Feedback Linearization of nonlinear distortion in Electrodynamic Loudspeakers

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Abstract

In this short paper it is theoretically shown that Kaizer's low-frequency model for nonlinear distortion in electrodynamic loudspeakers is feedback linearizable in current drive as well as voltage drive. This means there exists some nonlinear feedback mechanism such that the given system can be transformed into an exact linear system, provided one has full state information and there would be no parametric uncertainty or noise upon the nonlinear loudspeaker dynamics.

Keywords. Electrodynamic loudspeakers, low-frequency nonlinear distortion, nonlinear control, feedback linearization, inverse dynamics, Lie derivative.

1 Introduction

In [3], Kaizer described a nonlinear model for electrodynamic loudspeakers at low frequency, taking into account nonlinearities in the motor part (magnet system/voice coil) and in the mechanical part. These nonlinearities cause unwanted harmonics and intermodulation products in the response of the loudspeaker. Among the approaches for distortion reduction that are taken up till now are e.g. Kaizer's nonlinear inversion circuit based on Volterra series expansions [3] and Klippel's mirror filter [6].

In this paper a feedback linearization method from nonlinear control theory [2][8] is applied to Kaizer's model for distortion reduction in current drive and voltage drive. In closed loop a perfectly linear system is obtained, provided the nonlinear loudspeaker model would be exact and all the state variables of the model are measurable. The aim of this paper is precisely to show that under certain conditions it is at least theoretically possible to obtain exact linearization, according to nonlinear control theory. On the other hand aspects of robustness and implementation are not considered here.

This paper is organized as follows. In Section 2 we briefly review Kaizer's model. In Section 3 the feedback linearization method is explained in general. In Section 4 feedback linearization is applied to Kaizer's model.

2 Kaizer's low-frequency nonlinear model for electrodynamic loudspeakers

According to Kaizer [3] or [10], in current drive and for low frequencies an electrodynamic loudspeaker can be modelled as an analog mass-spring system:

$$b\,i = m\,\ddot{x}_p + r_m\,\dot{x}_p + k\,x_p \tag{1}$$

where $x_p(t)$ is the voice-coil excursion influenced by the voice-coil current $i(t)$. The parameters b, m, r_m, k are respectively the force factor, moving mass, mechanical damping and stiffness of the mass-spring system. The parameters b and k depend upon the displacement

x_p and are modelled approximately as:

$$\begin{aligned} b(x_p) &= b_0 + b_1\, x_p + b_2\, x_p^2 \\ k(x_p) &= k_0 + k_1\, x_p + k_2\, x_p^2 \end{aligned} \tag{2}$$

Written in the general nonlinear state space form

$$\begin{cases} \dot{x} &= f(x) + g(x)\, u \\ y &= h(x) \end{cases} \tag{3}$$

with $u(t) \in \mathbb{R}^m$ the input vector, $y(t) \in \mathbb{R}^p$ the output vector and $x(t) \in \mathbb{R}^n$ the state of the system, one has for the loudspeaker

$$f(x) = \begin{bmatrix} x_2 \\ -\frac{k(x_1)}{m}\, x_1 - \frac{r_m}{m}\, x_2 \end{bmatrix},\ g(x) = \begin{bmatrix} 0 \\ \frac{b(x_1)}{m} \end{bmatrix},\ h(x) = x_1 \tag{4}$$

where $x = [x_1\, x_2]^T$, $x_1 = x_p$ and $x_2 = \dot{x}_p$, $u = i$.

In voltage drive the electrodynamic loudspeaker is modelled by a set of two coupled differential equations [3]

$$\begin{cases} v_e &= r_e\, i + \frac{d(l(x_p)\, i)}{dt} + b(x_p)\, \dot{x}_p \\ b(x_p)\, i &= m\, \ddot{x}_p + r_m\, \dot{x}_p + k(x_p)\, x_p \end{cases} \tag{5}$$

where l is the inductivity of the voice coil, which depends upon x_p and can be modelled as

$$l(x_p) = l_0 + l_1\, x_p + l_2\, x_p^2 \tag{6}$$

and $v_e(t)$ is the generator voltage, r_e is the electrical resistance of the voice coil. By introducing the state variables $x_1 = x_p$, $x_2 = \dot{x}_p$ and $x_3 = l(x_p)\, i$, (5) can be written in state-space form (3) as

$$f(x) = \begin{bmatrix} x_2 \\ -\frac{k(x_1)}{m}\, x_1 - \frac{r_m}{m}\, x_2 + \frac{b(x_1)}{m\, l(x_1)}\, x_3 \\ -b(x_1)\, x_2 - \frac{r_e}{l(x_1)}\, x_3 \end{bmatrix},\ g(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ h(x) = x_1 \tag{7}$$

and $u = v_e$.

