded.

However, in both of cases, these small charged lepton mixing angles will not markedly effect atmospheric neutrino oscillations [19], which appear to require essentially maximal mixing $\sin^2 2\theta_{atm} \simeq 1$. It follows then that the large neutrino mixing responsible for atmospheric neutrino oscillations should mainly come from the U_ν matrix associated with the neutrino mass matrix in (30). This requires a different mass matrix texture for the neutrinos compared to the charged fermions; in this connection a number of interesting models have been suggested [21], using different mechanisms for neutrino mass generation. Remarkably, there appears to be no need in our case for some different mechanism to generate the observed mixing pattern of neutrinos: they can get physical masses and mixings via the usual "see-saw" mechanism [20], using the proposed LFMG mechanism for their primary Dirac and Majorana masses.

Let us consider this pattern in more detail. According to the "see-saw" mechanism the effective mass-matrix M_ν for physical neutrinos has the form [20]

$$M_\nu = -M_N^T M_{NN}^{-1} M_N \tag{33}$$

where M_N is their Dirac mass matrix, while M_{NN} is the Majorana mass matrix of their right-handed components. Therefore, one must first choose which of the two alternatives I or II works for the neutrino mass-matrix (33), particularly for its Dirac and Majorana ingredients, M_N and M_{NN} respectively. There are in fact four possible cases: (i) alternative I works for both the matrices M_N and M_{NN} ; (ii) alternative I works for M_N, while alternative II works for M_{NN} ; (iii) alternative I works for M_{NN} , while alternative II works for M_N ; (iv) alternative II works for both the matrices M_N and M_{NN}. It is easy to confirm that the cases (iii) and (iv) are completely excluded, since they do not lead to any significant $\nu_\mu - \nu_\tau$ mixing as is required by the SuperKamiokande data [19]. However the first two cases, (i) and (ii), happen to be of actual experimental interest when an appropriate hierarchy is proposed between the matrix elements in the matrices M_N and M_{NN}. So, we also have two possible scenarios in the lepton sector.

2.2.1 Scenario A*: "all the lightest lepton Dirac and Majorana masses running along the diagonal"

Let us enlarge first on the case (i) considered recently in ref. [22]. The eigenvalues of the neutrino Dirac mass matrix M_N are taken to have a hierarchy similar to that for the charged leptons (and down quarks)

$$M_{N3} : M_{N2} : M_{N1} \simeq 1 : y^2 : y^4, \quad y \approx 0.1 \tag{34}$$

and the eigenvalues of the Majorana mass matrix M_{NN} are taken to have a stronger hierarchy

$$M_{NN3} : M_{NN2} : M_{NN1} \simeq 1 : y^4 : y^6 \tag{35}$$

One then readily determines the matrices M_N and M_{NN} to be of the type

$$M_N \simeq M_{N3} \begin{pmatrix} 0 & y^3 & 0 \\ y^3 & y^2 & y^2 \\ 0 & y^2 & 1 \end{pmatrix} \tag{36}$$

and

$$M_{NN} \simeq M_{NN3} \begin{pmatrix} 0 & y^5 & 0 \\ y^5 & y^4 & y^3 \\ 0 & y^3 & 1 \end{pmatrix} \qquad (37)$$

Substitution in the seesaw formula (33) generates an effective physical neutrino mass matrix M_ν of the form:

$$M_\nu \simeq -\frac{M_{N3}^2}{M_{NN3}} \begin{pmatrix} 0 & y & 0 \\ y & 1 + (y - y^2)^2 & 1 - (y - y^2) \\ 0 & 1 - (y - y^2) & 1 \end{pmatrix} \qquad (38)$$

The physical neutrino masses are then given with an accuracy of $O(y^2)$ by:

$$\begin{aligned} m_{\nu 1} &\simeq (\frac{1}{2} - \frac{\sqrt{3}}{2})\frac{M_{N3}^2}{M_{NN3}} \cdot y, \\ m_{\nu 2} &\simeq (\frac{1}{2} + \frac{\sqrt{3}}{2})\frac{M_{N3}^2}{M_{NN3}} \cdot y, \quad m_{\nu 3} \simeq (2 - y)\frac{M_{N3}^2}{M_{NN3}} \end{aligned} \qquad (39)$$

Taking the Dirac mass parameter M_{N3} to approximately equal the top quark mass m_t, the Majorana mass parameter can be determined to be $M_{NN3} \simeq 1 \cdot 10^{15}$ GeV from the experimentally observed value of $\Delta m_{atm}^2 \simeq 3 \cdot 10^{-3}$ eV^2 [19] (one may neglect the changes in the lepton masses and mixings coming from running from the top mass scale to the MeV scale).

The mass matrix (38) gives essentially maximal $\nu_\mu - \nu_\tau$ mixing in agreement with the atmospheric neutrino data [19]. Also it is in agreement, while marginal, with the known LMA (large mixing angle MSW) fit to the solar neutrino experiments [23]. The standard two flavour atmospheric and solar neutrino oscillation parameters are expressed in terms of the leptonic CKM matrix U as follows:

$$\begin{aligned} \sin^2 2\theta_{sun} &= 4 |U_{e1}|^2 |U_{e2}|^2, \quad \sin^2 2\theta_{atm} = 4 |U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \quad (40) \\ \Delta m_{sun}^2 &= m_{\nu 2}^2 - m_{\nu 1}^2, \quad \Delta m_{atm}^2 = m_{\nu 3}^2 - m_{\nu 2}^2 \end{aligned}$$

assuming the approximate decoupling condition $/U_{e3}/^2 \ll 1$ (see below). The predicted values of these parameters in scenario A* are [22]:

$$\sin^2 2\theta_{atm} \simeq 1, \quad \sin^2 2\theta_{sun} \simeq \frac{2}{3}, \quad U_{e3} \simeq \frac{1}{2\sqrt{2}}y, \quad \frac{\Delta m_{sun}^2}{\Delta m_{atm}^2} \simeq \frac{\sqrt{3}}{4}y^2 \quad (41)$$

to be well compared with the experimentally allowed regions [19, 23] corresponding to the LMA solution for the solar neutrino problem

$$
\begin{aligned}
0.82 &\leq \sin^2 2\theta_{atm} \leq 1, \quad 0.65 \leq \sin^2 2\theta_{sun} \leq 1 \\
|U_{e3}|^2 &\leq 0.05, \quad 3 \cdot 10^{-3} \leq \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}} \leq 5 \cdot 10^{-2}
\end{aligned}
\tag{42}
$$

2.2.2 Scenario B*: "the lightest Majorana mass walking around the corner, while the lightest Dirac masses run along the diagonal"

We now turn to case (ii), which appears to lead to another case of neutrino oscillations where together with essentially maximal mixing of atmospheric neutrinos small mixing angle solution MSW (SMA) naturally appears for solar neutrinos. We again take the alternative I form (36) for the Dirac mass matrix M_N, while for the Majorana mass matrix M_{NN} we take the alternative II form

$$
M_{NN} \simeq M_{NN3} \begin{pmatrix} 0 & 0 & y^q \\ 0 & y^p & 0 \\ y^q & 0 & 1 \end{pmatrix}
\tag{43}
$$

with an arbitrary eigenvalue hierarchy of the type

$$
M_{NN3} : M_{NN2} : M_{NN1} \simeq 1 : y^p : y^{2q}
\tag{44}
$$

where p and q are positive integers. The seesaw formula (33) then generates an effective physical neutrino mass matrix M_ν which for any $p \geq 2q-1$, $q > 1$ has a particularly simple form (inessential higher order terms are omitted)

$$
M_\nu \simeq -\frac{M^2_{N3}}{M_{NN3}} y^{4-p} \begin{pmatrix} y^2 & y & y \\ y & 1 - y^{p-2q+2} & 1 + y^{p-q-1} \\ y & 1 + y^{p-q-1} & 1 \end{pmatrix}
\tag{45}
$$

One can see that the matrix (45) automatically leads to the large (maximal) $\nu_\mu - \nu_\tau$ mixing and small $\nu_e - \nu_\mu$ mixing for any hierarchy in the Majorana mass matrix (43) satisfying the condition $p \geq 2q-1$, $q > 1$ mentioned above. Taking, for an example, $p = 5$ and $q = 3$ one can find for neutrino masses the values

12

$$m_{\nu 1} \simeq \frac{M_{N3}^2}{M_{NN3}} \cdot \frac{y^2}{3}, \qquad m_{\nu 2} \simeq -\frac{3}{2}\frac{M_{N3}^2}{M_{NN3}}, \qquad m_{\nu 3} \simeq 2\frac{M_{N3}^2}{M_{NN3}} \cdot y^{-1} \quad (46)$$

while the following predictions for the two flavour oscillation parameters naturally appears

$$\sin^2 2\theta_{atm} \simeq 1, \quad \sin^2 2\theta_{sun} \simeq \frac{2}{9}y^2, \quad U_{e3} \simeq \frac{1}{\sqrt{2}}y, \quad \frac{\Delta m_{sun}^2}{\Delta m_{atm}^2} \simeq \frac{9}{16}y^2 \quad (47)$$

Again taking $y \approx 0.1$, our predictions (47) turn out to be inside of the experimentally allowed intervals [23] for the SMA solution for solar neutrino oscillations:

$$
\begin{aligned}
0.82 &\leq \sin^2 2\theta_{atm} \leq 1, \quad 10^{-3} \leq \sin^2 2\theta_{sun} \leq 10^{-2} \\
|U_{e3}|^2 &\leq 0.05, \quad 5 \cdot 10^{-4} \leq \frac{\Delta m_{sun}^2}{\Delta m_{atm}^2} \leq 9 \cdot 10^{-3}
\end{aligned}
\quad (48)
$$

Note that in contrast to the LMA case (41), one must include now even the small contributions stemming from the charged lepton sector (see Eq. (31) into the solar neutrino oscillations.

So, one can see that the LFMG mechanism works quite successfully in the lepton sector as well as in the quark sector. Remarkably, the same mechanism results simultaneously in small quark mixing and large lepton mixing (in scenario B* for the second and third families). On the other hand, there could be different sources of the large neutrino mixing. An attractive alternative method for generating large neutrino mixing within the supersymmetric Standard Model is via R-parity violating interactions, which can give radiatively induced neutrino masses and mixing angles in just the area required by the current observational data [24, 25].

3 The SU(3)$_F$ Theory of Flavour Mixing

3.1 Motivation

The choice of a local chiral $SU(3)_F$ symmetry [8] as the underlying flavour symmetry beyond the Standard Model is based on the following three "pillars":

(i) It provides a natural explanation of the number three of observed quark-lepton families, correlated with three species of massless or light ($m_\nu < M_Z/2$) neutrinos contributing to the invisible Z boson partial decay width [16];

(ii) Its local nature could be expected by analogy with the other fundamental symmetries of the Standard Model, such as weak isospin symmetry $SU(2)_W$ or colour symmetry $SU(3)_C$. A stronger motivation follows from superstrings (at least it is one of the most important model-independent results derived from perturbative string theory); there are no global non-abelian symmetries in the 4D string models (for a review see [26]);

(iii) Any family symmetry should be completely broken at low energies in order to conform with reality. This symmetry should be chiral, rather than a vectorlike one under which left-handed and right-handed fermions have the same group-theoretical features. A vectorlike symmetry would not provide any mass protection and would give degenerate rather than hierarchical quark and lepton family mass spectra. Interestingly, both known examples of local vectorlike symmetries, electromagnetic $U(1)_{EM}$ and colour $SU(3)_C$, appear to be exact symmetries. Also, in standard GUT models fermions and antifermions can lie in the same irreducible representation; so a $GUT \otimes SU(3)_H$ model necessarily has a chiral family symmetry.

So, if one takes these naturality criteria seriously, all the candidates for flavour symmetry can be excluded except for local chiral $SU(3)_F$ symmetry. The chiral $U(1)$ symmetries [2, 3, 4] do not satisfy the criterion (i) and are in fact applicable to any number of quark-lepton families. Also, the chiral $SU(2)$ symmetry can contain, besides two light families treated as its doublets, any number of additional (singlets or new doublets of $SU(2)$) families. All the global non-abelian symmetries are excluded by criterion (ii), while the vectorlike ones are excluded by the last criterion (iii).

Among the applications of the $SU(3)_F$ symmetry, the most interesting ones are the description of the quark and lepton masses and mixings in the Standard Model and GUTs [8], neutrino masses and oscillations [27] and rare processes [28]. Remarkably, the $SU(3)_F$ invariant Yukawa coupling are

always accompanied by an accidental global chiral $U(1)$ symmetry [28, 29], which can be identified with the Peccei-Quinn symmetry [30] provided it is not explicitly broken in the Higgs sector, thus giving a solution to the strong CP problem. In the SUSY context [31], the $SU(3)_F$ model leads to a special relation between fermion masses and the soft SUSY breaking terms at the GUT scale, so that all the dangerous supersymmetric flavour-changing processes are naturally suppressed [32].

