**Has the U.S. Economy Become More Stable?**
**A Bayesian Approach Based on a Markov-Switching**
**Model of Business Cycle**

by

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**Abstract**

We hope to be able to provide answers to the following questions: 1) Has there been a structural break in postwar U.S. real GDP growth toward more stabilization? 2) If so, when would it have been? 3) What's the nature of the structural break?

For this purpose, we employ a Bayesian approach to dealing with structural break at an unknown changepoint in a Markov-switching model of business cycle. Empirical results suggest that there has been a structural break in U.S. real GDP growth toward more stabilization, with the posterior mode of the break date around 1984:1. Furthermore, we find a narrowing gap between growth rates during recessions and booms is at least as important as a decline in the volatility of shocks.

**Key Words:** Bayes Factor, Gibbs sampling, Marginal Likelihood, Markov-Switching, Stabilization, Structural Break.

JEL Classifications: C11, C12, C22, E32.

## 1. Introduction

In the literature, the issue of postwar stabilization of the U.S. economy relative to the prewar period has mainly been delivered in the context of volatility stabilization or the duration stabilization. Burns (1960) and Diebold and Rudebusch (1992) are the representative examples. The results in these and other related papers, namely the postwar stabilization, are questioned by Romer (1986a, b, 1991) and Watson (1994) based on inconsistency of data between the two periods and the unreliability of prewar reference dates relative their postwar counterparts. The relative scarcity and poor quality of prewar data or unreliable prewar reference cycles seem to make the comparison of the two periods difficult.

This paper deviates from existing literature on the stabilization issue in at least three respects. First, we focus on the postwar period and avoid the problem of data inconsistency. A related question would be: Has there been a structural break in the postwar U.S. economy toward more stabilization? Second, unlike the comparison of the prewar and postwar periods, the date of the potential structural break is not assumed known. We ask: if there has been a structural break in the postwar U.S. economy, when would it have been? Third, unlike existing literature, we explicitly take into account the asymmetric nature of the business cycle in our analysis. That is, while McConnell and Quiros (1999) document a recent structural decline in the volatility of real GDP growth within a linear model, we additionally investigate the possibility of a structural break in real GDP toward narrowing gap between the growth rates during booms and recessions.

Parameter changes that are thought to be recurrent and endogenous have been modeled as Markov-switching processes in Goldfeld and Quandt (1973), Hamilton (1989), and Kim (1994). [1]. For example, Hamilton (1989) models recessions and booms as switches

[1] Goldfeld and Quandt (1973) introduced Markov-switching models for serially uncorrelated data, and Hamilton (1989) for serially correlated data. Kim (1994) extends the

in the growth rate of real output between high and low states, governed by a Markov process. However, we would also like to be able to detect non-recurrent changes that can be thought of as shifts in the hyper parameters of these models that are otherwise assumed fixed. We refer such changes as "structural." In this paper, we deal with the issues related to recent stabilization of the U.S. economy within the context of Hamilton's (1989) Markov-switching model of the business cycle, focusing on the possibility of one-time structural break in the mean growth rates of real output in each of the two unobserved regimes, expansion and recession, as well as the variance of the disturbance terms.

Section 2 presents model specifications and a Bayesian approach to making inferences. In Section 3, we present a procedure for comparing models under consideration. For this purpose, we build on ideas set forth in Chib (1995, 1998). In Section 4, we apply the proposed models and the model selection procedure to postwar U.S. real GDP to investigate whether the economy has become more stable during the postwar period. Section 5 concludes the paper and provides a suggestion for further research.

## 2. Modeling Structural Break in a Markov-Switching Model and the Problem Setup

Consider the following version of a Markov-switching model with a structural break in the hyper parameters:

$$\phi(L)(y_t - \mu^*_{S_t}) = e_t, \quad e_t \sim i.i.d.N(0, \sigma_t^2), \tag{1}$$

$$\mu_{S_t} = \mu^*_{0t}(1 - S_t) + \mu^*_{1t} S_t, \tag{2}$$

$$\mu^*_{0t} < \mu^*_{1t}, \tag{3}$$

approach to general state-space models. For more general treatment of the Markov-switching models, refer to Kim and Nelson (1999) where $y_t$ is the demeaned real output growth rate; $\mu^*_{0t}$ and $\mu^*_{1t}$ are the short run deviations of output growth from its long-run growth during recession and boom, respectively; roots of $\phi(L) = 0$ lie outside the complex unit circle; and $S_t$ is an unobserved two-state Markov-switching variable that evolves according to the transition probabilities given below:

$$Pr[S_t = 1|S_{t-1} = 1] = p_{11}, \tag{4}$$

$$Pr[S_t = 0|S_{t-1} = 1] = 1 - p_{11}, \tag{5}$$

$$Pr[S_t = 0|S_{t-1} = 0] = p_{00}, \tag{6}$$

$$Pr[S_t = 1|S_{t-1} = 0] = 1 - p_{00}, \tag{7}$$

$$0 < p_{00} < 1, \quad 0 < p_{11} < 1. \tag{8}$$

We consider a possibility that the two shift parameters, $\mu^*_{0t}$ and $\mu^*_{1t}$, as well as the variance of $e_t$, $\sigma^2_t$, are subject to one-time structural break with unknown changepoint ($\tau$). In order to incorporate this possibility, we specify these parameters as follows:

$$\mu^*_{0t} = \mu_0 + \mu_{00} D_t, \tag{9}$$

$$\mu^*_{1t} = \mu_1 + \mu_{11} D_t, \tag{10}$$

$$\mu_0 < \mu_0 + \mu_{00}, \quad \mu_1 > \mu_1 + \mu_{11} \tag{11}$$

$$\sigma^2_t = (1 - D_t)\sigma^2_0 + D_t \sigma^2_1, \tag{12}$$

$$\sigma^2_0 > \sigma^2_1, \tag{13}$$

where

$$D_t = 0 \ for \ 1 \leq t \leq \tau \ \ and \ \ D_t = 1 \ for \ \tau < t \leq T - 1, \tag{14}$$

$$D_1 = 0, \ D_T = 1, \tag{15}$$

and where $D_t$ is independent of $S_t$.

Like the latent variable $S_t$ that determines the recurrent business cycle phases, the latent variable $D_t$ can also be modeled as a two-state Markov process, as suggested by Chib (1998). This is done by appropriately constraining the transition probabilities so that we have one-time permanent shift from $D_\tau = 0$ to $D_{\tau+1} = 1$ at an unknown changepoint $\tau$. For example, the transition probabilities should be constrained such that, conditional on $D_t = 0$ there always exists non-zero probability that $D_{t+1}$ may be 1, but conditional on $D_{\tau+1} = 1$ the probability that $D_{\tau+2} = 0$ should always be 0, so that we have $D_t = 1$ for $t \geq \tau + 1$. The following specification for the transition probabilities achieves this goal:

$$Pr[D_t = 0|D_t = 0] = q_{00};\ \Pr[D_t = 1|D_t = 0] = 1 - q_{00}, \tag{16}$$

$$Pr[D_t = 1|D_t = 1] = 1;\ \Pr[D_t = 0|D_t = 1] = 0, \tag{17}$$

$$0 < q_{00} < 1, \tag{18}$$

where the expected duration of $D_t = 0$, or the expected duration of a regime before a structural break occurs, is given by $E(\tau) = \frac{1}{1-q_{00}}$.

