

# The study of dynamic singularities of seismic signals by the generalized Langevin equation

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## Abstract

Analytically and quantitatively we reveal that the GLE equation, based on a memory function approach, in which memory functions and information measures of statistical memory play fundamental role in determining the thin details of the stochastic behavior of seismic systems, naturally conduces to a description of seismic phenomena in terms of strong and weak memory. Due to a discreteness of seismic signals we use a finite - discrete form of GLE. Here we studied some cases of seismic activities of Earth ground motion in Turkey with consideration of complexity, nonergodicity and fractality of seismic signals.

*Key words:* Generalized Langevin Equation, seismic systems, nonergodicity, fractality

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## 1 Introduction

Specific stochastic dynamics occur in a large variety of systems, such as supercooled liquids, seismic systems, human brain, finance, meteorology and

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granular matter. These systems are characterized by an extremely rapid increase or a slowdown of relaxation times and by a non-exponential decay of time-dependent correlation functions [1,2].

The canonical theoretical framework for stochastic dynamics of complex systems is the time-dependent generalized Langevin equation (GLE) [3,4,5,6,7,14,15]. It successfully describes the phenomenon of statistical memory, whereby the relaxation time for order parameter fluctuations scales as a power of the correlation length. An obvious question to ask would be whether this framework can be adapted to describe seismic phenomena. Analytically and quantitatively we show that the GLE equation, based on a memory function approach, where the memory functions and information measures of statistical memory play fundamental role in determining the thin details of the stochastic behavior of seismic systems, naturally leads to a description of seismic phenomena in a terms of a strong and weak memory. Due the discreteness of a seismic signals we use a finite - discrete form of GLE. Here we study some cases of seismic activities of Earth ground motion in last years in Turkey with consideration of complexity, irregularity and metastability of seismic signals.

## 2 Some extraction from the theory of discrete stochastic processes

The GLE analytical model was originally proposed for displaying stochastic behavior of signals in complex systems [3,4,5,6,7], one of which identifies memory effects with diverse memory time scales in manifold of signal's correlation so that the arbitrary seismic state is characterized by the set of memory time length scale in a system.

Here we consider data of seismic signals recording as a time series  $\xi$ :

$$\xi = \{\xi_0, \xi_1, \xi_2, \dots, \xi_{N-1}\} = \{\xi(0), \xi(\tau), \xi(2\tau), \dots, \xi([N-1]\tau)\}. \quad (1)$$

Here  $\tau$  is a discretization time of seismic signals,  $N$  is a total number of signals. A set of fluctuations  $\delta\xi$  is an initial dynamic variable  $W_0$  :

$$W_0 = \{\delta\xi_0, \delta\xi_1, \delta\xi_2, \dots, \delta\xi_{N-1}\}, \quad \delta\xi_j = \xi_j - \langle\xi\rangle, \quad \langle\xi\rangle = \frac{1}{N} \sum_{j=0}^{N-1} \xi_j. \quad (2)$$

The Gram-Schmidt orthogonalization procedure

$$\langle W_n, W_m \rangle = \delta_{n,m} \langle |W_n|^2 \rangle \quad (3)$$

leads to the set of the following orthogonal dynamic variables:

$$\begin{cases} W_0 = \delta\xi, \\ W_1 = \mathcal{L}W_0 = \frac{d}{dt}\delta\xi, \\ W_2 = \mathcal{L}W_1 - \Lambda_1W_0, \\ \dots, \\ W_{n+1} = \mathcal{L}W_n - \Lambda_nW_{n-1}, \quad n \geq 1, \end{cases} \quad (4)$$

where  $\mathcal{L} = (\Delta - 1)/\tau$  is the Liouville's quasioperator and  $\Lambda_n$  is the relaxation parameter of the  $n$ th order (where  $\Delta$  is the shift operator  $\Delta x_j = x_{j+1}$  and  $\tau$  is the discretization time).

Within the framework of statistical theory and Zwanzig-Mori's theoretical-functional procedure of projection operators one can receive following recurrent relation as a finite-difference kinetic equation:

$$\Delta M_n(t) = \tau\lambda_{n+1}M_n(t) - \tau^2\Lambda_{n+1} \sum_{j=0}^{m-1} M_{n+1}(t - j\tau)M_n(j\tau), \quad n = 0, 1, 2, \dots \quad (5)$$

Here we introduce a Liouville's quasioperator eigenvalue  $\lambda_{n+1}$ , a relaxation parameter  $\Lambda_{n+1}$  and a memory function  $M_n(t)$  of the  $n$ th order respectively

$$\lambda_n = \frac{\langle W_{n-1}\mathcal{L}W_{n-1} \rangle}{\langle |W_{n-1}|^2 \rangle}, \quad \Lambda_n = \frac{\langle |W_n|^2 \rangle}{\langle |W_{n-1}|^2 \rangle}, \quad M_n(t) = \frac{\langle W_n(t)W_n \rangle}{\langle |W_n|^2 \rangle}. \quad (6)$$

For analysis of relaxation time scales of underlying processes we use the frequency-dependent statistical non-Markovity parameter  $\varepsilon_n(\omega)$ :

$$\varepsilon_n(\omega) = \left\{ \frac{\mu_{n-1}(\omega)}{\mu_n(\omega)} \right\}^{1/2}. \quad (7)$$

Here  $\mu_n(\omega)$  is a frequency power spectrum for the memory function of the  $n$ th order:

$$\mu_n(\omega) = \left| \tau \sum_{j=0}^{N-1} M_n(j\tau) \cos(j\tau\omega) \right|^2. \quad (8)$$

Using of Eqns. (1) - (8) we can study all specific singularities of the statistical memory effects in an underlying system. Non-Markovity parameter and its

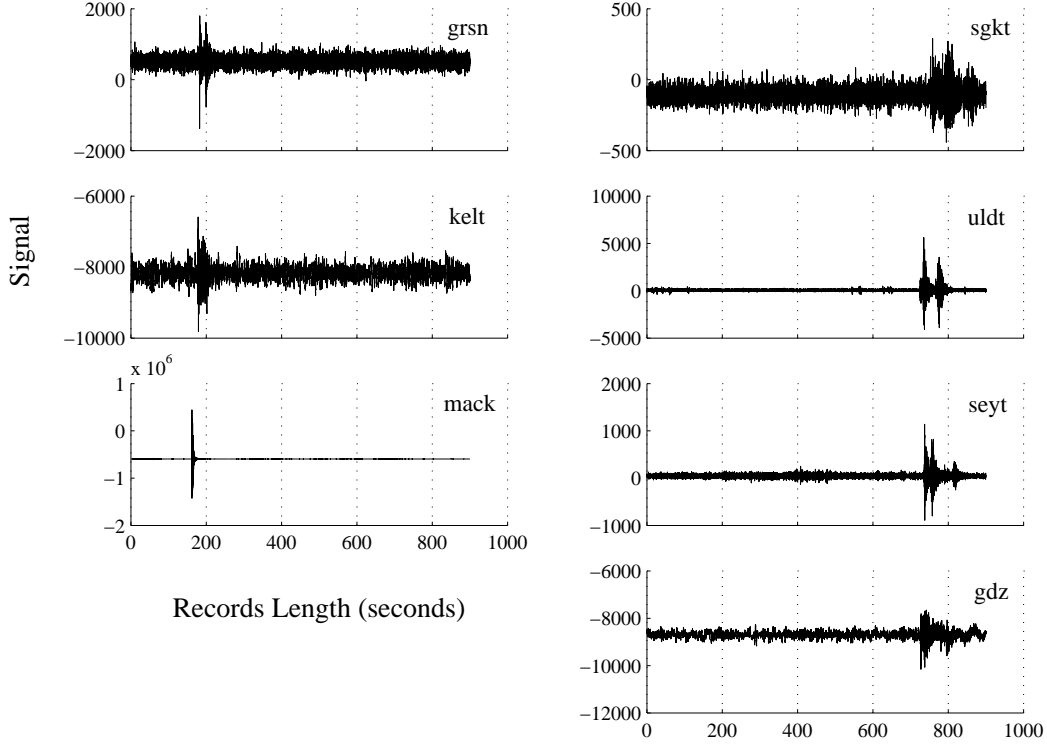


Fig. 1. An initial row data of seismic signals from 7 areas of seismic signals: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, *gdz*. The discretization time is  $\tau = 0.02$  sec.

statistical spectrum were introduced by Yulmetyev et al. in [8]. It is worthy of mentioning that non-Markovian character of seismic data was discussed by Varotsos et al. [9]. One of the first proofs of non-Markovity of empirical random processes was given in Refs. [10]. Stochastic origins of the long-range correlations of ionic current fluctuations in membrane channels with non-Markovian behavior were studied in [11].

### 3 An analysis of results

Fig. 1 presents the initial time series of seismic signals for 7 seismic origins: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, *gdz*. Discretization time is  $\tau = 0.02$  sec. From the Figures we can see that all time series have distinctive features.

Fig. 2 demonstrates the frequency dependence of the first point of non-Markovity parameter for 7 seismic origins from Turkey  $\varepsilon_1(\omega)$ : *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, *gdz*. Since the nature of each seismic source is unknown to us, it would be interesting to establish its character. It seems possible that the signals can be distributed to 3 groups: **group A** (*kelt*, *gdz*), **group B** (*grsn*, *sgkt*, *uldt*, *seyt*) and **group C** (*mack*).

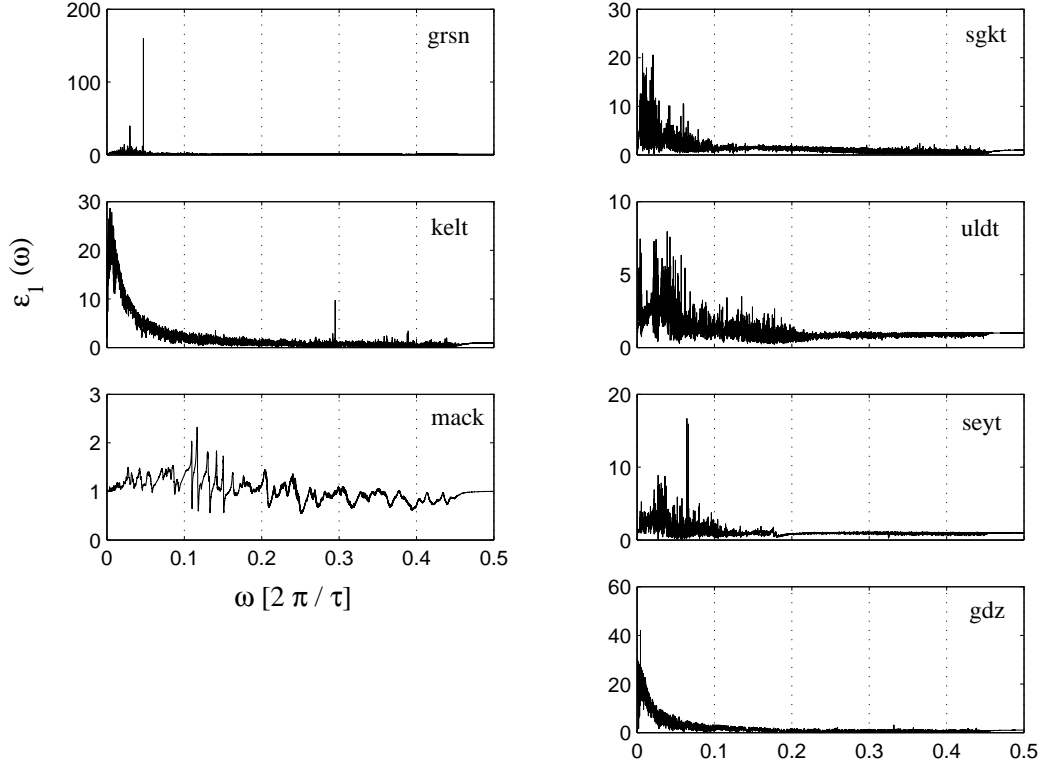


Fig. 2. The frequency dependence of the first point of non-Markovity parameter  $\varepsilon_1(\omega)$  for the each seismic origin: *grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, *gdz*.

**Signals of Group A** are characterized by the more regular structure and smooth decay of the function  $\varepsilon_1(\omega)$ .

There is a frequency dependence of non-Markovity parameter for seismic origins: *gdz* *kelt* in Fig. 3. General power dependence  $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$  is submitted by a continuous line with parameters  $\omega_0 = 0.2$ ,  $\alpha = 1.05$ .

**Signals of Group B** are characterized by the irregular frequency structure and by the frequency bursts on the distinct frequencies. Spectra have a noisy character.

There is a frequency dependence of  $\varepsilon_1(\omega)$  for 4 seismic origins: *grsn*, *sgkt*, *uldt*, *seyt* in double log-log scale has been presented in Fig. 4. Discretization time is  $\tau = 0.02$  sec. General power dependence  $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$  is submitted by a continuous line with parameters  $\omega_0 = 0.2$ ,  $\alpha = 0.6$ .

**Signals of group C** can not be attributed to one of the above groups. The parameter  $\varepsilon_1(\omega)$  fluctuate strongly between 1 and 10 in full frequency scale. That testifies existence of strong memory effects in the long-range time correlation. A possible origin of the similar signals is due to strong Earthquake. More careful and detailed analysis of the signal structure on the various time scales and relaxation levels (with the taking into account of the long-range correla-

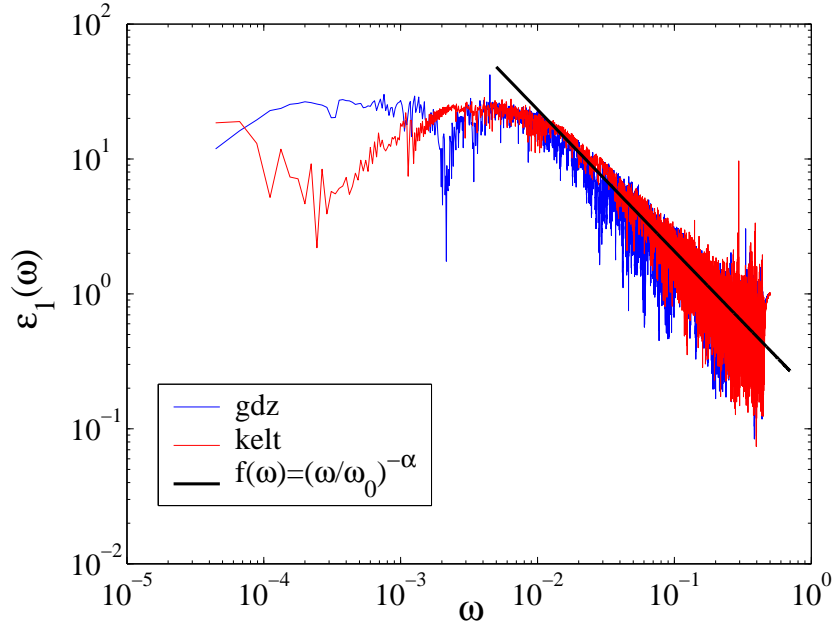


Fig. 3. (Color online) The frequency dependence of  $\varepsilon_1(\omega)$  in double log-log scale for 2 seismic origins: *gdz* *kelt*. The time discretization is  $\tau = 0.02$  sec. The general power dependence of  $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$  is submitted by a continuous line with parameters  $\omega_0 = 0.2$ ,  $\alpha = 1.05$ .

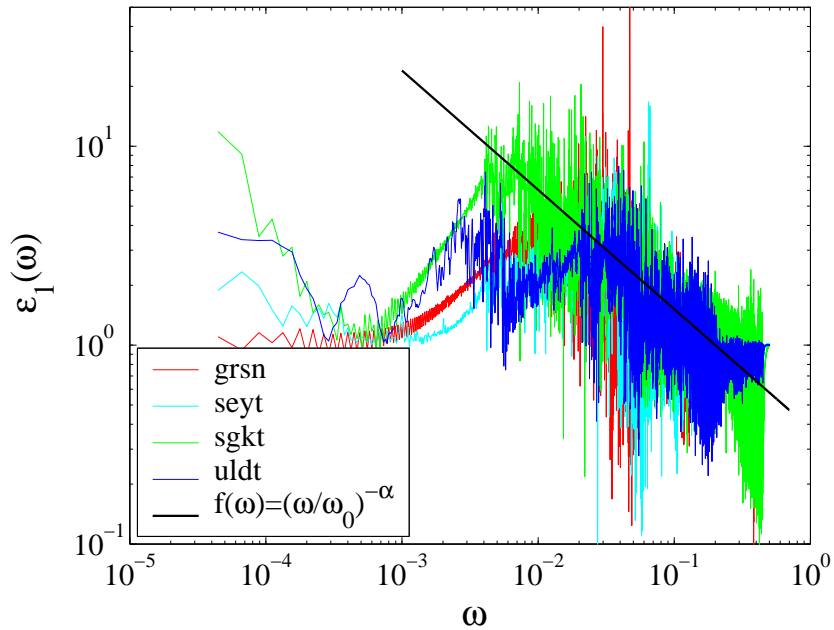


Fig. 4. (Color online) The frequency dependence of  $\varepsilon_1(\omega)$  for 4 seismic origins: *grsn*, *sgkt*, *uldt*, *seyt* in double log-log scale. The time discretization is  $\tau = 0.02$  sec. The general power dependence of  $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$  is submitted by a continuous line with parameters  $\omega_0 = 0.2$ ,  $\alpha = 0.6$ .

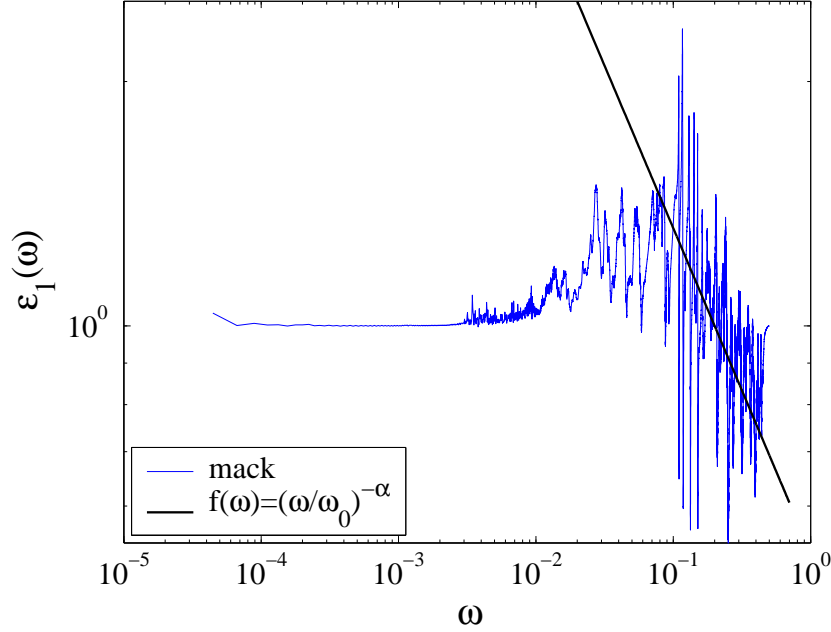


Fig. 5. (Color online) The frequency dependence of non-Markovity parameter  $\varepsilon_1(\omega)$  for seismic origin *mack* in the double log-log scale. The time discretization is  $\tau = 0.02$  sec. The general power dependence of  $\varepsilon_0(\omega) = (\omega/\omega_0)^{-\alpha}$  is submitted by a continuous line with parameters  $\omega_0 = 0.2$ ,  $\alpha = 0.4$ .

tion, memory effects, nonergodicity and metastability of underlying system) is required.

Fig. 5 displays the frequency dependence of  $\varepsilon_1(\omega)$  in double log-log scale for *mack*. Discretization time is  $\tau = 0.02$  sec. General power dependence  $\varepsilon_1(\omega) = (\omega/\omega_0)^{-\alpha}$  is submitted by a continuous line with parameters  $\omega_0 = 0.2$ ,  $\alpha = 0.4$ .

The analysis of all spectra shows that all the signals can be classified into three different groups in the order of the breaking of fractal behavior of high frequency dependence of  $\varepsilon_1(\omega)$ . Signals for the **group A** can be characterized by stronger fractality with the exponents  $\alpha = 1.05$ . A linear trend with the small fluctuation has been simultaneously observed in the spectrum. We can see range of the diversity  $10 < \varepsilon_1(\omega) < 100$ . Signals for the **group B** are characterized by the breaking of fractality with the exponents  $\alpha = 0.6$ . A nonlinear oscillating trend with big fluctuation has been observed here. Spectra of signals for **group C** are characterized by a weak fractality with the exponent  $\alpha = 0.4$  and  $\varepsilon_1(\omega) \sim 1$ .

The auto-correlation functions (ACF)

$$C(t) = \frac{\langle \xi(0)\xi(t) \rangle}{\langle |\xi(0)|^2 \rangle} \quad (9)$$

for the signals of different groups have been presented in Fig. 6.

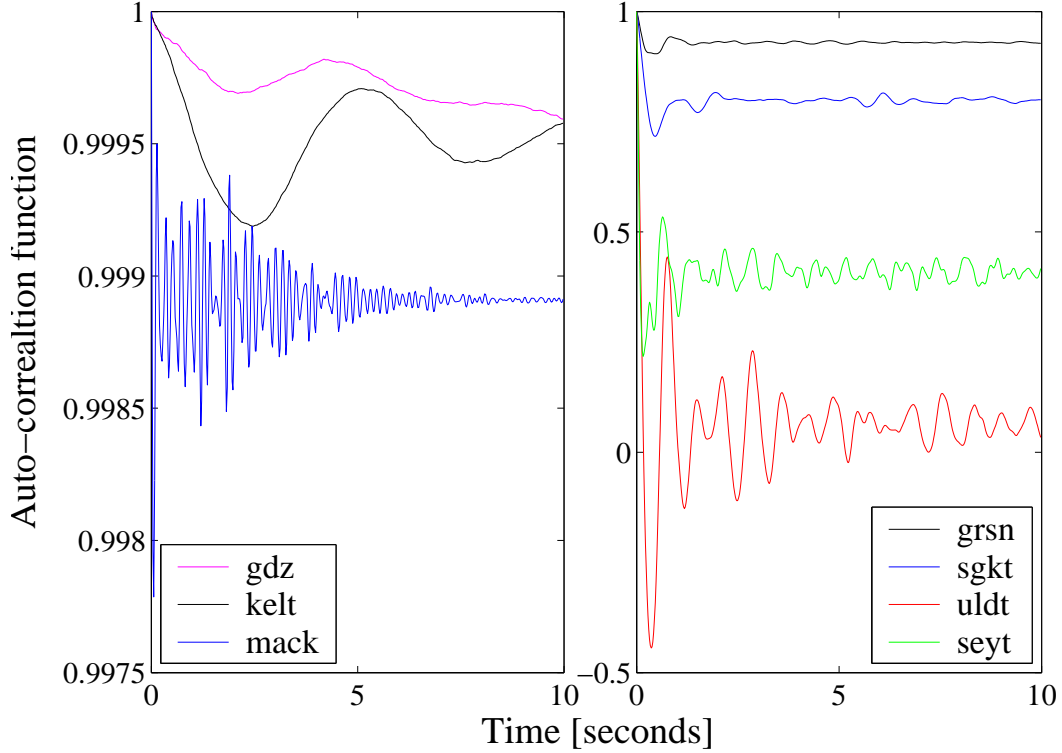


Fig. 6. (Color online) The auto-correlation function of the signals.

The left panel includes ACF for the signals of **group A** and **group C**, (*mack*) and (*kelt*, *gdz*), while the right panel contains ACF for the signals of **group B** (*grsn*, *sgkt*, *uldt*, *seyt*). The brackets here note averaging in time iterations. It is seen from the Figures, that auto-correlation of the signals has a pronounced nonergodic character (undamped behavior of the time correlation function at time  $t \rightarrow \infty$ ):

$$\lim_{t \rightarrow \infty} C(t) \neq 0. \quad (10)$$

According to recent works on the ergodic hypothesis Eqns. (9), (10) would imply violation of ergodicity. Net results [12] suggest the breaking of ergodicity for a class of generalized, Brownian motion, obeying a non-Markovian dynamics being driven by a generalized Langevin equation. This very feature originates from vanishing of the effective friction. Khinchins theorem of ergodicity is examined [13] by means of linear response theory. The resulting ergodic condition shows that, contrary to the theorem, irreversibility is not a sufficient condition for ergodicity.

Similar behavior of the time correlation functions is characteristic for the supercooled and glass states of condensed matter [14,15]. A higher level of nonergodicity corresponds with the signals of **group A** and **group C**, whereas signals of **group B** are characterized by the minor nonergodicity. Signals from



Table 1  
The frequency relaxation parameters for seismic signals.

Object	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\Lambda_1$	$\Lambda_2$
grsn	-0.0306	-0.2004	-0.8945	0.0497	0.0396
kelt	-0.0667	-1.1399	-1.0485	-0.0193	0.2954
mack	-0.3938	-0.5808	-1.0747	0.4374	-0.0342
sgkt	-0.0306	-0.7490	-1.0849	0.0156	-0.2967
uldt	-0.0349	-0.2605	-0.7609	0.0526	0.1243
seyt	-0.0745	-0.2434	-0.8427	0.1173	0.0590
gdz	-0.0646	-0.8836	-0.9650	0.0155	0.2586

the object *uldt* are rigorously ergodic. Therefore one can suppose that these signals cannot belong to Earthquake.

Table 1 presents a set of relaxation parameters  $\lambda_1, \lambda_2, \lambda_3, \Lambda_1$  and  $\Lambda_2$  for seismic signals from: *grsn, kelt, mack, sgkt, uldt, seyt, gdz*. The set of these parameters characterizes some peculiarities of relaxation processes on the low relaxation levels of seismic systems (1, 2 and 3). It has seen from the Table that these parameters don't have a clear distinction vs distinctions unlike those visible from the frequency dependence of  $\varepsilon_1(\omega)$ . Therefore thinner and more sensitive techniques are needed for studying of dynamic processes in examined signals.

Figure 7 depicts the comparison of seismic data with results of computer simulation for the metallic glass  $Al_{50}Cu_{50}$ . On top, auto-correlation functions of the seismic events (*grsn, sgkt, uldt* and *seyt*) are shown. At the bottom, auto-correlation functions for incoherent scattering of copper atoms are shown for comparison. The data are received with the help of statistical averaging on ensembles of statistical systems. Each curve was received by time averaging. Full sample consists of 45000 points. The single calculation was carried out for each separate time window of 500 points size with time step of 0.02 s. Further this window was displaced to one step to the right up to the end of time sampling.

On comparison data for seismic phenomena with the results of computer simulations for the glassy system it is visible, that in behaviour of correlation functions for EQ's nonergodic effects, characteristic for glass-like behaviour of dense systems are distinctly observed.

To illustrate the general picture of nonergodic singularities in chaotic seismic systems the parameter of nonergodicity  $f = \lim_{t \rightarrow \infty} c(t)$  for a set of seismic events has been calculated. Parameter of nonergodicity for a set of seismic

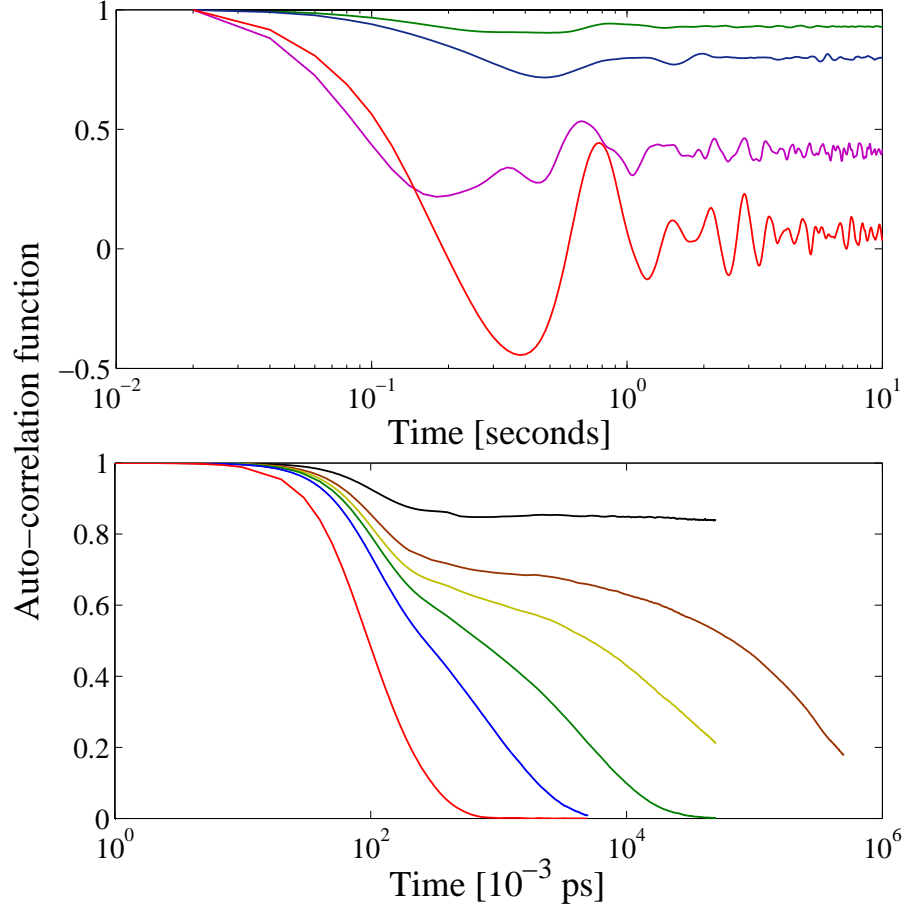


Fig. 7. (Color online) On top, auto-correlation functions of the seismic events *grsn*, *sgkt*, *uldt* and *seyt* in time log-scale. At the bottom: temperature dependence of incoherent scattering function  $F_s(k, t)$  for the *Cu* - component in metallic glass system  $Al_{50}Cu_{50}$ -alloy for the wave vector  $k = 3.05\text{\AA}^{-1}$  at the temperatures:  $T=2000$  K, 1000 K, 600 K, 500 K, 400 K, 200 K (bottom up).

events appeared equal: 0.9995 (*gdz*), 0.9995 (*kelt*), 0.99885 (*mack*), 0.9310 (*grsn*), 0.7241 (*sgkt*), 0.3965 (*uldt*), 0.0603 (*seyt*). The resulted data are evidence of strong singularity of seismic phenomena in 5 sources (*gdz*), (*kelt*), (*mack*), (*grsn*), (*sgkt*), they show moderate nonergodicity for a source (*uldt*) and weak singularity for a source (*seyt*). All taken together received data well speak about wide variety of effects of nonergodicity in the seismic phenomena. Similar variety of nonergodicity effects can be very useful and extremely effective for the classification of wide variety of seismic phenomena. Note, that the notions of fractality and non-Markovity have quite extensively been explored in the past, see, for example, [16,17,18,19].

## 4 Summary

In this work we have presented the results of statistical analysis of seismic signals in Turkey for 7 objects (*grsn*, *kelt*, *mack*, *sgkt*, *uldt*, *seyt*, *gdz*). Our study was made in the context of statistical theory on the discrete non-Markov processes, which is based on the Generalized Langevin Equation (GLE). It allows to take into account the effects of the statistical memory, metastability and space-time nonlocality. We have shown with the theory that all considered signals can be divided into three groups in order of breaking of the fractal behavior in high frequency zone of spectrum of non-Markovity parameter. Signals from A group (*kelt*, *gdz*) can be characterized by the pronounced fractality, signals from the B group (*grsn*, *sgkt*, *uldt*, *seyt*) can be characterized by the moderate fractality and signals from the C group (*mack*) correspond to weak fractality and powerful non-Markov processes. From the analysis of the time correlation function we can confidently certify hypothesis Abe [1] of nonergodic “glass-like nature” of seismic signals for Earth activity. On the other hand aforementioned testifies an information concerning a wide variety of metastability in seismic phenomena.

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