Another sector of applications is related with a new type of topological defect—non-abelian cosmic strings appearing during the spontaneous breaking of the $SU(3)_F$—considered as a possible candidate for the cold dark matter in the Universe [33]. And the last point worthy of note is that the local chiral $SU(3)_F$ symmetry has been applied to GUTs not only in a direct product form, such as $SU(5) \otimes SU(3)_F$ [8], $SO(10) \otimes SU(3)_F$ [34] or $E(6) \otimes SU(3)_F$ [34], but also as a subgroup of the family unified $SU(8)$ GUT model [35].

3.2   The Matter and Higgs Supermultiplets

In the MSSM extended by the local chiral $SU(3)_F$ symmetry the quark and lepton superfields are supposed to be $SU(3)_F$ chiral triplets, so that their left-handed (weak-doublet) components

$$L_{\alpha f} = \left[\begin{pmatrix} U_L \\ D_L \end{pmatrix}_\alpha, \begin{pmatrix} N_L \\ E_L \end{pmatrix}_\alpha \right] = \tag{49}$$

$$= \left[\left(\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \right), \left(\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right) \right]$$

are triplets of $SU(3)_F$, while their right-handed (weak-singlet) components

$$\begin{aligned} R_{uf}^\alpha &= [U_R^\alpha, N_R^\alpha] = [(u, c, t)_R, (N_e, N_\mu, N_\tau)_R] \\ R_{df}^\alpha &= [D_R^\alpha, E_R^\alpha] = [(d, s, b)_R, (e, \mu, \tau)_R] \end{aligned} \tag{50}$$

are anti-triplets (or vice versa). For completeness we have included the right-handed neutrino superfields N_R^α as well. Here α is a family symmetry index ($\alpha = 1, 2, 3$), while the index f simply stands for the type of basic fermion, quark ($f = q$) or lepton ($f = l$).

The matter sector also includes some set of right-handed states, which are needed to cancel all the $SU(3)_F$ triangle anomalies. While being singlets under the Standard Model $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ gauge symmetry, these states can be chosen as sixteen right-handed triplets r_α^n ($n = 1, .., 16$) or some other combination of $SU(3)_F$ multiplets properly compensating the triangle anomalies. All of them receive heavy masses of order the family symmetry breaking scale M_F and are practically decoupled from the low-energy particle spectrum. In fact, they look like right-handed heavy neutrinos and, as such, can contribute to the ordinary light neutrino masses and mixings via the "see-saw" mechanism [20] (see below).

In addition to the standard weak-doublet scalar supermultiplets H and $\overline{H}$ of the MSSM, the Higgs sector contains the $SU(3)_F$ symmetry-breaking horizontal scalars. These scalars are anti-triplets $\overline{\eta}^i$ and sextets χ^j of $SU(3)_F$ (indices i and j number the scalar multiplets, $i = 1, 2, ..., j = 1, 2, ...$), which transform under $SU(3)_F$ like matter bi-linears so as to give fermion masses through the symmetry-allowed general effective Yukawa couplings:

$$
\begin{aligned}
W_Y \;=\; & \overline{L}_f^\alpha R_{uf}^\beta H_u [A_{uf}^i \cdot \frac{\chi_{\{\alpha\beta\}}^i}{M_F} + B_{uf}^i \cdot \frac{\overline{\eta}_{[\alpha\beta]}^i}{M_F}] + \\
& + \overline{L}_f^\alpha R_{df}^\beta \overline{H}_d [A_{df}^i \cdot \frac{\chi_{\{\alpha\beta\}}^i}{M_F} + B_{df}^i \cdot \frac{\overline{\eta}_{[\alpha\beta]}^i}{M_F}] + \\
& + N_R^\alpha N_R^\beta [A_{NN}^i \cdot \chi_{\{\alpha\beta\}}^i]
\end{aligned}
\tag{51}
$$

for the up and down fermions, quarks ($f = q$) and leptons ($f = l$). Also Majorana couplings are included for the right-handed neutrinos, where only the symmetrical parts are allowed to appear. The constants $A_{uf,df,N}^i$ and $B_{uf,df,N}^i$ ($A_{uq}^i \equiv A_U^i$, $A_{ul}^i \equiv A_N^i$ and so on) are the dimensionless Yukawa constants associated with the symmetric and anti-symmetric couplings in family space respectively (we have here used the anti-symmetric feature of triplets in $SU(3)$: $\overline{\eta}_{[\alpha\beta]} \equiv \epsilon_{\alpha\beta\gamma}\overline{\eta}^\gamma$). These couplings normally appear via the well-known see-saw type mechanism [2, 8] due to the exchange of a special set of heavy (of order the flavour scale M_F) vectorlike fermions or directly through the gravitational interactions, being suppressed in the latter case by inverse powers of the Planck scale M_{Pl} [36]. One can even make the Yukawa couplings (51) renormalizable, by introducing additional scalars with both electroweak and horizontal indices. The VEVs of the horizontal scalars $\overline{\eta}^i$

16

and χ^i, taken in general as large as M_F (or M_{Pl}), are supposed to be hierarchically arranged along the different directions in family space, so as to properly imitate the observed quark and lepton mass and mixing hierarchies even with coupling constants $A^i_{uf,df,N}$ and $B^i_{uf,df,N}$ taken of the order $O(1)$. The Majorana couplings of the right-handed neutrinos N^α_R do not, of course, involve the Weinberg-Salam Higgs field H_u and are thus naturally of order $O(M_F)$. As was mentioned above, the Yukawa couplings (51) admit an extra global chiral symmetry $U(1)$ identified with the Peccei-Quinn symmetry[2] $U(1)_{PQ}$ [30]. However, in general, one must also consider the Higgs superpotential (see below), in order to determine whether there exists global $U(1)$ or discrete Z_N symmetry(ies) in the model.

There is a more economic way of generating effective Yukawa couplings, by using just horizontal triplets η^i_α and anti-triplets $\overline{\eta}^i_{[\alpha\beta]}$ and imitating sextets by the pairing of triplet scalars. Instead of the couplings (51), in this case [28] one has:

$$
\begin{aligned}
W_Y &= \overline{L}^\alpha_f R^\beta_{uf} H_u [A^{ij}_{uf} \cdot \boldsymbol{\eta}^i_\alpha \boldsymbol{\eta}^j_\beta \mathbf{I}^{k^{ij}_{uf}} + B^i_{uf} \cdot \overline{\boldsymbol{\eta}}^i_{[\alpha\beta]} \mathbf{I}^{n^i_{uf}}] + \\
&\quad + \overline{L}^\alpha_f R^\beta_{df} \overline{H}_d [A^{ij}_{df} \cdot \boldsymbol{\eta}^i_\alpha \boldsymbol{\eta}^j_\beta \mathbf{I}^{k^{ij}_{df}} + B^i_{df} \cdot \overline{\boldsymbol{\eta}}^i_{[\alpha\beta]} \mathbf{I}^{n^i_{df}}] + \\
&\quad + N^\alpha_R N^\beta_R [A^{ij}_{NN} \cdot \boldsymbol{\eta}^i_\alpha \boldsymbol{\eta}^j_\beta \mathbf{I}^{k^{ij}_{NN}}]
\end{aligned}
\tag{52}
$$

where all the horizontal "bold" scalar fields $\boldsymbol{\eta}^i$ are taken now to be scaled by the $SU(3)_F$ symmetry breaking scale M_F, $\boldsymbol{\eta}^i_\alpha \equiv \eta^i_\alpha/M_F$. The hierarchy in the corresponding mass matrices (including the up-down hierarchy) depends solely on the powers $k^{ij}_{u,d,N}$ and $n^i_{u,d,N}$ of the specially introduced hierarchy-making singlet scalar field $\mathbf{I}$ ($\mathbf{I} \equiv I/M_F$) in (52). This description turns out to be well fitted to the supersymmetric case where both types of scalar, η^i_α and its SUSY counter-part $\overline{\eta}^{i\alpha} \equiv \epsilon^{\alpha\beta\gamma} \overline{\eta}^i_{[\beta\gamma]}$, appear automatically.

The basic Yukawa couplings (51) (or (52)) can lead to various types of fermion mass matrices, depending on the vacuum configurations developed by the horizontal triplets η^i, anti-triplets $\overline{\eta}^i$ and sextets χ^i involved. Remarkably, a texture-zero structure emerges as one of the generic features of a typical $SU(3)_F$ symmetry breaking pattern. This is controlled by the antisymmetric (anti-triplet) VEVs $< \overline{\eta}^i >$ giving off-diagonal elements in the mass matrices,

_____

[2]Even in the non-supersymmetric Standard Model which contains only one Higgs doublet this symmetry can be successfully implemented [8, 29], by assigning the $U(1)_{PQ}$ charges directly to the horizontal scalars η^i and χ^i.

while the symmetric (sextet) ones $< \chi^i >$ generate the dominant diagonal elements. Therefore, any of the texture-zero models catalogued in literature (see[10]) are readily available in the framework of the $SU(3)_F$ model.

For example, the original Fritzsch ansatz [14] with the zero mass-matrix elements, $M_{11} = M_{22} = M_{13} = 0$, while presently excluded by experiment, appears from a basic vacuum configuration of the Higgs potential [8] (or superpotential [31]) with a simple set of horizontal scalars—two anti-triplets $\overline{\eta}_{[\alpha\beta]}$ and $\overline{\xi}_{[\alpha\beta]}$ and one sextet $\chi_{\{\alpha\beta\}}$ developing VEVs along the following directions in family space[3]:

$$\overline{\eta}_{[12]} = -\overline{\eta}_{[21]}, \quad \overline{\xi}_{[23]} = -\overline{\xi}_{[32]}, \quad \chi_{\{33\}} \tag{53}$$

The "improved" Fritzsch ansatz (see [1]) with a non-zero M_{22} element appears, when a second sextet $\omega_{\{\alpha\beta\}}$ is included[4], which develops a VEV $\omega_{\{22\}}$. Unfortunately, this ansatz is not completely predictive of the CKM matrix in terms of quark mass ratios (which would require no more than three independent parameters for each of the mass matrices). In particular it gives no possibility of predicting the V_{cb} element in the CKM matrix.

As matter of fact the only ansatz left, which successfully predicts all the CKM matrix elements in terms of quark mass ratios (not to mention here some eclectic approaches using ad hoc GUT and string arguments, which are already largely excluded by experiment) is that suggested by the LFMG mechanism. In the next subsection, we show how the LFMG can be derived within the $SU(3)_F$ model.

3.3 Minimal flavour mixing from family symmetry

3.3.1 Basic vacuum configurations

We start with an essential question: how does one get the proportionality condition (1) underlying the LFMG in the framework of the $SU(3)_F$ model? In terms of the Yukawa superpotential (51), this condition (1) means that

---

[3]Interestingly, the VEVs χ_{33} and $\overline{\eta}_{[12]}$ in the coherent basis correspond to the flavour democracy picture[1] in the mass-matrices of quarks and leptons with symmetrical and antisymmetrical "pairing forces" respectively, while the VEV $\xi_{[23]}$ breaks this picture properly.

[4]Both of these ansätze, original and "improved", readily appear in a pure triplet realization (52) as well, with three and four scalar triplets respectively.

not only the VEVs of the horizontal supermultiplets, sextets $\chi^i_{\{\alpha\beta\}}$ and anti-triplets $\overline{\eta}^i_{[\alpha\beta]}$ of $SU(3)_F$, but also their Yukawa coupling constants $A^i_{u,d,N}$ and $B^i_{u,d,N}$ must be properly arranged. Since the coupling constants are in general quite independent even in the framework of $SU(3)_F$, this in turn means that the number of horizontal supermultiplets involved in the Yukawa couplings should be restricted. At the same time, some of these scalars have to develop VEVs along several directions in the family space in order to satisfy the proportionality condition (1).

We take the set of scalar supermultiplets in the model to consist of three pairs of triplets and anti-triplets and two pairs of sextets and anti-sextets of $SU(3)_F$:

$$\eta_\alpha(\overline{\eta}^\alpha), \quad \xi_\alpha(\overline{\xi}^\alpha), \quad \zeta_\alpha(\overline{\zeta}^\alpha);$$
$$\chi_{\{\alpha\beta\}}(\overline{\chi}^{\{\alpha\beta\}}), \quad \omega_{\{\alpha\beta\}}(\overline{\omega}^{\{\alpha\beta\}}) \tag{54}$$

We will see later that just the triplet scalars determine masses and mixing quarks and leptons, while the sextets govern the basic vacuum configuration in the family space leading to the LFMG picture as such for flavour mixing.