3 Theory of feedback linearization

In this Section we will briefly sketch some theoretical aspects of feedback linearization theory for the determination of a nonlinear static state feedback law in order to linearize a given nonlinear system [2][8]. The procedure consists basically of two steps (see Figure 1):

1. First one transforms the given MIMO nonlinear system into an integrator-decoupled linear system by means of nonlinear static state feedback.

2. In a second step pole placement can be applied in order to assign the desired linear dynamics to the system.

Let us consider again the nonlinear system (3)

$$\begin{cases} \dot{x} &= f(x) + g(x)\,u \\ y &= h(x) \end{cases}$$

Suppose the number of inputs equals the number of outputs ($m = p$). In the static state-feedback input-output decoupling problem, one searches for a state-feedback law of the form

$$u = \sigma(x) + \rho(x)\,v \tag{8}$$

with $v \in \mathbb{R}^m$ a vector of reference inputs, σ an analytic vector function and ρ a nonsingular matrix for all x, such that the closed-loop system

$$\begin{cases} \dot{x} &= f(x) + g(x)\,\sigma(x) + g(x)\rho(x)\,v \\ y &= h(x) \end{cases} \tag{9}$$

is input-output decoupled. This means that each new input component v_i influences only the corresponding component of the output y_i and none of the other output components y_j ($j \neq i$).

In order to obtain this, the following definitions are essential. For analytic functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $\phi : \mathbb{R}^n \to \mathbb{R}$, the Lie derivative is defined as $L_f\phi$ as $L_f\phi(x) =< \nabla\phi, f >= \sum_{i=1}^n f_i(x)\frac{\partial\phi}{\partial x_i}(x)$ and iteratively $L_f^k\phi = L_f(L_f^{k-1}\phi)$ and $L_f^0\phi = \phi$. The characteristic numbers d_i ($i = 1, ..., m$) are the smallest integers for which the function $L_g L_f^{d_i} h_i \neq 0$. The $(d_i + 1)$-th time derivative of y_i is the first time derivative that explicitly depends on u

($d_i + 1$ is called the relative degree of the system). Taking the derivatives of the output vector with respect to time, it can be shown that

$$\begin{bmatrix} y_1^{(d_1+1)} \\ \vdots \\ y_m^{(d_m+1)} \end{bmatrix} = \begin{bmatrix} L_f^{d_1+1} h_1(x) \\ \vdots \\ L_f^{d_m+1} h_m(x) \end{bmatrix} + \begin{bmatrix} L_g L_f^{d_1} h_1(x) \\ \vdots \\ L_g L_f^{d_m} h_m(x) \end{bmatrix} u \tag{10}$$

Eqn (10) is defined as

$$y^{(d+1)} = \phi(x) + \psi(x)\, u$$

Then it can be proven that the static state-feedback input-output decoupling problem is solvable if and only if the $(m \times m)$-matrix $\psi(x)$ is *nonsingular* for all x. The decoupling state-feedback law is then given by

$$u = (\psi(x))^{-1} (-\phi(x) + v) \tag{11}$$

which leaves the original system in the *integrator-decoupled* form:

$$y^{(d+1)} = v \tag{12}$$

with corresponding state-space form in the case of SISO systems

$$\dot{z} = A\, z + B\, v \tag{13}$$

and

$$A = \begin{bmatrix} O_{d\times 1} & I_d \\ 0 & O_{1\times d} \end{bmatrix},\ B = \begin{bmatrix} O_{d\times 1} \\ 1 \end{bmatrix}$$

where I_d denotes the $d \times d$ identity matrix.