Under the null hypothesis that there is no structural break in the real output growth, we have $\mu_{00} = \mu_{11} = 0$ and $\sigma_0^2 = \sigma_1^2$ and the above model collapses to the benchmark Hamilton model of business cycle. To investigate the nature of a potential structural break in the real output growth, we consider various null and alternative hypotheses, and they result in the following four competing models:

**Model I**: A Benchmark Markov-switching model with no structural Break [$\mu_{00} = \mu_{11} = 0$, $\sigma_1^2 = \sigma_2^2$]

**Model II**: A Model with Structural Break in the shift parameters [$\mu_{00} \neq 0$, $\mu_{11} \neq 0$, $\sigma_1^2 = \sigma_2^2$]

**Model III**: A Model with Structural Break in the variance [$\mu_{00} = 0$, $\mu_{11} = 0$, $\sigma_1^2 \neq \sigma_2^2$]

**<u>Model IV</u>**: A Model with Structural Break in both the shift parameters and the variance [$\mu_{00} \neq 0$, $\mu_{11} \neq 0$, $\sigma_1^2 \neq \sigma_2^2$]

The standard test for a structural break at a known date is the Chow (1960) test. When the date, called changepoint ($\tau$), is not known, it could be treated as a parameter to be estimated, but it is a nuisance parameter that exists only under the alternative hypothesis that structural change occurred. Thus, the problem is non-standard, and though Quandt (1958, 1960) proposed a test with unknown changepoint, the classical asymptotic results do not hold for the test statistic. Only recently have asymptotic theories for appropriate test statistics been investigated in the classical framework: the 'supremum' tests of Andrews (1993) and 'average' and 'exponential' tests of Andrews and Ploberger (1994) and Andrews, Lee, and Ploberger (1992). [2] In testing for the null hypothesis of a linear model against the alternative hypothesis of a Markov-switching model, the transition probabilities are the nuisance parameters that exist only under the alternative model. Such tests for Markov switching has recently been proposed by Hansen (1992b) and Garcia (1998) within the classical framework.

As we specify one-time structural break in the hyper-parameters of the model by treating $D_t$ as a Markov-switching process, the transition probability $q_{00}$ is a nuisance parameter that does not exist under the null hypothesis of no structural break. We could

[2] Other related parametric approaches to such tests in the classical framework include James, James, and Siegmund (1987), Hawkins (1987), and Kim and Siegmund (1989). Nonparametric approaches to tests of parameter instability with unknown changepoint in the classical framework include the CUSUM test by Brown, Durbin, and Evans (1975) and fluctuation tests by Ploberger, Kramer, and Kontrus (1989). Banerjee et al. (1992), Zivot and Andrews (1992), Chu and White (1992), and Hansen (1992a) consider an unknown changepoint with nonstationary regressors. For more a comprehensive survey, refer to Maddala and Kim (1996).

therefore apply the testing procedure proposed by Hansen (1992b) and Garcia (1998). However, we cast the problem into a Bayesian framework in order to take advantage of the important features of the Bayesian approach. First, unlike the classical approach, the nuisance parameters that exists only under the alternative but not under the null does not pose any special problem. The main issue in a Bayesian model selection comes down to calculating the marginal likelihood for each model under consideration and the resulting Bayes factors, and these are obtained by integrating the nuisance parameters out of the joint density, whether they exist only under the alternative or under both hypotheses.

Second, the hierarchical nature of the models under consideration allows us to easily employ the Markov chain Monte Carlo (MCMC) integration method of Gibbs sampling in obtaining the marginal likelihoods and the Bayes factors, or marginal posterior distributions of interest for inference.

Third, test results within the Bayesian framework embody sample information about the distribution of the unknown changepoint. In addition, as a byproduct of the Bayesian test of structural change we get the posterior distribution of the unknown changepoint, which would be the ultimate goal of the research given structural change. Within the classical framework, on the contrary, sample information is not appropriately used in integrating the changepoint out of an appropriate test statistic, as Koop and Potter (1996) noted. For example, Andrews and Ploberger (1994) consider an average of the test statistic over different values of the the changepoint. Alternatively, they consider an arbitrary distribution to integrate the changepoint out of the test statistic. The resulting classical test statistics fail to appropriately include sample information about the changepoint.

The next section deals with Bayesian inferences of the model and the Bayesian model selection procedure.

## 3. Bayesian Inference and Model Selection Procedure

### 3.1. Bayesian Inference of the Model

For Bayesian inference of the model, given appropriate priors we need the marginal posterior distributions for the followings: $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]'$; $\tilde{\phi} = [\,\phi_1 \quad \dots \quad \phi_k\,]'$; $\tilde{\sigma^2} = [\,\sigma_0^2 \quad \sigma_1^2\,]'$; $\tilde{D}_T = [\,D_1 \quad \dots \quad D_T\,]'$; $\tilde{S}_T = [\,S_1 \quad \dots \quad S_T\,]'$; $\tilde{p} = [\,p_{00} \quad p_{11}\,]'$; and $q_{00}$. These marginal posterior distributions may be obtained from the joint posterior distribution,

$$p(\tilde{\mu}, \tilde{\phi}, \tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{p}, q_{00} | \tilde{Y}_T), \tag{19}$$

where $\tilde{Y}_T = [\,y_1 \quad \dots \quad y_T\,]'$.

However, the hierarchical nature of the model allows us to easily employ Gibbs sampling in obtaining the marginal posterior distributions of interest. This is done by successively sampling from the full conditional densities. The following describes the Gibbs sampling procedure:

i) Generate $\tilde{\mu}$ from $p(\tilde{\mu}|\tilde{\phi}, \tilde{\sigma}^2, \tilde{D}_T, \tilde{S}_T, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$ and $\tilde{S}_T$, $\tilde{\mu}$ is independent of $\tilde{p}$ and $q_{00}$;

ii) Generate $\tilde{\phi}$ from $p(\tilde{\phi}|\tilde{\mu}, \tilde{\sigma}^2, \tilde{D}_T, \tilde{S}_T, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$ and $\tilde{S}_T$, $\tilde{\phi}$ is independent of $\tilde{p}$ and $q_{00}$;

iii) Generate $\tilde{\sigma}^2$ from $p(\tilde{\sigma}^2|\tilde{\mu}, \tilde{\phi}, \tilde{D}_T, \tilde{S}_T, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$ and $\tilde{S}_T$, $\tilde{\sigma^2}$ is independent of $\tilde{p}$ and $q_{00}$;

iv) Generate $\tilde{D}_T$ from $p(\tilde{D}_T|\tilde{\mu}, \tilde{\phi}, \tilde{\sigma}^2, \tilde{S}_T, q_{00}, \tilde{Y}_T)$, where, conditional on $\tilde{S}_T$, $\tilde{D}_T$ is independent of $\tilde{p}$;

v) Generate $\tilde{S}_T$ from $p(\tilde{S}_T|\tilde{\mu}, \tilde{\phi}, \tilde{\sigma}^2, \tilde{D}_T, \tilde{p}, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$, $\tilde{S}_T$ is independent of $q_{00}$;

vi) Generate $\tilde{p}$ from $p(\tilde{p}|\tilde{S}_T)$, where, conditional on $\tilde{S}_T$, $\tilde{p}$ is independent of the other variates;

vii) Generate $q_{00}$ from $p(q_{00}|\tilde{D}_T)$, where, conditional on $\tilde{D}_T$, $q_{00}$ is independent of the other variates.

Notice that the above procedure is a straightforward extension of Albert and Chib's (1993) Bayes inference via Gibbs sampling of autoregressive time series subject to Markov mean and variance shifts. Also notice that as a byproduct of generating $\tilde{D}_T$ in iv), we can get the marginal posterior distribution of the changepoint, $\tau$, such that $D_1 = \ldots = D_\tau = 0$ and $D_{\tau+1} = \ldots = D_T = 0$.

### 3.2. Model Comparison: Calculating the Marginal Likelihood

Let $\omega$ be the model indicator parameter. Thus, when $\omega = i$, $i = 1, 2, 3, 4$, we assume that data $\tilde{Y}_T$ have arisen from **Model i** defined in Section 2, according to a probability function (marginal likelihood) $m(\tilde{Y}_T|\omega = j)$. Within the Bayesian framework, the Bayes factor has been widely used for model comparison. It is defined as the ratio of marginal likelihoods for models under consideration:

$$B_{ij} = \frac{m(\tilde{Y}_T|\omega = i)}{m(\tilde{Y}_T|\omega = j)}, \quad i, j = 1, 2, 3, 4; \; i \neq j, \tag{20}$$

where $B_{ij}$ refers to the Bayes factor in favor of **Model i**.

Various ways of Bayesian model comparison or calculating the Bayes factor have been proposed in the literature. For example, Carlin and Polson (1991), George and McCulloch (1993), Geweke (1996), and Carlin and Chib (1995) provide a procedure for model comparison based on the sensitivity of the posterior probability of the model indicator parameter $\omega$ to the prior probability. Verdinelli and Wasserman (1995) and Koop and Potter (1999) suggest a way to indirectly calculating the Bayes factor using the 'Savage-Dickey' density ratio for the nested models. Alternatively, Chib (1995) suggests a procedure for directly calculating the marginal likelihoods based on the Gibbs output. [3]

In this section, we present a procedure for directly calculating the marginal likelihoods for models under our consideration, by extending Chib's (1995) procedure in a straightforward way. As the other three models are nested within **Model IV** in Section 2, the procedure is described only within the context of **Model IV**. Thus, the model indicator parameter $\omega$ is suppressed throughout the discussion.

Define $\tilde{\theta} = [\,\tilde{\mu}' \quad \tilde{\phi}' \quad \tilde{p}' \quad q_{00} \quad \tilde{\sigma^2}' \quad \tilde{D}_T' \quad \tilde{S}_T'\,]'$ to be a vector of the parameters of the model. Then, as in Chib (1995) the marginal density of $\tilde{Y}_T = [\,y_1 \quad \ldots \quad y_T\,]'$, by virtue of being the normalizing constant of the posterior density, can be written as:

$$m(\tilde{Y}_T) = \frac{f(\tilde{Y}_T|\tilde{\theta})\pi(\tilde{\theta})}{\pi(\tilde{\theta}|\tilde{Y}_T)}, \tag{21}$$

where the numerator is the product of the sampling density and the prior, with all integrating constants included, and the denominator is the posterior density of $\tilde{\theta}$. As the above identity holds for any $\tilde{\theta}$, we may evaluate $m(\tilde{Y}_T)$ at the posterior mean $\tilde{\theta}^*$. Taking the logarithm of the above equation for computational convenience, we have:

$$ln\ m(\tilde{Y}_T) = ln\ f(\tilde{Y}_T|\tilde{\theta}^*) + ln\ \pi(\tilde{\theta}^*) - ln\ \pi(\tilde{\theta}^*|\tilde{Y}_T) \tag{22}$$

The log likelihood function and the log of the prior density at $\tilde{\theta} = \tilde{\theta}^*$ can be evaluated relatively easily. First, the log likelihood function is given by:

$$ln\ f(\tilde{Y}_T|\tilde{\theta}^*) = \sum_{t=k+1}^{T} ln(\sum_{V_t=1}^{4} \ldots \sum_{V_{t-k}=1}^{4} p(V_t,\ldots,V_{t-k}|\tilde{Y}_{t-1},\tilde{\theta}^*) f(y_t|\tilde{Y}_{t-1},V_t,\ldots,V_{t-k},\tilde{\theta}^*)), \tag{23}$$

where $V_t = 1$ if $S_t = 0$ and $D_t = 0$; $V_t = 2$ if $S_t = 0$ and $D_t = 1$; $V_t = 3$ if $S_t = 1$ and $D_t = 0$; and $V_t = 4$ if $S_t = 1$ and $D_t = 1$. Second, the log of prior density is given by:

[3] For a general discussion of Bayesian model comparison and the issues related to the calculation of the Bayes factors, readers are referred to Kass and Raftery (1995).

$$ln\ \pi(\tilde{\theta}^*) = ln\ \pi(\tilde{\mu}^*) + ln\ \pi(\tilde{\phi}^*) + ln\ \pi(\tilde{\sigma^2}^*) + ln\ \pi(\tilde{p}^*, q_{00}^*), \tag{24}$$

where it is a priori assumed that $\tilde{\mu}$, $\tilde{\phi}$, $\tilde{\sigma^2}$, $\tilde{p}$, and $q_{00}$ are independent of one another.

Evaluation of the posterior density at $\tilde{\theta} = \tilde{\theta}^*$ is more demanding, but we can take advantage of the approach proposed by Chib (1995). For this purpose, consider the following decomposition of the posterior density:

$$\pi(\tilde{\theta}^*|\tilde{Y}_T) = \pi(\tilde{\mu}^*|\tilde{Y}_T)\pi(\tilde{\phi}^*|\tilde{\mu}^*, \tilde{Y}_T)\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{Y}_T)\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{Y}_T), \tag{25}$$

where

$$\begin{aligned}&\pi(\tilde{\mu}^*|\tilde{Y}_T)\\ &= \int \pi(\tilde{\mu}^*, |\tilde{\phi}, \tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{p}, q_{00}, \tilde{Y}_T)\pi(\tilde{\phi}, \tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{p}, q_{00}|\tilde{Y}_T)d\tilde{\phi}d\tilde{\sigma^2}d\tilde{D}_Td\tilde{S}_Td\tilde{p}dq_{00},\end{aligned} \tag{26}$$

$$\begin{aligned}&\pi(\tilde{\phi}^*|\tilde{\mu}^*, \tilde{Y}_T)\\ &= \int \pi(\tilde{\phi}^*|\tilde{\mu}^*, \tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{p}, q_{00}, \tilde{Y}_T)\pi(\tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{p}, q_{00}, |\tilde{\mu}^*, \tilde{Y}_T)d\tilde{\sigma^2}d\tilde{D}_Td\tilde{S}_Td\tilde{p}dq_{00},\end{aligned} \tag{27}$$

$$\begin{aligned}&\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{Y}_T)\\ &= \int \pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{D}_T, \tilde{S}_T, \tilde{p}, q_{00}, \tilde{Y}_T)\pi(\tilde{D}_T, \tilde{S}_T|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{p}, q_{00}, \tilde{Y}_T)d\tilde{D}_Td\tilde{S}_Td\tilde{p}dq_{00},\end{aligned} \tag{28}$$

and

$$\begin{aligned}&\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)\\ &\qquad = \int \pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*\tilde{D}_T, \tilde{S}_T, \tilde{Y}_T)\pi(\tilde{D}_T, \tilde{S}_T|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)d\tilde{D}_Td\tilde{S}_T\end{aligned} \tag{29}$$

The above decomposition of the posterior density suggests that $\pi(\tilde{\mu}^*|\tilde{Y}_T)$ can be calculated based on draws from the full Gibbs run, and $\pi(\tilde{\phi}^*|\tilde{\mu}^*, \tilde{Y}_T)$, $\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{Y}_T)$, and $\pi(p_{00}^*, p_{11}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)$ can be calculated based on draws from the reduced Gibbs runs. The following explains how each of these can be calculated based on output from appropriate Gibbs runs:

$$\hat{\pi}(\tilde{\mu}^*|\tilde{Y}_T) = \frac{1}{G}\sum_{g=1}^{G}\pi(\tilde{\mu}^*, |\tilde{\phi}^g, \tilde{\sigma^2}^g, \tilde{D}_T^g, \tilde{S}_T^g, \tilde{p}^g, q_{00}^g, \tilde{Y}_T), \tag{30}$$

$$\hat{\pi}(\tilde{\phi}^*|\tilde{\mu}^*, \tilde{Y}_T) = \frac{1}{G}\sum_{g_1=1}^{G}\pi(\tilde{\phi}^*|\tilde{\mu}^*, \tilde{\sigma^2}^{g_1}, \tilde{D}_T^{g_1}, \tilde{S}_T^{g_1}, \tilde{p}^{g_1}, q_{00}^{g_1}, \tilde{Y}_T), \tag{31}$$

$$\hat{\pi}(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{Y}_T) = \frac{1}{G}\sum_{g_2=1}^{G}\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{D}_T^{g_2}, \tilde{S}_T^{g_2}, \tilde{p}^{g_2}, q_{00}^{g_2}, \tilde{Y}_T), \tag{32}$$

$$\begin{aligned}\hat{\pi}(p_{00}^*, p_{11}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, &\tilde{\sigma^2}^*, \tilde{Y}_T) \\ &= \frac{1}{G}\sum_{g_3=1}^{G}\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{D}_T^{g_3}, \tilde{S}_T^{g_3}, \tilde{Y}_T),\end{aligned} \tag{33}$$

where the superscript $g$ refers to the $g-th$ draw of the full Gibbs run and the superscript $g_i$, $i = 1, 2, 3$, refers to the $g_i - th$ draw from the appropriate reduced Gibbs runs. Thus, apart from the usual $G$ iterations for the full Gibbs run, we need additional $3 \times G$ iterations for the appropriate reduced Gibbs run. In order to calculate $\pi(p_{00}^*, p_{11}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)$, for example, we need output from an additional $G$ iterations for the following reduced Gibbs run: i) Generate $\tilde{p}$ and $q_{00}$ from $p(\tilde{p}, q_{00}|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{D}_T, \tilde{S}_T, \tilde{Y}_T)$; ii) Generate $\tilde{D}_T$ from $p(\tilde{D}_T|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{S}_T, \tilde{p}, q_{00}, \tilde{Y}_T)$; iii) Generate $\tilde{S}_T$ from $p(\tilde{S}_T|\tilde{\mu}^*, \tilde{\phi}^*, \tilde{\sigma^2}^*, \tilde{D}_T, \tilde{p}, q_{00}\tilde{Y}_T)$. Notice that throughout the reduced Gibbs run, $\tilde{\mu}$, $\tilde{\phi}$, and $\tilde{\sigma^2}$ are not generated and they are set equal to $\tilde{\mu}^*$, $\tilde{\phi}^*$, $\tilde{\sigma^2}^*$, respectively.

## 4. Empirical Results: Has the U.S. Economy Become More Stable?

### 4.1. Data

Data employed is quarterly U.S. real GDP growth that covers the sample period of 1953.II - 1997.I. In order to take into account the post-1973 'great productivity slowdown', the pre-1973 subsample and the post-1973 subsample have been demeaned separately. However, it would be worth while to mention that the empirical results were not qualitatively different from the case in which the post-1973 productivity slowdown was not taken into account.

### 4.2. Prior Specifications

We employ Normal priors for $\tilde{\mu}$ and $\tilde{\phi}$; inverted Gamma distributions for $\sigma_0^2$ and $\sigma_1^2$; and finally, Beta distributions for $p_{00}$, $p_{11}$, and $q_{00}$. In order to analyze the sensitivity of the empirical results to prior specifications for the parameters of the model, we employ the following three alternative sets of priors: [4]

**<u>Prior #1:</u>** $\phi_1 \sim N(0,4)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', 4I_4)$; $1/\sigma_i^2 \sim Gamma(1,1)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(4,1)$; $q_{00} \sim Beta(80,0.1)$

**<u>Prior #2:</u>** $\phi_1 \sim N(0,2)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', 2I_4)$; $1/\sigma_i^2 \sim Gamma(2,2)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(9,1)$; $q_{00} \sim Beta(80,0.1)$

**<u>Prior #3:</u>** $\phi_1 \sim N(0,1)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', I_4)$;
$1/\sigma_i^2 \sim Gamma(4,4)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(9,1)$; $q_{00} \sim Beta(80,0.1)$,

where $Beta(.,.)$ and $Gamma(.,.)$ refer to the Beta distribution and Gamma distribution, respectively.

### 4.3. Results

[4] We employ an AR(1) specification for the autoregressive parameter, as in McConnell and Quiros (1999). Thus, $\tilde{\phi} = \phi_1$. In addition, the priors are described in terms of Model IV.

We first compare Bayesian inferences from the four alternative models considered in Section 2. Throughout this section, all inferences are based on 10,000 Gibbs simulations, after discarding the initial 2,000 Gibbs simulations in order to mitigate the effects of initial conditions. Tables I-IV summarize the marginal likelihoods as well as the prior moments and the posterior moments of the parameters for each of the models. Notice that the prior moments correspond to Prior #1 in Section 4.2. A comparison of the log of marginal likelihoods suggests that the benchmark Hamilton model with no structural break (Model I) is clearly dominated by the other three models with a structural break. Furthermore, in Tables II and IV (for Models II and IV), a decrease in the posterior mean of the growth rate during booms ($\mu_1$ versus $\mu_1 + \mu_{11}$) is sizable, even though an increase in the posterior mean of the growth rate during recessions ($\mu_0$ versus $\mu_0 + \mu_{00}$) is less so. In Tables III and IV (for Models II and IV), we also notice that a decrease in the posterior mean for the variance of the disturbance terms ($\sigma_0^2$ versus $\sigma_1^2$) is sizable. Thus, we conclude that there has been a structural break in the U.S. economy toward more stabilization: a narrowing gap between the mean growth rates during booms and recessions and a decline in the volatility of real GDP growth.

To further investigate the nature of the structural break in real GDP growth, we need to compare Models II, III, and IV. Model II with a structural break in the mean growth rates is most preferred, suggesting that a narrowing gap between the mean growth rates during booms and recessions is at least as important as a decline in the volatility of real GDP growth. [5] Notice that within the context of a linear model, a narrowing gap between the mean growth rates would show up as a decline in the volatility. Thus,

[5] By just comparing Models III and IV, one may be more inclined to infer that a structural break in the mean growth rates is not as important as a decline in the volatility, as is the case with McConnell and Quiros (1999). However, this does not seem to be the case.

within the linear model, one may not be able to distinguish between the two sources of stabilization in real GDP growth.

In Figures 2.A, 3.A, and 4.A, the posterior distributions of the changepoint ($\tau$) for Models II, III, IV are depicted against real GDP growth. In all three cases, the posterior mode of the Changepoint is 1984.I, as in McConnell and Quiros (1999), even though the posterior distribution is more widely spread for model II than for the other models.

Inferences about recession probabilities from alternative models may provide us with further insights into the nature of the structural break in real GDP growth. Figure 1 depicts recession probabilities from the benchmark Hamilton model (Model I) with no structural break. Figure 2.B depicts recession probabilities from Model II with a structural break in the shift parameters. For the period before 1984:I, inferences about recession probabilities are much sharper in Figure 2.B than in Figure 1. However, for the period after 1984:1, they are worse in Figure 2.B than in Figure 1. As we allow for a structural break only in the variance term in Figure 3.B (Model III), such pattern is reversed. That is, for the period before 1984:I, inferences about recession probabilities are much worse in Figure 3.B than in Figure 1. However, for the period after 1984:1, they are sharper in Figure 3.B than in Figure 1. These observations suggest that a structural break in neither the shift parameters nor the variance terms may be ignored. When we allow for a structural break in the shift parameters as well as the variance terms (Model IV), inferences about recession probabilities in Figure 4.B improve over those in Figure 1. However, the improvement seems to be only marginal. This explains why the log of marginal likelihood for Model IV is lower than that for Model II or Model III.

Within the Bayesian framework, inferences are sometimes dependent upon the priors employed. In order to check the robustness of our results, we try two more alternative sets of priors as described in Section 4.2 [Prior #2 and Prior #3]. [6] Table 5 summarizes

[6] We also checked whether the results are sensitive to different priors for the $q_{00}$ param-

the sensitivity of the marginal likelihood (in log scale) to alternative priors employed. Even though the marginal likelihood for Models I, II, IV are not very sensitive to the priors employed, the marginal likelihood for Model III seems to be quite sensitive. In general, Table 5 suggests the followings: First, Model II with a structural break in the shift parameters is always most preferred regardless of the priors employed. Second, depending on the priors employed, either Model III or Model IV is second most preferred. Third, depending on the priors employed, either Model I or Model III is least preferred. The sensitivity analysis leads us to conclude that, out of the two important sources of stabilization in recent U.S. real GDP growth, namely a narrowing gap between the mean growth rates during recessions and booms and a decline in volatility, sample evidence is stronger for the former.

## 5. Summary and Conclusion

In this paper, we characterize the nature of recent structural break in U.S. real GDP growth toward more stabilization. For this purpose, we present a Bayesian approach to dealing with structural break at an unknown changepoint in a Markov-switching model of business cycle. Within the context of a Markov-switching model, one can distinguish between two important sources of stabilization in real GDP growth: a decline in the variance of shocks and a narrowing gap between growth rates during booms recessions. Within the context of a linear model, one cannot distinguish between the two sources, and a narrowing gap between growth rates would show up as a decline in volatility.

Empirical results suggest both sources of stabilization may not be ignored, even though we find stronger sample evidence in favor of a narrowing gap between growth rates during booms and recessions. In addition, the posterior mode of the break date turned out to be

eter. However, the results were not sensitive.

1984:I. This is consistent with McConnell and Quiros (1999), who document a structural decline in the volatility of U.S. real GDP growth within the context of a linear model and within a classical framework.

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**Table 1. Bayesian Inference [Model I: Benchmark Hamilton Model with No Structural Break]**

| | Prior | | Posterior | | | |
|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | MD | 90% Bands |
| $\phi_1$ | 0 | 2 | 0.258 | 0.105 | 0.262 | (0.077, 0.420) |
| $\sigma_0^2$ | 1 | 1 | 0.776 | 0.136 | 0.771 | (0.570, 1.007) |
| $\sigma_1^2$ | – | – | – | – | – | – |
| $\mu_0$ | -0.5 | 2 | -0.817 | 0.620 | -0.800 | (-1.811, -0.042) |
| $\mu_1$ | 0 | 2 | 0.297 | 0.343 | 0.234 | (0.035, 0.666) |
| $\mu_0 + \mu_{00}$ | – | – | – | – | – | – |
| $\mu_1 + \mu_{11}$ | – | – | – | – | – | – |
| $p_{11}$ | 0.8 | 0.163 | 0.840 | 0.152 | 0.899 | (0.504, 0.975) |
| $p_{00}$ | 0.8 | 0.163 | 0.706 | 0.161 | 0.711 | (0.429, 0.968) |
| $q_{00}$ | – | – | – | – | – | – |

**Log of Marginal Likelihood ($ln\ m(\tilde{Y}_T)$): -267.63**

1. SD and MD refer to standard deviation and median, respectively.
2. 90% Bands refers to 90% posterior probability bands.
3. Out of 12,000 Gibbs simulations, the first 2,000 are discarded and inferences are based on the remaining 10,000 Gibbs simulations.
4. Prior Distributions employed:
$\phi_1 \sim N(0,4)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1\,]' \sim N((-0.5 \quad 0)', 4I_2)$; $1/\sigma_0^2 \sim Gamma(1,1)$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(4,1)$

**Table 2. Bayesian Inference [Model II: Structural Break in the Shift Parameters]**

| | Prior | | Posterior | | | |
|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | MD | 90% Bands |
| $\phi_1$ | 0 | 2 | 0.210 | 0.105 | 0.212 | (0.035, 0.382) |
| $\sigma_0^2$ | 1 | 1 | 0.688 | 0.121 | 0.670 | (0.516, 0.914) |
| $\sigma_1^2$ | – | – | – | – | – | – |
| $\mu_0$ | -0.5 | 2 | -1.052 | 0.395 | -1.025 | (-1.736, -0.422) |
| $\mu_1$ | 0 | 2 | 0.559 | 0.378 | 0.486 | (0.201, 1.111) |
| $\mu_0 + \mu_{00}$ | -0.5 | $\sqrt{8}$ | -0.274 | 0.338 | -0.195 | (-0.959, 0.127) |
| $\mu_1 + \mu_{11}$ | 0 | $\sqrt{8}$ | 0.153 | 0.262 | 0.137 | (-0.159, 0.476) |
| $p_{11}$ | 0.8 | 0.163 | 0.819 | 0.119 | 0.850 | (0.579, 0.945) |
| $p_{00}$ | 0.8 | 0.163 | 0.706 | 0.132 | 0.713 | (0.468, 0.911) |
| $q_{00}$ | 0.988 | 0.0013 | 0.984 | 0.028 | 0.993 | (0.942, 0.999) |

**Log of Marginal Likelihood ($ln\ m(\tilde{Y}_T)$): -247.01**

1. SD and MD refer to standard deviation and median, respectively.
2. 90% Bands refers to 90% posterior probability bands.
3. Out of 12,000 Gibbs simulations, the first 2,000 are discarded and inferences are based on the remaining 10,000 Gibbs simulations.
4. Prior Distributions employed:
$\phi_1 \sim N(0,4)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', 4I_4)$; $1/\sigma_0^2 \sim Gamma(1,1)$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(4,1)$; $q_{00} \sim Beta(80,0.1)$;

**Table 3. Bayesian Inference [Model III: Structural Break in the Variance]**

| | Prior | | Posterior | | | |
|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | MD | 90% Bands |
| $\phi_1$ | 0 | 2 | 0.298 | 0.098 | 0.308 | (0.120, 0.447) |
| $\sigma_0^2$ | 1 | 1 | 1.151 | 0.168 | 1.142 | (0.888, 1.438) |
| $\sigma_1^2$ | 1 | 1 | 0.259 | 0.063 | 0.252 | (0.167, 0.372) |
| $\mu_0$ | -0.5 | 2 | -0.446 | 0.518 | -0.218 | (-1.401, -0.016) |
| $\mu_1$ | 0 | 2 | 0.234 | 0.384 | 0.143 | (0.017, 0.730) |
| $\mu_0 + \mu_{00}$ | – | – | – | – | – | – |
| $\mu_1 + \mu_{11}$ | – | – | – | – | – | – |
| $p_{11}$ | 0.8 | 0.163 | 0.809 | 0.174 | 0.870 | (0.447, 0.986) |
| $p_{00}$ | 0.8 | 0.163 | 0.752 | 0.165 | 0.765 | (0.455, 0.985) |
| $q_{00}$ | 0.988 | 0.0013 | 0.992 | 0.008 | 0.994 | (0.977, 0.999) |

**Log of Marginal Likelihood ($ln\ m(\tilde{Y}_T)$): -253.70**

1. SD and MD refer to standard deviation and median, respectively.
2. 90% Bands refers to 90% posterior probability bands.
3. Out of 12,000 Gibbs simulations, the first 2,000 are discarded and inferences are based on the remaining 10,000 Gibbs simulations.
4. Prior Distributions employed:

$\phi_1 \sim N(0,4)$; $\tilde{\mu} = [\mu_0 \quad \mu_1]' \sim N((-0.5 \quad 0)', 4I_2)$; $1/\sigma_i^2 \sim Gamma(1,1),\ i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(4,1)$; $q_{00} \sim Beta(80, 0.1)$

**Table 4. Bayesian Inference [Model IV: Structural Break in Both the Shift Parameters and the Variance]**

| | Prior | | Posterior | | | |
|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | MD | 90% Bands |
| $\phi_1$ | 0 | 2 | 0.245 | 0.120 | 0.254 | (0.040, 0.425) |
| $\sigma_0^2$ | 1 | 1 | 1.015 | 0.192 | 1.003 | (0.717, 1.344) |
| $\sigma_1^2$ | 1 | 1 | 0.254 | 0.064 | 0.246 | (0.164, 0.371) |
| $\mu_0$ | -0.5 | 2 | -0.910 | 0.565 | -0.881 | (-1.892, -0.130) |
| $\mu_1$ | 0 | 2 | 0.387 | 0.323 | 0.327 | (0.087, 0.811) |
| $\mu_0 + \mu_{00}$ | -0.5 | $\sqrt{8}$ | -0.413 | 0.432 | -0.295 | (-1.240, 0.092) |
| $\mu_1 + \mu_{11}$ | 0 | $\sqrt{8}$ | 0.126 | 0.224 | 0.097 | (-0.078, 0.371) |
| $p_{11}$ | 0.8 | 0.163 | 0.847 | 0.137 | 0.895 | (0.543, 0.978) |
| $p_{00}$ | 0.8 | 0.163 | 0.717 | 0.145 | 0.733 | (0.451, 0.939) |
| $q_{00}$ | 0.988 | 0.0013 | 0.992 | 0.008 | 0.994 | (0.976, 0.999) |

**Log of Marginal Likelihood ($ln\ m(\tilde{Y}_T)$): -260.19**

1. SD and MD refer to standard deviation and median, respectively.
2. 90% Bands refers to 90% posterior probability bands.
3. Out of 12,000 Gibbs simulations, the first 2,000 are discarded and inferences are based on the remaining 10,000 Gibbs simulations.
4. Prior Distributions employed:

$\phi_1 \sim N(0,4)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', 4I_4)$; $1/\sigma_i^2 \sim Gamma(1,1)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(4,1)$; $q_{00} \sim Beta(80, 0.1)$

**Table 5. Sensitivity of the Log of Marginal Likelihood to Alternative Priors**

| | Prior #1 | Prior #2 | Prior #3 |
|---|---|---|---|
| Model I | -267.63 | –265.81 | -266.67 |
| Model II | -247.01 | -246.09 | -245.25 |
| Model III | -253.70 | -289.94 | -256.77 |
| Model IV | -260.19 | -257.76 | -258.97 |

1. Prior Distributions employed:

Prior #1: $\phi_1 \sim N(0,4)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', 4I_4)$; $1/\sigma_i^2 \sim Gamma(1,1)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(4,1)$; $q_{00} \sim Beta(80,0.1)$

Prior #2: $\phi_1 \sim N(0,2)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', 2I_4)$; $1/\sigma_i^2 \sim Gamma(2,2)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(9,1)$; $q_{00} \sim Beta(80,0.1)$

Prior #3: $\phi_1 \sim N(0,1)$; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \mu_{00} \quad \mu_{11}\,]' \sim N((\,-0.5 \quad 0 \quad 0 \quad 0\,)', I_4)$; $1/\sigma_i^2 \sim Gamma(4,4)$, $i = 0,1$; $p_{00} \sim Beta(4,1)$; $p_{11} \sim Beta(9,1)$; $q_{00} \sim Beta(80,0.1)$

Figure 1 Probability of a Recession from Model I [Benchmark Hamilton Model with No Structural Break]

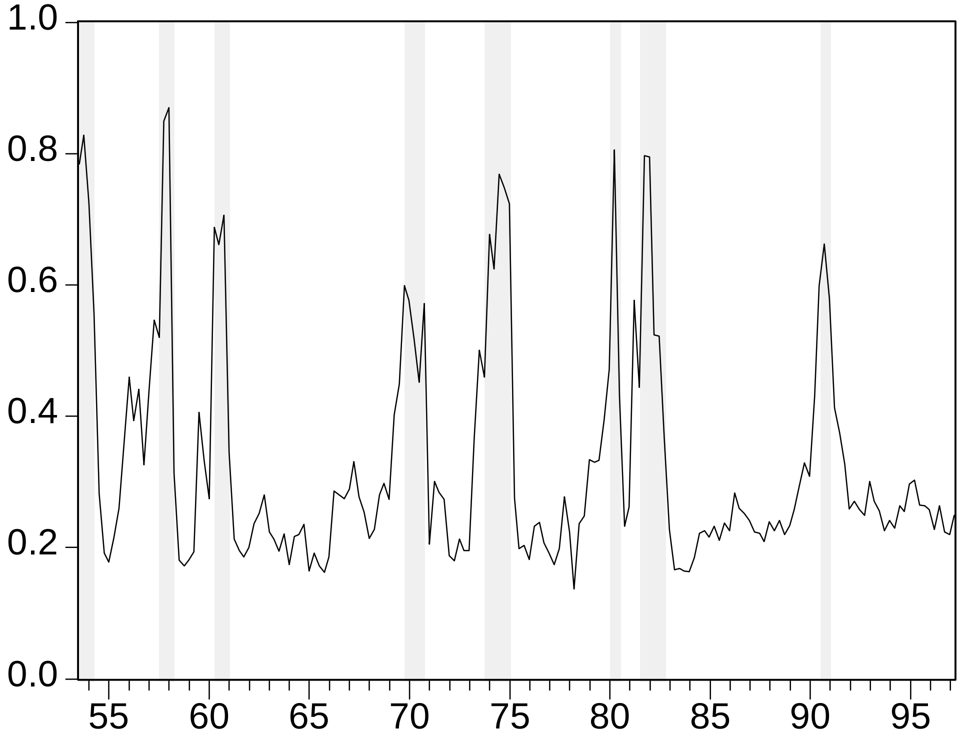

Figure 2.A. Real GDP Growth and Posterior Distribution of Changepoint from Model II
[Structural Break in the Shift Parameters]

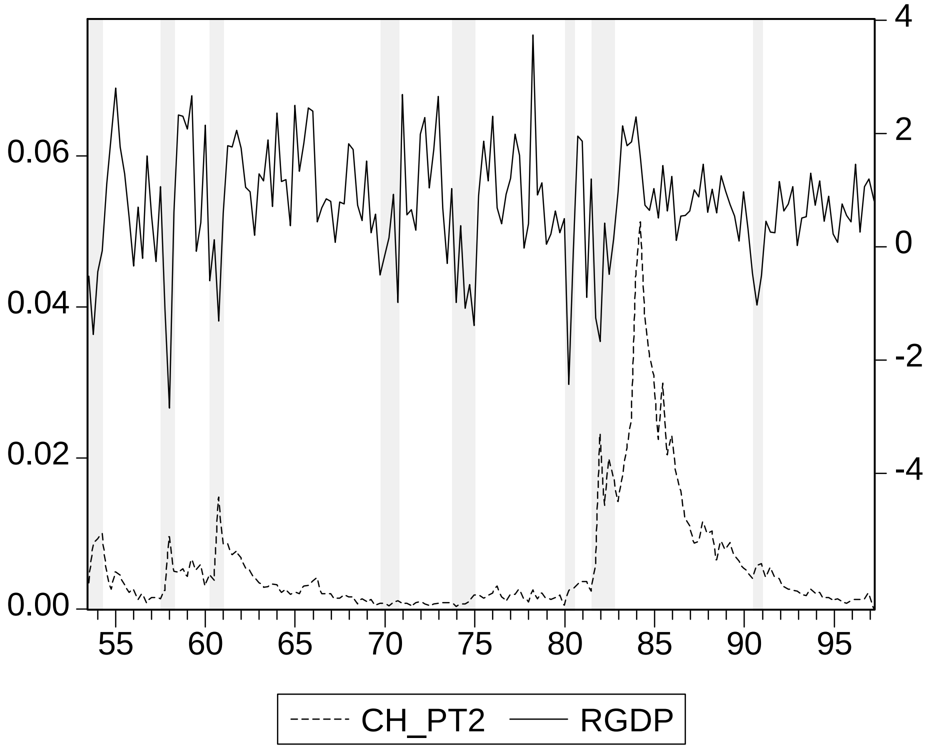


Figure 2.B. Probability of a Recession from Model II [Structural Break in the Shift Parameters ]

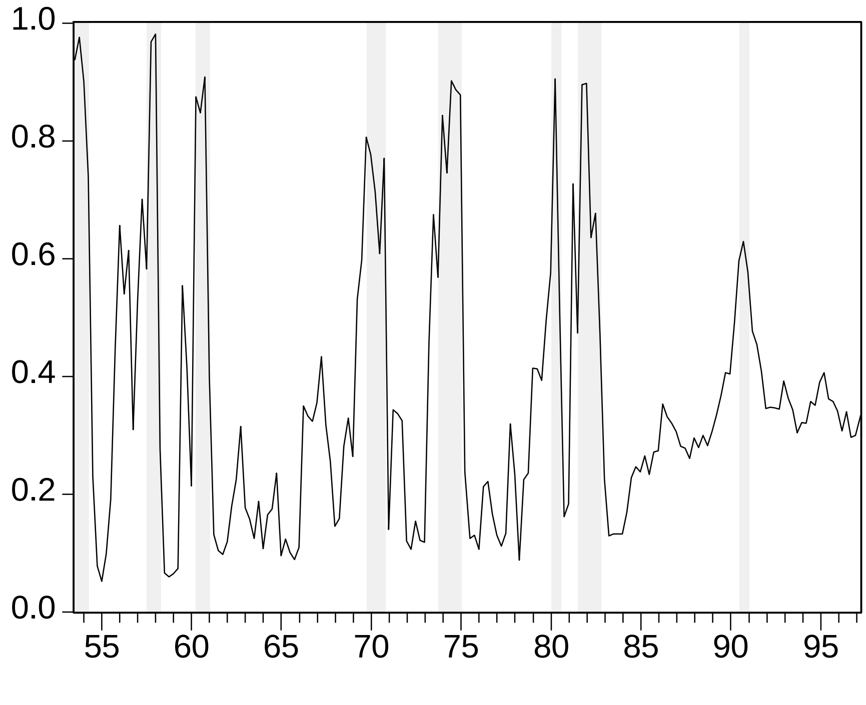

Figure 3.A. Real GDP Growth and Posterior Distribution of Changepoint from Model III [Structural Break in the Variance]

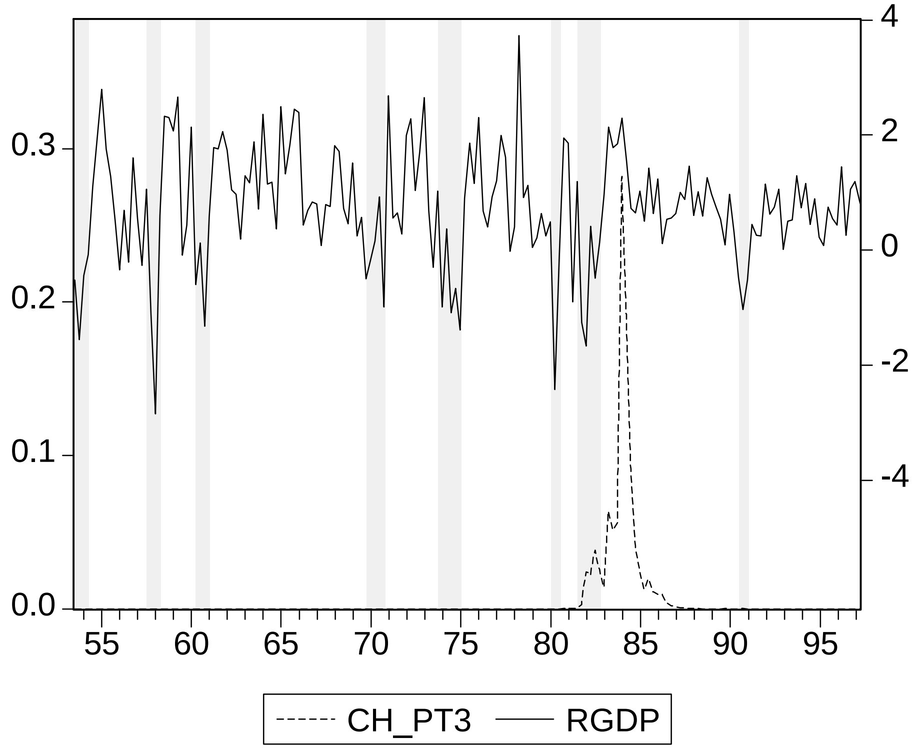


Figure3.B. Probability of a Recession from Model III [Structural Break in the the Variance ]

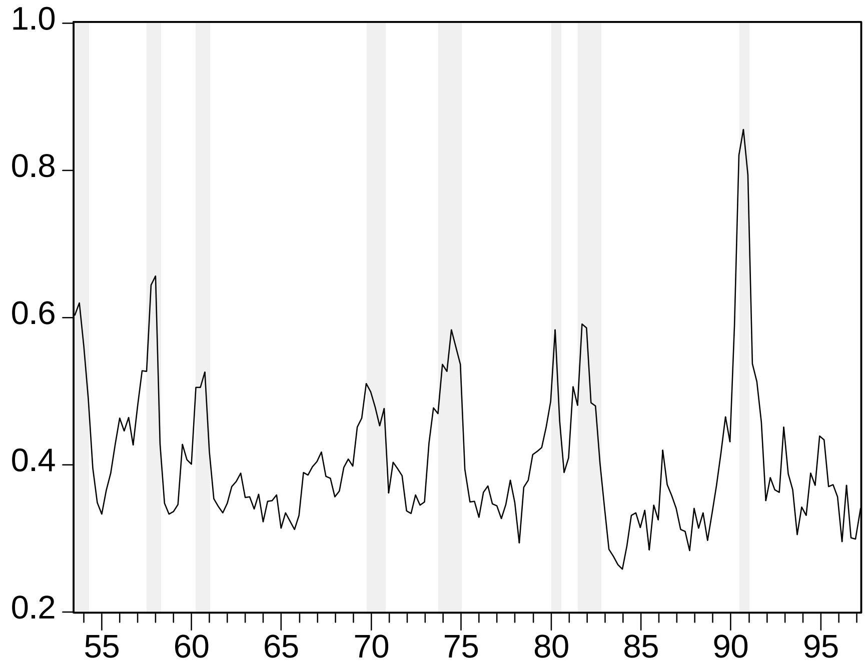

Figure 4.A. Real GDP Growth and Posterior Distribution of Changepoint from Model IV
[Structural Break in both the Shift Parameters and  the Variance]

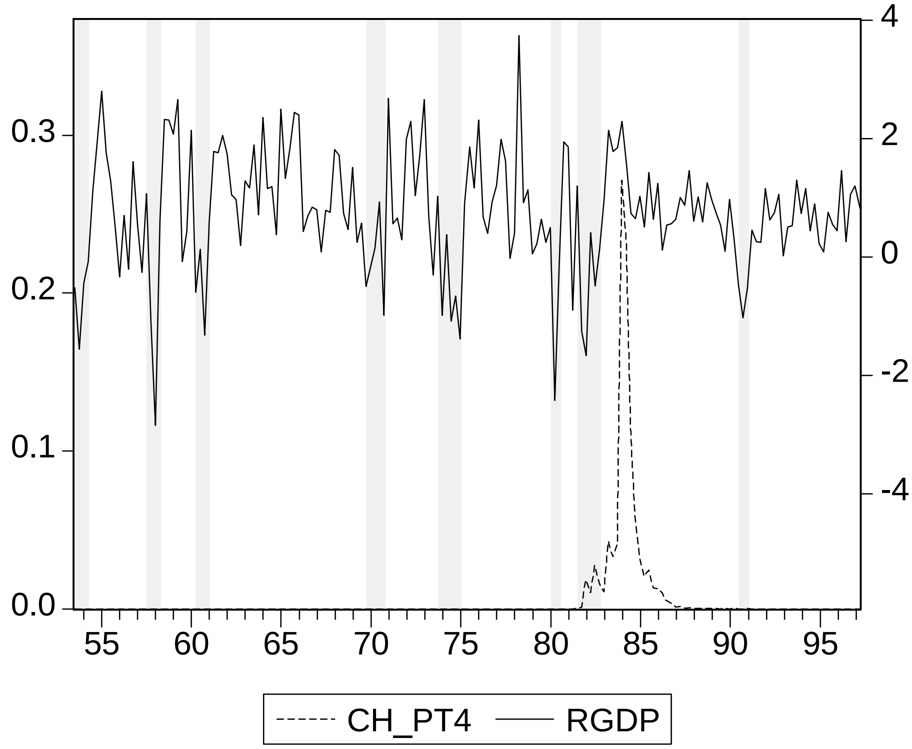


Figure4.B. Probability of a Recession from Model IV   [Structural Break in both the Shift Parameters and the  Variance  ]