We take the Higgs superpotential to have the form

$$
\begin{aligned}
W \quad = \quad & M_\eta \eta\overline{\eta} + M_\xi \xi\overline{\xi} + M_\zeta \zeta\overline{\zeta} + M_\chi \chi\overline{\chi} + M_\omega \omega\overline{\omega} + \\
& + f \cdot \epsilon^{\alpha\beta\gamma} \eta_\alpha \xi_\beta \zeta_\gamma + \overline{f} \cdot \epsilon_{\alpha\beta\gamma} \overline{\eta}^\alpha \overline{\xi}^\beta \overline{\zeta}^\gamma \\
& + F \cdot \epsilon^{\alpha\beta\gamma} \epsilon^{\alpha'\beta'\gamma'} \chi_{\{\alpha\alpha'\}} \chi_{\{\beta\beta'\}} \omega_{\{\gamma\gamma'\}} + \overline{F} \cdot \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'} \overline{\chi}^{\{\alpha\alpha'\}} \overline{\chi}^{\{\beta\beta'\}} \overline{\omega}^{\{\gamma\gamma'\}}
\end{aligned}
\tag{55}
$$

with masses $M_{\eta,\xi,\zeta}$ and $M_{\chi,\omega}$ and coupling constants f, $\overline{f}$, F and $\overline{F}$. One can readily notice that all the trilinear couplings of triplets and sextets in the superpotential W are very specific just to the $SU(3)$ symmetry. These couplings by themselves induce, as we will see later, strictly orthogonal (to each other) VEVs for the triplets and sextets involved. However, as was argued above, some of these scalars also have to develop hierarchically small parallel VEVs in the family space, so as to get the proportionality condition (1) satisfied. For this purpose we include in the superpotential W the additional terms

$$\Delta W_1 = a \cdot \left(\frac{S}{M_F}\right)^r M_F \overline{\eta}\zeta \tag{56}$$

19

with a particular coupling of triplets $\overline{\eta}$ and ζ which, through their mixing, will cause some extra small parallel VEVs for them whose magnitudes will be determined by the power r of the VEV of the singlet scalar S. This hierarchy-making scalar S also appears in the Yukawa couplings (see below). We emphasize here that, besides the $SU(3)_F$ symmetry, the superpotential W has a global $U(1)$ symmetry. This global $U(1)$ is in substance the custodial symmetry strictly protecting the form of the superpotential W (55) considered[5].

Let us come now to a general analysis of the non-trivial supersymmetric vacuum configurations of the superpotential W. One can quickly find from the vanishing F-terms of the triplet supermultiplets involved,

$$
\begin{aligned}
M_\eta \eta_\alpha + \overline{f} \cdot \epsilon_{\alpha\beta\gamma} \overline{\xi}^\beta \overline{\zeta}^\gamma + a \cdot \left(\frac{S}{M_F} \right)^r M_F \zeta_\alpha &= 0, \quad M_\eta \overline{\eta}^\alpha + f \cdot \epsilon^{\alpha\beta\gamma} \xi_\beta \zeta_\gamma = 0, \\
M_\xi \xi_\beta + \overline{f} \cdot \epsilon_{\alpha\beta\gamma} \overline{\eta}^\alpha \overline{\zeta}^\gamma &= 0, \quad M_\xi \overline{\xi}^\beta + f \cdot \epsilon^{\alpha\beta\gamma} \eta_\alpha \zeta_\gamma = 0, \\
M_\zeta \zeta_\gamma + \overline{f} \cdot \epsilon_{\alpha\beta\gamma} \overline{\eta}^\alpha \overline{\xi}^\beta &= 0, \\
M_\zeta \overline{\zeta}^\gamma + f \cdot \epsilon^{\alpha\beta\gamma} \eta_\alpha \xi_\beta + a \cdot \left(\frac{S}{M_F} \right)^r M_F \overline{\eta}^\gamma &= 0,
\end{aligned}
\tag{57}
$$

that they develop VEVs satisfying the equations

$$
\begin{aligned}
\eta_\alpha \overline{\eta}^\alpha &= \frac{M_\xi M_\zeta}{f\overline{f}}, \quad \xi_\alpha \overline{\xi}^\alpha = \frac{M_\eta M_\zeta}{f\overline{f}}, \quad \zeta_\alpha \overline{\zeta}^\alpha = \frac{M_\eta M_\xi}{f\overline{f}} \\
\epsilon^{\alpha\beta\gamma} \eta_\alpha \xi_\beta \zeta_\gamma &= -\frac{M_\eta M_\xi M_\zeta}{f\overline{f}^2}, \quad \epsilon_{\alpha\beta\gamma} \overline{\eta}^\alpha \overline{\xi}^\beta \overline{\zeta}^\gamma = -\frac{M_\eta M_\xi M_\zeta}{\overline{f}^2 f}
\end{aligned}
\tag{58}
$$

All but one of the scalar products between pairs of triplets vanish

$$
\begin{aligned}
\eta_\alpha \overline{\xi}^\alpha &= \xi_\alpha \overline{\eta}^\alpha = \xi_\alpha \overline{\zeta}^\alpha = \zeta_\alpha \overline{\eta}^\alpha = \zeta_\alpha \overline{\xi}^\alpha = 0 \\
\eta_\alpha \overline{\zeta}^\alpha &= -\frac{a}{f\overline{f}} \cdot \left(\frac{S}{M_F} \right)^r M_F M_\xi
\end{aligned}
\tag{59}
$$

<hr />

[5]Let us note that this $U(1)$ may be identified with a superstring-inherited anomalous $U(1)_A$ gauge symmetry [37] broken at a high scale through the Fayet-Iliopoulos (FI) D-term [38]. One can take just this scale as the family symmetry scale M_F. While for the heterotic strings this scale normally lies only a few orders of magnitude below the Planck mass M_{Pl}, in the orientifold case, where the FI term appears moduli-dependent [39], it can be made to locate much lower.

and their non-zero components can be chosen as follows:

$$\begin{aligned}
\eta_\alpha &= (X,0,0), & \overline{\eta}^\alpha &= (\overline{X},0,\overline{x}), \\
\xi_\alpha &= (0,Y,0), & \overline{\xi}^\alpha &= (0,\overline{Y},0), \\
\zeta_\alpha &= (z,0,Z), & \overline{\zeta}^\alpha &= (\overline{z},0,\overline{Z}).
\end{aligned} \tag{60}$$

The components $\overline{x}$, z and $\overline{z}$ which appear in them are small and just arise due to the $\overline{\eta}\zeta$ mixing term in the superpotential W mentioned above. We propose here that the VEV of the singlet scalar S responsible for this mixing is typically somewhat smaller than the basic VEVs of the triplets (58), so as to have some input hierarchy parameter $\frac{S}{M_F}$ in the model (see below).

The vanishing of the D-terms

$$(\Phi, T^A\Phi) = 0, \tag{61}$$

where all the scalars fields (triplets and sextets) are grouped in the vector Φ (T^A are generators of $SU(3)_F$ acting on the reducible representation given by the vector Φ), results in further limitations on the possible supersymmetric vacuum configurations. Particularly, for the generator T^{4+i5}, equation (61) leads to an additional non-trivial condition on the VEVs of the triplets $\eta_\alpha(\overline{\eta}^\alpha)$ and $\zeta_\alpha(\overline{\zeta}^\alpha)$

$$-\overline{X}^*\overline{x} + Z^*z - \overline{Z}^*\overline{z} = 0, \tag{62}$$

When this condition is combined with the basic F-term equations (57), the values of the small VEV components $\overline{x}$, z and $\overline{z}$ can be expressed in terms of the VEV of the scalar singlet S (the bold letter $\mathbf{S}$ stands for S/M_F) and the ratios of the big VEV components as follows:

$$\begin{aligned}
\overline{x} &= -a\mathbf{S}^r M_F \frac{\overline{X}}{M_\zeta} \frac{Y}{M_\eta} \frac{\overline{Z}^*}{Z^*} \frac{1}{1+\mid f\frac{Y}{M_\eta}\mid^2}, \\
z &= -a\mathbf{S}^r M_F \frac{\overline{X}}{M_\zeta} \frac{\overline{Z}^*}{Z^*} \frac{1}{1+\mid f\frac{Y}{M_\eta}\mid^2}, \\
\overline{z} &= -a\mathbf{S}^r M_F \frac{\overline{X}}{M_\zeta}
\end{aligned} \tag{63}$$

The VEVs slide by themselves into the valleys determined by the F- and D-term conditions (57,61) conserving supersymmetry.

Let us turn now to the sextet superfields in the superpotential W (55). From their vanishing F-term equations

$$M_\chi \chi_{\{\alpha\beta\}} + \overline{F}\epsilon_{\alpha\gamma\sigma}\epsilon_{\beta\delta\rho}\overline{\omega}^{\{\gamma\delta\}}\overline{\chi}^{\{\sigma\rho\}} = 0, \quad M_\chi \overline{\chi}^{\{\alpha\beta\}} + F\epsilon^{\alpha\gamma\sigma}\epsilon^{\beta\delta\rho}\omega_{\{\gamma\delta\}}\chi_{\{\sigma\rho\}} = 0,$$
$$M_\omega \omega_{\{\alpha\beta\}} + \overline{F}\epsilon_{\alpha\gamma\sigma}\epsilon_{\beta\delta\rho}\overline{\chi}^{\{\gamma\delta\}}\overline{\chi}^{\{\sigma\rho\}} = 0, \quad M_\chi \overline{\omega}^{\{\alpha\beta\}} + F\epsilon^{\alpha\gamma\sigma}\epsilon^{\beta\delta\rho}\chi_{\{\gamma\delta\}}\chi_{\{\sigma\rho\}} = 0,$$

$$(64)$$

and the corresponding D-term equation (61), one can confirm that the sextets develop strictly orthogonal vacuum configurations of the type:

$$\chi_{\{\alpha\beta\}} = \chi \cdot diag(1,1,0)_{\alpha\beta}, \quad \overline{\chi}^{\{\alpha\beta\}} = \overline{\chi} \cdot diag(1,1,0)^{\alpha\beta} \tag{65}$$
$$\omega_{\{\alpha\beta\}} = \omega \cdot diag(0,0,1)_{\alpha\beta}, \quad \overline{\omega}^{\{\alpha\beta\}} = \overline{\omega} \cdot diag(0,0,1)^{\alpha\beta}$$

In order to show this explicitly, one must first rotate away all the components of the sextet $\chi_{\{\alpha\beta\}}$, except for χ_{11} and χ_{22}, by the appropriate $SU(3)_F$ transformations. Then, successively using the F-term equations (64), one can see that the sextet $\omega_{\{\alpha\beta\}}$ and the anti-sextet $\overline{\omega}^{\{\alpha\beta\}}$ inescapably develop the VEVs given in equation (65) with

$$\omega\overline{\omega} = \frac{M_\chi^2}{\overline{F}F} \tag{66}$$

while the anti-sextet $\overline{\chi}^{\{\alpha\beta\}}$ also develops VEVs on the same components, (11) and (22), as the sextet $\chi_{\{\alpha\beta\}}$ does. Furthermore, a relation appears between them of the form

$$\chi_{11}\overline{\chi}^{11} = \chi_{22}\overline{\chi}^{22} = \frac{M_\chi M_\omega}{\overline{F}F} \tag{67}$$

Finally, using the D-term equation (61) with generators T^3 and T^8 and assuming that the $\chi_{\alpha\beta}$ and $\overline{\chi}^{\alpha\beta}$ VEVs are large compared with all the other scalar VEVs contributing to (61), one unavoidably comes to the equalities

$$|\chi_{11}| = |\overline{\chi}^{11}| = |\chi_{22}| = |\overline{\chi}^{22}| \tag{68}$$

Thus we are led to the VEVs for the scalars $\chi_{\alpha\beta}$ and $\overline{\chi}^{\alpha\beta}$ of the form given in equation (65) with $|\chi| = |\overline{\chi}|$. These VEVs by themselves break

the family $SU(3)_F$ symmetry to the plane $SO(2)_F$ symmetry, acting in the family subspace of the first and second families of quarks and leptons. We will see later that just this vacuum configuration turns out to be essential for generating Yukawa couplings, which follow our minimal mixing pattern for quarks and leptons. As mentioned above, we assume the VEVs of the sextets $\chi_{\alpha\beta}$ and $\overline{\chi}^{\alpha\beta}$ are the largest in the model, so that this vacuum configuration of the sextets is not disturbed by the VEVs of the triplets. However, it should be noted that this assumption in no way influences the hierarchy in masses of the quarks and leptons, which is completely determined by the triplet scalar VEVs given above. The relative alignment assigned to the VEVs of the triplet (60) and the sextet (65) fields is arranged by introducing a coupling between the triplet and sextet fields in the superpotential W of the form

$$\Delta W_2 = b \cdot \omega \overline{\zeta} \zeta \left(\frac{S}{M_F} \right)^p \tag{69}$$

so as not to disturb the above vacuum configurations. This term also excludes an accidental global symmetry $U(3)_{triplets} \otimes U(3)_{sextets}$ in the superpotential W which might, otherwise, induce the appearance of extra goldstones (familons) after symmetry breaking. Finally, one has the total symmetry $SU(3)_F \otimes U(1)$ for the superpotential $W + \Delta W_1 + \Delta W_2$ considered.

3.3.2 Yukawa couplings

One can write down a plethora of $SU(3)_F$ invariant Yukawa couplings. However, by the use of the additional protecting symmetry $U(1)$, one can choose them in a special form which leads to (at least) one of the four experimentally allowed scenarios for the minimal flavour mixing of quarks and leptons. These four scenarios consist of the following four pairs of combined quark and lepton flavour mixings (see section 2): A+A*, A+B*, B+A* and/or B+B*.

Since we have all the triplet superfields needed in our chosen set (54), the most economic choice of Yukawa couplings would be the pure triplet realization of the form (52) discussed in section 3.2. However it does not lead to the desired strict proportionality condition (1) or (6) between mass-matrix elements of the basic (alternative I) LFMG. So the pure triplet Yukawa couplings can only be used for the up quarks in scenario B (with its direct 1-3 mixing) for quark mixing. The up quark Yukawa couplings then take the

23

form[6]:

$$W_{Y1} = \overline{U}_L^\alpha U_R^\beta H_u [A_{U1} \cdot \boldsymbol{\zeta}_\alpha \boldsymbol{\zeta}_\beta + A_{U2} \cdot \boldsymbol{\xi}_\alpha \boldsymbol{\xi}_\beta \mathbf{I}^k + B_U \cdot \overline{\boldsymbol{\xi}}_{[\alpha\beta]} \mathbf{I}^n] \qquad (70)$$

which contains two symmetrical couplings related with the bi-linears for the triplet scalars $\boldsymbol{\zeta}_\alpha$ and $\boldsymbol{\xi}_\alpha$ and one antisymmetric coupling related with the anti-triplet $\overline{\boldsymbol{\xi}}_{[\alpha\beta]}$ from our scalar set (54). We recall here that the "bold" horizontal scalar fields $\boldsymbol{\zeta}$, $\boldsymbol{\xi}$, $\overline{\boldsymbol{\xi}}$ and $\mathbf{I}$ are scaled by the $SU(3)_F$ symmetry mass scale parameter M_F (which can generally be as large as the Plank mass M_{Pl}), so that the coupling constants $A_{U,NN}$ and $B_{U,NN}$ are all dimensionless and assumed to be of order unity. The hierarchy in the corresponding mass matrix of the up quarks depends, as in the general case (52), solely on the powers k and n of the hierarchy-making singlet scalar field $\mathbf{I}$ whose values are determined by the $U(1)$ symmetry imposed. According to the triplet VEVs (60), the first symmetrical coupling in W_{Y1} giving masses to the heaviest third family are not suppressed, while all other couplings (including the direct 1-3 mixing term) are suppressed by powers of the scalar $\mathbf{I}$. As in the general case (52), by introducing additional scalars with electroweak and horizontal indices combined one could make the Yukawa couplings (70) renormalizable.

The other possible Yukawa couplings, which can be written with the triplets (54), are those having the form

$$\begin{aligned}
W_{Y2} &= \overline{L}_f^\alpha R_{uf}^\beta H_u [A_{uf} \cdot \overline{\boldsymbol{\eta}}_{[\alpha\sigma]} \overline{\boldsymbol{\eta}}_{[\beta\rho]} + B_{uf} \cdot \epsilon_{\alpha\beta\sigma} \boldsymbol{\eta}_\rho \mathbf{S}^l] \overline{\boldsymbol{\chi}}^{\{\sigma\rho\}} + \qquad (71) \\
&+ \overline{L}_f^\alpha R_{df}^\beta \overline{H}_d [A_{df} \cdot \overline{\boldsymbol{\eta}}_{[\alpha\sigma]} \overline{\boldsymbol{\eta}}_{[\beta\rho]} + B_{df} \cdot \epsilon_{\alpha\beta\sigma} \boldsymbol{\eta}_\rho \mathbf{S}^l] \overline{\boldsymbol{\chi}}^{\{\sigma\rho\}} + \\
&+ N_R^\alpha N_R^\beta [A_{NN} \cdot \overline{\boldsymbol{\eta}}_{[\alpha\sigma]} \overline{\boldsymbol{\eta}}_{[\beta\rho]}] \overline{\boldsymbol{\chi}}^{\{\sigma\rho\}}
\end{aligned}$$

for the up and down fermions, quarks ($f = q$) and leptons ($f = l$), and right-handed neutrinos. They contain only one triplet scalar $\boldsymbol{\eta}_\alpha$ (and its SUSY counterpart $\overline{\boldsymbol{\eta}}_{[\alpha\beta]} \equiv \epsilon_{\alpha\beta\gamma} \overline{\boldsymbol{\eta}}^\gamma$) developing VEVs (60) along both the first and third directions at the same time. They also include one of the sextets $\overline{\boldsymbol{\chi}}^{\{\alpha\beta\}}$, so as to be properly arranged in the family space. We remark that the couplings W_{Y1} (70), like the general Yukawa couplings (52), use horizontal triplets for their symmetric parts and anti-triplets for the antisymmetric

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[6]We do not consider here the mass matrix of right-handed neutrinos to which the scenario B+B* is applicable as well. This will be done later in section 3.3.5.

parts. However the couplings W_{Y2} (71), on the contrary, prefer to use anti-triplets for the symmetric parts and triplets for the antisymmetric parts. Just those couplings collected in W_{Y2} turn out to correspond, as we will see later, to the minimal mixing of quarks and leptons with the proportionality condition (1) between their mass matrix elements. Note that the couplings in W_{Y2} include their own hierarchy making singlet scalar field **S**. This scalar **S** determines (see Eq.(63)) the small components in the VEVs of the triplets η ($\overline{\eta}$) and ζ ($\overline{\zeta}$) and thereby underlie the basic hierarchy between the diagonal elements M_{22} and M_{33} in the quark and lepton mass matrices (see later). Thus, it looks natural that the hierarchy in their off-diagonal elements is also dependent solely on the power (l) of the same scalar field **S** and not on the other scalar **I** introduced in the couplings W_{Y1} (70). Here again all the horizontal scalar fields $\boldsymbol{\eta}$, $\boldsymbol{\overline{\eta}}$, $\boldsymbol{\overline{\chi}}$ and **S** are scaled by the $SU(3)_F$ symmetry mass scale parameter M_F, so that the coupling constants $A_{uf,df,NN}$ and $B_{uf,df}$ are all dimensionless and assumed to be of order unity. According to the $U(1)$ symmetry, this hierarchy should be the same for all the fermions involved in W_{Y2}. The symmetric couplings containing the bi-linears of the $\boldsymbol{\eta}$ scalar give masses to the fermions of the heaviest third family and have no suppression, apart from the contributions coming from the small components in its VEV configuration (60). For simplicity, we also propose that the up-down hierarchy in the Yukawa couplings (71) is entirely given by the ratio of the VEVs of the ordinary Higgs doublets H_u and $\overline{H}_d$ of the MSSM, $\tan\beta = v_u/v_d$, thus considering the large $\tan\beta$ case. One can, of course, include instead extra powers of the singlet scalar fields **S** in the Yukawa couplings of the down fermions to generate the up-down mass hierarchy.

The Yukawa couplings (71), when taken for all the quarks and leptons involved, correspond in fact to the combined scenario A+A* for flavour mixing. In this case there is only one (natural) hierarchy parameter $y_S = \frac{<S>}{M_F}$ for all the fermion mass-matrices. However, in the combined scenario B+B* where the masses and mixings of the up quarks are determined by the Yukawa couplings W_{Y1} (70), while those of the down quarks and leptons are determined by the Yukawa couplings W_{Y2} (71), generally two independent hierarchy parameters $y_I = \frac{<I>}{M_F}$ and $y_S = \frac{<S>}{M_F}$ appear in the model. In general, one can hardly expect that the different Yukawa superpotentials, W_{Y1} and W_{Y2}, should lead to a similar hierarchy pattern for the corresponding fermions. We shall assume (see section 3.3.4) that the hierarchy making scalars I and S develop VEVs which are close in value, leading to close values of the hi-

erarchy parameters y_I and y_S. However they must have quite different $U(1)$ charges, in order to provide the required power-like hierarchy in the quark and lepton mass matrices.

The couplings W_{Y1} (70) and W_{Y2} (71) are the only Yukawa couplings which give masses to the quarks and leptons. Other possible $SU(3)_F$ couplings containing the same fermion and scalar superfields are, in fact, disallowed by the extra $U(1)$ symmetry mentioned above (see more detail in section 3.3.4).

3.3.3   Mass-matrices of quarks and leptons

We show now that, from the four presently (experimentally) allowed LFMG scenarios for the combined flavour mixing of quarks and leptons, the $SU(3)_F$ theory admits only one; namely the B+B* scenario which, fortunately, seems to be the most preferable with regard to the experimental situation in quark mixing (the present value of the V_{ub} element, see section 2.1) and in the solar neutrino problem (see section 2.2).

Towards this end let us first consider the Yukawa couplings W_{Y2} (71) to understand why they, while being good for all the other fermions, cannot work for the up quarks and right-handed neutrinos. Substituting all the VEV values of the horizontal triplets (60) and sextets (65), as well as those of the ordinary MSSM doublets H_u and $\overline{H}_d$, into the Yukawa couplings W_{Y2} (71), one obtains the following mass matrices for the quarks and leptons:

$$M_{uf} \;=\; m_u^0 \begin{pmatrix} A_{uf}\overline{\mathbf{x}}^2 & 0 & A_{uf}\overline{\mathbf{x}}\overline{\mathbf{X}} \\ 0 & A_{uf}\overline{\mathbf{x}}^2 & B_{uf}\mathbf{X}\mathbf{S}^l \\ A_{uf}\overline{\mathbf{x}}\overline{\mathbf{X}} & -B_{uf}\mathbf{X}\mathbf{S}^l & A_{uf}\overline{\mathbf{X}}^2 \end{pmatrix}, \tag{72}$$

$$M_{df} \;=\; m_d^0 \begin{pmatrix} A_{df}\overline{\mathbf{x}}^2 & 0 & A_{df}\overline{\mathbf{x}}\overline{\mathbf{X}} \\ 0 & A_{df}\overline{\mathbf{x}}^2 & B_{df}\mathbf{X}\mathbf{S}^l \\ A_{df}\overline{\mathbf{x}}\overline{\mathbf{X}} & -B_{df}\mathbf{X}\mathbf{S}^l & A_{df}\overline{\mathbf{X}}^2 \end{pmatrix} \tag{73}$$

$$M_{NN} = m_{NN}^0 \begin{pmatrix} A_{NN}\overline{\mathbf{x}}^2 & 0 & A_{NN}\overline{\mathbf{x}}\overline{\mathbf{X}} \\ 0 & A_{NN}\overline{\mathbf{x}}^2 & 0 \\ A_{NN}\overline{\mathbf{x}}\overline{\mathbf{X}} & 0 & A_{NN}\overline{\mathbf{X}}^2 \end{pmatrix} \tag{74}$$

Here the mass parameters m_u^0, m_d^0 and m_{NN}^0 include all the other VEV factors appearing with the Yukawa couplings in W_{Y2} :

$$m_u^0 = <H_u> \overline{\mathbf{X}}, \quad m_d^0 = <\overline{H}_d> \overline{\mathbf{X}}, \quad m_{NN}^0 = M_F \overline{\mathbf{X}} \tag{75}$$

(the bold letters stand everywhere for the properly scaled VEVs, for example $\mathbf{X} \equiv X/M_F$ etc). Now, after diagonalization of the 1-3 blocks in the mass matrices M_{uf}, M_{df} and M_{NN}, one immediately comes to the LFMG form (2) for the mass-matrices of all the fermions involved (specifying those for the up and down quarks, neutrinos, charged leptons and right-handed neutrinos, respectively):

$$M_U = m_u^0 \begin{pmatrix} 0 & B_U \frac{\overline{\mathbf{x}}}{\overline{\mathbf{X}}} \mathbf{X S}^l & 0 \\ -B_U \frac{\mathbf{x}}{\mathbf{X}} \mathbf{X S}^l & A_U \overline{\mathbf{x}}^2 & B_U \mathbf{X S}^l \\ 0 & -B_U \mathbf{X S}^l & A_U \overline{\mathbf{X}}^2 \end{pmatrix}, \tag{76}$$

$$M_D = m_d^0 \begin{pmatrix} 0 & B_D \frac{\overline{\mathbf{x}}}{\overline{\mathbf{X}}} \mathbf{X S}^l & 0 \\ -B_D \frac{\mathbf{x}}{\mathbf{X}} \mathbf{X S}^l & A_D \overline{\mathbf{x}}^2 & B_D \mathbf{X S}^l \\ 0 & -B_D \mathbf{X S}^l & A_D \overline{\mathbf{X}}^2 \end{pmatrix}, \tag{77}$$

$$M_N = m_u^0 \begin{pmatrix} 0 & B_N \frac{\overline{\mathbf{x}}}{\overline{\mathbf{X}}} \mathbf{X S}^l & 0 \\ -B_N \frac{\mathbf{x}}{\mathbf{X}} \mathbf{X S}^l & A_N \overline{\mathbf{x}}^2 & B_N \mathbf{X S}^l \\ 0 & -B_N \mathbf{X S}^l & A_N \overline{\mathbf{X}}^2 \end{pmatrix}, \tag{78}$$

$$M_E = m_d^0 \begin{pmatrix} 0 & B_E \frac{\overline{\mathbf{x}}}{\overline{\mathbf{X}}} \mathbf{X S}^l & 0 \\ -B_E \frac{\mathbf{x}}{\mathbf{X}} \mathbf{X S}^l & A_E \overline{\mathbf{x}}^2 & B_E \mathbf{X S}^l \\ 0 & -B_E \mathbf{X S}^l & A_E \overline{\mathbf{X}}^2 \end{pmatrix} \tag{79}$$

$$M_{NN} = m_{NN}^0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & A_{NN} \overline{\mathbf{x}}^2 & 0 \\ 0 & 0 & A_{NN} \overline{\mathbf{X}}^2 \end{pmatrix} \tag{80}$$

with the desired proportionality condition (1) or (6) between their diagonal and off-diagonal elements. This diagonalization, as one can readily see, is reached by a physically irrelevant rotation (being the same for the up and down fermions) with a hierarchically small angle $\Theta_{13}^{u,d} \simeq \frac{\overline{\mathbf{x}}}{\overline{\mathbf{X}}} = \frac{\overline{x}}{\overline{X}} \approx (\frac{S}{M_F})^r$ (see (63))[7].

_____

[7]We also neglected the small addition of the order of $O(S^2)$ to the new (33)-elements appearing in all these mass-matrices after the diagonalization of the 1-3 blocks.

This is, in fact, a crucial point for deriving the LFMG picture within the $SU(3)_F$ symmetry framework. On the one hand, the scalar η in the Yukawa couplings W_{Y2} (71) must develop VEVs along the first and third directions in family space, in order that the proportionality condition (6) be satisfied. On the other, these VEVs are allowed to induce at most only non-physical extra elements in the mass-matrices M_{uf} and M_{df} (72, 73) that could be rotated away by some rotation with $\Theta_{13}^u = \Theta_{13}^d$. Otherwise, one would have large physical mixing of the order of $O(y)$ (where $y \approx 0.1$ is the hierarchy parameter introduced in our discussion of neutrino oscillations in section 2.2.1) between the first and third families of fermions that is actually excluded, at least for quarks where this mixing angle is in fact of order of $O(y^3)$, as follows from the V_{ub} element value in the observed CKM matrix (see (29)).

There seems to be no other way of getting the LFMG in the $SU(3)_F$ model, when the same form W_{Y2} (71) is taken for the Yukawa couplings of both the up and down fermions . If so, however, all the matrices M_{uf}, M_{df} and M_{NN} appear, as one can readily see from their explicit forms (76-80), to be largely proportional to each other. This results in an equality of the mass ratios for the heavier leptons and quarks

$$\frac{m_c}{m_t} \simeq \frac{m_s}{m_b} \simeq \frac{m_\mu}{m_\tau} \simeq \frac{M_{N2}}{M_{N3}} \simeq \frac{M_{NN2}}{M_{NN3}} \tag{81}$$

which should hold with an accuracy of a few percent (given in fact by the ratios of the masses of the first and second families). While such an equality approximately works for the down quarks and charged leptons and also might be taken for the Dirac masses of neutrinos, it is certainly unacceptable for the up quarks [16]. This means that we are left with just one possible scenario B for the minimal quark mixing in the $SU(3)_F$ context, according to which the up quarks follow the pattern of direct (1-3) mixing (the alternative II, see general discussion in section 2.2), while the down quark mixing is given by a mass matrix of the above type (77) corresponding to the alternative I.

As to the lepton mixing, both of the experimentally allowed scenarios A^* and B^* (section 2.2) follow the alternative I for the Dirac mass-matrices of leptons. Thus M_N and M_E should have the form given above (78, 79), resulting in the proportionality condition (81) between the Dirac masses of neutrinos and charged leptons. This condition should be necessarily kept into account when analyzing neutrino oscillations (as we have done in section

2.2.), since it allows us to predict the hierarchy parameter y appearing in the neutrino Dirac mass matrix: $y = \sqrt{m_\mu/m_\tau}$. If we take (as we actually did) that the neutrino Majorana mass matrix also depends on the same parameter y then all the characteristic feature of the neutrino oscillations can be predicted according to basic equations (41) in scenario A* and equations (47) in scenario B*.

However, scenario A* presupposes the stronger hierarchy (35) between the second and third eigenvalues of the neutrino Majorana mass matrix M_{NN} to get the observed large mixings (see section 2.2) than the hierarchy (34) appearing in the Dirac mass matrix M_N of neutrinos. The requirement for such a hierarchy to be in the matrix M_{NN} is certainly in conflict with the mass proportionality condition (81), which appears in scenario A* within the $SU(3)_F$ framework. Another drawback of this scenario A* in the $SU(3)_F$ framework is the absence of the antisymmetric mixing term in the Yukawa couplings for the right-handed neutrinos (see (71)) that leads to one massless Majorana state in M_{NN} (80). Thus, scenario A* of lepton flavour mixing should be dropped in favour of scenario B*, with direct (1-3) mixing in the matrix M_{NN} just as it appears for the up quarks in scenario B (see section 3.3.5 for further discussion of the matrix M_{NN}). So, we can conclude that the combined scenario B+B* is in substance the only one admitted in the $SU(3)_F$ theory. For this case the mass proportionality condition (81) takes the reduced form:

$$\frac{m_s}{m_b} \simeq \frac{m_\mu}{m_\tau} \simeq \frac{M_{N2}}{M_{N3}} \tag{82}$$

which seem to work better than the whole equation (81), however, not yet in a quite satisfactory way. Actually, this relation between the masses of the down quarks and leptons is similar to the usual GUT relation, while arising in a different way. However, by enlarging the set of horizontal scalars, one can rearrange or completely decouple the masses of the quarks and leptons.

Now in scenario B, after family symmetry breaking according to the VEVs of the horizontal triplets (60), one is immediately led from the Yukawa couplings collected in W_{Y1} to the mass matrix for the up quarks:

$$M_U = m_U^0 \begin{pmatrix} A_{U1}\mathbf{z}^2 & 0 & A_{U1}\mathbf{z}\mathbf{Z} - B_U\overline{\mathbf{Y}}\mathbf{I}^n \\ 0 & A_{U2}\mathbf{Y}^2\mathbf{I^k} & 0 \\ A_{U1}\mathbf{z}\mathbf{Z} + B_U\overline{\mathbf{Y}}\mathbf{I}^n & 0 & A_{U1}\mathbf{Z}^2 \end{pmatrix} \tag{83}$$

where the mass factor m_U^0 is given by the electroweak symmetry scale involved

$$m_U^0 = < H_u > \tag{84}$$

Further, after diagonalization of the symmetric terms in the 1-3 block, one comes to the familiar form

$$M_U = m_U^0 \begin{pmatrix} 0 & 0 & -B_U\overline{\mathbf{Y}}\mathbf{I}^n \\ 0 & A_{U2}\mathbf{Y}^2\mathbf{I}^k & 0 \\ B_U\overline{\mathbf{Y}}\mathbf{I}^n & 0 & A_{U1}\mathbf{Z}^2 \end{pmatrix} \tag{85}$$

discussed in section 2 (see (22)).

Let us note that the above partial diagonalization of the mass matrix (83) was in fact reached by a rotation for the up quarks with the hierarchically small angle $\Theta_{13}^U \simeq -\frac{\mathbf{z}}{\mathbf{Z}} = \frac{z}{Z} \approx (\frac{S}{M_F})^r$ (see (63)). It is crucial for the scenario B+B* considered that this rotation turns out to be physically irrelevant: it is in fact the same as for the down quarks and leptons, despite the latter having mass-matrices of another type (73) or (77). However, the orthogonality (59) of the VEVs of the scalars involved keeps the rotations in both sectors strictly equivalent to one another

$$\overline{\eta}^\alpha \zeta_\alpha = 0 \rightarrow \frac{\overline{\mathbf{X}}}{\overline{\mathbf{X}}} + \frac{\mathbf{z}}{\mathbf{Z}} = 0 \rightarrow \Theta_{13}^{D,N,E} = \Theta_{13}^U \tag{86}$$

As a result, one can simultaneously transform the primary ("unrotated") matrices for the up quarks (83), on the one hand, and those of the down quarks (73) and leptons (neutrinos (72) and charged leptons 73)), on the other, into the final ("rotated") matrices (85) and (77-79) respectively. These matrices are just those which exactly correspond to the combined scenario B+B* for flavour mixing of quarks and leptons. These matrices follow, as we have seen above, from the Yukawa couplings W_{Y1} (70) and W_{Y2} (71) when the family symmetry $SU(3)_F$ spontaneously breaks.

3.3.4   Hierarchies in masses and mixings

We saw in the previous sections that the extra $U(1)$ symmetry, requiring the same powers k, n and l of the hierarchy making scalars I and S in similar Yukawa couplings of quarks and leptons, leads in the considered scenario B+B* to similar hierarchical mass matrices for the down quarks (77) and the leptons (78, 79). As a result, we obtained the mass relations (82) for the quarks and leptons in the scenario B+B*.

However, besides this general conclusion the $U(1)$ symmetry turns out, as we now show, to generate the required hierarchy in the mass matrices of the quarks and leptons. In order to show this in the most general form, we start with a scale-invariant superpotential which, in its only difference with the above-used Higgs superpotential W (55), contains no mass parameter for any of the scalar superfields, triplets and sextets, involved. Instead, we assume that their masses are forbidden by the $U(1)$ symmetry and are generated by the VEV of some singlet scalar field T from the following allowed couplings:

$$W_T = T(A_\eta \eta \overline{\eta} + A_\xi \xi \overline{\xi} + A_\zeta \zeta \overline{\zeta} + A_\chi \chi \overline{\chi} + A_\omega \omega \overline{\omega}) + P(I, S, T) \qquad (87)$$

So the masses of the scalars, which were directly introduced above (see section 3.3.1), are now given by

$$M_{\eta,\xi,\zeta,\chi,\omega} = A_{\eta,\xi,\zeta,\chi,\omega} \cdot <T> \qquad (88)$$

through the VEV of the scalar field T and the coupling constants $A_{\eta,\xi,\zeta,\chi,\omega}$. Thus, the VEV of the scalar field T gives the basic flavour scale M_F in the model, which is proposed now to appear dynamically rather than being introduced by hand. This, in substance, should stem from the polynomial $P(I, S, T)$ which includes all three singlet scalars; one of which (T) gives the basic mass scale in the model, while the other two (I and S) determine the mass and mixing hierarchy of the quarks and leptons. The hierarchy parameters are basically given by the characteristic VEV ratios I/T and S/T determined by the proper minimum of the polynomial $P(I, S, T)$ (see below).

Based on the above comments, we can now write the new total Higgs superpotential (see Eqs. (55, 56, 69, 87)) in the form

31

$$
\begin{aligned}
W_{tot} \;=\;& T(A_\eta \eta\overline{\eta} + A_\xi \xi\overline{\xi} + A_\zeta \zeta\overline{\zeta} + A_\chi \chi\overline{\chi} + A_\omega \omega\overline{\omega}) + \\
&+ f \cdot \epsilon^{\alpha\beta\gamma} \eta_\alpha \xi_\beta \zeta_\gamma + \overline{f} \cdot \epsilon_{\alpha\beta\gamma} \overline{\eta}^\alpha \overline{\xi}^\beta \overline{\zeta}^\gamma + \\
&+ F \cdot \epsilon^{\alpha\beta\gamma} \epsilon^{\alpha'\beta'\gamma'} \chi_{\{\alpha\alpha'\}} \chi_{\{\beta\beta'\}} \omega_{\{\gamma\gamma'\}} + \overline{F} \cdot \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'} \overline{\chi}^{\{\alpha\alpha'\}} \overline{\chi}^{\{\beta\beta'\}} \overline{\omega}^{\{\gamma\gamma'\}} + \\
&+ a \cdot \overline{\eta}\zeta \mathbf{S}^r M_F + b \cdot \omega\overline{\zeta}\zeta \mathbf{S}^p
\end{aligned}
\tag{89}
$$

with $\mathbf{I}$ and $\mathbf{S}$ standing for I/M_F and S/M_F and the polynomial $P(I, S, T)$ being as yet omitted. One can now quickly derive, from the new total Higgs superpotential W_{tot} and the Yukawa couplings W_{Y1} (70) and W_{Y2} (71), that the $U(1)$ charges of all the horizontal scalar fields involved are given as follows in terms of the charges of the singlet scalars S and I:

$$
\begin{aligned}
Q_\eta \;&=\; -\frac{5l}{3} Q_S + \frac{8n - 2k}{3} Q_I, \\
Q_\xi \;&=\; -\frac{2l}{3} Q_S + \frac{5n - 2k}{3} Q_I, \\
Q_\zeta \;&=\; -\frac{2l}{3} Q_S + \frac{10n - k}{6} Q_I, \\
Q_\chi \;&=\; -\frac{4l + 3p}{3} Q_S + \frac{14n - 5k}{6} Q_I, \\
Q_\omega \;&=\; -\frac{l + 3p}{3} Q_S + \frac{8n + k}{6} Q_I, \\
Q_T \;&=\; -l Q_S + \frac{4n - k}{2} Q_I
\end{aligned}
\tag{90}
$$

The $U(1)$ charges for their supersymmetric counterparts $\overline{\eta}$, $\overline{\xi}$, $\overline{\zeta}$, $\overline{\chi}$ and $\overline{\omega}$ are given by

$$
Q_{\overline{\eta},\overline{\xi},\overline{\zeta},\overline{\chi},\overline{\omega}} = -Q_{\eta,\xi,\zeta,\chi,\omega} + 2Q_T
\tag{91}
$$

In fact, the $U(1)$ charges of all the horizontal scalars can be expressed in terms of one independent charge Q_S, as one can readily see from the relation

$$
Q_I = \frac{2l + r}{3n - k} Q_S
\tag{92}
$$

arising from the $\overline{\eta}\zeta$ mixing term in the total superpotential W_{tot}. Here the numbers k, l and n are just the powers of the singlet scalars I and S in the

Yukawa couplings W_{Y1} and W_{Y2}, while the numbers r and p appear as the powers of S in the triplet-triplet and triplet-sextet mixing terms respectively in the superpotential W_{tot} (89).

When the above horizontal superfields undergo a $U(1)$ transformation, according to the charges (90), the total superpotential W_{tot} acquires a physically inessential overall phase[8]

$$W_{tot} \to e^{i3\alpha_T} W_{tot} \qquad (93)$$

where α_T is the transformation phase corresponding to the singlet scalar superfield T. Under the same transformation, the Yukawa couplings W_{Y1} (70) and W_{Y2}(71) are left invariant.

As to the matter superfields, the $U(1)$ charges of the quarks

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix}_\alpha, \quad U_R^\alpha, \quad D_R^\alpha \qquad (94)$$

leptons

$$\begin{pmatrix} N_L \\ E_L \end{pmatrix}_\alpha, \quad N_R^\alpha, \quad E_R^\alpha \qquad (95)$$

(see (49, 50)) and the ordinary Higgs doublets, H_u and $\overline{H}_d$, are in substance arbitrary[9] and can be chosen in many possible ways.

We will now investigate which powers k, l, n and r are required to get the observed mass and mixing hierarchy from the quark and lepton mass matrices. First of all one should make sure that no other matrix elements are allowed, by the $U(1)$ symmetry and the chosen powers, than those required by the scenario B+B* in all four mass matrices considered; namely for the up quarks (M_U (85)), down quarks (M_D 77)), neutrinos (M_N (78)) and charged leptons (M_E (79)). One can readily check that for an even value of r and an odd value of k with

$$n > k \qquad (k \text{ is odd}) \qquad (96)$$

---

[8]This means that the $U(1)$ considered is in fact the global R-symmetry $U(1)_R$ of the superpotential W_{tot} (89).

[9]One can immediately check that, in the B+ B* scenario, the Yukawa couplings W_{Y1} (70) and W_{Y2} (71) only give five equations for the eight charges (or phases) of the six independent fermion (94, 95) and two Higgs (H_u, $\overline{H}_d$) multiplets involved.

the undesired horizontal scalar combinations of type $\eta\eta$, $\eta\xi$, $\eta\zeta$ and $\xi\zeta$ (accompanied by any powers of the singlet scalars I and S) in the Yukawa couplings W_{Y1} (70), as well as the combinations $\overline{\zeta\zeta\chi}$, $\overline{\xi\xi\chi}$ and $\zeta\overline{\chi}$ in the Yukawa couplings W_{Y2} (71) are strictly prohibited to appear. This is important since their presence might crucially destroy scenario B+B*. The only allowed extra combination is $\xi\overline{\chi}$ which appears in W_{Y2}, thus giving an additional Yukawa coupling

$$\overline{L}_f^\alpha R_{df}^\beta \overline{H}_d [B'_{df} \cdot \epsilon_{\alpha\beta\sigma} \boldsymbol{\xi}_\rho \mathbf{I}^n] \overline{\boldsymbol{\chi}}^{\{\sigma\rho\}} \tag{97}$$

This actually turns out to be vanishingly small as compared to the other couplings for the down quarks and leptons ($f = D, N, E$) involved. So, one can see that the form of all the mass matrices considered are highly protected by the $U(1)$ symmetry from any scalar contributions, other than those allowed by the B+B* scenario of the minimal mixing of quarks and leptons.

Let us consider now the mass matrices M_D, M_N and M_E (77-79). All of them have a similar structure with the proportionality (1) between their diagonal and off-diagonal matrix elements. The natural orders of magnitude of these mass matrix elements are given by powers of the VEV of the singlet scalar S scaled by the basic mass M_F ($y = \frac{<S>}{M_F}$):

$$(M_f)_{33} \approx m^0, \ (M_f)_{22} \approx m^0\overline{\mathbf{x}}^2 \approx m^0 y^{2r}, \ (M_f)_{23} \approx m^0 y^l, \ (M_f)_{12} \approx m^0 y^{l+r} \tag{98}$$

($f = D, N, E$) as directly follows from equations (77-79) and (63). Since for the down quarks the approximate equality $(M_D)_{22} \approx (M_D)_{23}$ (or equivalently $m_s \approx \sqrt{m_d m_b}$) is known to work well phenomenologically, one is led to the following relation between the powers l and r:

$$l = 2r \tag{99}$$

Thus, according to the scenario B+B* considered, all three matrices M_D, M_N and M_E should have a similar hierarchical structure of the above type (98) with $l = 2r$ where r is as yet an arbitrary even number.

We turn now to the mass matrix M_U (85) for the up quarks. The natural orders of magnitude of its elements are given by the appropriate powers of the VEV of another singlet I (scaled by M_F). Taking for simplicity $y_I \approx y_S \equiv y$ ($< I > \approx < S >$), one readily finds for the matrix elements of M_U:

$$(M_U)_{33} \approx m_U^0, \quad (M_U)_{22} \approx m_U^0 y^k, \quad (M_f)_{13} \approx m_U^0 y^n \qquad (100)$$

For consistency with the phenomenological observation [16] concerning the quark masses of the second and third families, $(m_c/m_t)^2 \approx (m_s/m_b)^3$, comparison of the diagonal matrix elements of M_U and M_D gives, for the case of an even value for r and an odd value of k (see eq. (96)), a relation of the type

$$k = 3r + 1 \qquad (101)$$

And, finally, according to the inequality (96), the minimal choice for the power n is

$$n = 3r + 2 \qquad (102)$$

One can see that the above relations for the powers k, l and n lead to approximately the same hierarchy in the mass matrices of quarks and leptons for any (even) value of the number r. However, we will proceed further with $r = 2$, thus finally establishing the values

$$k = 7, \quad l = 4, \quad n = 8 \quad (r = 2) \qquad (103)$$

for the other powers.

According to these values, the quark and lepton mass matrices (77-79) and (85) in the scenario B+B* acquire the hierarchical forms:

$$M_f = \mathbf{m}_f \begin{pmatrix} 0 & \lambda \rho_f y^6 & 0 \\ -\lambda \rho_f y^6 & \lambda^2 y^4 & \rho_f y^4 \\ 0 & -\rho_f y^4 & 1 \end{pmatrix}, \quad f = D, N, E \qquad (104)$$

and

$$M_U = \mathbf{m}_U \begin{pmatrix} 0 & 0 & \sigma y^8 \\ 0 & \alpha y^7 & 0 \\ \sigma y^8 & 0 & 1 \end{pmatrix} \qquad (105)$$

Here the parameters ρ_f, α and σ, being ratios of Yukawa coupling constants, are all proposed to be of order unity, while the mass scale factors $\mathbf{m}_f$ and $\mathbf{m}_U$ are as yet "unrotated" masses of the heaviest third family of fermions involved. Also, we have parameterized the common VEV ratio $\frac{\bar{x}}{X}$ contained in the matrices $M_{D,N,E}$ as $\frac{\bar{x}}{X} = \lambda y^2$ ($\lambda \approx 1$) according to the behaviour of the

small components of the VEVs appearing on the horizontal triplets η and ζ (see (63)). Note that, in the matrices M_f (104), we have not included the vanishingly small matrix element $(M_f)_{13} = O(y^8)$ stemming generally from the "extra" Yukawa coupling (97).

One can see that, due to the high values of the powers in equation (103), even a very smooth starting hierarchy[10] $y \approx (m_s/m_b)^{1/4} \approx 0.4$ leads, as is apparent in the mass matrices (104) and (105), to the observed hierarchy in the masses and mixings of the quarks and leptons. Apart from the values of the weak mixing angles (27) which, as was already shown in section 2.1, are in a good agreement with experiment, there appear characteristic relations between the quark (and lepton) masses of the type:

$$\frac{m_u}{m_c} \approx \left(\frac{m_c}{m_t}\right)^{9/7}, \ \frac{m_d}{m_s} \approx \frac{m_s}{m_b}, \ \frac{m_u}{m_c} \approx \left(\frac{m_d}{m_s}\right)^{9/4}, \ \frac{m_c}{m_t} \approx \left(\frac{m_s}{m_b}\right)^{7/4} \qquad (106)$$

which seem to be successfull phenomenologically. Actually, one needs to know only the heaviest mass for each family of fermions—the other masses are then given in terms of this mass and the hierarchy parameter y. At the same time it is well to bear in mind that all these mass relations appear at the flavour scale M_F which could be as high as the GUT scale or even the Planck scale. Doing so one then needs to project these relations down to laboratory energies to compare them with experiment[40].

We consider finally the form the polynomial $P(I, S, T)$ introduced above (see Eq. (87) and how the VEVs of the basic singlet scalars I, S and T are determined. Note first of all that it is not immediately evident that, for the $U(1)$ charges of the scalars I, S and T already determined from W_{tot}, W_{Y1} and W_{Y2} (see Eqs. (90, 92)) and from the hierarchy requirements (99,101,102), such a non-trivial $U(1)$ invariant polynomial exists. Remarkably, however, just for these charges of the three singlet scalars I, S and T a polynomial of the type

$$P(I, S, T) = a_{ABC} T^A I^B S^C \qquad (107)$$

actually exists, where the summation over all the allowed integer powers A,

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[10]To avoid confusion we should note that the hierarchy parameter y introduced here differs from those used in section 2.2 (and denoted by the same letter): the old y appears to be the square of the new one.

B and C is imposed[11]. This polynomial determines the basic mass scale $M_F \simeq <T>$ in the model, as well as the characteristic hierarchies related with the two other VEVs $<I>$ and $<S>$ (all the high-order terms in P are scaled by appropriate powers of the mass M_F contained in the coupling constants a_{ABC}).

Actually, in order for a term in this invariant polynomial to exist, the $U(1)$ symmetry condition

$$AQ_T + BQ_I + CQ_S = 3Q_T \tag{108}$$

for the charges of the scalars I, S and T should be satisfied. Substitution of the values of these charges, expressed (see (Eqs. 90, 92, 99, 101, 102)) in terms of the power r of the scalar S in the total Higgs superpotential W_{tot} (89), in this condition gives the equation

$$\frac{A-3}{2}(7r+5) + \frac{5}{3}B + C\frac{6r+5}{3r} = 0 \tag{109}$$

There are only a finite number of positive integer solutions of this equation for the powers A, B and C. For the case $r=2$ taken in our model, only the following four terms appear in $P(I,S,T)$

$$P(I,S,T) = a_{300}T^3 + a_{241}T^2I^4S + a_{182}T^1I^8S^2 + a_{0123}I^{12}S^3 \tag{110}$$

while not disturbing the other parts of the total Higgs superpotential W_{tot} (89). In this case one can show from the corresponding F-term equations (together with those stemming from W_{tot}) that in general non-zero VEVs appear for all the singlet scalars I, S and T. Even a very smooth hierarchy between them could then lead, as was argued above, to the observed hierarchy in masses and mixings of the quarks and leptons.

3.3.5 Neutrino masses and oscillations

We have not yet considered the masses of the right-handed neutrinos and the Majorana mass matrix M_{NN} needed to construct the effective mass matrix M_ν for the ordinary neutrinos (33), using the see-saw mechanism [20].

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[11]Non-integer or negative values are of course not allowed for the powers in the Higgs superpotential or in the Yukawa couplings, due to their generic analyticity.

According to the B+B* scenario, the right-handed neutrinos should have a mass matrix similar to that of the up quarks, having symmetric Yukawa couplings with the same structure as W_{Y1} (70), but the antisymmetric couplings are necessarily absent for Majorana neutrinos. As a result, with symmetric Yukawa couplings only contained in the W_{Y1} one is unavoidably led to one massless right-handed neutrino. Therefore, some additional symmetric couplings are actually needed[12]. Fortunately, such a coupling generating right-handed neutrino masses automatically appears in the W_{Y1} once the triplet-sextet degeneracy lifting coupling ΔW_2 (69) is included in the total superpotential W_{tot} (89). One can immediately confirm, if so, that the horizontal sextet ω with the proper $U(1)$ charge value given in (90) is allowed then to contribute to the W_{Y1} in a way as the triplet pairs $\zeta\zeta$ and $\xi\xi$ do. Thus, for the right-handed neutrinos the total Yukawa potential reads as

$$W_{Y11} = N_R^\alpha N_R^\beta [A_{NN0} \cdot \omega_{\{\alpha\beta\}} \mathbf{S}^p \mathbf{T} + A_{NN1} \cdot \boldsymbol{\zeta}_\alpha \boldsymbol{\zeta}_\beta + A_{NN2} \cdot \boldsymbol{\xi}_\alpha \boldsymbol{\xi}_\beta \mathbf{I}^k] \quad (111)$$

The first term in the W_{Y11} should be, of course, included also in the the Yukawa couplings W_{Y1} (70) for the up quarks. However, this term, while being crucial for the neutrino mass formation, is turned out to be inessential for the up quarks (for the high values of the S singlet power number p as is appeared below).

After substituting the VEVs of the sextet $\omega_{\alpha\beta}$ (65) and the triplets ζ and ξ (60, 63) into the Yukawa coupling W_{Y11} (111), one immediately comes to the Majorana mass matrix for the right-handed neutrinos:

$$M_{NN} = M_F \begin{pmatrix} A_{NN1}\mathbf{z}^2 & 0 & A_{NN1}\mathbf{zZ} \\ 0 & A_{NN2}\mathbf{Y}^2\mathbf{I}^\mathbf{k} & 0 \\ A_{NN1}\mathbf{zZ} & 0 & A_{NN1}\mathbf{Z}^2 + A_{NN0}\boldsymbol{\omega}\mathbf{S}^p\mathbf{T} \end{pmatrix} \quad (112)$$

This matrix can be written in a convenient hierarchical form, like the other mass matrices (104, 105), using the already established hierarchy indices (see (103)) and keeping the power p as yet undefined:

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[12]One might imagine introducing symmetric mixing term containing the non-diagonal horizontal triplet combination $\eta\zeta$ in the Yukawa coupling W_{Y1} (70). However, as we have seen in the above, such a term is strictly prohibited by the $U(1)$ symmetry.

$$M_{NN} = \mathbf{m}_{NN} \begin{pmatrix} y^4 & 0 & y^2 \\ 0 & \gamma y^7 & 0 \\ y^2 & 0 & 1 + \beta y^p \end{pmatrix} \qquad (113)$$

Here $\mathbf{m}_{NN}$, β and γ are appropriate combinations of the masses and coupling constants in (112) and the value $\mathbf{T} \equiv \frac{T}{M_F} \simeq 1$ has been used. The physical masses of the Majorana neutrinos are then given by the equations

$$M_{NN1} \simeq \mathbf{m}_{NN}\beta y^{4+p}, \quad M_{NN2} \simeq \mathbf{m}_{NN}\gamma y^7, \quad M_{NN3} \simeq \mathbf{m}_{NN} \qquad (114)$$

The mass matrix M_ν for the physical neutrinos can now readily be constructed according to the see-saw formula (33), with the neutrino Dirac mass matrix M_N (104) taken in its "unrotated" form (72)

$$M_N = \mathbf{m}_N \begin{pmatrix} \lambda^2 y^4 & 0 & \lambda y^2 \\ 0 & \lambda^2 y^4 & \rho_N y^4 \\ \lambda y^2 & -\rho_N y^4 & 1 \end{pmatrix} \qquad (115)$$

and the Majorana mass matrix M_{NN} given above (113). The inverse of the Majorana mass matrix M_{NN} is readily formed

$$M_{NN}^{-1} = \frac{1}{\beta y^{4+p}\mathbf{m}_{NN}} \begin{pmatrix} 1 + \beta y^p & 0 & -y^2 \\ 0 & \beta\gamma^{-1}y^{-3+p} & 0 \\ -y^2 & 0 & y^4 \end{pmatrix}, \qquad (116)$$

and gives rise to an effective light neutrino mass matrix M_ν. To simplify its form we take for all the order-one parameters λ, ρ_N and β contained in the matrices M_N and M_{NN} a condition of the type

$$|\Delta - 1| \lesssim y^4 \qquad (\Delta \equiv \lambda, \rho_N, \beta) \qquad (117)$$

according to which they are supposed to be equal to unity with a few percent accuracy. After this simplification and the choice $p = 8$, the matrix M_ν takes a transparent symmetrical form (ignoring some inessential higher order terms)

$$M_\nu = \mathbf{m}_\nu \begin{pmatrix} y^4 & -y^2 & -y^2 \\ -y^2 & 1 + \gamma y^{-1} & 1 + \gamma y^{-1}\delta \\ -y^2 & 1 + \gamma\delta y^{-1} & 1 + \gamma y^{-1}(1 + \delta^2) \end{pmatrix} \qquad (118)$$

where the matrix M_ν has been brought to the appropriate physical basis, by making a rotation in the (1-3) block (equal to that already applied to the matrix of the charged leptons M_E so as to bring it to the form (104)). Here the mass parameter

$$\mathbf{m}_\nu = -\frac{\mathbf{m}_N^2}{\mathbf{m}_{NN}}\gamma^{-1}y \tag{119}$$

corresponds in fact to the mass of the heaviest neutrino, while the parameter $\delta \equiv \frac{|\lambda-1|}{y^4} \leq 1$ according to the equation (117).

One can see that the matrix M_ν is crucially dependent on the parameter γ related with the triplet-triplet term $\xi\xi\mathbf{I}^k$ in the Yukawa potential W_{Y11} (111). While all the dimensionless coupling constants in W_{Y11} (as for the other Yukawa and Higgs coupling constants in the model) are proposed to be of the natural order $O(1)$, the parameter γ should be considerably smaller to have an experimentally acceptable mass interval between the masses of the second and third family neutrinos. In particular, if we take $\gamma \approx y^4$ (which is just of the order $O(m_s/m_b)$, see the matrices (104)), we come to an attractive picture for neutrino masses and oscillations. Actually, after diagonalization of the matrix M_ν, one can readily see that the ratios of the neutrino masses are[13]

$$m_{\nu 1} : m_{\nu 2} : m_{\nu 3} = \frac{1}{4}y^7 : \frac{1}{2}y^3 : 1 \tag{120}$$

and the small-mixing angle MSW solution (SMA) for neutrino oscillations naturally emerges, with the following predictions for the basic parameters:

$$\sin^2 2\theta_{atm} \simeq 1, \quad \sin^2 2\theta_{sun} \lesssim 2y^4, \quad U_{e3} \simeq \frac{y^2}{\sqrt{2}}, \quad \frac{\Delta m_{sun}^2}{\Delta m_{atm}^2} \simeq \frac{y^6}{4} \tag{121}$$

Here, in deriving the limit on $\sin^2 2\theta_{sun}$, we have used the inequality (117) for the $(M_\nu)_{12}$ element obtained after successive (2-3) and (1-3) block diagonalizations.

For the known value of the hierarchy parameter $y \approx (m_s/m_b)^{1/4} \approx 0.4$ in the model, our predictions (121) turn out to be inside of the experimentally

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[13]We take, for simplicity, $\delta = 1$. For an opposite limiting case, $\delta = 0$, the mass ratious is a little changed, $m_{\nu 1} : m_{\nu 2} : m_{\nu 3} = \frac{1}{2}y^7 : \frac{1}{4}y^3 : 1$.

allowed intervals for the SMA solution (48). Note that for the case when the above parameters λ, ρ_N and β are strictly equal to unity the (1-2) mixing completely disappears, while the (2-3) mixing determining the $\sin^2 2\theta_{atm}$ is left maximal. However, in the SMA case one must expect, as we mentioned above (see section 2.2), a sizable contribution to $\sin^2 2\theta_{sun}$ stemming from the charged lepton sector (see Eqs.(31)). Remarkably, this contribution taken alone

$$\sin^2 2\theta_{sun} \simeq 4\sin^2\theta_{e\mu} = 4\frac{m_e}{m_\mu} \qquad (122)$$

gives a good approximation to the SMA solution.

So, one can conclude that the $SU(3)_F$ theory of flavour successfully describes the neutrino masses and mixings as well. Remarkably, just the neutrino oscillation phenomena already observed could give the first real hint as to where the scale M_F of the flavour symmetry $SU(3)_F$ might be located. Actually, if the right-handed neutrinos are not sterile under $SU(3)_F$ (as they are under all other gauge symmetries) then their masses should be directly related with the flavour scale, just like the masses of the quarks and leptons are related with an electroweak scale. We treat the right-handed neutrinos, like the other quarks and leptons, as triplets N_R^α under $SU(3)_F$ (see (50)) which acquire their masses from the $SU(3)_F$ invariant couplings W_{Y11} (111), when the horizontal scalars develop VEVs and flavour symmetry breaks. Due to the chiral nature of $SU(3)_F$, there appear no mass terms for the Majorana neutrinos N_R^α when the symmetry is unbroken. Thus, it is apparent that the mass of the heaviest Majorana neutrino, appearing from the Yukawa coupling W_{Y11}, should be of the same order as the flavour scale M_F, since the basic Yukawa coupling constants are all proposed to be of order unity. Using now equation (119), with the heaviest Dirac mass of the neutrino $\mathbf{m}_N$ taken as large as the top quark mass m_t, the heaviest physical neutrino mass $\mathbf{m}_\nu$ determined from the experimentally observed value of Δm_{atm}^2 [19], $\mathbf{m}_\nu \simeq \sqrt{\Delta m_{atm}^2} \simeq 0.07$ eV and the hierarchy parameter $y \approx (m_s/m_b)^{1/4}$, one comes to an approximate value for the flavour scale

$$M_F \approx \mathbf{m}_{NN} = -\frac{\mathbf{m}_N^2}{\mathbf{m}_\nu}\gamma^{-1}y \simeq \frac{m_t^2}{\mathbf{m}_\nu}\left(\frac{m_b}{m_s}\right)^{3/4} \simeq 10^{16} \text{ GeV} \qquad (123)$$

from which the masses and mixings of the quarks and leptons are basically generated. Due to the enhancement factor $\gamma^{-1}y$ appearing in the $SU(3)_F$

model considered, this scale turns out to be as large as the well-known SUSY GUT scale [41]. The high scale M_F of the flavour symmetry found above (see (123)) would mean that the flavour symmetry $SU(3)_F$ could be part of some extended SUSY GUT, either in a direct product form like $SU(5) \otimes SU(3)_F$ [8], $SO(10) \otimes SU(3)_F$ [34] and $E(6) \otimes SU(3)_F$ [34] or even in the form of a family unifying simple group like $SU(8)$ GUT [35].

3.3.6 Spontaneous CP violation

It seems that the whole picture of flavour mixing of quarks and leptons, including the hierarchical pattern of the masses and mixing angles, can arise as a consequence of the spontaneous breakdown of the family symmetry $SU(3)_F$ considered. We show, in this section, that even CP violation can be spontaneously induced in the $SU(3)_F$ theory of flavour. Furthermore, when it occurs, one is unavoidably led to maximal CP violation.

The essential point is that, in general, the horizontal scalars (54) develop complex VEVs (60, 65) leading to CP violation even if all the coupling constants in the superpotential W_{tot} (89) and the Yukawa couplings in W_{Y1} (70) and W_{Y2} (71) are taken to be real. Let us consider this in more detail. For simplicity, one can always choose real VEVs for the singlet scalars I, S and T, among the possible VEV solutions stemming from the invariant polynomial $P(I, S, T)$ (107). Then, as follows from the triplet VEV equations (58, 63), the phases of the large and small components (60) of their VEVs should satisfy the conditions

$$\delta_X + \delta_{\overline{X}} = 0, \quad \delta_Y + \delta_{\overline{Y}} = 0, \quad \delta_Z + \delta_{\overline{Z}} = 0, \quad \delta_X + \delta_Y + \delta_Z = \pi \quad (124)$$
$$\delta_{\overline{x}} = \pi - \delta_X + \delta_Y + 2\delta_Z, \quad \delta_z = \pi - \delta_X + 2\delta_Z, \quad \delta_{\overline{z}} = \pi - \delta_X$$

There is one more non-trivial condition, stemming from the imaginary part of the triplet F-term equations (57) containing also the singlet scalar S. When one uses the previous conditions (124), this extra condition reads as follows:

$$-\delta_X + \delta_Y + 3\delta_Z = \pi \cdot \mathbf{k}, \quad \mathbf{k} = 0, 1 \quad (125)$$

Note now that, due to the exact $U(1)$ symmetry of the total superpotential W_{tot} (89) and the polynomial $P(I, S, T)$ (107), one can always choose the phase of the $U(1)$ transformation in such a way as to make one of the phases

δ_X, δ_Y or δ_Z in the equations (124, 125) vanish. Thus, taking $\delta_Y = 0$, one finally comes to two non-trivial solutions for the phases of the large and small components of the triplet VEVs:

$$\begin{aligned}
\delta_X &= \pi/4, \quad \delta_Z = 3\pi/4, \\
\delta_{\overline{x}} &= \pi/4, \quad \delta_z = \pi/4, \quad \delta_{\overline{z}} = 3\pi/4
\end{aligned} \qquad (126)$$

and

$$\begin{aligned}
\delta_X &= \pi/2, \quad \delta_Z = \pi/2, \\
\delta_{\overline{x}} &= 3\pi/2, \quad \delta_z = 3\pi/2, \quad \delta_{\overline{z}} = \pi/2
\end{aligned} \qquad (127)$$

depending on the two possible values of $\mathbf{k}$, $\mathbf{k} = 0$ and $\mathbf{k} = 1$ respectively. We show below by applying these triplet VEV phase values to the quark and lepton mass matrices that, while the second solution (127) is CP-conserving, the first one (126) leads to maximal CP-violation. There are no other solutions for the phases of the triplet VEVs in the $SU(3)_F$ model considered[14].

Let us consider the CP-conserving case (127) first. As one can quickly confirm, the mass matrices M_U (85) and M_D (77) of the up and down quarks, in this case containing the phases of the horizontal triplet VEVs given by equation (127), are strictly Hermitian with real diagonal elements and pure imaginary off-diagonal ones. Such matrices M_U and M_D, with five and three texture zero structure respectively, cannot lead to CP violation—the phase of the corresponding Jarlskog determinant [42] vanishes identically.

By contrast, the CP-violating case (126) results in non-Hermitian mass matrices M_U and M_D. One can find their explicit form, in terms of the masses of the up and down quarks and the triplet VEV phases (126), by forming their Hermitian products $M_U M_U^+$ and $M_D M_D^+$ and then using the conservation of their traces, determinants and sums of principal minors. Doing so, we find for the matrices M_U and M_D:

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[14]As to the VEVs of the horizontal sextets χ and ω they always can be chosen real. Actually, by a proper choice of the phase of the $SU(3)_F$ transformations (particularly its λ^8 subgroup transformation) one can take the phase of $< \chi >$ to be zero. Then, as follows from the sextet F-term equations (64), the phase of $< \omega >$ turns out to vanish as well.

$$M_U = \begin{pmatrix} 0 & 0 & -\sqrt{m_u m_t}e^{i\pi/2} \\ 0 & m_c e^{i\pi/2} & 0 \\ \sqrt{m_u m_t}e^{i\pi/2} & 0 & m_t - m_u \end{pmatrix} \qquad (128)$$

and

$$M_D = \begin{pmatrix} 0 & -\sqrt{m_d m_s}e^{-i3\pi/4} & 0 \\ \sqrt{m_d m_s}e^{-i3\pi/4} & m_s & \sqrt{m_d m_b}e^{i3\pi/4} \\ 0 & -\sqrt{m_d m_b}e^{i3\pi/4} & m_b - m_d \end{pmatrix} \qquad (129)$$

where we have removed the inessential common phase factors.

Notably, while the diagonalization of the non-Hermitian matrices M_U and M_D requires different unitary transformations for the left-handed and right-handed quarks, the Hermitian constructions $M_U M_U^+$ and $M_D M_D^+$ are diagonalized by unitary transformations acting on just the left-handed quarks. These are precisely the transformations which contribute to the weak interaction mixings of quarks collected in the CKM matrix. Thus, one must diagonalize the $M_U M_U^+$ and $M_D M_D^+$ first and then construct the CKM matrix. Proceeding in such a manner, we immediately find that the diagonalization of $M_U M_U^+$ and $M_D M_D^+$ is carried out by unitary matrices of the type

$$V_U = R_{13}^U \Phi^U, \qquad V_D = R_{12}^D R_{23}^D \Phi^D \qquad (130)$$

They in fact contain the same plane rotations R_{13}^U ($s_{13} \simeq \sqrt{m_u/m_t}$), R_{12}^D ($s_{12} \simeq \sqrt{m_d/m_s}$) and R_{23}^D ($s_{23} \simeq \sqrt{m_d/m_b}$) as those used for the diagonalization of the phenomenological Hermitian mass matrices M_U (22) and M_D (2) considered in section 2.2. However, the phase matrices Φ^U and Φ^D now become

$$\Phi^U = \begin{pmatrix} e^{i\pi/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Phi^D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i3\pi/4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (131)$$

Finally, substituting all the above rotations and phases into the CKM matrix $V_{CKM} = V_U V_D^\dagger$ (14) and properly rephasing the c quark field ($c \to e^{i3\pi/4}c$), one comes to

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$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} - is_{12}s_{13}s_{23} & -s_{12}c_{13} - is_{13}s_{23}c_{12} & -is_{13}c_{23} \\ s_{12}c_{23} & c_{12}c_{23} & -s_{23} \\ -s_{13}c_{12} - is_{13}s_{23}c_{13} & s_{13}s_{12} - is_{23}c_{13} & -ic_{13}c_{23} \end{pmatrix} \quad (132)$$

While this is in substance a new parameterization for the CKM matrix, it is quite close to the standard parameterization[16], as one can show by comparing the moduli of the matrix elements in them both. In such a way, one readily concludes that the CP violation phase in V_{CKM} (132) is in fact maximal, $\delta = \pi/2$, while all the mixing angles are rather small being basically determined by the masses of the lightest quarks u and d (see Eqs. (27)), as was expected.

4   Conclusions

The present observational status of quark flavour mixing, as described by the CKM matrix elements [16], shows that the third family t and b quarks are largely decoupled from the lighter families. At first sight, it looks quite surprising that not only the 1-3 "far neighbour" mixing (giving the V_{ub} element in V_{CKM}) but also the 2-3 "nearest neighbour" mixing (V_{cb}) happen to be small compared with the "ordinary" 1-2 Cabibbo mixing (V_{us}) which is determined, according to common belief, by the lightest u and d quarks. This led us to the idea that all the other mixings, and primarily the 2-3 mixing, could also be controlled by the masses m_u and m_d and that the above-mentioned decoupling of the third family t and b quarks is determined by the square roots of the corresponding mass ratios $\sqrt{\frac{m_u}{m_t}}$ and $\sqrt{\frac{m_d}{m_b}}$ respectively. So, in the chiral symmetry limit $m_u = m_d = 0$, not only does CP violation vanish, as argued in [1], but all the flavour mixings disappear as well.

In such a way the Lightest Family Mass Generation (LFMG) mechanism for flavour mixing of quarks was formulated in section 2.1 with two possible scenarios A and B. We found that the LFMG mechanism reproduces well the values of the already well measured CKM matrix elements and gives distinctive predictions for the yet poorly known ones in both scenarios A and B. One could say that, for the first time, there are compact working formulae (especially compact in scenario B) for all the CKM angles in terms of quark mass ratios. The only unknown parameter is the CP violating phase

δ. Taking it to be maximal ($\delta = \frac{\pi}{2}$), we obtain a full determination of the CKM matrix that is numerically summarized in section 2.1. The LFMG mechanism was extended to the lepton sector in section 2.2, again with two possible scenarios A* and B*. These were shown to be quite successful phenomenologically as well, naturally leading to a consistent description of the presently observed situation in solar and atmospheric neutrino oscillations . Remarkably, the same mechanism results simultaneously in small quark and large lepton mixing.

From the theoretical point of view, the basic LFMG mechanism arises from the generic proportionality condition (1) between diagonal and off-diagonal elements of the mass matrices. One might think that this condition suggests some underlying flavour symmetry, probably a non-abelian $SU(N)$ symmetry, treating the N families in a special way. Indeed, for $N = 3$ families, we have found that the local $SU(3)_F$ chiral family (or horizontal) symmetry [8], considered in detail in section 3, seems to be a good candidate. The $SU(3)_F$ family symmetry is accompanied by an additional global $U(1)$ symmetry, which seems to be very important for the recovery of the final picture of flavour mixing of quarks and leptons. There is a whole plethora of $SU(3)_F$ invariant Yukawa couplings available. However, due to the extra protecting symmetry $U(1)$, one is able to choose them in a special form which leads to the minimal flavour mixing of quarks and leptons. There are four pairs of experimentally allowed scenarios (see section 2) for the combined quark and lepton flavour mixings: A+A*, A+ B*, B+A* and/or B+B*. We found that the $SU(3)_F$ theory only admits the scenario B+B* which, fortunately, seems to provide the best fit to the experimental data on quark mixing. It must though be admitted that the detailed $SU(3)_F$ model led to the SMA solution for the solar neutrino problem. Besides this crucial role in selecting the right pattern of flavour mixing among the four scenarios involved, the $U(1)$ symmetry turns out, as we have shown, to determine the form of the hierarchy in the quark and lepton mass matrices[15]. Actually, one only needs to know the heaviest mass for each family of fermions—the other masses are then given terms of this mass and the input hierarchy parameter y.

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[15]Note, however, that we had to take the parameter γ in the Majorana mass matrix (113) to be small ($\gamma \approx y^4$), while all the parameters in the quark and lepton mass matrices are supposed to of the order $O(1)$.

The high scale M_F of the flavour symmetry found here from neutrino oscillations means that the local flavour symmetry $SU(3)_F$ might be interpreted as part of some extended SUSY GUT; this could appear in a direct product form like $SU(5) \otimes SU(3)_F$, $SO(10) \otimes SU(3)_F$ and $E(6) \otimes SU(3)_F$ or even in the form of a simple group like the $SU(8)$ GUT unifying all families.

And the last point, which is especially noteworthy, is that the symmetry-breaking horizontal scalar fields, triplets and sextets of $SU(3)$, in general develop complex VEVs and, in cases linked to the LFMG mechanism, transmit a maximal CP violating phase $\delta = \frac{\pi}{2}$ to the effective Yukawa couplings involved. Apart from the direct predictability of δ, the possibility that CP symmetry is broken spontaneously like other fundamental symmetries of the Standard Model seems very attractive—both aesthetically and because it gives some clue to the origin of maximal CP violation in the CKM matrix.

So, an $SU(3)$ family symmetry seems to be a good candidate for the basic theory underlying our proposed LFMG mechanism, although we do not exclude the possibility of other interpretations as well. Certainly, even without a theoretical derivation of Eq. (1), the LFMG mechanism can be considered as a successful predictive ansatz in its own right. Its further testing could shed light on the underlying flavour dynamics and the way towards the final theory of flavour.

Acknowledgements

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