The second stage of the method consists of prescribing the desired linear dynamics to the linearized system. This is done as follows

$$v = -\sum_{j=0}^{d} P_j\, y^{(j)} + P_{d+1}\, w \tag{14}$$

Here w is a new external control input. Pole placement is done by carefully choosing the $m \times m$ diagonal matrices P_k. The maximum number of poles that can be placed with a nonlinear inverse dynamics control law depends upon the number $\sum_{i=1}^{m} d_i$. In the case that

$\sum_{i=1}^{m} d_i = n$ all system poles can be placed and closed-loop stability can be guaranteed if closed-loop observability can be proven [7]. For cases where $\sum_{i=1}^{m} d_i < n$ closed-loop stability can be guaranteed only locally by showing that the modes made unobservable by the control law have stable dynamics over the regions of interest in the state space.

4 Feedback linearization theory applied to Kaizer's model

Here we apply the theory of feedback linearization to the electrodynamic loudspeaker model in current drive as well as voltage drive. As a result exact linearization of the nonlinear model is obtained.

4.1 Current drive

The loudspeaker model (4) is a SISO system. Although decoupling of a SISO system is trivial, it still makes sense to apply the same theory. Let us first derive the characterstic number d. Therefore we compute the derivatives of the output y with respect to time

$$\begin{aligned} \dot{y} &= x_2 \\ \ddot{y} &= -\tfrac{k(x_1)}{m}\, x_1 - \tfrac{r_m}{m}\, x_2 + \tfrac{b(x_1)}{m}\, u \end{aligned} \tag{15}$$

Clearly $d = 1$ because $\ddot{y}$ is the first time derivative that explicitly depends upon the input signal u. The relative degree is 2 which is equal to the system order and hence we don't have to consider zero dynamics [2]. The state-feedback law (11) becomes

$$u = (\psi(x_1))^{-1}(-\phi(x_1) + v) \tag{16}$$

with

$$\begin{aligned} \phi(x_1) &= -\tfrac{k(x_1)}{m}\, x_1 - \tfrac{r_m}{m}\, x_2 \\ \psi(x_1) &= \tfrac{b(x_1)}{m} \end{aligned}$$

The integrator-decoupled form (12) exists if $b(x_1) \neq 0$. Equation (14) becomes

$$\begin{aligned} v &= -p_0\, y - p_1\, \dot{y} + p_2\, w \\ &= -p_0\, x_1 - p_1\, x_2 + p_2\, w \end{aligned} \tag{17}$$

The overall system is then

$$\left\{ \begin{array}{rcl} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} & = & \begin{bmatrix} 0 & 1 \\ -p_0 & -p_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ p_2 \end{bmatrix} w \\ y & = & [1\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{array} \right. \tag{18}$$

Pole placement can now be done by choosing the parameters p_0, p_1, p_2. In order to obtain a perfect linearization of (1)(2), the parameters are respectively chosen as $\frac{k_0}{m}$, $\frac{r_m}{m}$ and $\frac{b_0}{m}$. Remark that the given nonlinear system together with the inverse dynamic system forms the following double integrator system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \tag{19}$$

4.2 Voltage drive

In the voltage drive case we have a third order nonlinear system. In order to linearize the system we determine again the characteristic number d by taking the derivatives of the output y with respect to time

$$\begin{array}{rcl} y & = & x_1 \\ \dot{y} & = & x_2 \\ \ddot{y} & = & \dot{x}_2 = -\frac{k(x_1)}{m}\, x_1 - \frac{r_m}{m}\, x_2 + \frac{b(x_1)}{m\, l(x_1)}\, x_3 \\ y^{(3)} & = & \ddot{x}_2 = -\frac{d}{dt}\left(\frac{k(x_1)}{m}\, x_1\right) - \frac{r_m}{m}\, \dot{x}_2 + \frac{d}{dt}\left(\frac{b(x_1)}{m\, l(x_1)}\right) x_3 + \frac{b(x_1)}{m\, l(x_1)}\, \dot{x}_3 \end{array} \tag{20}$$

or

$$y^{(3)} = \alpha(x) + \frac{b(x_1)}{m\, l(x_1)}\, u \tag{21}$$

with

$$\begin{array}{rcl} \alpha(x) & = & -\frac{k_{x_1}}{m}\, x_1 - \frac{k(x_1)}{m}\, x_2 + \frac{r_m\, k(x_1)}{m^2}\, x_1 + \frac{r_m^2}{m^2}\, x_2 - \frac{r_m\, b(x_1)}{m^2\, l(x_1)}\, x_3 + \\ & & \frac{b_{x_1}\, l(x_1) - b(x_1)\, l_{x_1}}{m\, l(x_1)^2}\, x_2 x_3 + \frac{b(x_1)}{m\, l(x_1)}\left(-b(x_1)\, x_2 - \frac{r_e}{l(x_1)}\, x_3\right) \end{array}$$

