

# The “*Lorenz* gauge” is named in honour of Ludwig Valentin Lorenz!

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### **Abstract**

The letter reminds the historical fact that the known “Lorenz gauge” (or “Lorenz condition/relation”) is first mentioned in a written form by and named after Ludwig Valentin Lorenz and not by/after Hendrik Antoon Lorentz.

## 1. Introduction

Let  $\varphi$  and  $\mathbf{A}$  be respectively the scalar and vector potentials of the classical electromagnetic field [1]. Since they contain some arbitrariness, they can be connected via different relations, called gauges or gauge conditions/relations. A particular example of them being the “Lorenz gauge”, viz.

$$\nabla \cdot \mathbf{A} + \frac{1}{c} \frac{\partial \varphi}{\partial t} = 0,$$

where  $c$  is the velocity of light in vacua,  $\nabla := \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$  and  $x, y, z$  and  $t$  are the space and time coordinates, respectively. In invariant form, this gauge reads

$$\partial^\mu A_\mu = 0$$

with  $A_\mu$ ,  $\mu, \nu = 0, 1, 2, 3$  being the components of the 4-vector potential and  $\partial^\mu = \eta^{\mu\nu} \frac{\partial}{\partial x^\nu}$ , where  $\eta^{\mu\nu}$  are the components of the standard metric tensor of the Minkowski spacetime and  $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$ .

More generally, if  $A_\mu^a$ ,  $a = 1, \dots, n \in \mathbb{N}$ , are the 4-potential of an arbitrary classical gauge field, they can be subjected to the Lorenz relation in the form

$$\partial^\mu A_\mu^a = 0.$$

The importance of the Lorenz gauge comes from its relativistic invariance, from a simplification of many calculations in it, etc.

A partial analysis of the Lorenz gauge in quantum electrodynamics can be found in [2, ch. II, § 12] or [3, ch. 9, § 2] (see also [4, ch. I, § 5]).

## 2. The historical truth

The Lorenz condition/relation and gauge are named in honour of the Danish theoretical physicist Ludwig Valentin Lorenz (1829–1891), who has first published it in 1867 [5, 6] (see also [7, pp. 268–269, 291] and [8, 9]). However this condition was first introduced in lectures by Bernhard G. W. Riemann in 1861 as pointed in [7, p. 291].

It should be noted that the *Lorenz* condition/gauge is quite often erroneously referred as the *Lorentz* condition/gauge after the name of the Dutch theoretical physicist Hendrik Antoon Lorentz (1853–1928) as, e.g., in [10, p. 18], [11, p. 45] and [12, pp. 421–422, 426, 542].

The table below represents some results of searching over the Internet for “Lorenz gauge” and “Lorentz gauge.” We see that the situation is quite sad in favour of the wrong term, but there is a slight improvement during the last 3 years.

It is remarkable that in the book [13, p. 58, footnote 1] is mentioned the Lorenz (gauge) condition, but the author continues to call it Lorentz (gauge) condition.

The reasons for the error in referring to the “Lorenz gauge” as “Lorentz gauge” are explained and analyzed, for instance, in [14, pp. 670–671].

## 3. On the geometry of the gauge conditions

It is known that the gauge potentials of a gauge field, in particular of the electromagnetic one, are coefficients of a linear connection on a vector bundle from geometrical point of view [11, 15]. If a gauge field is given, its potentials are fixed in any frame of reference. For that reason, the imposition of some relations between them, in particular of the Lorenz gauge, leads to restrictions on the reference frames one can invoke. To be more precise, the class of frames in the total bundle space that can be used is narrowed so that the corresponding gauge relations to be satisfied.

Table 2.1: Number <sup>a</sup> of found search results for “Lorentz gauge” and “Lorenz gauge”

Web database <sup>b</sup>	Date	“Lorentz gauge”	“Lorenz gauge”	Ratio
arXiv full record (Physics)	<i>Jul 2005</i>	89	<b>17</b>	5.25
arXiv full record (Physics)	Feb 2008	85	<b>25</b>	3.40
Google	<i>Jul 2005</i>	13700	<b>558</b>	24.55
Google	Feb 2008	22600	<b>5500</b>	4.11
Google Scholar	<i>Jul 2005</i>	2640	<b>216</b>	12.22
Google Scholar	Feb 2008	5030	<b>499</b>	10.08
arXiv exp. full text (Physics)	Feb 2008	2273	<b>345</b>	6.59
arXiv exp. full text (Math.)	Feb 2008	133	<b>28</b>	4.75
Yahoo	Feb 2008	21700	<b>4460</b>	4.87
AOL	Feb 2008	6020	<b>944</b>	6.38
Ask	Feb 2008	1600	<b>658</b>	2.43

<sup>a</sup> The number of found results depends on many factors and may be variable even during one day.

<sup>b</sup>The URL of arXiv is <http://arxiv.org>, of Google is <http://www.google.com>, of Google Scholar is <http://scholar.google.com>, of Yahoo is <http://search.yahoo.com>, of AOL is <http://search.aol.com>, and of Ask is <http://www.ask.com>.

## 4. Appeal instead of a conclusion

The Lorenz gauge is in current usage and seems to be in use in future. For that reason, let us recognize the contribution of Ludwig Valentin Lorenz in this field of physics and restore the historical truth by terming this gauge/relation after him!

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