Here k_{x_1}, b_{x_1}, l_{x_1} denote the derivatives of k, b and l with respect to x_1 respectively. Hence the characteristic number d is equal to 2 and again we don't have to consider zero dynamics.

The nonlinear static state feedback is given by

$$u = \frac{m\, l(x_1)}{b(x_1)}\, (-\alpha(x) + v) \tag{22}$$

A necessary condition is that $b(x_1) \neq 0$ like in the current control case, together with $l(x_1) \neq 0$. This control signal leaves the nonlinear system in the integrator-decoupled form

$$y^{(3)} = v \tag{23}$$

The next step in the design is to impose the desired linear dynamics by means of pole placement

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= v = -p_0\, z_1 - p_1\, z_2 - p_2\, z_3 + p_3\, w \end{cases} \tag{24}$$

where the parameters p_0, p_1, p_2, p_3 are to be chosen. Moreover $z_1 = x_1$, $z_2 = x_2$ and $z_3 = -\frac{k(x_1)}{m}\, x_1 - \frac{r_m}{m}\, x_2 + \frac{b(x_1)}{m\, l(x_1)}\, x_3$. Finally, in order to obtain the desired linear loudspeaker dynamics

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k_0}{m}\, x_1 - \frac{r_m}{m}\, x_2 + \frac{b_0}{m\, l_0}\, x_3 \\ \dot{x}_3 &= -b_0\, x_2 - \frac{r_e}{l_0}\, x_3 + u \end{cases} \tag{25}$$

the parameters are chosen as follows:

$$p_0 = \frac{k_0\, r_e}{m\, l_0},\ p_1 = \frac{r_m\, r_e}{m\, l_0} + \frac{b_0^2}{m\, l_0} + \frac{k_0}{m},\ p_2 = \frac{r_m}{m} + \frac{r_e}{l_0},\ p_3 = \frac{b_0}{m\, l_0} \tag{26}$$

5 Conclusion

In this paper the general theory of feedback linearization was applied in order to reduce nonlinear distortion in an electrodynamic loudspeaker low-frequency lumped parameter model. Both in the case of current drive and voltage drive, it is at least theoretically possible to exactly linearize the given nonlinear model. Of course this is only valid under the strong assumption that the loudspeaker model is time invariant, exact and noiseless. Nevertheless the method may contribute towards a better insight into nonlinear distortion reduction schemes in general.

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Caption of Figure

Figure 1. Transforming a given nonlinear dynamical systems into a desired linear dynamical system, using feedback linearization theory.

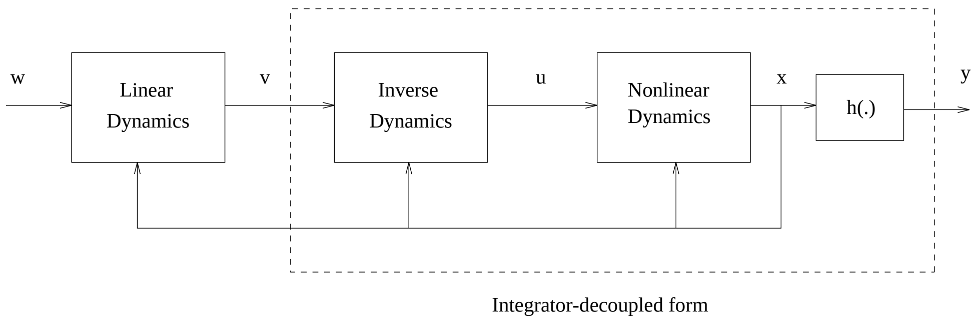
w
Linear
Dynamics
v
Inverse
Dynamics
u
Nonlinear
Dynamics
x
h(.)
y
Integrator-decoupled form

Figure 1: