List of axioms and axiom systems

oldalszámokat ellenőrizni!

Convention: In this list the axiom systems (i.e. theories) to be recalled will be boxed in. The only purpose of this is to make searching in the list easier.

(1) Main axiom systems

$\boxed{\textbf{Basax}} \stackrel{\text{def}}{=} \{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}, \mathbf{Ax4}, \mathbf{Ax5}, \mathbf{Ax6}, \mathbf{AxE} \,\}$ (cf. p.51), where:

Ax1 $G = \mathsf{Eucl}(n, \mathbf{F})$, p.45.

Ax2 $Obs \cup Ph \subseteq Ib$, p.48.

Ax3 $(\forall h \in Ib)(\forall m \in Obs)\ tr_m(h) \in G$, p.48.

Ax4 $(\forall m \in Obs)\ tr_m(m) = \bar{t}$, p.48.

Ax5 $(\forall m \in Obs)(\forall \ell \in G) \Big(ang^2(\ell) < 1 \ \Rightarrow\ (\exists k \in Obs)\, \ell = tr_m(k)$ and

$$ang^2(\ell) = 1 \ \Rightarrow\ (\exists ph \in Ph)\, \ell = tr_m(ph)\Big),\ \text{p.50.}$$

Ax6 $(\forall m, k \in Obs)\ Rng(w_m) = Rng(w_k)$, p.50.

AxE $(\forall m \in Obs)(\forall ph \in Ph)\ v_m(ph) = 1$, p.51.

$\boxed{\textbf{Newbasax}} \stackrel{\text{def}}{=} (\mathbf{Basax} \setminus \{\, \mathbf{Ax6}, \mathbf{Ax3}, \mathbf{AxE} \,\}) \cup \{\, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}}, \mathbf{Ax3_0}, \mathbf{AxE_0} \,\} = \{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{Ax4}, \mathbf{Ax5}, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}}, \mathbf{AxE_0} \,\}$ (cf. p.191), where:

$\mathbf{Ax6_{00}}$ $(\forall m, k \in Obs)\ w_m[tr_m(k)] \subseteq Rng(w_k)$, p.190.

Intuitively, observer k sees all those events which are seen by another observer m on k's life-line.

$\mathbf{Ax6_{01}}$ $(\forall m, k \in Obs)\ Dom(\mathsf{f}_{mk}) \in Open$, p.190.

$\mathbf{Ax3_0}$ $(\forall h \in Ib)\ (tr_m(h) \in G \cup \{\emptyset\} \ \wedge\ (\exists k \in Obs) tr_k(h) \neq \emptyset)$, p.191.

$\mathbf{AxE_0}$ $(\forall m \in Obs)(\forall ph \in Ph)(tr_m(ph) \neq \emptyset \ \Rightarrow\ v_m(ph) = 1)$, p.191.

$\boxed{\mathbf{Bax}} \stackrel{\text{def}}{=} (\mathbf{Newbasax} \setminus \{\, \mathbf{Ax5}, \mathbf{AxE_0} \,\}) \cup \{\, \mathbf{Ax5^{Obs}}, \mathbf{Ax5^{Ph}}, \mathbf{AxE_{00}}, \mathbf{AxE_{01}} \,\} =$
$\{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{Ax4}, \mathbf{Ax5^{Obs}}, \mathbf{Ax5^{Ph}}, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}}, \mathbf{AxE_{00}}, \mathbf{AxE_{01}} \,\}$ (cf. p.219), where:

$\mathbf{Ax5^{Obs}}$ $(\exists ph)(\forall \ell)\Big(m \stackrel{\odot}{\to} ph \ \wedge\ [ang^2(\ell) < v_m(ph) \ \Rightarrow\ (\exists k)\ell = tr_m(k)]\Big)$, p.218

$\mathbf{Ax5^{Ph}}$ $ang^2(\ell) = v_m(ph) \ \Rightarrow\ (\exists ph)\ell = tr_m(ph)$, p.219.

$\mathbf{AxE_{00}}$ $(m \stackrel{\odot}{\to} ph_1, ph_2) \ \Rightarrow\ v_m(ph_1) = v_m(ph_2)$, p.218.

$\mathbf{AxE_{01}}$ $v_m(ph) \neq 0$, p.218.

$\boxed{\mathbf{Flxbasax}} \stackrel{\text{def}}{=} \mathbf{Bax} + \mathbf{AxE_{02}} =$
$\{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{Ax4}, \mathbf{Ax5^{Obs}}, \mathbf{Ax5^{Ph}}, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}}, \mathbf{AxE_{00}}, \mathbf{AxE_{01}}, \mathbf{AxE_{02}} \}$ (cf. p.428), where:

$\mathbf{AxE_{02}}$ $(\forall m, k \in Obs)(\forall ph, ph_1 \in Ph)$
$[(m \stackrel{\odot}{\to} ph \ \wedge\ k \stackrel{\odot}{\to} ph_1) \ \Rightarrow\ v_m(ph) = v_k(ph_1)]$, p.427.

$\boxed{\mathbf{Bax^-}} \stackrel{\text{def}}{=} (\mathbf{Bax} \setminus \{\, \mathbf{Ax5^{Obs}}, \mathbf{Ax5^{Ph}}, \mathbf{AxE_{00}} \,\}) \cup \{\, \mathbf{Ax5_{Obs}}, \mathbf{Ax5_{Ph}}, \mathbf{AxP1} \,\} =$
$\{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{Ax4}, \mathbf{Ax5_{Obs}}, \mathbf{Ax5_{Ph}}, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}}, \mathbf{AxP1}, \mathbf{AxE_{01}} \,\}$ (cf. p.479), where:

AxP1 Intuitively, starting out from one point p of space-time, in every direction (forwards) there is at most one "speed of light" (i.e. photon-trace), formally:

$(\forall m \in Obs)(\forall ph_1, ph_2 \in Ph)(\forall d \in \mathsf{directions})$[1256] $\Big((ph_1$ and ph_2 are moving forwards in direction d as seen by m and $tr_m(ph_1) \cap tr_m(ph_2) \neq \emptyset) \ \Rightarrow$
$tr_m(ph_1) = tr_m(ph_2)\Big)$, p.472.

$\mathbf{Ax5_{Ph}}$ Intuitively, from any point p of space-time in any direction there is a photon moving forwards in that direction, cf. Fig.138 (p.477), formally:

$(\forall m \in Obs)(\forall p \in {}^nF)(\forall d \in \mathsf{directions})(\exists ph \in Ph)$
$[p \in tr_m(ph) \ \wedge\ (ph$ is moving forwards in direction d as seen by $m)]$, p.477.

[1256] Let us recall that directions are (nonzero) space-vectors, i.e. $\mathsf{directions} = {}^{n-1}F \setminus \{\bar{0}\}$, cf. p.470.

Ax5$_{\mathbf{Obs}}$ Intuitively: Let us fix an observer m, a direction d, and a point p of space-time. We will speak about things moving forwards in direction d through point p as seen by m (without mentioning all this data). Assume there is a photon moving in direction d. Then there is a photon in the same direction which is limiting in the following sense: For all speeds slower than this limiting photon, there is an observer moving with this speed, cf. Fig.139 (p.478). Formally:

$(\forall m \in Obs)(\forall p \in {}^nF)(\forall d \in \mathsf{directions})$
$\Big(\Big[(\exists ph \in Ph)(p \in tr_m(ph) \ \wedge \ (ph \text{ is moving forwards in } d \text{ as seen by } m)\Big] \ \Rightarrow$
$\Big[(\exists ph \in Ph)\Big(p \in tr_m(ph) \ \wedge \ (ph \text{ is moving forwards in } d \text{ as seen by } m) \ \wedge$
$(\forall \lambda \in F)(0 \leq \lambda < v_m(ph) \ \Rightarrow \ (\exists k \in Obs)(p \in tr_m(k) \ \wedge \ v_m(k) = \lambda \ \wedge$
$(k \text{ is moving forwards in direction } d \text{ as seen by } m))\Big)\Big]\Big)$, p.477.

$\boxed{\mathbf{Pax}} \stackrel{\text{def}}{=} \{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{Ax4}, \mathbf{Ax5_{Obs}}^{--}, \mathbf{Ax6_{00}}, \mathbf{Ax6_{01}} \,\}$ (cf. p.482) where:

§4-ben is kijavítani az intuitív szöveget! esetleg a formulaban is P síkot használni!

Ax1, Ax2, Ax3$_0$, Ax4, Ax6$_{00}$, Ax6$_{01}$ have already been listed.

Ax5$_{\mathbf{Obs}}^{--}$ Intuitively, for each direction d there is a positive λ such that through any point there are observers moving forwards in direction d with all speeds smaller than λ. More precisely, for any observer m and for any plane P parallel with $\bar{t}$ there is $\lambda \in {}^+F$ such that for any straight line ℓ in P, with $ang^2(\ell) < \lambda$, ℓ is the trace of an observer (as seen by m, of course). In other words:

$(\forall m \in Obs)(\forall d \in \mathsf{directions})(\forall p \in {}^nF)(\exists \lambda \in {}^+F)$
$(\forall q \in {}^nF)\Big[space(p) - space(q) = \delta \cdot d \text{ for some } \delta \in F \ \Rightarrow \ (\forall 0 \leq \varepsilon < \lambda)$
$(\exists k \in Obs)(k \text{ moves forwards in direction } d \text{ with speed } \varepsilon \text{ and } q \in tr_m(k))\Big]$, p.481.

* * *

Assume **Ax1**, **Ax2**, **Ax3$_0$**, **AxP1**. Let $m \in Obs$. Then

$$c_m : {}^nF \times \mathsf{directions} \stackrel{\circ}{\longrightarrow} F \cup \{\infty\}$$

is a partial function such that $c_m(p, d)$ is defined iff m sees a photon at point p moving forwards in direction d, <u>and</u> $c_m(p, d)$ is the speed of this photon,[1257] cf. pp. 473, 535.

[1257] There is only one such speed because of **AxP1**.

Let Th be one of our theories such that $Th \models \{\mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{AxP1}\}$. Then $\boxed{Th^{\oplus}} \stackrel{\text{def}}{=} Th + c_m(p,d) < \infty$, p.643.

Next, we turn to listing the Reichenbachian versions of our theories. For this we recall some notation.

Assume $\mathbf{Bax^-}$. By Thm.4.3.17 (p.488), the speed $c_m(p,d)$ does not depend on p. This motivates the following:

$$\mathsf{c}_m(d) \stackrel{\text{def}}{=} c_m(\bar{0}, d),$$

cf. p.488. Intuitively, $\mathsf{c}_m(d)$ is the (square of the) speed of light in direction d as seen by observer m.

speed of light vagy square of the speed of light?

<u>*Notation:*</u> Let $m \in Obs$ and $d \in \mathsf{directions}$. Then

$$T_m(d) \stackrel{\text{def}}{=} \begin{cases} 1/\sqrt{\mathsf{c}_m(d)} & \text{if } 0 \neq \mathsf{c}_m(d) < \infty, \\ \infty & \text{if } \mathsf{c}_m(d) = 0, \\ 0 & \text{if } \mathsf{c}_m(d) = \infty, \end{cases}$$

cf. p.555. $T_m(d)$ is the reciprocal of the "speed of light", i.e. it is the time needed for a photon to cover the unit distance in direction d (as seen by observer m).

$\boxed{\mathbf{Reich_0(Bax)}} \stackrel{\text{def}}{=} \mathbf{Bax^-} + \mathbf{R(AxE_{00})}$ (cf. p.562), where

$\mathbf{R(AxE_{00})}$ $\mathbf{Ax}(\sqrt{\ })$ and
$(\forall d, d_1 \in \mathsf{directions})[T_m(d) + T_m(-d) = T_m(d_1) + T_m(-d_1)]$, p.557.

$\boxed{\mathbf{Reich_0(Flxbasax)}} \stackrel{\text{def}}{=} \mathbf{Bax^-} + \mathbf{R(AxE_{02})}$ (cf. p.562), where

$\mathbf{R(AxE_{02})}$ $(\forall m, k \in Obs)(\forall d, d_1 \in \mathsf{directions})$
$T_m(d) + T_m(-d) = T_k(d_1) + T_k(-d_1)$, and $\mathbf{Ax}(\sqrt{\ })$, p.557.

$\boxed{\mathbf{Reich_0(Newbasax)}} \stackrel{\text{def}}{=} \mathbf{Bax^-} + \mathbf{R(AxE)}$ (cf. p.562), where

$\mathbf{R(AxE)}$ $\mathbf{Ax}(\sqrt{\ })$ and
$(\forall m \in Obs)(\forall d \in \mathsf{directions}) T_m(d) + T_m(-d) = 2$, p.557.

$\boxed{\mathbf{Reich_0(Basax)}} \stackrel{\text{def}}{=} \mathbf{Reich_0(Newbasax)} + \mathbf{Ax6}$, p.562.

Let $Th \in \{\mathbf{Bax}, \mathbf{Flxbasax}, \mathbf{Newbasax}, \mathbf{Basax}\}$. Then
$\boxed{\mathbf{Reich}(Th)} \stackrel{\text{def}}{=} \mathbf{Reich_0}(Th) + \mathbf{R}_\Delta(E)$ (cf. p.576), where

$\mathbf{R}_\Delta(E)$ $\quad (\forall m \in Obs)(\exists r \in F)(\forall d_1, d_2, d_3 \in \mathsf{directions})\Big[d_1 + d_2 + d_3 = \bar{0} \;\Rightarrow$

$$\frac{|d_1| \cdot T_m(d_1) + |d_2| \cdot T_m(d_2) + |d_3| \cdot T_m(d_3)}{|d_1| + |d_2| + |d_3|} = r\Big], \text{ p.574.}$$

(2) Axioms concerning the direction of flow of time

The binary relation $\uparrow \subseteq Obs \times Obs$ is defined as follows.

$$m \uparrow k \;\overset{\text{def}}{\Longleftrightarrow}\; (\mathsf{f}_{km}(1_t)_t - \mathsf{f}_{km}(\bar{0})_t > 0), \text{ p.296.}$$

Intuitively, $m \uparrow k$ means that m sees k's clock running forwards. Further, if $m, k \in Obs$ then $m\ STL\ k$ means that m sees k moving slower than light (cf. Def.4.2.6 on p.460).

$\mathbf{Ax(\uparrow)}$ $\quad (\forall m, m' \in Obs)\ (\ tr_m(m') = \bar{t} \quad \Rightarrow \quad m \uparrow m'\ )$, p.296.

$\mathbf{Ax(\uparrow\uparrow)}$ $(\forall m, k \in Obs) m \uparrow k$, p.426.

$\mathbf{Ax(\uparrow\uparrow_0)}$ $(\forall m, k \in Obs)\,(m \overset{\odot}{\to} k \;\to\; m \uparrow k)$, p.790.

$\mathbf{Ax(\uparrow\uparrow_{00})}$ $(\forall m, k \in Obs)\,(m\ STL\ k \;\to\; m \uparrow k)$, p.790.

(3) Auxiliary axioms

Recall that

$$Triv = \{\, f \;:\; f \text{ is an isometry of } {}^n\mathbf{F} \text{ and } f(1_t) - f(\bar{0}) = 1_t \,\},$$

cf. p.135.

$\mathbf{Ax}(Triv)$ $\quad (\forall m \in Obs)(\forall f \in Triv)(\exists k \in Obs)\ \mathsf{f}_{mk} = f$, p.135.

$\mathbf{Ax}(Triv_t)$ $\quad (\forall m \in Obs)(\forall f \in Triv)\ \Big(f[\bar{t}\,] = \bar{t} \quad \Rightarrow \quad (\exists k \in Obs)\ \mathsf{f}_{mk} = f\Big)$, p.135.

$\mathbf{Ax}(Triv_t)^-$ Assume we are given an observer m and a $Triv$ transformation f that leaves the time-axis fixed. Then m has a brother, call it k, such that m thinks that (i) the coordinate axes of k are the f-images of the original coordinate axes $\bar{x}_i$, and (ii) the clock of k runs forwards, formally:

$(\forall m \in Obs)(\forall f \in Triv)\,[\,f[\bar{t}\,] = \bar{t} \quad \Rightarrow$
$\qquad (\exists k \in Obs)(\forall i \in n)(\,\mathsf{f}_{km}[\bar{x}_i] = f[\bar{x}_i] \;\;\wedge\;\; m \uparrow k\,)\,]$, p.762.

Ax(||) $(\forall m, k \in Obs)\Big(tr_m(k) \parallel \bar{t} \quad \Rightarrow \quad (\mathsf{f}_{mk} \text{ is an isometry})\Big)$, p.136.

Ax(||)$^-$ $(\forall m, k \in Obs \cap Ib)$
$[\, tr_m(k) = \bar{t} \quad \Rightarrow \quad (\mathsf{f}_{mk} = h \circ I, \quad \text{for some expansion } h \text{ and isometry } I)\,]$, p.778.

Ax($\sqrt{\ }$) $(\forall 0 < x \in F)(\exists y \in F)\ y^2 = x$, p.91.

Ax(rc) (Axiom schema for real-closed fields)

Ax($\sqrt{\ }$) $+ \{\, \phi_{2n+1} \ :\ n \in \omega \,\}$, where

$(\phi_n) \qquad \forall x_0 \ldots \forall x_n \exists y\, [x_n \neq 0 \ \rightarrow\ (x_0 + x_1 \cdot y + \ldots + x_n \cdot y^n = 0)]$,

p.301.

Ax(diswind) (Axiom of disjoint windows)

$(\forall m, k \in Obs \cap Ib)\ [(m \overset{\odot}{\rightarrow} ph \ \wedge\ k \overset{\odot}{\rightarrow} ph) \ \Rightarrow\ m \overset{\odot}{\rightarrow} k]$, p.762.

(4) Axioms concerning measuring distances

Ax(eqtime) Observers with common life-line agree on time-like distances, i.e.
$(\forall m, m' \in Obs)$
$\Big(tr_m(m') = \bar{t} \quad \Rightarrow \quad (\forall p, q \in \bar{t}\,)\ |p - q| = |\mathsf{f}_{mm'}(p) - \mathsf{f}_{mm'}(q)|\Big)$, p.127.

Ax(eqspace) Observers agree on spatial distances, i.e.
$(\forall m, k \in Obs)(\forall p, q \in {}^nF)$
$\Big(\,(p_t = q_t \ \wedge\ \mathsf{f}_{mk}(p)_{\,t} = \mathsf{f}_{mk}(q)_{\,t}) \quad \Rightarrow \quad |p - q| = |\mathsf{f}_{mk}(p) - \mathsf{f}_{mk}(q)|\,\Big)$,
p.136.

Ax(eqm) Inertial observers agree on distances, i.e.
$(\forall m, k \in Obs \cap Ib)(\forall i, j \in n)(\forall p, q \in \bar{x}_i)(\forall p', q' \in \bar{x}_j)$
$\Big([w_m(p) = w_k(p') \ \wedge\ w_m(q) = w_k(q')] \ \Rightarrow\ |p - q| = |p' - q'|\Big)$, p.746.

(5) Axiom systems $\mathbf{Pax}^{+}$, $\mathbf{Pax}^{++}$, $\mathbf{Pax}^{+}_{+}$, $\mathbf{Pax}^{++}_{+}$, $\mathbf{Wax}$, $\mathbf{Wax}^{+}$

Ax(Bw) $(\forall m, k \in Obs)[m \overset{\odot}{\to} k \;\Rightarrow\; (\mathsf{f}_{mk}$ is betweenness preserving$)\,]$, p.978.

Ax♡ $B = Obs \cup Ph$, p.296.

Ax(∞ph) $(\forall m \in Obs)(\forall ph, ph' \in Ph)\Big([\,\bar{0} \in tr_m(ph) \cap tr_m(ph') \;\wedge\; (\,ph$ and ph' move in the same direction as seen by $m\,) \wedge v_m(ph) = \infty\,] \;\to\; v_m(ph') = \infty\Big)$, p.978.

sorrend?

Intuitively, no observer can emit simultaneously in the same direction two photons one with infinite speed and the other one with finite speed.

Ax(ext) $(\forall m, k \in Obs)\,[w_m = w_k \;\Rightarrow\; m = k] \;\wedge$
$(\forall b, b_1 \in B \setminus Obs)(\forall m \in Obs)\,[tr_m(b) = tr_m(b_1) \;\Rightarrow\; b = b_1]$, p.298.

Ax(Ph) $(\forall m \in Obs)(\forall p \in {}^{n}F)(\exists ph_1, ph_2 \in Ph)\, tr_m(ph_1) \cap tr_m(ph_2) = \{p\}$, p.1023.

$\boxed{\mathbf{Pax}^{+}} \overset{\text{def}}{=} \mathbf{Pax} + \mathbf{AxE_{01}} + \mathbf{Ax(Bw)} + \mathbf{Ax}(\infty ph) +$
$\Big([\,\mathbf{Ax(eqtime)} \;\wedge\; (\forall m, k \in Obs)(\forall 0 < i \in \omega)\, tr_m(k) \neq \bar{x}_i\,] \quad\vee\quad \mathbf{Ax(eqm)}\Big)$, p.979.

$\boxed{\mathbf{Pax}^{++}} \overset{\text{def}}{=} \mathbf{Pax}^{+} + \mathbf{Ax(eqm)} + \mathbf{Ax(ext)} + \mathbf{Ax}\heartsuit$, p.1031.

$\boxed{\mathbf{Pax}^{+}_{+}} \overset{\text{def}}{=} \mathbf{Pax}^{+} + \mathbf{Ax(diswind)}$, p.1036.

$\boxed{\mathbf{Pax}^{++}_{+}} \overset{\text{def}}{=} \mathbf{Pax}^{++} + \mathbf{Ax(diswind)}$, p.1043.

$\boxed{\mathbf{Wax}} \overset{\text{def}}{=} \{\,\mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}, \mathbf{Ax4}, \mathbf{Ax6}, \mathbf{Ax(Bw)}, \mathbf{Ax}(Ph)\,\}$, p.1023.

$\boxed{\mathbf{Wax}^{+}} \overset{\text{def}}{=} \mathbf{Wax} + \mathbf{Ax(ext)} + \mathbf{Ax}\heartsuit + \mathbf{Ax}(\infty ph) + (\forall m, k)(\mathsf{f}_{mk} \in Aftr)$, p.1031.

(6) Symmetry axioms

Ax(symm$_0$) $(\forall m,k \in Obs)(\exists m',k' \in Obs)$
$$\Big(tr_m(m') = tr_k(k') = \bar{t} \quad \wedge \quad \mathsf{f}_{mk} = \mathsf{f}_{k'm'}\Big),$$ p.124.

Ax(symm) is defined to be **Ax(symm$_0$)** + **Ax(eqtime)**, p.127.

Ax(syt$_0$) $(\forall m,k \in Obs)\Big(tr_m(k) \neq \emptyset \quad \Rightarrow$
$$(\forall p \in \bar{t}\,)\ |\mathsf{f}_{mk}(p)_{\,t} - \mathsf{f}_{mk}(\bar{0})_{\,t}| = |\mathsf{f}_{km}(p)_{\,t} - \mathsf{f}_{km}(\bar{0})_{\,t}|\Big),$$ p.134.

Ax(syt$_{00}$) $(\forall m,k \in Obs)\ [\mathsf{f}_{mk}(\bar{0}) = \bar{0} \ \Rightarrow\ |\mathsf{f}_{mk}(1_t)_t| = |\mathsf{f}_{km}(1_t)_t|]$, p.138.

Ax(syt)* $\mathsf{f}_{mk}(\bar{0}) = \bar{0} \ \Rightarrow\ \mathsf{f}_{mk}(1_t)_t = \mathsf{f}_{km}(1_t)_t$, p.671.

Ax(syx)* (m,k are in pre-standard configuration[1258]) $\Rightarrow$ $|\mathsf{f}_{mk}(1_x)_x| = |\mathsf{f}_{km}(1_x)_x|$, p.675.

Ax(speedtime) $(\forall m,k,m',k' \in Obs)\ \Big(v_m(k) = v_{m'}(k') \quad \Rightarrow$
$$(\forall p \in \bar{t}\,)\ |\mathsf{f}_{mk}(p)_{\,t} - \mathsf{f}_{mk}(\bar{0})_{\,t}| = |\mathsf{f}_{m'k'}(p)_{\,t} - \mathsf{f}_{m'k'}(\bar{0})_{\,t}|\Big),$$ p.137.

Ax$\Box$1 $(\forall m,k,m' \in Obs)(\exists k' \in Obs)\mathsf{f}_{mk} = \mathsf{f}_{m'k'}$, p.350.

Ax$\Box$2 $(\forall m,k,m',k' \in Obs)(tr_m(k) = tr_{m'}(k') \rightarrow$
there is an isometry N of ${}^n\mathbf{F}$ such that $N[\bar{t}\,] \parallel \bar{t}$ and $\mathsf{f}_{mk} = \mathsf{f}_{m'k'} \circ N$), p.350.

Ax$\triangle$1 $(\forall m,k \in Obs)(\exists k' \in Obs)(tr_m(k) = tr_m(k') \ \wedge\ \mathsf{f}_{mk'} = \mathsf{f}_{k'm})$, p.351.

Ax$\triangle$2 $(\forall m,k \in Obs)$ (there is an isometry N of ${}^n\mathbf{F}$ such that $N[\bar{t}\,] \parallel \bar{t}$ and $\mathsf{f}_{mk} = N \circ \mathsf{f}_{km} \circ N$), p.351.

Ax$(\omega)^0$ is defined to be the disjunction of the following symmetry axioms: **Ax(syt$_0$)**, **Ax(symm)**, **Ax(speedtime)**, **Ax$\triangle$1**+**Ax(eqtime)**, **Ax$\triangle$2**, **Ax$\Box$1**+**Ax(eqtime)**, **Ax$\Box$2**, p.794.

Ax$(\omega)^{00}$ is defined to be the disjunction of the following symmetry axioms: **Ax$(\omega)^0$**, **Ax(eqspace)**, **Ax(eqm)**+**Ax($Triv_t$)$^-$**, p.794.

Ax$(\omega)^\sharp$ is defined to be **Ax$(\omega)^0$**+**Ax($Triv_t$)$^-$**+**Ax($\surd$)**, p.794.

[1258] m and k are said to be in *pre-standard configuration* iff $\mathsf{f}_{mk}(\bar{0}) = \bar{0}$ and $\mathsf{f}_{mk}[\mathsf{Plane}(\bar{t},\bar{x})] = \mathsf{Plane}(\bar{t},\bar{x})$. Cf. Def.4.6.5 (p.602) and Fig.201 (p.603).

$\mathbf{Ax}(\omega)^{\sharp\sharp}$ is defined to be $\mathbf{Ax}(\omega)^{00} + \mathbf{Ax}(\mathit{Triv}_t)^{-} + \mathbf{Ax}(\sqrt{\ })$, p.794.

$\mathbf{Ax(symm)}^{\dagger}$ is defined to be $\mathbf{Ax(symm)} + \mathbf{Ax}(\mathit{Triv}) + \mathbf{Ax}(\|)$, p.151.

$\mathbf{Ax}(\omega)$ $\mathbf{Ax\Box 1} \wedge \mathbf{Ax\Box 2} \wedge \mathbf{Ax\triangle 1} \wedge \mathbf{Ax\triangle 2}$, p.351.

$\mathbf{Ax}(\omega^{-})$ $\mathbf{Ax\Box 1} \vee \mathbf{Ax\Box 2} \vee \mathbf{Ax\triangle 1} \vee \mathbf{Ax\triangle 2}$, p.351.

(7) Symmetry axioms adequate for Reichenbachian theories

$\mathbf{R}^{+}$(Ax eqsp) Intuitively, the thickness of spaceships do not change in the direction orthogonal to movement (cf. pp. 608–614), formally:

Assume $m, k \in \mathit{Obs}$ such that $m \overset{\odot}{\to} k$. Assume P, Q are parallel planes of ${}^{n}F$ such that they are parallel with both $\bar{t}$ and $tr_m(k)$. Then

$$\begin{aligned} \mathsf{Eudist}(P,Q) &= \mathsf{Eudist}(\mathsf{f}_{mk}[P], \mathsf{f}_{mk}[Q]), \text{ p.614, where} \\ \mathsf{Eudist}(H,H_1) &\overset{\text{def}}{=} \inf\{\, \|p-q\| \,:\, p \in H \text{ and } q \in H_1 \,\}. \end{aligned}$$

R(Ax eqsp) Intuitively, the thickness of spaceships do not change in the direction orthogonal to movement (cf. pp. 608–614), formally:

Assume m and k are in pre-standard configuration[1259]. Let P be a (2-dimensional) plane parallel with $\mathsf{Plane}(\bar{t}, \bar{x})$. Then the distance between P and $\mathsf{Plane}(\bar{t}, \bar{x})$ is the same as the distance between $\mathsf{f}_{mk}[P]$ and $\mathsf{f}_{mk}[\mathsf{Plane}(\bar{t}, \bar{x})]$. Formally,

$$\begin{aligned} \mathsf{Eudist}(P, \mathsf{Plane}(\bar{t}, \bar{x})) &= \mathsf{Eudist}(\mathsf{f}_{mk}[P], \mathsf{f}_{mk}[\mathsf{Plane}(\bar{t}, \bar{x})]), \text{ p.611, where} \\ \mathsf{Eudist}(H,H_1) &\overset{\text{def}}{=} \inf\{\, \|p-q\| \,:\, p \in H \text{ and } q \in H_1 \,\}, \text{ cf. p.609.} \end{aligned}$$

See Fig.205 on p.611.

R(Ax $\mathbf{syt_0}$) Intuitively m and k *literally* see, via photons, each other's clocks slowing down with the same rate, see Fig.207 (p.616), formally:

$$\begin{aligned} &(\forall m, k \in \mathit{Obs})[\mathsf{f}_{mk}(\bar{0}) = \bar{0} \;\Rightarrow \\ &\qquad (\forall p \in \bar{t}\,)|\mathit{view}_m(\mathsf{f}_{km}(p))| = |\mathit{view}_k(\mathsf{f}_{mk}(p))|] \text{ (cf. p.615),} \end{aligned}$$

where $\mathit{view}_m \overset{\text{def}}{=} \{\, \langle p, q\rangle \in {}^{n}F \times \bar{t} \,:\, p_t \leq q_t \text{ and } (\exists ph \in Ph)\; p, q \in tr_m(ph) \,\}$, cf. Fig.206 (p.615).

R(sym) is defined to be **R(Ax eqsp)**+ **R(Ax $\mathbf{syt_0}$)**, p.616.

[1259] Cf. footnote 1258 on p.1206 for the notion of a pre-standard configuration.

(8) Twin paradox

Let $m, k \in Obs$. Then $m\ STL\ k$ means that m sees k moving slower than light, cf. Def.4.2.6 on p.460 for details.

Ax(TwP) $(\forall m, k_1, k_2 \in Obs)(\forall p, q, r \in {}^nF)$
$\Big([\, m\ STL\ k_1 \quad \wedge \quad m\ STL\ k_2 \quad \wedge \quad p_t < q_t < r_t \quad \wedge$
$\{p\} = tr_m(m) \cap tr_m(k_1)\ \wedge\ \{q\} = tr_m(k_1) \cap tr_m(k_2)\ \wedge\ \{r\} = tr_m(m) \cap tr_m(k_2)\,] \Rightarrow$
$|p_t - r_t| > |\mathsf{f}_{mk_1}(p)_t - \mathsf{f}_{mk_1}(q)_t| + |\mathsf{f}_{mk_2}(q)_t - \mathsf{f}_{mk_2}(r)_t| \Big)$, p.460
(cf. Fig.43 on p.141).

The <u>*existential version*</u> **Ax(∃TwP)** of the twin paradox is defined as follows. First, let us notice that **Ax(TwP)** is a formula of the pattern

$$(\forall m \ldots)(\forall p \ldots)\Big([\ldots] \ \Rightarrow |\ldots| > |\ldots|\Big).$$

Let ψ_1, ψ_2 be formulas such that **Ax(TwP)** is the formula

$$(\forall m, k_1, k_2 \in Obs)(\forall p, q, r \in {}^nF)(\psi_1 \Rightarrow \psi_2).$$

§4.2 **Ax(∃TwP)** def-ját kijavítani: $\forall p, q, r$ helyett $\exists p, q, r$ kellene hogy legyen.

Now we define **Ax(∃TwP)** as follows.

Ax(∃TwP) $(\exists m, k_1, k_2 \in Obs)(\exists p, q, r \in {}^nF)[\psi_1 \ \wedge\ \psi_2]$, p.461.

(9) Axiom systems Specrel, Flxspecrel, BaCo, Compl, NewtK⁻, NewtK

$\boxed{\textbf{Specrel}} \stackrel{\text{def}}{=} \textbf{Basax} + \textbf{Ax(symm)}^\dagger$, p.151.

$\boxed{\textbf{Flxspecrel}} \stackrel{\text{def}}{=} \textbf{Bax} + \textbf{Ax6} + \textbf{Ax(symm)}^\dagger + \textbf{AxE}_{\mathbf{02}}$, p.428.

$\boxed{\textbf{Compl}} \stackrel{\text{def}}{=} \{\, \textbf{Ax(symm)}, \textbf{Ax}\heartsuit, \textbf{Ax}(\uparrow), \textbf{Ax5}^+, \textbf{Ax(ext)}, \textbf{Ax}(\mathit{Triv}_t)\,\}$ (cf. p.298), where

$\textbf{Ax5}^+ \quad \ell \in \mathsf{SlowEucl} \quad \Rightarrow \quad (\exists k \in Obs)\,(\ell = tr_m(k) \ \wedge\ m \uparrow k)$, p.297.

$\boxed{\textbf{BaCo}} \stackrel{\text{def}}{=} \textbf{Basax} + \textbf{Compl}$, p.298.

$\boxed{\textbf{NewtK}^-} \stackrel{\text{def}}{=} \textbf{Bax} + \textbf{Ax6} + \textbf{Ax(symm)}^\dagger + (\forall m \in Obs) c_m = \infty$ (cf. p.426), where c_m is the speed of light for observer m, assuming **Bax**.

$\boxed{\textbf{NewtK}} \stackrel{\text{def}}{=} \textbf{NewtK}^- + \textbf{Ax}(\uparrow\uparrow) + \textbf{Ax}\Box\mathbf{1}$, p.426.

(10) Geometrical axioms and axiom systems

Axioms $\mathbf{A_0}$–$\mathbf{A_4}$ and $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{Pa}$ below apply to geometries with reducts $\langle Mn;\, Bw\rangle$ or $\langle Mn;\, coll\rangle$. In the case of "$\langle Mn;\, Bw\rangle$" *coll* is a defined relation, cf. p.768. The new universe (or sort) *lines* is (explicitly) defined over $\langle Mn;\, coll\rangle$ on p.987. For $H \subseteq Mn$, $Plane'(H)$ is intuitively the "n-long closure of H under *coll*" (cf. Def.6.2.15 on p.769), where throughout n is the dimension of our geometry and $n \geq 2$.

$\mathbf{A_0}$ $(\forall a, b, c \in Mn)[\, coll(a, b, c) \leftrightarrow (\exists \ell \in lines)\, a, b, c \in \ell\,]$, p.988.

$\mathbf{A_1}$ $(\forall a, b \in Mn)(\, a \neq b \;\rightarrow\; (\exists! \ell \in lines)\, a, b \in \ell\,)$, p.988.

$\mathbf{A_2}$ Intuitively, if H is a less than $n+2$ element subset of Mn then the "n-long closure" $Plane'(H)$ of H under *coll* will be closed under *coll*, hence the plane $Plane(H)$ generated by H coincides with $Plane'(H)$ (cf. Def.6.2.15, p.769), formally:

$(\forall H \subseteq Mn)$
$\Big(\big(\, |H| \leq n+1 \;\wedge\; a, b \in Plane'(H) \;\wedge\; coll(a, b, c)\,\big) \;\rightarrow\; c \in Plane'(H)\Big)$, p.989.

$\mathbf{A_3}$ Intuitively, if $i \leq n$ and H is an $i+1$ element independent subset[1260] of Mn then there is exactly one i-dimensional plane[1261] that contains H, formally:

$(\forall H, H' \subseteq Mn)\Big(\big(\, |H| = |H'| \leq n+1 \;\wedge\; (\text{both } H \text{ and } H' \text{ are independent}) \;\wedge$
$H \subseteq Plane'(H')\,\big) \;\rightarrow\; Plane'(H) = Plane'(H')\Big)$, p.989.

$\mathbf{A_4}$ Mn is an n dimensional plane, p.989.

In connection with axioms $\mathbf{P_1}$, $\mathbf{P_2}$ below we note that the relation of parallelism $\parallel$ on *lines* is defined the usual way in Def.6.6.19 on p.989.

$\mathbf{P_1}$ (Euclid's axiom)

$(\forall \ell \in lines)(\forall a \in Mn)(\exists! \ell' \in lines)(a \in \ell' \;\wedge\; \ell \parallel \ell')$, p.990.

$\mathbf{P_2}$ $(\ell \parallel \ell' \;\wedge\; \ell' \parallel \ell'') \;\rightarrow\; \ell \parallel \ell''$, p.990.

[1260] Let $H \subseteq Mn$. H is called *independent* iff $(\forall e \in H)\, e \notin Plane'(H \setminus \{e\})$, cf. p.989.

[1261] Let $P \subseteq Mn$. P is called an i-dimensional iff there is an $i+1$ element independent subset H of Mn such that $Plane'(H) = P$, where for the notion of an independent subset cf. footnote 1260. Cf. Def.6.6.18(ii) on p.989.

$\boxed{\mathbf{ag}} \stackrel{\text{def}}{=} \{\mathbf{A_0}, \mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}, \mathbf{A_4}, \mathbf{P_1}, \mathbf{P_2}\}$, p.990. **ag** is the axiom system for <u>*affine geometries*</u>.

For $a, b, c, d \in Mn$, the abbreviation $\langle a, b\rangle \parallel \langle c, d\rangle$ means that $a \neq b$, $c \neq d$, and there are $\ell, \ell' \in \mathit{lines}$ such that $a, b \in \ell$, $c, d \in \ell'$ and $\ell \parallel \ell'$, cf. p.992.

Pa (Pappus-Pascal property)

$(\forall \ell, \ell' \in \mathit{lines})(\forall a, b, c \in \ell \setminus \ell')(\forall a', b', c' \in \ell' \setminus \ell)$

$$[\,(\,\langle a, b'\rangle \parallel \langle a', b\rangle \;\wedge\; \langle a, c'\rangle \parallel \langle a', c\rangle\,) \;\rightarrow\; \langle b, c'\rangle \parallel \langle b', c\rangle\,],$$

see Fig.304, p.992.

$\boxed{\mathbf{pag}} \stackrel{\text{def}}{=} \mathbf{ag} + \mathbf{Pa}$, p.992. **pag** is the axiom system for <u>*Pappian affine geometries*</u>.

Axioms $\mathbf{B_1}$–$\mathbf{B_3}$ below apply to geometries with reducts $\langle Mn; Bw\rangle$. (*coll* is a defined relation.)

$\mathbf{B_1}$ $Bw(a, b, c) \;\rightarrow\; (\,a \neq b \neq c \neq a \;\wedge\; Bw(c, b, a) \;\wedge\; \neg Bw(b, a, c)\,)$, p.993.

$\mathbf{B_2}$ $a \neq b \;\rightarrow\; (\exists c) Bw(a, b, c)$, p.993.

$\mathbf{B_3}$ (Pasch's Law)

Intuitively, if a line ℓ lies in the plane determined by a triangle abc, and passes between a and b but not through c, then ℓ passes between a and c, or between b and c, formally:

$(\,\neg \mathit{coll}(a, b, c) \;\wedge\; \ell \subseteq \mathit{Plane}'(\{a, b, c\}) \;\wedge\; (\exists d \in \ell) Bw(a, d, b)\,) \;\rightarrow$
$(\exists e \in \ell)(Bw(a, e, c) \;\vee\; Bw(b, e, c))$, p.993 (cf. Fig.305 on p.994).

$\boxed{\mathbf{opag}} \stackrel{\text{def}}{=} \mathbf{pag} + \{\mathbf{B_1}, \mathbf{B_2}, \mathbf{B_3}\}$, p.994. **opag** is the axiom system for <u>*ordered Pappian affine geometries*</u>.

Axioms $\mathbf{L_1}, \ldots, \mathbf{L_{10}}$ below apply to geometries with reducts

$$\langle Mn, L;\, L^T, L^{Ph}, L^S, \in, \prec, Bw, \perp_r, eq\rangle.$$

Further, *coll* is a defined relation and it is defined over $\langle Mn; Bw\rangle$, cf. p.768, and *lines* is the new sort firts-order defined from *coll*.

$\mathbf{L_1}$ $L \subseteq \mathit{lines}$, p.1021.

$\mathbf{L_2}$ $(\forall a \in Mn)(\exists \ell, \ell' \in L^{Ph})\, \ell \cap \ell' = \{a\}$, p.1021.

$\boxed{\textbf{lopag}} \stackrel{\text{def}}{=} \textbf{opag} + \mathbf{L_1} + \mathbf{L_2}$, p.1021.

$\mathbf{L_3}$ $(\,[\,a \prec b \;\wedge\; (\,Bw(a,b,c) \vee Bw(a,c,b)\,)\,] \;\rightarrow\; a \prec c\,) \;\wedge$
$(\,[\,a \prec b \;\wedge\; (\,Bw(c,a,b) \vee Bw(a,c,b)\,)\,] \;\rightarrow\; c \prec b\,)$, p.1026.

$\mathbf{L_4}$ Intuitively, eq is (very) symmetric, formally:
$\langle a,b\rangle \; eq \; \langle c,d\rangle \quad \rightarrow \quad (\,\langle c,d\rangle \; eq \; \langle a,b\rangle \;\wedge\; \langle b,a\rangle \; eq \; \langle c,d\rangle \;\wedge\; \langle a,a\rangle \; eq \; \langle c,c\rangle\,)$, p.1026.

$\mathbf{L_5}$ eq is transitive, i.e.
$(\,\langle a,b\rangle \; eq \; \langle c,d\rangle \;\wedge\; \langle c,d\rangle \; eq \; \langle e,f\rangle\,) \quad \rightarrow \quad \langle a,b\rangle \; eq \; \langle e,f\rangle$, p.1026.

$\mathbf{L_6}$ (For the intuitive meaning of this axiom see Fig.311 on p.1026.)
$(\forall \ell, \ell' \in L)(\forall o, e, e', a, a' \in Mn)\Big([\,\ell \cap \ell' = \{o\} \;\wedge\; e, a \in \ell \;\wedge\; e', a' \in \ell' \;\wedge$
$\langle e,e'\rangle \parallel \langle a,a'\rangle \;\wedge\; \langle o,e\rangle \; eq \; \langle o,e'\rangle\,] \quad \rightarrow \quad \langle o,a\rangle \; eq \; \langle o,a'\rangle\Big)$, p.1026.

$\mathbf{L_7}$ (For the intuitive meaning of this axiom see Fig.312 on p.1027.)
$(\forall \ell \in L^T \cup L^S)(\forall a, b, c, d, e, f \in Mn)\,[\,(\,a, b, c, d \in \ell \;\wedge$
$\langle a,b\rangle \parallel \langle e,f\rangle \parallel \langle c,d\rangle \;\wedge\; \langle a,e\rangle \parallel \langle b,f\rangle \;\wedge\; \langle c,e\rangle \parallel \langle d,f\rangle\,) \;\rightarrow\; \langle a,b\rangle \; eq \; \langle c,d\rangle\,]$, p.1026.

$\mathbf{L_8}$ $\perp_r$ is symmetric, i.e.
$(\forall \ell, \ell' \in L)\,(\ell \perp_r \ell' \;\rightarrow\; \ell' \perp_r \ell)$, p.1026.

$\mathbf{L_9}$ $\perp_r$ is closed under parallelism, i.e.
$(\forall \ell, \ell_1, \ell_2 \in L)\,[\,(\,\ell \perp_r \ell_1 \;\wedge\; \ell_1 \parallel \ell_2\,) \quad \rightarrow \quad \ell \perp_r \ell_2\,]$, p.1026.

$\mathbf{L_{10}}$ $\perp_r$ is closed under taking limits, p.1027.

$\boxed{\textbf{lopag}^+} \stackrel{\text{def}}{=} \textbf{lopag} + \mathbf{L_3} + \mathbf{L_4} + \mathbf{L_5} + \mathbf{L_6} + \mathbf{L_7} + \mathbf{L_8} + \mathbf{L_9} + \mathbf{L_{10}}$, p.1031

(11) "Speed of light free" axiom systems for relativity (axioms and axiom systems used in Chapter 5)

$\boxed{\mathbf{Relnoph_0}} \stackrel{\text{def}}{=} (\mathbf{Ax1}\text{–}\mathbf{Ax4})^{1262} + \mathbf{Ax6} + \mathbf{Ax\Box 1} + \mathbf{Ax\triangle 1} + \mathbf{Ax}(\sqrt{\ }) + \mathbf{Ax}(\mathit{Triv}) + \mathbf{Ax}(\|)$, p.655.

Ax(5nop) $\forall m, k\ (\forall \lambda \in {}^{+}F)[\lambda < v_m(k) \ \Rightarrow\ \exists k'\ (v_m(k') = \lambda)]$, p.656.

The intuitive idea of **Ax(5nop)** is that if a certain speed is realized by some observer then the *smaller speeds* are also realized by some observers.

$\boxed{\mathbf{Relnoph}} \stackrel{\text{def}}{=} \mathbf{Relnoph_0} + \mathbf{Ax(5nop)}$, p.657.

Ax(group$^+$) $(\forall m, k, m', k' \in \mathit{Obs})(\exists k'' \in \mathit{Obs})\mathsf{f}_{mk} \circ \mathsf{f}_{m'k'} = \mathsf{f}_{mk''}$, p.410.

Ax(syt)* $\mathsf{f}_{mk}(\bar{0}) = \bar{0} \ \Rightarrow\ \mathsf{f}_{mk}(1_t)_t = \mathsf{f}_{km}(1_t)_t$, p.671.

Ax(syx)* (m, k are in pre-standard configuration)[1263] $\Rightarrow$ $|\mathsf{f}_{mk}(1_x)_x| = |\mathsf{f}_{km}(1_x)_x|$, p.675.

($*1$)–($*3$) below are also potential axioms[1264] (or principles) connected with **Relnoph**.

($*1$) The sum of finitely many small positive velocities is not infinite, formally: Let $j \in \omega$. Let $m_0, \ldots, m_j \in \mathit{Obs}$. Assume m_i, m_{i+1} are in strict standard configuration[1265] and $v_{m_i}(m_{i+1}) < 1$, for all $i < j$. Then $v_{m_0}(m_j) \neq \infty$, p.660.

($*2$) The sum of finitely many small positive velocities is nonnegative, formally: Let $j \in \omega$. Let $m_0, \ldots, m_j \in \mathit{Obs}$. Assume m_i, m_{i+1} are in strict standard configuration and $v_{m_i}(m_{i+1}) < 1$, for all $i < j$. Then m_0 sees m_j moving forwards in direction 1_x, p.660.

[1262] **Ax1**, **Ax2**, **Ax3**, **Ax4**.

[1263] Cf. footnote 1258 on p.1206 for the notion of a pre-standard configuration.

[1264] ($*1$)–($*3$) are schemas of formulas.

[1265] Assume $m, k \in \mathit{Obs}$. Then m and k are defined to be in *strict standard configuration* if they are in standard configuration, m sees k moving forwards in direction 1_x, k sees m moving backwards in direction 1_x, further $[v_m(k) = 0 \ \Rightarrow\ \mathsf{f}_{km}(1_t)_t \cdot \mathsf{f}_{km}(1_x)_x > 0]$ and $[v_m(k) = \infty \ \Rightarrow\ (\mathsf{f}_{km}(1_t)_x > 0\ \&\ \mathsf{f}_{km}(1_x)_t < 0)]$. Cf. Def.5.0.22 (p.659) and Remark 5.0.23 (p.659).

(∗3) The sum of finitely many small positive velocities is nonzero, formally: Let $0 < j \in \omega$. Let $m_0, \ldots, m_j \in Obs$. Assume m_i, m_{i+1} are in strict standard configuration and $0 < v_{m_i}(m_{i+1}) < 1$, for all $i < j$. Then $v_{m_0}(m_j) \neq 0$, p.660.

"**Flxspecrel**", the notation $\mathfrak{M} \models$ **"Flxspecrel"** was introduced on p.658.

Ax(natu) $(*1) \vee (*2) \vee (*3)$,[1266] p.702

Ax(natu)$^+$ $(\forall m, k, k' \in Obs)\sqrt{v_m(k')} \leq \sqrt{v_m(k)} + \sqrt{v_k(k')}$, p.703.

Ax(∃body) $\forall m, k\ (\forall \ell \in \mathsf{Eucl})[(v_m(k) > 0\ \wedge\ \mathsf{f}_{mk}[\ell] = \ell)\ \Rightarrow\ (\exists b \in B) tr_m(b) = \ell]$, p.707.

Ax(5nop)$^-$ $\forall m\ (\exists c \in {}^+F)(\forall \lambda \in {}^+F)[\lambda < c\ \Rightarrow\ (\exists k) v_m(k) = \lambda]$, p.711.

$\boxed{\textbf{Relnoph}^-}$ is obtained from **Relnoph** by replacing **Ax(5nop)** with **Ax(5nop)$^-$**, p.711.

$\boxed{\textbf{Bax}^-_{\textbf{nobs}}} \stackrel{\text{def}}{=} \textbf{Bax}^- \setminus \{\textbf{Ax5}_{\textbf{Obs}}\} + \textbf{Ax(5nop)}^-$, p.712.

$\boxed{\textbf{Relnoph}^{--}} \stackrel{\text{def}}{=} \textbf{Relnoph}^- \setminus \{\textbf{Ax}\triangle\textbf{1}\} + \textbf{Ax(symm)}$, p.714.

egyezetetni §4.3-ben levő definicióval

(12) The cone-smooth versions of our theories

Assume **Ax1**, **Ax2**, **Ax3$_0$**, **AxP1**. Let $m \in Obs$. Then

$$c_m : {}^nF \times \mathsf{directions} \xrightarrow{\circ} F \cup \{\infty\}$$

is a partial function such that $c_m(p, d)$ is defined iff m sees a photon at point p moving forwards in direction d, <u>and</u> $c_m(p, d)$ is the speed of this photon,[1267] cf. pp. 473, 535. Further, for any $m \in Obs$ and $d \in \mathsf{directions}$

$$\mathsf{c}_m(d) \stackrel{\text{def}}{=} c_m(\bar{0}, d),$$

[1266] **Ax(natu)** is a schema of formulas.

[1267] There is only one such speed because of **AxP1**.

cf. p.488. Hence, c_m : directions $\overset{\circ}{\longrightarrow} F \cup \{\infty\}$ is a partial function. (**Ax(consm)** below will imply that it is not partial. The notion of strongly continuous functions, partial derivatives etc. used in **Ax(consm)** below are formulated in our frame language on pp. 536, 518.) Now,

$\mathbf{Ax(consm)} \overset{\text{def}}{=} \mathbf{Ax(cnsm_0)} + \mathbf{Ax(cnsm_1)} + \mathbf{Ax(cnsm_2)}$ (cf. p.518), where

Ax(cnsm$_0$) c_m is a strongly continuous function defined on directions, p.519.

Ax(cnsm$_1$) For all $0 < i < n$, the partial derivative $(\partial_i \mathsf{c}_m)$: directions $\longrightarrow F$ is everywhere defined on the domain directions, p.519.

Ax(cnsm$_2$) For all $0 < i < n$, $\partial_i \mathsf{c}_m$ is strongly continuous on the domain directions, p.519.

Let Th be one of our theories such that $Th \models \{\, \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3_0}, \mathbf{AxP1} \,\}$. Then $\boxed{Th_\partial} \overset{\text{def}}{=} Th + \mathbf{Ax(consm)}$ is called the <u>*cone-smooth version*</u> of the theory Th, cf. p.521.

(13) Axioms and axiom systems not listed in this list

$\mathbf{Ax}_{\mathrm{OF}}$, p.30.
$\mathbf{Ax}_{\mathrm{G}}$, p.31. ($\mathbf{Ax}_{\mathrm{OF}}$ and $\mathbf{Ax}_{\mathrm{G}}$ are assumed throughout the present work.)
Ax1', p.45.
$\mathbf{AxE_1}$, p.223.
$\mathbf{AxE_2}$, p.223.
$\mathbf{Ax2_1}$, p.223.
$\mathbf{Ax5_1}$, p.223.
$\boxed{\mathbf{Relphax}}$, p.223.
Ax□1', p.354.
Ax△2*, p.359.
Ax△3, p.406.
Ax(isotropy), p.399.
Ax(isotropy'), p.400.
Ax(isotropy$^-$), p.401.
homogeneity **Ax(homogenity)**, p.405.
Ax(group$^-$), p.410.
Ax(group), p.410.
$\boxed{\mathbf{Flxspecrel^+}}$, p.425.

$\mathbf{Ax2^n}$, p.435.
$\mathbf{Ax5^n}$, p.435.
$\mathbf{AxE^n}$, p.435.
$\boxed{\mathbf{NewtK^n}}$, p.435.
$\mathbf{Ax5^c}$, p.429.
$\mathbf{AxE_0^c}$, p.429.
$\mathbf{Ax5^f}$, p.431.
$\mathbf{AxE_0^f}$, p.431.
$\mathbf{Ax}(E_{ess})$, p.424.
$\mathbf{AxP1^-}$, p.529.
$\mathbf{Ax5^-_{Ph}}$, p.530.
$\mathbf{Ax5^-_{Obs}}$, p.530.
$\boxed{\mathbf{Bax^{--}}}$, p.531
$\mathbf{AxP1^-_1}$, p.532.
$\mathbf{AxP1^-_2}$, p.532.
$\mathbf{AxP1^-_3}$, p.534.
$\boxed{\mathbf{Bax^{--}_1}}$, p.533.
$\boxed{\mathbf{Bax^{--}_2}}$, p.533.
$\boxed{\mathbf{Bax^{--}_3}}$, p.534.
$\boxed{\mathbf{Bax^{--}_+}}$, p.538.
$\mathbf{Ax(i)}$, p.540.
$\mathbf{Ax(ii)}$, p.540.
$\mathbf{Ax(ii)}^+$, p.542.
$\boxed{\mathbf{Bax^{--}_{++}}}$, p.544.
$\boxed{\mathbf{Bax(P1)}}$, p.544.
$\mathbf{R(AxE)}^-$, p.557.
$\mathbf{R(AxE_{02})}^-$, p.559.
$\mathbf{R(AxE_{00})}^-$, p.559.
$\boxed{\mathbf{Reich}_0(Th)^-}$, p.563.
$\mathbf{R}_k\mathbf{(AxE)}$, p.574.
$\boxed{\mathbf{Reich}'(Th)}$, p.580.
$\mathbf{AxR^-}$, p.584.
$\mathbf{AxR^{1/2}}$, p.584.
$\mathbf{AxR^+}$, p.584.
$\mathbf{AxR^{--}}$, p.585.

Ax(sy), p.628.
Ax(sy$_0$), p.629.
Ax(5nop)$^{-+}$, p.713.
Ax(cont), p.716.
Ax(cont)$^{+}$, p.717.
Ax($\exists\uparrow$), p.717.
Ax(fun), p.718.
Det, p.942.
det, p.942.
Ax(mild), p.1017.
G$_1$, p.1122.
G$_2$, p.1122.
G$_3$, p.1122.
G$_4$, p.1122.
G$_5$, p.1123.
$\boxed{\textbf{busg}}$, p.1122.

Credits

Figures 243, 246, 318 are created from works by M. C. Escher, which appeared in D. R. Hofstadter: "Gödel, Escher, Bach", Hungarian translation, Typotex Kiadó, 1998. Figure 340 (representing Gödel's rotating universe) and Figure 275 are created from figures in S. W. Hawking and G. F. R. Ellis [122]. Figure 266 (view from near a black hole) is created from a figure in K. S. Thorne [247].

Index

$(*1)$, 660
$(*2)$, 660
$(*3)$, 660
$(\mathfrak{G}^R_{\mathfrak{M}})^{\equiv}$, 750
$(\perp_0)_\mu$, 828
$(\perp_0)_m$, 832
$(\exists! x)\psi(x)$, 897
(**TwP**), 38
(x, y), interval, 1128
$+_{oe}$, 996
0, 30
1, 30
1/2-simultaneous, 582
1_i, 69
$1_t, 1_x, 1_y, 1_z$, 69
$< \infty$, 477
$=|\models$, 193
B, 29
$LG_{\mathfrak{M}}$, 408
N_{pq}, 268
$R[X]$, 27
$R \circ S$, 26
R^{-1}, 27
S, 54, 208
S, space-part, 470
$S''(a, b)$, 789
$S'(H)$, 788
$S(a)$, 444
$SL_{\mathfrak{M}}$, 408
S_1, 492
T_0, a subbase for the topology $\mathcal{T}$, 747
T'_0, a subbase for the topology $\mathcal{T}'$, 789
T''_0, a subbase for the topology $\mathcal{T}''$, 789
T^R_0, 750
$T_m(d)$, 555
T_{pq}, 268
$VL_{\mathfrak{M}}$, 420
$[oe$, 995
$[x]_I$, 449
ag, 990
$\mathfrak{A} \rightarrowtail \mathfrak{B}$, 958
$\mathbf{A_1}$, 988
$\mathbf{A_3}$, 989
$\mathbf{A_2}$, 989
$\mathbf{A_4}$, 989
$\mathbf{A_0}$, 988
$Aut^F(\mathfrak{G}_{\mathfrak{M}})$, 865
$Aut(\mathfrak{A})$, 154, 161
$Aut(\mathfrak{F})$, 154
BaCo, 298
Basax, 51
Basax geometry, 749
Bax, 219
$\mathbf{Bax^-}$, 479
$\mathbf{Bax^-_{nobs}}$, 712
$\mathbf{B_1}$, 993
$\mathsf{Betw}(p, r, q)$, 740
$\mathsf{Betw}(p, r, q)$, 492
$\mathbf{B_3}$, 993
$\mathbf{B_2}$, 993
Bw, 740
Bw_μ, 810
Bw_m, 831
$Ch(H)$, convex hull of $H \subseteq Mn$, 788
$Co_{\langle o, e_0, \ldots, e_{n-1}\rangle}$, 1001
$coll(a, b, c)$, 768
$coll_{\mathbf{F}}$, $coll_{\mathbf{D}}$, 990
Compl, 298
$Dom(R)$, 26
EC_Δ, 451
$\mathsf{Eucl} = \mathsf{Eucl}(n, \mathfrak{F})$, 45
FM, 35, 956

Flxbasax, 428
Flxspecrel, 428
Flxspecrel$^+$, 425
Fm, 28
$Fm(Th)$, 970
$Fm(\mathsf{K})$, 912
${}^n\mathbf{F}$, 42, 43
${}^n\mathbf{F}_1$, 42
${}^n\mathbf{F}_2$, 42
GEO, 1020
$\mathcal{G}$, 957
$\mathcal{G} : \mathsf{Mod}(Th) \longrightarrow \mathsf{Ge}(Th)$, 958
GM, 269
$GT^i_{\mathfrak{M}}$, $-2 \leq i \leq 2$, 875
$GT_{\mathfrak{M}}$, 873
$\mathbf{G_1}$, 1122
$\mathsf{Ge}^{inc}(Th)$, 1125
$\mathbb{G}\mathsf{e}(Th)$, 1035
$\mathfrak{G}$-parallel, 740
$\mathfrak{G}''_{\mathfrak{M}}$, 796
$\mathfrak{G}'_{\mathfrak{M}}$, 796
$\mathfrak{G} \restriction N$, 832
$\mathfrak{G} \restriction^* N$, 1080
$\mathfrak{G} \restriction^+ N$, 832
$\mathfrak{G}_0 \dot{\cup} \mathfrak{G}_1$, 821
$\mathfrak{G}^7_{\mathfrak{M}}$, 1079
$\mathfrak{G}^R_{\mathfrak{M}}$, 749
$\mathfrak{G}^{\perp_0}_{\mathfrak{M}}$, 821
$\mathfrak{G}^{\equiv}_{\mathfrak{M}}$, 749
$\mathfrak{G}_m$, 830
$\mathfrak{G}^{\perp_0}_m$, 831
$\mathfrak{G}_{\mathfrak{M}}$, 737
$\mathfrak{G}^*_{\mathfrak{M}}$, 1061
$\mathfrak{G}^0_{\mathfrak{M}}$, 1020, 1074
$\mathfrak{G}^1_{\mathfrak{M}}$, 1074
$\mathfrak{G}^2_{\mathfrak{M}}$, 1074
$\mathfrak{G}^3_{\mathfrak{M}}$, 1074
$\mathfrak{G}^4_{\mathfrak{M}}$, 1075
$\mathfrak{G}^5_{\mathfrak{M}}$, 1075
$\mathfrak{G}^6_{\mathfrak{M}}$, 1079
$\mathbf{G_3}$, 1122
$\mathfrak{G}^{inc}_{\mathfrak{M}}$, 1125
$\mathbf{G_2}$, 1122
$\mathbf{G_4}$, 1122
$\mathcal{G}o, \mathcal{M}o$, 1022
$\mathbf{G_5}$, 1123
Ib, 30
Id, 27
Id_A, 27
$\mathsf{LG}(Th, \mathbf{n})$, 409
lopag, 1021
lopag$^+$, 1031
$\mathbf{L_1}$, 1021
$\mathbf{L_3}$, 1026
$\mathbf{L_6}$, 1026
$\mathbf{L_7}$, 1026
G, 30
$\mathbf{L_2}$, 1021
$\mathbf{L_9}$, 1026
$\mathbf{L_4}$, 1026
$\mathbf{L_8}$, 1026
$Lor = Lor(n, \mathfrak{F})$, 152
$\mathbf{L_5}$, 1026
$\mathbf{L_{10}}$, 1027
$\mathcal{M}$, 1004
$\mathbb{M}\mathsf{og}(TH)$, 1035
$\mathsf{MS}(\mathfrak{M}, m)$, 89
MS, 89
$\mathbb{M}\mathsf{od}(Th)$, 1035
$\mathfrak{M}'$ is obtained from $\mathfrak{M}$ by Step (2.1), 897
$\mathfrak{M}'$ is obtained from $\mathfrak{M}$ by Step (2.2), 900
$\mathfrak{M}'$ is obtained from $\mathfrak{M}$ by step (1), 896
$\mathfrak{M} \models \psi[\bar{a}]$, 895
$\mathfrak{M} \cup \mathfrak{N}$, 819
$\mathfrak{M} \restriction Voc$, 882
$\mathfrak{M}/P$, 569

$\mathfrak{M} \dot\cup \mathfrak{N}$, 818, 819
$\mathfrak{M} \subseteq \mathfrak{N}$, 161
$\mathfrak{M}^M_{\mathfrak{F}}$, 331
$\mathfrak{M}^Q_{\mathfrak{F}}$, 325
Mn, 737
$Mn_{\mathfrak{M}}$, 740
M, 28
$\mathsf{Mog}(TH)$, 1021
$\mathsf{Mor}\,\mathbb{C}$, 1035
Newbasax, 191
$Triv = Triv(n, \mathfrak{F})$, 135, 247, 601
NewtK, 426
$\mathbf{NewtK^n}$, 435
$\mathbf{NewtK^-}$, 426
$Triv_0 = Triv_0(n, \mathfrak{F})$, 247
$\mathfrak{N} \models$ "Th", 658
opag, 994
$\mathsf{Ob}\,\mathbb{C}$, 1035
Obs, 30
$Open = Open(n, \mathfrak{F})$, 190
Pa, 992
pag, 992
$\mathsf{PG}(Th, \mathbf{n})$, 408
$\mathsf{PG}_{\mathfrak{M}}$, 408
Pax, 482
$\mathbf{Pax}^+_+$, 1036
$\mathbf{Pax}^{++}_+$, 1043
$\mathbf{Pax}^+$, 979
$\mathbf{Pax}^{++}$, 1031
$\mathbf{P_1}$, 990
Ph, 30
$\mathsf{PhtEucl} = \mathsf{PhtEucl}(n, \mathfrak{F})$, 58
$\mathbf{P_2}$, 990
$Plane'(H)$, 770
$Plane(H)$, 769
$Plane(\ell_1, \ell_2)$, 770
$Poi = Poi(n, \mathfrak{F})$, 153
$\mathfrak{R}$, 26
Relnoph, 657
$\mathbf{Relnoph^-}$, 711
$\mathbf{Relnoph^{--}}$, 714
$\mathbf{Relnoph_0}$, 655
Relphax, 223
$Rhomb$, 74
$Rhomb^M = Rhomb^M(n, \mathfrak{F})$, 332
$Rhomb^Q = Rhomb^Q(n, \mathfrak{F})$, 327
$Rng(R)$, 26
$\mathsf{SL}(Th, \mathbf{n})$, 409
$SLor = SLor(n, \mathfrak{F})$, 153
SM, 249
$\mathsf{SlowEucl} = \mathsf{SlowEucl}(n, \mathfrak{F})$, 58
Specrel, 151
TH, 451
$\mathcal{T}'$, alternative version of the topology part $\mathcal{T}$ of observer-independent geometry, 789
$\mathcal{T}''$, alternative version of the topology part $\mathcal{T}$ of observer-independent geometry, 789
$\mathcal{T}^R$, 750
$\mathcal{T}_\mu$, 811
$\mathcal{T}_m$, 831
Th geometry, 749
Th-potential law (of nature), 1059, 1060
Th-simultaneity-stable, 578
Th^{+-}, 778
$Th^{\oplus}$, 643
$Th_1 \models$ "Th", 658
Th_∂, 521
$Tran = Tran(n, \mathfrak{F})$, 152
Tr_k, 917
Tr, 916, 917
$Uv(\mathfrak{M})$, 879
$Var(U_i)$, 914
$\|-\|$, 1046
$\|p\|$, 189
$Voc(\mathfrak{M})$, 881

$Voc(\mathsf{K})$, 882
$Voc \cap Voc'$, 883
$Voc \cup Voc'$, 883
Wax, 1023
$\mathbf{Wax}^{+}$, 1031
W, 32
Wtm, 60
Det, 942
det, 942
$Aftr = Aftr(n, \mathfrak{F})$, 152
$\mathbf{Ax}(\omega)$, 351
$\mathbf{Ax}(\omega)^{\sharp}$, 794
$\mathbf{Ax}(\omega)^{\sharp\sharp}$, 794
$\mathbf{Ax}(\omega^{-})$, 351
$\mathbf{Ax}(\omega)^{00}$, 794
Ax(cont), 716
$\mathbf{Ax(cont)}^{+}$, 717
AxE, 51
Ax(∃body), 707
$\mathbf{AxE_1}$, 223
$\mathbf{AxE^n}$, 435
$\mathbf{AxE_2}$, 223
$\mathbf{AxE_0}$, 191
$\mathbf{AxE_0^c}$, 429
$\mathbf{AxE_{01}}$, 218
$\mathbf{AxE_0^f}$, 431
$\mathbf{AxE_{02}}$, 427
$\mathbf{AxE_{00}}$, 218
Ax(eqm), 397
$\mathbf{Ax}_{\mathrm{G}}$, 31
Ax(group), 410
$\mathbf{Ax(group^{-})}$, 410
$\mathbf{Ax(group^{+})}$, 410
$\mathbf{Ax3_0}$, 191
$\mathbf{Ax6_{01}}$, 190
$\mathbf{Ax6_{00}}$, 190
Ax△1, 351
Ax△2, 351
$\mathbf{Ax\triangle 2}^{*}$, 359
Ax△3, 406
Ax(homogenity), 405
Ax(isotropy), 399
$\mathbf{Ax(isotropy^{-})}$, 401
$\mathbf{Ax(\uparrow\uparrow_0)}$, 790
$\mathbf{Ax(\uparrow\uparrow_{00})}$, 790
$\mathbf{Ax2_1}$, 223
$\mathbf{Ax2^n}$, 435
Ax(natu), 702
$\mathbf{Ax(natu)}^{+}$, 703
Ax(∃ ↑), 717
Ax□1, 350, 426
Ax□2, 350
Ax□1', 354
Ax(↑), 296
$\mathbf{Ax}_{\mathrm{OF}}$, 30
$\mathbf{Ax5^{Obs}}$, 218
$\mathbf{Ax5^{Ph}}$, 219
$\mathbf{Ax5^c}$, 429
$\mathbf{Ax5_1}$, 223
$\mathbf{Ax5^f}$, 431
$\mathbf{Ax5^n}$, 435
Ax(5nop), 656
$\mathbf{Ax(5nop)^{-}}$, 711
$\mathbf{Ax5_{Obs}}$, 477
$\mathbf{Ax5_{Obs}}^{--}$, 481
$\mathbf{Ax5}^{+}$, 297
$\mathbf{Ax5_{Ph}}$, 477
AxP1, 472
Ax(||), 136, 457
$\mathbf{Ax(||)^{-}}$, 778
$\mathbf{Ax}(\infty ph)$, 978
$\mathbf{Ax(symm)}^{\dagger}$, 426
$\mathbf{Ax(symm_0)}$, 124
$\mathbf{Ax(sy_0)}$, 629
Ax(syt), 386, 457
$\mathbf{Ax(syt)}^{*}$, 671
$\mathbf{Ax(syt_0)}$, 134, 457
$\mathbf{Ax(syx)}^{*}$, 675

Ax♡, 296
Ax($\sqrt{\ }$), 91
Ax(E_{ess}), 424
Ax(Bw), 978
Ax(TwP), 140
Ax(*Triv*), 135, 297, 601
Ax($Triv_t$), 135, 297, 601
Ax(*Ph*), 1023
Ax(diswind), 762
Ax(eqm), 746
Ax(eqspace), 136, 406
Ax(eqtime), 127
Ax(ext), 298
Ax(fun), 718
Ax(i), 540
Ax(ii), 540
Ax(ii)$^+$, 542
Ax(isotropy'), 401
Ax(mild), 1017
Ax(rc), 301
Ax(speedtime), 137
Ax(sy), 628
Ax(symm), 127
Ax(symm)†, 151
Ax1, 45
Ax1$'$, 45
Ax2, 48
Ax3, 48
Ax4, 48
Ax5, 50
Ax6, 50
Ax7, 264
$\boldsymbol{\mathcal{M}}$, $\boldsymbol{\mathcal{G}}$, $\boldsymbol{\mathcal{M}o}$, $\boldsymbol{\mathcal{G}o}$, 1037
$\bar{0}$, 42
$\bar{t}$, 43
$\bar{t}$, time-part, time-axis, 470
$\bar{t}$-axis, 43
$\bar{t}$-symmetric, 306
$\bar{x}$, 43
$\bar{x}_i$, 44
$\bar{y}$, 44
$\bar{z}$, 44
$\bigcup \mathsf{K}$, 819
$\perp$, 735, 740, 742
$\perp_r^\omega$, 771
$\perp_r^i$, 772
$\perp_0$, 741
$\perp_0$-version of the Minkowskian geometry, 828
$\perp_\mu$, 810
$\perp_e$, 172
$\perp_m$, 831
$\perp_r$, 740, 742
$\perp_r'$, 760
$\perp_r''$, 760, 761
$\perp_r'''$, 772
$\cdot_{oe}$, 996
c_m, $\mathsf{c}_m(d)$, 488
code, 916, 917
Col, 941
Col^T, Col^{Ph}, Col^S, 948
$\mathsf{Cone}_{m,p}$, 473
$\cong$, relation of isomorphism between structures, 740
$\cong_I$, 447
$\equiv_\Delta^*$, 920
$\equiv_\Delta^w$, weak definitional equivalence, 936
$\equiv_\Delta$, 920
ε-neighborhood, 189
ε-neighborhood of an event in an observer-independent (or a relativistic) geometry, 747
$\longrightarrow\!\!\!\!\succ$, 933
$\mathbb{EC}_\Delta$, 452
$(\mathrm{E5})^{+\exists}$, 649
$\ell \perp_e \ell'$, 172
$\ell \sim \ell'$, 844
$\ell_1 \parallel \ell_2$, 164

$\ell_1 \parallel \ell_2$ when $\ell_1, \ell_2 \in \mathit{lines}$, 989
τ_p, 152
eq, 745
eq_0^S, 852
eq_0^S-witness, 852
eq_μ, 810
eq_m, 831
eq_0, 743
eq_0-witness, 849
eq_i, 745
$\equiv^S$, 749
$\equiv^T$, 749
$\equiv^{Ph}$, 749
$\equiv_v$, 421
$\equiv_{ee}$, 272
Ax(∃TwP), 461
$\succ\!\!\longrightarrow$, 933, 958
$\succ\!\!\longrightarrow\!\!\succ$, 933
$\mathbf{F_1} = \langle F; 0, 1, +, \leq \rangle$, 738
$\mathbf{F}$, 30, 43
$\mathbf{F}^{\mathfrak{M}}$, 30
$\mathsf{Mod}_{\mathfrak{F}}(\Sigma)$, 194
$\mathbf{F_0} = \langle F; 0, +, \leq \rangle$, 1101
$Geom(Th)$, 748
$\mathsf{Ge}(Th)$, 748
$Linb = Linb(n, \mathfrak{F})$, 152
$Exp = Exp(n, \mathfrak{F})$, 153
$time(p)$, 470
$time$, 470
∞, 46, 477
ι_x, 333
$S(e, \varepsilon)$, 747
$S(p, \varepsilon)$, 189
$S^R(e, \varepsilon)$, 750
$\mathsf{LightCone}(\bar{0})$, 207
$\mathsf{LightCone}(p)$, 207
$\langle Points; Col \rangle$, 941
$\langle a, b \rangle \; eq \; \langle c, d \rangle$, 745
$\langle a, b \rangle \parallel \langle c, d \rangle$, 992
$\langle Points, Lines; \in \rangle = \langle Mn, L; \in \rangle$, 941
$\lambda < \infty$, 477
$\lambda \cdot -$, 1046
$\overset{\odot}{\rightarrow}$, 192
$\langle \, expr(x) \; : \; x \in D \, \rangle$, 27
$\leq_{\mathcal{M}o}, \; \leq_{\mathcal{G}o}, \; \leq_{\mathcal{M}}, \; \leq_{\mathcal{G}}$, 1032
$\leq_{oe}$, 996
lines, defined sort of $\langle Mn; Bw \rangle$ or $\langle Mn; coll \rangle$, 987
L, 739
L^R, 749
L^S, 738
L_m^S, 831
L^T, 738
L_m^T, 830
L^{Ph}, 738
L_m^{Ph}, 830
$L_{\mathfrak{M}}$, 740
L_μ, 809
L_μ^S, 810
L_μ^T, 809
L_μ^{Ph}, 810
L_m, 830
$\longleftarrow\!\!\prec$, 958
(vel), 437
(abspace), 438
(abstime), 437
(asynch), 436
(clock), 436
(clock)$^=$, 438
(meter), 436
(meter)$^=$, 438
(meter)$^\geq$, 438
(meter)$^\leq$, 438
$\mathbf{Th}_1 \models\!\!\!\models \mathbf{Th}_2$, 193
geod, 52
directions, 470
BA, 967, 968
Eudist, 609

$\mathsf{Ge}^i(Th)$, 1075
$\mathsf{Ordinals}$, 741
inf, 451
sup, 451
$Lines = L$
cf. footnote 931 on p, 941
$Points = Mn$
cf. footnote 931 on p, 941
STL, 110, 139, 460
$expr(-)$, 1046
$\mathsf{Mod}_{\mathrm{OFG}}(Th)$, 35
$\mathsf{Mod}(\Sigma)$, 35
$\mathsf{Mod}(Th)$, 956
min, see footnote 638 on p, 746
$\mathsf{Mod}_{\mathsf{Arch}}(Th)$, 707
$\models$, 28, 35, 161
$A \xrightarrow{f} B$, 26
$\not\models$, see footnote 205 on p, 197
$\models^{\mathrm{OFG}}$, 35
ω, 26
$\ominus^{\mathfrak{M}} v$, 420
$\overline{pq}$, 68
$\overline{pq}$, 164
$\|$, 164, 992
$\|_{\mathfrak{S}}$, 740, 749
$\mathbf{Asim}(\mathsf{K})$, 570
$\mathbf{Asim}(Th)$, 570
$\mathbf{Bax}(\mathbf{P1})$, 544
$\mathbf{Reich}'(Th)$, 580
$\mathbf{Reich}(Th)$, 576
$\mathbf{Reich}_0(Th)$, 562
$\mathbf{Reich}_0(Th)^-$, 563
$\mathbf{Reich}_0(Th)_\partial$, 562
$\mathbf{R}(\mathbf{AxE_{02}})$, 557
$\mathbf{R}(\mathbf{AxE_{02}})^-$, 559
$\mathbf{R}(\mathbf{AxE_{00}})$, 557
$\mathbf{R}(\mathbf{AxE_{00}})^-$, 559
$\mathbf{R}(\mathbf{AxE_0})$, 557
$\mathbf{R}(\mathbf{AxE})$, 557
$\mathbf{R}(\mathbf{AxE})^-$, 557
$\mathbf{Mink}(n)$, 1109
$\mathbf{Mink}(n, rc)$, 1066
$\mathsf{Planes}(n, \mathbf{F})$, 532
$Euclgeom^0(n, \mathfrak{F})$, $Euclgeom^0(\mathfrak{F})$, 1079
$Euclgeom(n, \mathfrak{F})$, $Euclgeom(\mathfrak{F})$, 1079
$Mink(\mathfrak{F})$, 809
$Mink(n, \mathfrak{F})$, 809
$Mink_{nonE}(\mathfrak{F})$, 1110
$Mink_{nonE}(n, \mathfrak{F})$, 1110
$Mink^{\perp_0}(\mathfrak{F})$, 828
$\partial_i f$, 518
$\mathbb{C}_m$, 505
pj_i, 897
$\mathsf{Plane}(\bar{t}, \bar{x})$, 69
$\mathsf{Plane}(\ell_1, \ell_2)$, 164
$\mathcal{P}(H)$, 26
PT, 265
$PT^M = PT^M(n, \mathfrak{F})$, 332
$PT^Q = PT^Q(n, \mathfrak{F})$, 327
$\prec$, 739
$\prec_\mu$, 810
$\prec_m$, 831
$\preccurlyeq$, 1122
$\preceq$, 1120
$\psi(x/\tau)$, 895
$\psi(y)$, 895
$\xrightarrow{\circ}$, $f : A \xrightarrow{\circ} B$, 517
$\mathfrak{F}$, 30, 43
$\mathfrak{F}$-categorical, 299
$\mathfrak{F}^{\mathfrak{M}}$, 30
$\mathfrak{F}^\infty$, 535
$\mathfrak{F}_{oe}$, 996
rep, 916, 917
$\mathbf{R}(\mathbf{Ax\ eqsp})$, 611
B, 29
G, 29
Q, 29
R, 26

$\mathbf{R}(\mathbf{sym})$, 616
$\mathbf{R}(\mathbf{Ax\ syt_0})$, 615
$\mathsf{T}(Axi)$, theory generated by the axiom system $Axi \subseteq Fm$, 977, 1061
suc, 889
$\sim$, 768, 844
$\simeq$, 1031
$\mathsf{simplexes}$, 788
$\dot{\bigcup}_{i\in I} \mathfrak{G}_i$, 821, 824
$\dot{\bigcup} \mathsf{K}$, 819, 820
$\succ^r_\mu$, 1116
$\succ^r$, 1106
$ang^2(\ell)$, 46
$\tau(-)$, 1046
$space(p)$, 470
$spacep$, 438
$space$, 470
F, 30, 43
F^∞, 535
F_{oe}, 996
$\mathsf{Th}(\mathfrak{M})$, 29
$\mathsf{Th}(\mathfrak{F})$, 29, 301
$\mathsf{f}_{mk}(p)$, 61
f_{mk}, 56
$tr_m(b)$, 47
$\mathbb{TH}$, 452
σ_S, 359
$\sigma_{\bar{t}}$, 306
σ_ℓ, 173, 367
$\vec{\ell}_{o,e}$, 841
$\vec{v}_m(b)$, 48
$view_m$, 615
$v_m(b)$, 47
$\widehat{\mathsf{f}_{mk}}$, 865
$\widetilde{\varphi}$, 154
$\subseteq_{\mathsf{w}}$, 958
$\mathsf{f}_{mk} \equiv_v \mathsf{f}_{m'k'}$, 421
w_m, 32
$w_m : {}^nF \xrightarrow{\circ} (??)$, cf. footnote 196 on p, 188, 1156
a-simultaneity, 444
c, 428
c_m, 535
$c_m(p,d)$, 473, 535
$c_m(p,d) < \infty$, 491
$c^f_m(p,d)$, 542
$d \parallel d_1$, when $d, d_1 \in {}^{n-1}F$, 471
$f : A \longrightarrow B$, 26, 958
f', derivative of f, 517
$f(x)$, 26
$f[X]$, 27
$f[\mathfrak{A}]$, 1035
$f \circ g$, 26
$f \restriction C$, 27
$f^2 := f \circ f$, if f is a function, 961
g, 747
g, pseudo-metric, 746
$g(-,y,z), g(x,-,z), g(x,y,-)$, 518, 1046
$g(e,e_1)$, 746, 747
$g^R(e,e_1)$, 750
$g^\prec$, 1120
g_μ, 810
g^2_μ, 152
$g^2_\mu(p,q)$, 152
g_m, 831
i-dimensional plane in $\langle Mn; Bw\rangle$, 989
$int(p,q)$, 509
j-dimensional plane, 164
k sees p, 482
k-way speed of light, 573
m-space like, 566
$m \xrightarrow{\odot} b$, 192
$m \uparrow k,\ m \downarrow k$, 296
n, 29, 44
$p \perp_e q$, 172
p_i, 42

p_t, 44
p_x, 44
p_y, 44
p_z, 44
$v \oplus^{\mathfrak{M}} w$, 420
$\mathbf{G}_{Ta}$, 941
$\mathbf{G}_{We}$, 941
$\mathbf{G}_m$, 1155
$\mathbf{G}_{\mathfrak{M}}$, 737
$\mathbf{X} = \langle X, \mathcal{O} \rangle$, $\langle Mn, \mathcal{T} \rangle$, topological space, 820
$\mathcal{T}$, 747
$\mathbb{C}_{m,t}$, 505
$\mathbf{I}$, 735
IK, 735
$\mathsf{Ge}''(Th)$, 797
$\mathsf{Ge}'(Th)$, 797
$\mathsf{Ge}^0(Th)$, 1020
$\mathsf{Ge}^R(Th)$, 1120
$\mathsf{Ge}^{\prec}(Th)$, 1124
$\mathsf{Ge}^{\perp_0}(Th)$, 821
Ge_{Ta}, 943
Ge_{We}, 943
$\mathsf{G}_{\mathfrak{M}}$, 737
$\mathsf{K} \restriction Voc$, 883
$ang^2(d, d')$, 538
${}^{+}F$, 189
${}^{n}H$, 26
${}^{n}F$, 32, 43
${}^{({}^{n}F)}\,{}^{n}F$, 135
$|a|$, 30
$|p|$, 100
(Ei)(*fast*), 640
(Ei)(*or*), 640
(P1), 522
(P2), 522
(P3), 522
$\mathbf{Bax}_{\mathbf{1}}^{--}$, 533
$\mathbf{Bax}_{\mathbf{3}}^{--}$, 534
$\mathbf{Bax}_{\mathbf{2}}^{--}$, 533
$\mathbf{Bax}_{\partial}^{-}$, 521
$\mathbf{Bax}^{--}$, 531
$\mathbf{Bax}_{+}^{--}$, 538
$\mathbf{Bax}_{++}^{--}$, 544
$E_m^{1/2}$, 584
$\mathbf{R}_{\Delta}(E)$, 574
Reich, 563
$\mathbf{R}_k(\mathbf{AxE})$, 574
$S_m^{1/2}$, 584
$S_m^{1/2}$-simultaneous, 584
$\mathbf{Ax}(\omega)^0$, 794
$\mathbf{Ax(consm)}$, 518
$\mathbf{Ax}(\uparrow\uparrow)$, 426
$\mathbf{Ax(5nop)}^{-+}$, 713
$\mathbf{Ax5}_{\mathbf{Obs}}^{-}$, 530
$\mathbf{Ax5}_{\mathbf{Ph}}^{-}$, 530
$\mathbf{AxP1}^{-}$, 529
$\mathbf{AxP1}_{\mathbf{1}}^{-}$, 531
$\mathbf{AxP1}_{\mathbf{3}}^{-}$, 533
$\mathbf{AxP1}_{\mathbf{2}}^{-}$, 532
$\mathbf{AxR}^{\mathbf{1/2}}$, 584
$\mathbf{AxR}^{-}$, 584
$\mathbf{AxR}^{--}$, 585
$\mathbf{AxR}^{+}$, 584
$\mathbf{Ax}(\sqrt{\ })$, 481
$\mathbf{Ax}(Triv_t)^{-}$, 762
$\mathbf{Ax(cnsm}_0)$, 519
$\mathbf{Ax(cnsm}_1)$, 519
$\mathbf{Ax(cnsm}_2)$, 519
$\mathbb{C}$, 492
(E1), 635
$\mathbb{EC}_{\Delta,0}$, 451
(E5), 638
$(\mathrm{E5})^{+}$, 641
(E6), 641
(E2), 635
(E3), 636

(E7), 641
(E4), 636
Planes, 532
Pth, 541
$\mathbb{TH}_0$, 451
"async", 652
"fat", 652
"noftl", 652
"shrink", 652
"slow", 652
"twin", 652
K is definable implicitly over L, 886
1/2-simultaneity, 582
1/2-simultaneous, 584
2-dimensional plane in a relativistic geometry, see footnote 771 on p, 807
3-sorted first-order language, 27, 29

(A)–(I), theorem schemas for duality theories, 962
absolute space, 438
absolute time, 437
abstract class of structures, see footnote 616 on p, 736, 748
abstract structure, see footnote 616 on p, 736
abstract/concrete distinction, cf. footnote 616 on p, 736
additive (geodesic, quasi geodesic), 1132
additive, $g \restriction D$ is additive, 1132
adequate, 607
adequate (a symmetry principle being adequate for a theory), 456
adequate for **Reich**(Th), 607
adjoint pair of functors, 1041
adjoint situation, 1041
affine geometry, 990
affine structures, 938
affine transformation, 152
affinity of space-time, 405
Alexandrov-Zeeman theorem, 1107
Alexandrov-Zeeman Theorem, cf. e.g. Goldblatt [105, Appendix B], 170
algebra, 735
algebraic element of $\mathfrak{F}$, 784
Algebraic Logic, 1055
algebraic structure, 160
analytic geometry = represented geometry i.e. Cartesian geometry, 1056
Archimedean field = Archimedean ordered field, 66
Archimedean geodesic, 1130
Archimedean ordered field, 446
Archimedean ordered field, see footnote 86 on p, 66
Archimedean ordered group, 1133
art-sim hull of K, 570
art-sim models of a theory, 570
art-sim version of a model, 569
artificial-simultaneity version of a model, 569
automorphism, 65, 160, 298
auxiliary axioms, 135, 297
auxiliary relation, 914
axiom of choice, 434
axiom of cone-smoothness, 519
axiom of continuity, 716
axiom of disjoint windows (**Ax(diswind)**), 762
axiom of functionality, 718

B-submodel of a frame model, 658
backwards, 471
base for a topology, see footnote 683,

759
basic equidistance (eq_0) of an observer-independent geometry, 743
basic orthogonality ($\perp_0$) of an observer-independent geometry, 741
basic paradigmatic effects, 635
between, 530
betweenness (Bw), a ternary relation of an observer-independent (or a relativistic) geometry, 740
big universe of a many-sorted model, 879
binary relation, 26
bisecting segments, 167
bodies, 29
Boolean (topological) spaces, 968
Boolean algebras, 967
boundary, 510
boundary point, 510
bounded, 509

C^*-algebras, 1050
Carnap, 731
Cartesian geometry over a field, 41, 1088
category, 1035
category of theory morphisms, 975
category theoretic convention for introducing functions like $f(-)$ or $g(-,p)$, 1045
causality pre-ordering relation, $\prec$, 739
changing simultaneities, 566
circle, cf. footnote 182, 176
class form of the axiom of choice, 933
clocks get out of synchronism, 90, 96, 436
clocks slow down, 90, 92, 436
clopen sets of a topology, 967
closed set of a topology, 820
closure operator up to isomorphism, 963
closure operator, cf. footnote 969 on p, 963
CMA-lattices = complemented modular algebraic lattices, 1049
codomain of a morphism, 1035
collinear points, 167, 168
collinearity relation (ternary relation on points), 167, 168
collineation, 65
common generalization of axiom systems or theories, 434
compact subset of a topology, cf. footnote 1077 on p, 1050
compact topological space, cf. footnote 981 on p, 968
complemented modular algebraic lattices = CMA-lattices, 1049
complete ordered field, 520
complete theory, 301
composition $\circ$ of a category, 1035
conceptual analysis of relativity, 469, 522
conceptual analysis, cf. footnote 2 on p, 8
concrete class of structures, see footnote 616 on p, 736, 748
cone-smooth, 519
cone-smooth version of a theory, 521
congruence transformation, 350
connectedness, ($\sim$) a binary relation on points of relativistic geometries, 768
conservative extension, 654
continuity, 716
continuous, 536
continuous weak geodesic, 1132

conventional, 581
convergence, 741
convex, 509
convex hull, 509
convex hull of a set of points of a relativistic geometry, 788
coordinate-system, 32, 470
coordinatization (of an ordered Pappian affine geometry), 1001
coproduct of topologies = sum of topologies, 820
core part of a theory, 465
core theory part of a theory, 454
coreflection, 1041
coreflection arrow, 1041

deductively closed theories, 451
definability, 893
definability of topological spaces, 759
definable, 895
definable implicitly, 886
definable implicitly up to isomorphism, 884, 885
definable implicitly with parameters, 885
definable implicitly without taking reducts, 884, 885
definable in, 880, 901
definable over, 880, 901
definable relation, 895
definitional expansion, 896, 900, 901
definitional expansion without taking reducts, 900
definitionally equivalent, 920
definitionally equivalent languages, cf. footnote 932 on p, 941
definitionally equivalent theories, 922
definitionally equivalent, weakly, 936
derivative f' of f, 517
Desargues Theorem, cf. e.g. Hilbert [129], 170
direction, 470
directional speed, 534
disjoint unions of frame models, 818, 819
disjoint unions of geometries, 821, 824
disjoint unions of non-body-disjoint models, 819
disjoint unions of non-disjoint geometries, 824
distance, 609
distance between simultaneities, 444
distance preserving transformation, 350
divisible geodesic, 1131
division ring, 990
domain of a morphism, 1035
drawing compositions of world-view transformations, 694
duality theory, duality theories, 953–957, 964–977, 1019, 1046–1057

Einstein's 1/2-simultaneity, 582
Einstein's Special Principle of Relativity, 123, 454
elementarily-equivalent, 303
elementary classes, 451
ellipse, 176
embeddable, 958
emit a photon, 472
empty model, cf. footnote 950 on p, 956
empty point, 510
equidistance (eq), a 4-ary relation of an observer-independent (or a relativistic) geometry, 743
equivalence of categories, 1044
equivalent categories, 1044

Euclid's axiom, 990
Euclidean case, 1085
Euclidean distance, 609
Euclidean field = Euclidean ordered field, 91
Euclidean geometry over $\mathfrak{F}$, 1079
Euclidean length, 100, 189
Euclidean ordered field, 91
Euclidean orthogonality, 172
Euclidean reduct of a relativistic geometry, 1098
Euclidean straight line, 45
Euclidean topology, 791
evaluation, 895, 913
evaluation, cf. footnote 1107 on p, 1060
event, 32, 737
existential version of the twin paradox, 460
expansion, 153
expansion of a class of models, 883
expansion of a model, 880
explicit definability with parameters, 900
explicit definition, 896, 900
explicit definition of $\mathfrak{N}$ over $\mathfrak{M}$, 900
explicit definition of type (1), 896
explicit definition of type (2.1), 897
explicit definition of type (2.2), 900
explicit definitional expansion, 896, 901
explicitly definable, 895, 900
explicitly definable in, 901
explicitly definable over, 901
explicitly definable relation, 895
explicitly definable without taking reducts, 900
explicitly rigidly definable, 901

field, 30
field reduct, 30
field-extension, 964
finite, 447
finite elements of a non-Archimedean field, 447
finitely nr-implicitly definable, 894
first-order definable meta-function, 933
first-order formulation of $\mathbf{Bax}_{+}^{--}$, 542
first-order language, cf. language, 27
first-order logic, 27
FOL, 7
forwards, 470
Fourier transformation, 1053
frame model, 35
frame theory, 35
frame-language, 29, 34
Friedman, 724, 726, 727
Friedman's conceptual analysis of relativity, 469
FTL, 110
full Reichenbachian version, 576
function, 26
function notation $f(-)$, $expr(-) \stackrel{\text{def}}{:=} \langle\, expr(x) \,:\, x \in A \,\rangle$, 1045
functionality, 718
functor, 1035

Gödel's rotating universe, 725, 731, 862, 1070
Galilean transformation, 439
Galois connection, 453, 1030
Galois group, 964, 977
Galois theory of Cylindric algebras, 977
Galois theory of fields, 964
gapy in $\mathfrak{F}$, 784
general case, 1086
general models, 269

generalization of an axiom system or a theory, cf. footnote ?? p, 423
generalized definitional expansion, 900, 901
generalized disjoint unions of frame models, 819
generalized Galilean transformation, 439
generalized manifold, 738
generalized Minkowski distance, 678
generalized Minkowski model, 676
generalized Poincaré transformation, 678
generalized rotation model, 680
$(\mathcal{G}, \mathcal{M})$-duality, 959, 962
Goldblatt-Tarski reduct $GT_{\mathfrak{M}}$ of $\mathfrak{G}_{\mathfrak{M}}$, 873
$(\mathcal{G}o, \mathcal{M}o)$-duality, 1022

half-line in a geometry $\langle Mn; Bw\rangle$ (defined by Bw), 995
half-line in a relativistic geometry (defined by L and Bw), 841
half-line with origin o and containing e (defined by Bw), 995
half-line with origin o and containing e (defined by L and Bw), 841
Hausdorff (i.e. T_2 space), cf. footnote 982 on p, 968
higher-order logic, 751
homeomorphism, see footnote 649 on p, 748
homogenity, 405
homogenity of space, 405
homogenity of space-time, 405
homomorphism, 160, 298
hyper-plane, 164, 566
hyper-plane in a relativistic geometry, 1079

ideal of infinitely small numbers in an ordered field, 447
imaginary observer, 753
implicit definition, 884, 886
implicitly definable, 884, 886
implicitly definable without taking reducts, 885
incidence geometry associated to $\mathfrak{M}$, 1125
incidence relation, 739
independent axiom system, 75
independent subset of Mn, 989
inertial bodies, 31
infimum, 451
infinitely large, 447
infinitely small, 447
instances of SPR, 349
interior, 510
interpret, 918
interpretation of one theory (or language) in another, Fig.291, 934, 970, 971, 973
interval of $\mathbf{F_0}$, 1128
isometry, 134, 349
isomorphism, 298
isomorphism as a distinguished morphism of a category, 1043
isomorphism between observer-independent (or relativistic) geometries, 748
isotropy, 219, 399, 468

Kant, 725, 731

language = first-order language = a language of first-order logic = similarity type, cf. any textbook on logic e.g. Monk [190, p.14] or Enderton [79], 27
language of first-order logic, 27

language, language of K, cf. also footnote 897, 912
Laplace transform, 1051
Laplace transformation, 1051
lattice of theories, 451
lattice, cf. footnote 975 (cf. also p.451), 965
laws of nature, 348, 727, 728, 1057
least common generalization of theories, 452
Leibniz, 725
Leibniz's principle of identity of indistinguishable concepts, cf. footnote 594 on p, 725
life-line, 47
light-cone, 207, 473
light-cone, $\mathsf{Cone}_{m,p}$, 473
light-point, 510
light-sphere, 505
limit, 741
limit of lines, 741
limit of sequences, 741
linear operator, cf. footnote 1084 on p, 1052
linear transformation, 65
linearly ordered field, 30, 43
lines, 29, 30
Lines $= L$
cf. footnote 931 on p, 941
lines (L) of an observer-independent (or a relativistic) geometry, 739
lines, see L, *lines*, Eucl e.g. on p, 739
local definability, 887, 893
locally additive geodesic, 1132
locally compact topological space, 1050
location, 32
logic, cf. item 1.1.(XI) and footnote 35 on pp, 13, 29
logical positivism, 724, 731
Lorentz Group, 408
Lorentz transformation, 152
Lorentzian metric, see footnote 641 on p, 747

manifold, 738
many-sorted approximation of higher-order logic see higher-order logic, 751
many-sorted first-order language, 27
many-sorted first-order logic, 27
many-sorted logic, 27
many-sorted logic, cf. item 1.1.(XI) and footnote 35 on pp, 13, 29
many-sorted model theory, 298
many-sorted models, cf. item 1.1.(XI) and footnote 35 on pp, 13, 29
many-sorted universal algebra, 298
mathematical logic, cf. item 1.1.(XI) and footnote 35 on pp, 13, 29
maximal definitional expansion, 901
maximal geodesic, 1131
median observer, 306
meter rods shrink, 436
$(\mathcal{M}, \mathcal{G})$-duality, 959, 962
midpoint of a segment, 166
Minkowski distance, 678, 810
Minkowski model, 331, 676
Minkowski-circles, 89
Minkowski-distance, 152
Minkowski-orthogonal lines, 810
Minkowski-sphere, 89
Minkowskian case, 1085
Minkowskian geometry, 809
Minkowskian geometry, $\perp_0$-version, 828
Minkowskian orthogonality ($\perp_\mu$), 810

mirror image w.r.t. a line, 173
model theory, 160, 298
$(\mathcal{M}o, \mathcal{G}o)$-duality, 1022
morphisms of a category, 1035
move backwards in direction d, 471
move forwards in direction d, 470
move in direction d, 470

natural number, 26
neighborhood, 189, 536
Newtonian kinematics, 423
nice automorphisms of $\mathfrak{G}_{\mathfrak{M}}$, 865
non-adequate, 607
non-body-disjoint models, 820
non-elementarily-equivalent models cf. elementarily-equivalent, 303
non-standard higher-order logic, 751
non-standard higher-order logic = many-sorted approximation of higher-order logic, see higher-order logic, 751
non-uniform definability, 893
nonstandard simultaneities, 554
nonstandard synchronization, 554
not adequate (a symmetry principle being not adequate for a theory), 455
nr-implicitly definable, 886

objects of a category, 1035
observational, 724
observational/theoretical, 724
observer-dependent geometry, 830
observer-independent geometry $\mathfrak{G}_{\mathfrak{M}}$, 736, 737
observer-point, 510
observers, 31
Occam, 725
Occam's razor, 123, 464
Occam's razor, cf. footnote 594 on p, 725
of nonzero speed, world-view transformation, 659
one-by-one (explicit) definability, 901
one-by-one definability, 887, 893
one-by-one nr-implicitly definable, 893
one-sorted vector space, 42
one-way speed of light, 553
open interval, $int(p, q)$, 509
open set of a topology, 820
ordered field, 30, 43
ordered field corresponding to an ordered Pappian affine geometry, 999
ordered field reduct of a frame model, 30, 43
ordered Pappian affine geometry, 994
orthogonal, 172
orthogonal in the Euclidean sense, 172
orthogonality ($\perp_r$, $\perp$), a relation of an observer-independent (or a relativistic) geometry, 740
orthogonality, Minkowskian ($\perp_\mu$), 810
orthogonality, relativistic, 740

P1, 522
P2, 522
P3, 522
Pappian affine geometry, 992
Pappus-Pascal Property, 991
paradigmatic effects, 90, 491, 635
paradigmatic theorems, 90
parallel lines, 989
parallel lines ($\ell \parallel_{\mathfrak{G}} \ell_1$) in an observer-independent (or in a relativistic) geometry, 740
parallel straight lines, 164
parallelogram, 167

parameterization, 1129
parsimonious version of a theory, 465
partial derivative, $\partial_i f$, 518
partial function, 517
parts of SPR, 349
Pasch's Law, 993
periodical body, 1067
photon-disjoint unions of frame models, 819
photon-free reduct of a frame model, 658
photon-free sub-reduct of a frame model, 658
photon-glued disjoint unions of geometries, 823, 824
photon-glued disjoint unions of non-disjoint geometries, 824
photon-like Archimedean geodesic, 1128
photon-like geodesic, 1128
photon-like lines, L^{Ph}, 738
photon-like quasi geodesic, 1128
photon-like separated events, $e \equiv^{Ph} e_2$, 749
photon-line, 58
photon-preserving affine transformations, 265
photon-preserving transformation, 62
photon-sphere, $\mathbb{C}_m$, 505
photons, 31
plane, 164
plane generated by a set of points of a relativistic geometry, 769
plane tangent to a light-cone in a relativistic geometry, cf. footnote 772 on p, 807
Poincaré Group, 408
Poincaré transformation, 153, 678
Points = *Mn*
cf. footnote 931 on p, 941
points (Mn) of an observer-independent geometry, 737
points of a topology, 820
poset, 451, 1047
potential laws (of nature), 727, 728, 1057
pre-standard configuration, 602
pre-standard symmetric configuration, 603
pre-standard-sym configuration, 603
prime ideal, cf. footnote 975 on p, 965
principle of absolute space, 438
principle of absolute time, 437
principle of isotropy, 219
principle of parsimony, 123, 464, 725
principles ($*1$), ($*2$), ($*3$), 660
projection functions, 897
pseudo-metric reduct of a relativistic geometry, 1128
pseudo-metric, g, 747

quantities, 29
quasi geodesic, 1129
quasi-standard configuration, 605
quasi-standard-sym configuration, 605

rank of a relation symbol, 881
real-closed field, 301
reduct, 30
reduct of a class of models, 882
reduct of a model, 880, 882
reference frames, 54
reflection, 1040, 1041
reflection arrow, 1041
reflection w.r.t. a line, 173
Reichenbach, 724, 725, 727, 731
Reichenbach-adequate, 608
Reichenbachian version of a theory, 562

Reichenbachian version of the observer-independent geometry, 749
Reichenbachizing a theory, 563
relation of connectedness, $\sim$, 768, 844
relative space-time, 470
relativistic addition of velocities, 420
relativistic effect, 491
relativistic geometry, 748
relativistic orthogonalities, $\perp_r'$, $\perp_r''$ (alternatives for $\perp_r$), 760
relativistic orthogonalities, $\perp_r'''$, $\perp_r^\omega$ (alternatives for $\perp_r$), 771
relativistic orthogonality, $\perp_r$, $\perp$, 740, 742
relativity based on Einstein's SPR only (i.e. not mentioning speed of light), 654
relativity without any connection with electrodynamics, 654
relativized model, 569
relativized models of a theory, 570
relativizing with artificial simultaneities, 569
representation theorems, 1056
residuated-residual pair, 1047
rhombus, 172
rhombus transformation, 72
rigidly definable, 901
Robb hyper-plane, 754, 1080
Robb plane, 1113
Robb's "after", 1106
rotating observers, 554
rotation model, 679
rotation, cf. footnote 557 on p, 679
round-trip of photons, 553

segment, 166
semantical, 581
sentence, see footnote 927 on p, 937
short geodesic, 1131
short space-like geodesic, 1131
short time-like geodesic, 1129
similar algebraic structures, cf. footnote 165 p, 161
similar models, 882
similar models, cf. footnote 165 p, 161
similar structures, 748
similarity type, 27
simple models, 249
simplifying principles part of a theory, 465
simultaneity-stable, 578
simultaneous events, 95
slow observer, 483
slow-line, 58
space component, 32, 43
space coordinates, 43
space location, 32
space part, 54
space part of the coordinate-system, 208
space-axes, 470
space-like Archimedean geodesic, 1130
space-like geodesic, 1130
space-like hyper-plane in a relativistic geometry, 1080
space-like lines, L^S, 738
space-like quasi geodesic, 1129
space-like separated events, 749
space-part, S, 470
space-time, 33, 470, 738, 752
space-time location, 32
spaceships shrink, 90, 100
spatial direction, 470
special case of an axiom system or a theory, cf. footnote **??** p, 423
special relativity with no photons, 654

speed, 47
speed of a body in a direction, 534
speed of light axiom, 564
speed of light axioms, 555
speed of light is finite, 491
SPR, 123, 347, 454, 523
SPR_0, 523
square of generalized Minkowski distance, 678
standard configuration, 71
Standard Lorentz Group, 408
standard Lorentz transformation, 152
standard model of $\mathbf{Reich}_0(Th)$, 570
standard models of a Reichenbachian theory, 570
Stone, 736, 965, 969
Stone duality theory, 965, 969
Stone representation theorem, cf. also Stone duality theory, 736
straight line, 45
streamlined partial metric = time-like-metric, 1119
streamlined partial metric reduct of $\mathfrak{G}_{\mathfrak{M}}$, 1120
strict standard configuration, 71, 659
strict standard world-view transformation, 659
strong embedding, 1035
strong geodesic, 1131
strong submodel, 161
cf. footnote 956 on p, 958
strong substructure, cf. strong submodel, 161
strongly continuous, 537
strongly non-inertial body, 1069
sub-vector space, cf. footnote 164 on p, 160
sub-vocabulary, 882
subalgebra, 160
subbase for a topology, cf. footnote 977 on p, 966
subcategory, 1040
submodel, 161
subspace, cf. footnote 164 on p, 160
substructure, 161
sum of topologies = coproduct of topologies, 820
sum topology, cf. footnote 739 p, 791
supremum, 451
surface, 594
symmetric version of **Reich**(**Basax**), 627
symmetric version of a theory, 454, 455
symmetry, 347
symmetry axioms, 123, 794, 860, 863, 1057
symmetry principle, 454, 455, 727
symmetry principle part of a theory, 454
synchronism, 90
syntactical, 581
syntax-semantics duality, 453, 970
(syntax, semantics)-duality, 970
synthetic geometry = "axiomatic geometry", 1056

T_2 (i.e. Hausdorff) space, cf. footnote 982 on p, 968
T_0-space, T_0 topological space, cf. footnote 979 on p, 967
taking limits, 741
terminology of universal algebra, 160, 161
theorem-schemas (A)–(I) for duality theories, 962
theoretical, 724
theory, 299
theory generated by an axiom system,

977, 1061
theory morphisms, 975
three-sorted first-order language = 3-sorted first-order language, 27
three-way speed of light, 572
time axis, 43
time component, 32, 43
time coordinate, 43
time-axis, 470
time-like Archimedean geodesic, 1130
time-like geodesic, 1130
time-like hyper-plane in a relativistic geometry, 1080
time-like lines, L^T, 738
time-like quasi geodesic, 1129
time-like separated events, $e \equiv^T e_1$, 749
time-like-metric $g^{\prec}$, 1120
time-like-metric geometry, 1120
time-like-metric reduct of a relativistic geometry, 1120
time-like-metric relativistic geometry, 1120
time-like-metric structure, 1120
time-part, $\bar{t}$, 470
toplogy generated by ..., see footnote 645 on p, 747
topological space, 820
topology ($\mathcal{T}$) of an observer-independent (or a relativistic) geometry, 747
trace, 47
transformation, 62
translation, 152
twin paradox, 38, 140, 460
two-dimensional plane in a relativistic geometry, see footnote 771 on p, 807
two-sorted vector space, 42
two-way speed of light, 553
type A transformations, 701
type B transformations, 701
type C transformations, 701
types A, B, C, 701

ultrapower, 1089
uniform (explicit) definability, 901
uniform implicit definition, 887
uniformly definable, 887
unions of geometries and models, 818
unit vector, 69
universal algebra, 160, 161, 298
universes of a three-sorted model, 33

vector, 38
vector space, 42
velocities add up, 437
velocity, 48
velocity structure, 420
velocity-equivalence, 421
visibility relation, 192
vocabulary, 27, 881, 912
vocabulary of a model, 881

weak geodesic, 1132
Weak Principle of Isotropy, 219
weak submodel, 958
weakly definitionally equivalent, 936
weakly standard configuration, 416
why-type questions, 469
window, 826
witness to eq_0^S, 852
witness to eq_0, 849
work (configurations work), 604
world-view, 52
world-view function, 32
world-view relation, 32
world-view transformation, 56, 863–865

world-view transformation of nonzero speed, 659
world-view transformations of types A, B, C, 701
WPI, 219

References

[1] J. Adámek. *Theory of mathematical structures.* Reidel, Dordrecht, Boston, Lancaster, 1983.

[2] J. Adámek, H. Herrlich, and G.E. Strecker. *Abstract and Concrete Categories.* A Wiley-interscience publication. John Wiley & Sons, Inc, New York, 1990.

[3] J. Adámek and S. Mac Lane, editors. *Categorical Topology and its relation to analysis, algebra and combinatorics.* World Scientific, Singapore, New Jersey, London, Hong Kong, 1989.

[4] A.D. Alexandrov. On Lorentz transformations. In *Sessions of the Mathematical Seminar of the Leningrad Section of the Mathematical Institute*, 15 September 1949. (abstract, in Russian).

[5] A.D. Alexandrov. A contribution to chronogeometry. *Canadian J. Math.*, 19:1119–1128, 1967.

[6] A. Andai. Category theoretic concepts for physics. available from www.math.bme.hu/˜andai/kutatas/kut.html.

[7] A. Andai. Lorentz-transformation for tachyons. Manuscript, Eötvös Loránd University Budapest, 2000.

[8] A. Andai. On the mathematical foundations of quantum mechanics (A kvantummechanika matematikai alapjairól). Master's thesis, Eötvös Loránd Univ., Budapest, 1998. (in Hungarian) available from www.math.bme.hu/˜andai/kutatas/kut.html.

[9] H. Andréka, S.D. Comer, and I. Németi. Epimorphisms in cylindric algebras. Manuscript, 1983.

[10] H. Andréka, S.D. Comer, and I. Németi. Surjectivness of epimorphisms of cylindric algebras. *Absts. Amer. Math. Soc.*, 4, 1983.

[11] H. Andréka, T. Gergely, and I. Németi. Model theoretical semantics for many-purpose languages and language hierarchies. *Computational linguistics* (Proc. 8th Intern. Conf. Tokio), pages 213–219, 1980.

[12] H. Andréka, T. Gergely, I. Németi, and I. Sain. Theory morphisms, stepwise refinement of program specifications, representation of knowledge, and cylindric algebras. Preprint, Mathematical Institute Budapest, 1980.

[13] H. Andréka, S. Givant, and I. Németi. Decision problems for varieties of relation algebras. *Memoirs of the AMS*, 126(604):126+xiv pages, March 1997.

[14] H. Andréka, V. Goranko, Sz. Mikulás, I. Németi, and I. Sain. Effective first order temporal logics. In *Time and Logic, a computational approach*, pages 51–129. Ed. by L. Bolc and A. Szałas. UCL Press Ltd., London, 1995.

[15] H. Andréka, R.J. Greechie, and Strecker G.E. On residuated approximations. In *Categorical Methods in Computer Science (With aspects from Topology)*, volume 393 of *Lecture Notes in Computer Science*, pages 333–339. Springer–Verlag, Berlin, 1989.

[16] H. Andréka, J.X. Madarász, and I. Németi. Logical analysis of special relativity. In J. Gerbrandy, M. Marx, M. de Rijke, and Y. Venema, editors, *JFAK. Essays Dedicated to Johan van Benthem on the Occasion of his 50th Birthday*, http://www.wins.uva.nl/˜ j50/cdrom, 1999. Vossiuspers, Amsterdam University Press. CD-ROM, ISBN: 90 5629 104 1.

[17] H. Andréka, J.X. Madarász, and I. Németi. Decidability, undecidability and Gödel incompleteness in relativity theories. Technical report, A. Rényi Institute of Mathematics, Budapest, 2001.

[18] H. Andréka, J.X. Madarász, and I. Németi. On the logical structure of relativity theories. Research report, Alfréd Rényi Institute of Mathematics, Hungar. Acad. Sci., Budapest, 2001. With contributions from A. Andai, G. Sági, I. Sain and Cs. Tőke.

[19] H. Andréka, J.X. Madarász, and I. Németi. Spaceships get distorted in Reichenbachian theories, December 1999.

[20] H. Andréka, J.X. Madarász, and I. Németi. Defining new universes in many-sorted logic, January 2001. A. Rényi Institute of Mathematics, Budapest.

[21] H. Andréka, J.X. Madarász, and I. Németi. A note on definability and definitional equivalence in model theory, June 2000. A. Rényi Institute of Mathematics, Budapest.

[22] H. Andréka, J.X. Madarász, I. Németi, and G. Sági. Accelerated observers, a continuation of On the logical structure of relativity theories, in preparation. A. Rényi Institute of Mathematics, Budapest.

[23] H. Andréka, J.X. Madarász, I. Németi, G. Sági, and I. Sain. Analyzing the logical structure of relativity theory via model theoretic logic. Lecture notes for A'dam course., 1998 December 12. http:www.math-inst.hu/pub/algebraic-logic/relativity1.ps, relativity2.ps.

[24] H. Andréka, J.X. Madarász, I. Németi, G. Sági, and I. Sain. Analyzing the logical structure of relativity theory via model theoretic logic. Lecture notes for A'dam course., 1998 early March. (This was distributed at CCSOM of Univ. of Amsterdam).

[25] H. Andréka and Sz. Mikulás. Lambek calculus and its relational semantics: completeness and incompleteness. *Journal of Logic, Language and Information*, 3(1):1–38, 1994.

[26] H. Andréka and I. Németi. Generalization of variety and quasivariety-concept to partial algebras through category theory. *Dissertationes Mathematicae (Rozprawy Math)*, 204:1–56, 1976.

[27] H. Andréka, I. Németi, and I. Sain. Craig property of a logic and decomposability of theories. In *Proceedings of the 9th Amsterdam Colloquium*. Institute for Logic Language and Computation, 1993.

[28] H. Andréka, I. Németi, and I. Sain. Applying algebraic logic to logic. In T. Rus M. Nivat, C. Rattray and G. Scollo, editors, *Algebraic Methodology and Software Technology* (AMAST'93, Proceedings of the Third International Conference on Algebraic Methodology and Software Technology, The Netherlands, 21-25 June 1993), Workshops in Computing series, pages 5–26. University of Twente, Springer-Verlag, London, 1994.

[29] H. Andréka, I. Németi, and I. Sain. Algebraic logic. In *Handbook of Philosophical Logic Vol. II, second edition*, page To appear. D. Gabbay and F. Guenthner (eds), D. Reidel Publ. Co., 2001.

[30] H. Andréka, J.A.F.K. van Benthem, and I. Németi. Modal languages and bounded fragments of predicate logic. *Journal of Philosophical Logic*, 27:217–274, 1998.

[31] M.A. Arbib and E.G. Manes. Adjoint machines, state-behavior machines and duality. *J. Pure Appl. Algebra*, 6:313–345, 1975.

[32] M.A. Arbib and E.G. Manes. A categorist's view of automata and systems. In *Lecture Notes in Computer Science*, volume 25, pages 51–64. Springer–Verlag, Berlin, 1975.

[33] R. Audi, editor. *The Cambridge Dictionary of Philosophy.* Cambridge University Press, 1995. ISBN 0-521-48328-X (pbk.).

[34] J. Ax. The elementary foundations of spacetime. *Found. Phys.*, 8(7–8):507–546, 1978.

[35] J. Baez and J. Dolan. Higher-dimensional algebra and topological quantum field theory. *Jour. Math. Phys.*, 36:6073–6105, 1995.

[36] J.C. Baez. Higher-dimensional algebra II: 2-Hilbert spaces. Research report, Department of Mathematics, University of California at Riverside, California, USA, 1996.

[37] P. Bahl, J. Cole, N. Galatos, P. Jipsen, and C. Tsinakis. Cancellative residuated lattices. Research report, Department of Mathematics, Vanderbilt University, Nashville, 2001.

[38] J.B. Barbour. *Absolute or relative motion?* Cambridge University Press, Cambridge, 1989.

[39] M. Barr and C. Wells. *Toposes, Triples and Theories.* A Series of Comprehensive Studies in Mathematics. Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1985.

[40] J. Barwise. *Admissible sets and structures.* Springer–Verlag, Berlin, 1975.

[41] J. Barwise. *Handbook of Mathematical Logic.* North-Holland, Amsterdam, 1977.

[42] J. Barwise and S. Feferman, editors. *Model-Theoretic Logics.* Springer–Verlag, Berlin, 1985.

[43] J. Bell and M. Machover. *A course in mathematical logic.* North–Holland, Amsterdam, 1977.

[44] J.L. Bell and A.B. Slomson. *Models and ultraproducts.* North-Holland, Amsterdam, 1969.

[45] N. Belnap. Branching space-time. *Synthese*, 92:385–434, 1992.

[46] G. Birkhoff. *Lattice theory*, volume 25 of *Colloq. Publ.* Amer. Math. Soc., 1967.

[47] P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic.* Kluwer, to appear.

[48] F. Borceaux. *Handbook of Categorical Algebra 1. Basic Caterory Theory.* Cambridge University Press, 1994.

[49] O. Bratteli and D.W. Robinson. *Operator algebras and quantum statistical mechanics 1.* Springer–Verlag, Berlin, 1979, 1987.

[50] W.L. Burke. *Spacetime, geometry, cosmology.* Books in Astronomy. University Science Books, Mill Valley, California, 1980.

[51] P. Burmeister. *A model theoretic oriented approach to partial algebras.* Akademi-Verlag, Berlin, 1986.

[52] S. Burris and H.P. Sankappanavar. *A course in universal algebra.* Graduate Texts in Mathematics. Springer Verlag, New York, 1981.

[53] H. Busemann. *The geometry of geodesics.* Academic Press, New York, 1955.

[54] H. Busemann. *Time like spaces.* Dissertationes Mathematicae (Rozprawy Math). Mathematical Istitute of Polish Academy of Sci., Warsaw, 1967.

[55] R. Carnap. *The logical structure of the world.* University of California Press, Berkeley, 1967. Translated by R. G. George (Original German edition published in 1928).

[56] G.J. Chaitin. *The Limits of Mathematics – Course Outline & Software.* IBM, Watson Center, Yorktown Heights, December 12, 1993.

[57] C.C. Chang and H.J. Keisler. *Model Theory.* North–Holland, Amsterdam, 1973, 1990.

[58] P.M. Cohn. *Universal algebra.* Harper and Row, New York, 1965.

[59] S.D. Comer. Galois-theory of cylindric algebras and its applications. *Trans. Amer. Math. Soc.*, 286:771–785, 1984.

[60] H.S.M. Coxeter. *Introduction to Geometry.* Wiley, New York, 1969.

[61] L. Crane. Clock and category: is quantum gravity algebraic? *Jour. Math. Phys.*, 36:6180–6193, 1995.

[62] L. Csirmaz. *Nemsztenderd Analizis.* Typotex, Budapest, 1999.

[63] G. Czédli. *Lattice Theory.* JATEPress, Szeged, 1999. (in Hungarian).

[64] N.C.A. da Costa, F.A. Doria, and J.A. de Berros. A Suppes predicate for general relativity and set theoretically generic spacetimes. *Internat. J. Theoret. Phys.*, 29:935–961, 1990.

[65] G. Darvas. Ontological levels and symmetry breaking. *Paideia, Philosophy of Science*, 1988. http://www.bu.edu/wcp/Papers/Scie/ScieDarv.htm.

[66] B.A. Davey and H.A. Priestley. *Introduction to Lattices and Order*. Cambridge mathematical textbooks. Cambridge University Press, 1990.

[67] Gy. Dávid. Personal communication with Németi on 1998 December 9, Math. Inst. seminar.

[68] Gy. Dávid. Personal communication with Németi on 1998 Nov. 2.

[69] Gy. Dávid. Relativity without speed of light. Manuscript (Dept. of General Physics, Eötvös Loránd Univ., Budapest), in preparation.

[70] P. Davies and J. Gribbin. *The matter myth*. Penguin books, 1991.

[71] J.W. Dawson, Jr. *Logical Dilemmas. The life and work of Kurt Gödel*. A. K. Peters, Ltd., 1997.

[72] G.D. Dimov and W. Tholen. A characterization of representable dualities. In Adámek and Mac Lane [3], pages 336–357.

[73] R. d'Inverno. *Introducing Einstein's Relativity*. Oxford University Press, 1983.

[74] H.D. Ebbinghaus, J. Flum, and W. Thomas. *Mathematical Logic*. Springer-Verlag, Berlin, 1984.

[75] J. Ehlers, F.A.E. Pirani, and A. Shild. The geometry of free fall and light propagation. In *General relativity, Papers in Honor of J.L. Synge*, pages 63–84. Claredon press, Oxford, 1972.

[76] H. Ehrig and B. Mahr. *Fundamentals of algebraic specification 1*. Springer–Verlag, Berlin, 1985.

[77] A. Einstein. *On the special and the general theories of relativity*. Verlag von F. Vieweg & Son, Braunschweig, 1921. in German.

[78] A. Einstein. The foundation of the general theory of relativity. In W. Perrett and G. B. Jeffery (trans.), editors, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity*. Dover, New York, 1923. (Original German version published in *Annalen der Physik* 49 [1916]).

[79] H.B. Enderton. *Introduction to Mathematical Logic*. North–Holland, Amsterdam, 1972.

[80] R. Engelking. *General topology*. PWN-Polish Scientific Publishers, Warszawa, 1977.

[81] M. Farkas, editor. *Concise Lexicon of Mathematics*. Műszaki kiadó, Budapest, 1972. (in Hungarian).

[82] H.H. Field. *Science without numbers*. Basil Blackwell, Oxford, 1980.

[83] J.D. Foulis. Baer *-semigroups. *Proc. Amer. Math. Soc.*, 11:648–654, 1960.

[84] D. Freed. Higher algebraic structures and quantization. *Commun. Math. Phys.*, 159:343–398, 1994.

[85] R. Freese. Free modular lattices. *Transactions of the American Mathematical Society*, 261:81–91, 1980.

[86] P.J. Freyd and A. Scedrov. *Categories, Allegories*. North-Holland, Amsterdam, New York, Oxford, Tokyo, 1990.

[87] M. Friedman. *Foundations of Space-Time Theories. Relativistic Physics and Philosophy of Science.* Princeton University Press, 1983.

[88] J. Fröhlich and T. Kerler. *Quantum groups, quantum categories and quantum field theory*, volume 1542 of *Springer Lecture Notes in Mathematics*. Springer–Verlag, Berlin, 1993.

[89] L. Fuchs. *Partially Ordered Algebraic Systems*. Pergamon Press, 1963.

[90] D. Gabbay and F. Guenthner (eds). *Handbook of Philosophical Logic*. D. Reidel Publishing Company, Dordrecht, 1984.

[91] D.M. Gabbay, editor. *What is a Logical System?* Oxford University Press, 1994.

[92] D.M. Gabbay. *Fibring logics*. Oxford University Press, 1998.

[93] D. Gallin. *Intensional and Higher-Order Modal Logic.* North Holland/-American Elsevier, Amsterdam-New York, 1975.

[94] P. Gärdenfors. *Knowledge in flux.* MIT Press, Cambridge USA, 1988.

[95] M. Gardner. *Relativity Simply Explained.* Dover, 1997.

[96] M. Gehrke and B. Jónsson. Monotone bounded distributive lattice expansions. *Mathematica Japonica*, 52(2), 2000.

[97] T. Gergely. Algebraic representation of language hierarchies. *Acta Cybernet.*, 5:307–323, 1980.

[98] R. Geroch. *General relativity from A to B.* The University of Chicago Press, 1978.

[99] S. Givant. Universal classes of simple relation algebras. *The Journal of Symbolic Logic*, 64:575–589, 1999.

[100] Kurt Gödel. *Collected Works. Volume II. Publications 1938-1974.* Oxford University Press, 1990. Editors: S. Feferman, J.W. Dawson, Jr., S.C. Kleene, G.H. Moore, R.M. Solovay, and J. van Heijenoort.

[101] Kurt Gödel. *Collected Works. Volume III. Unpublished Essays and Lectures.* Oxford University Press, 1995. Editors: S. Feferman, J.W. Dawson Jr., W. Goldfarb, and C. Parsons.

[102] J. Goguen and R. Burstall. Institutions: Abstract model theory for specifications and programming. *Journal of the ACM*, 39,1:95–146, 1992.

[103] R. Goldblatt. Diodorian modality in Minkowski spacetime. *Studia Logica*, XXXIX:219–236, 1980.

[104] R. Goldblatt. *Topoi. The Caterorial Analysis of Logic.* North-Holland, Amsterdam, New York, Oxford, 1984.

[105] R. Goldblatt. *Orthogonality and Spacetime Geometry.* Springer–Verlag, Berlin, 1987.

[106] R. Goldblatt. Algebraic polymodal logic: a survey. *Logic Journal of IGPL*, 8(4):393–450, 2000.

[107] E.C. Gootman and A.J Lazar. Quantum groups and duality. *Rev. Math. Phys.*, 5:417–451, 1993.

[108] G. Grätzer. *General lattice theory.* Akademie-Verlag, Berlin, 1978.

[109] G. Grätzer. *Universal Algebra, Second edition.* Springer–Verlag, New York, 1979.

[110] B. Greene. *The Elegant Universe.* W.W.Norton & Company, New York and London, 1999. ISBN 0-393-04688-5.

[111] B. Gruber and R.S. Millman. *Symmetries in science.* Plenum Press, New York and London, 1980. ISBN 0-306-40541-5.

[112] A. Grünbaum. Simultaneity by slow clock transport in the special theory of relativity. *Philosophy of Science*, 36:5–43, 1969.

[113] A. Grünbaum. *Philosophical problems of space and time.* D. Reidel Publishing Co., Boston, 1974.

[114] R. Guitart. On the geometry of computations. *II. Cahiers de Topologie et Géométrie Différentielle Catégoriques*, XXIX(4):297–326, 1988.

[115] P. Hájek and P. Pudlák. *Mathematics of First Order Arithmetic.* Springer–Verlag, Berlin, 1993.

[116] Gy. Hajós. *Bevezetés a geometriába.* Tankönyvkiadó, Budapest, 1971. In Hungarian.

[117] Gy. Hajós and J. Strohmajer. A geometria alapjai (The basics of geometry). Technical report, Eötöv Loránd University, Budapest, 1972. Tankönyvkiadó, Budapest (Hungary); (in Hungarian).

[118] P.R. Halmos. *Finite-dimensional vector spaces.* Springer–Verlag, Berlin, 1974.

[119] G. Hansoul. A duality for Boolean algebras with operators. *Algebra Universalis*, 17:34–49, 1983.

[120] S.O. Hansson. *A textbook of belief dynamics.* Kluwer Academic Publishers, 1999.

[121] M. Hausner. *A vector space approach to geometry.* Prentice-Hall Inc., Englewood Cliff N. J., 1965.

[122] S.W. Hawking and G.F.R. Ellis. *The large scale structure of space-time.* Cambridge University Press, 1973.

[123] G. Helzer. Special relativity with acceleration. *American Mathematical Monthly*, 107:219–237, 2000.

[124] L. Henkin. Completeness in the theory of types. *Journal of Symbolic Logic*, 15:81–91, 1950.

[125] L. Henkin, J.D. Monk, and A. Tarski. *Cylindric Algebras Parts I and II*. North–Holland, Amsterdam, 1971 and 1985.

[126] L. Henkin, P. Suppes, and A. Tarski, editors. *The axiomatic method with special reference to geometry and physics*, Studies in Logic and the Foundations of Mathematics. North–Holland, Amsterdam, 1959.

[127] N.J. Hicks. *Notes on differential geometry*. Van Nostrand Co. Inc., 1965.

[128] D. Hilbert. *Grundlagen der Geometrie*. Lepzig / B. G. Teubner Verlag, Stuttgart, 1899/1977.

[129] D. Hilbert and P. Bernays. *Grundlagen der Geometrie*. Teubner Verlag, Stuttgart, 1956.

[130] R. Hirsch and I. Hodkinson. *Relation algebras by games*. North-Holland, to appear.

[131] W. Hodges. *Model Theory*. Cambridge University Press, 1993.

[132] W.A. Hodges, I. Hodkinson, and H.D. Macpherson. Omega-categoricity, relative categoricity and coordinatisation. *Annals of Pure and Applied Logic*, 46(4):169–199, 1990.

[133] E. Hoogland. *Definability and interpolation. Model-theoretic investigations*. PhD thesis, Amsterdam, 2001. University of Amsterdam, ILLC Dissertation Series DS-2001-05.

[134] P. Hraskó. Introduction to general relativity. Technical report, Technical University, Budapest, 1997. (in Hungarian).

[135] W.v. Ignatowsky. *Phys. Zeitsch.*, 11:972, 1910.

[136] G. James and R.C. James. *Mathematical Dictionary*. Van Nostrand, 1942.

[137] G. Janelidze. Galois theory in categories: the new example of differentiable fields. In Adámek and Mac Lane [3], pages 369–380.

[138] A. Jánossy, Á. Kurucz, and Á.E. Eiben. Combining algebraizable logics. *Notre Dame J. of Formal Logic*, 37(2):366–380, 1996.

[139] P. Jipsen, B. Jónsson, and J. Rafter. Adjoining units to residuated Boolean algebras. *Algebra Universalis*, 34:118–127, 1995.

[140] S.A. Jones, A. Morris and K.R. Pearson. *Abstract algebra and famous impossibilities*. Universitext, Springer-Verlag, Berlin, 1991.

[141] B. Jónsson. Lattice-theoretic approach to projective and affine geometry. In Henkin et al. [126].

[142] B. Jónsson. The preservation theorem for canonical extensions of Boolean algebras with operators. In K.A. Baker and Wille R., editors, *Lattice theory and its applications*, pages 121–130. Heldermann Verlag, 1995.

[143] B. Jónsson and A. Tarski. Boolean algebras with operators. In *Alfred Tarski Collected Papers Vol. 3*, pages 369–420. Birhauser Verlag, 1986.

[144] B. Jónsson and C. Tsinakis. Relation algebras as residuated Boolean algebras. *Algebra Universalis*, 30:469–478, 1993.

[145] A. Joyal and R. Street. An introduction to Tannaka duality and quantum groups. In *Category theory (Como, 1990)*, volume 1488 of *Lecture Notes in Mathematics*, pages 413–492. Springer–Verlag, Berlin, 1991.

[146] I. Kant. Kritik der reinen Vernunft. Johann Friedrich Hardknoch, Riga, 1781. Second revised edition: Druck und Verlag von Philipp Reclam jun, Leipzig, 1787. Later edition: Akademie Verlag (Kant's Schriften), Berlin, 1910.

[147] I. Kant. *Critique of Pure Reason*, volume XLII, 48 of *Great Books of the Western World*. Encyclopaedia Britannica Inc., Chicago, London, Toronto, 1952. Original German edition is [146].

[148] C. Kassel. *Quantum Groups*. Springer-Verlag, New York, 1995.

[149] A. Kirillov. *Elements of the theory of representations*. Springer–Verlag, Berlin, 1976.

[150] A.I. Kostrikin and Yu.I. Manin. *Linear Algebra and Geometry*. Gordon and Breach, New York, 1988.

[151] Á. Kurucz. *Decision problems in algebraic logic.* PhD thesis, Math. Inst. Hungar. Acad. Sci., Budapest, 1997. available from www.math-inst.hu/pub/algebraic-logic/kuagthes.ps.

[152] A. Kurusa. *Introduction to differential geometry (in Hungarian).* Polygon (Bolyai Institute of University of Szeged), Hungary, 1999.

[153] L.D. Landau and E.M. Lifsic. *Theoretical Physics Part II (theory of fields).* Nauka, Moscow, 1973. in Russian.

[154] R.W. Latzer. Non-directed light signals and the structure of time. In P. Suppes, editor, *Space, Time, and Geometry.* D.Reidel, 1973.

[155] F.W. Lawvere. Adjointness in foundations. *Dialectica*, 23(3-4):281–296, 1969.

[156] F.W. Lawvere. Metric spaces, generalized logic, and closed categories. Rendiconti Seminario Matematico e Fisico di Milano 43, 1973.

[157] F.W. Lawvere. Variable quantities and variable structures in topoi. In *Algebra, topology, and category theory. A collection of papers in honor of Samuel Eilenberg*, pages 101–131. Academic Press, New York, 1976.

[158] F.W. Lawvere and S.H. Schanuel, editors. *Categories in continuum physics*, volume 1174 of *Lecture Notes in Mathematics.* Springer–Verlag, Berlin, 1986.

[159] H. Lenz. Fastgeordnete Körper. *Arch. Math.*, 11:333–338, 1960.

[160] L. Lipshitz. The undecidability of the word problems for projective geometries and modular lattices. *Transactions of the American Mathematical Society*, 193:171–180, 1974.

[161] J. Loose. *A historical introduction to the Philosophy of science.* Oxford Univ. Press, 1980.

[162] H. Lugowski. *Foundations of universal algebra.* Teubner-Verlag, Leipzig, 1976. In German.

[163] S. Mac Lane. *Categories for the working mathematician.* Springer–Verlag, Berlin, 1971.

[164] J.X. Madarász. Epimorphisms in discriminator varieties. In *Proc. of Workshop on Abstract Algebraic Logic*, 1998. Centre de Recerca Matemàtica, Bellaterra (Spain), Quaderns nùm 10/ gener 1998, pp. 106-108.

[165] J.X. Madarász. Interpolation in algebraizable logics; semantics for non-normal multi-modal logic. *Journal of Applied Non-Classical Logics*, 8(1-2):67–105, 1998.

[166] J.X. Madarász. On faster than light observers, Part II. Manuscript, Budapest, 1998.

[167] J.X. Madarász. Interpolation and amalgamation; pushing the limits (Parts I,II). *Studia Logica*, 61,62(3,1):311–345, 1–19, 1998, 1999.

[168] J.X. Madarász. The symmetry axioms are equivalent in Basax, 2000.

[169] J.X. Madarász. Surjectivity of epimorphisms in varieties of algebraic logic. Preprint, Alfréd Rényi Institute of Mathematics, Budapest, 2001.

[170] J.X. Madarász and I. Németi. On faster than light observers, Part I. Manuscript, Budapest, 1998.

[171] J.X. Madarász and I. Németi. FTL observers are at most two-dimensional. Manuscript in preparation, 1999.

[172] J.X. Madarász and I. Németi. On FTL observers in a generalized version of first-order relativity (The generalization is in the direction of partiality coming from general relativity). Manuscript in preparation, 1999.

[173] J.X. Madarász and T. Sayed-Ahmed. Amalgamation, interpolation, epimorphisms: solutions of all problems from Pigozzi's paper. Manuscript, Alfréd Rényi Institute of Mathematics, Budapest, 2001.

[174] M. Makkai. Stone duality for first-order logic. *Adv. in Math.*, 65:97–170, 1987.

[175] M. Makkai. Towards a categorical foundation of mathematics. In *Proc. Logic Coll*, Haifa, 1995.

[176] M. Makkai. The structuralist program in mathematics. Preprint, McGill Univ. Canada, 1998. (also available: makkai@triples.math.mcgill.ca).

[177] J. Makowsky and I. Sain. On the equivalence of weak second order and non-standard time semantics for various program verification systems. In *Logic in Computer Science (Proc. Conf. Cambridge USA 1986)*, 1986. IEEE Computer Society Press; pp.293-300.

[178] K.L. Manders. First-order logical systems and set-theoretic definability. Preprint, University of Pittsburgh, 1983.

[179] E.G. Manes. *Algebraic theories*, volume 26 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, Heidelberg, Berlin, 1976.

[180] M. Manzano. Introduction to many-sorted logic. In Meinke and Tucker [187], pages 3–89.

[181] N. Marti-Oliet and J. Meseguer. General logics and logical frameworks. In Gabbay [91], pages 355–392.

[182] M. Marx. *Algebraic Relativization and Arrow Logic*. PhD thesis, University of Amsterdam, appeared as ILLC Dissertation Series 1995–3. ILLC, Plantage Muidergracht 24, 1018 TV Amsterdam, e-mail: illc@fwi.uva.nl, 1995.

[183] M. Marx and Y. Venema. *Multi-dimensional Modal Logic*. Kluwer Academic Press, 1997.

[184] T. Matolcsi. *Spacetime without reference frames*. Akadémia Kiadó, Budapest, 1993.

[185] T. Matolcsi. Spacetime without reference frames: an application to synchronizations on a rotating disk. Eötvös Loránd University, Budapest, preprint, submitted, 1999.

[186] R.N. McKenzie, G.F. McNulty, and W.F. Taylor. *Algebras, Lattices, Varieties, Vol I*. The Wadsworth and Brooks/Cole Mathematics Series. Monterey, California, 1987.

[187] K. Meinke and J.V. Tucker, editors. *Many-sorted Logic and its Applications*. John Wiley & Sons Ltd, 1993.

[188] Sz. Mikulás. *Taming Logics*. PhD thesis, University of Amsterdam, appeared as ILLC Dissertation Series 1995–12. ILLC, Plantage Muidergracht 24, 1018 TV Amsterdam, e-mail: illc@fwi.uva.nl, 1995.

[189] C.W. Misner, K.S. Thorne, and J.A. Wheeler. *Gravitation*. W. H. Freeman, San Francisco, 1973.

[190] J.D. Monk. *Mathematical Logic*. Springer Verlag, 1976.

[191] R. Montague. Deterministic theories. In *Decisions, Value and Groups, 2*, 1962. Oxford, Pergamon Press; pp.325-370.

[192] J. Mundy. The physical content of Minkowski geometry. *Britisch J. Philos. Sci.*, 37(1):25–54, 1986.

[193] D. Myers. An interpretive isomorphism between binary and ternary relations. In *Structures in logic and computer science*, 1997. ed. Jan Mycielski, Grzegorz Rozenberg, and Arto Salomaa, Lecture Notes in Computer Science 1261, Springer, Berlin 1997, pp.84-105.

[194] G.L. Naber. *Spacetime and singularities, an introduction.* Cambridge Univ. Press, 1988.

[195] L.D. Nel. Categorical differential calculus and Banach-Steinhaus. In Adámek and Mac Lane [3], pages 149–162.

[196] I. Németi. Foundations for stepwise refinement of program specifications via cylindric algebra theory. *Diagrammes* (Paris), 8:1–24, 1982.

[197] I. Németi. On varieties of cylindric algebras with applications to logic. *Annals of Pure and Applied Logic*, 36:235–277, 1987.

[198] I. Németi. Algebraization of quantifier logics, an introductory overview. available from www.math-inst.hu/pub/algebraic-logic/survey.ps, 1997. (A shorter, incomplete version of this appeared in Studia Logica, 50(3/4)[a special issue devoted to Algebraic Logic, eds.: W.J. Blok and D.L. Pigozzi]:485–570, 1991).

[199] I. Németi and I. Sain, editors. *Special issue of Logic Journal of IGPL on Algebraic Logic.* Logic J. of IGPL 8(4), 2000.

[200] B. O'Neil. *The geometry of Kerr black holes.* A. K. Peters, 1995. ISBN: 1-56881-019-9.

[201] I. Ozsváth and E.L. Schücking. The finite rotating universe. *Annals of Physics*, 55:166–204, 1969.

[202] L.A. Pars. *Philos. Mag.*, 42:249, 1921.

[203] J.W. Pelletier and J. Rosický. Generating the equational theory of C^*-algebras and related categories. In Adámek and Mac Lane [3], pages 163–180.

[204] R. Penrose. *The Emperors's New Mind.* Oxford University Press, 1989.

[205] R. Penrose. *Shadows of the mind.* Oxford University Press, Oxford, New-York, 1994.

[206] A. Pillay and S. Shelah. Classification theory over a predicate I. *Notre Dame Journal of Formal Logic*, 26(4):361–376, 1985.

[207] M.B. Pour-El and J.I Richards. *Computability in analysis and physics.* Perspectives in mathematical logic. Springer–Verlag, Berlin, 1987.

[208] S. Pulmanová. Quantum measurements and quantum logics. *Journal of Mathematical Physics*, 35:1555–1572, April, 1994.

[209] W.V.O. Quine. *Philosophy of Logic.* Prentice-Hall, Englewood Cliffs, NJ., 1970.

[210] M. Rédei. *Introduction to Quantum Logic.* Eötvös University Press, Budapest, 1995.

[211] M. Rédei. *Quantum Logic in Algebraic Approach*, volume 91 of *Fundamental Theories of Physics.* Kluwer Academic Publishers, Dordrecht, Boston and London, 1998. pp. x+238.

[212] M. Reed and B. Simon. *Methods of modern mathematical physics, Volumes I–IV.* Academic Press, New York, 1975.

[213] H. Reichenbach. *The theory of relativity and a priori knowledge.* University of California Press, 1960. Translated by M. Reichenbach. (Original German edition published in 1920).

[214] H. Reichenbach. *Axiomatization of the theory of relativity.* University of California Press, Berkeley, 1969. Translated by M. Reichenbach. Original German edition published in 1924.

[215] W. Rindler. *Essential relativity. Special, general and cosmological.* Springer-Verlag, Berlin, 1969, 1977.

[216] A.A. Robb. *A Theory of Time and Space.* Cambridge University Press, 1914. Revised edition, *Geometry of Time and Space*, published in 1936.

[217] F. Rohlich. *From paradox to reality (our basic concepts of the physical world).* Cambridge University Press, 1987.

[218] H.L. Royden. Remarks on primitive notions for elementary Euclidean and non-Euclidean plane geometry. In Henkin et al. [126], pages 86–96.

[219] P. Rózsa. *Linear algebra and its applications.* Tankönyvkiadó, Budapest, 1991. (in Hungarian).

[220] B. Russel. *The problems of philosophy.* Williams and Norgate, London, 1990.

[221] G. Sági. Lecture notes for logic and relativity. Manuscript, 1998.

[222] I. Sain. *Dynamic logic with nonstandard model theory.* PhD thesis, Hungarian Academy of Sciences, Budapest, 1986. In Hungarian.

[223] I. Sain. There are general rules for specifying semantics: observations on abstract model theory. *CL&CL*, 13:195–250, 1990.

[224] W.C. Salmon. The philosophical significance of the one-way speed of light. *NOÛS*, 11:253–292, 1977.

[225] S. Sankaran. Hochschild-Tannaka duality theorem for homogeneous spaces. *Rend. Sem. Mat. Univ. Politec. Torino*, 36:59–85, 1979.

[226] P. Schauenburg. *Tannaka duality for arbitrary Hopf algebras*, volume 66 of *Algebra Berichte.* Verlag Reinhard Fischer, Munich, 1992. ii+57 pp.

[227] J.W. Schutz. *Independent axioms for Minkowski space-time.* Longoman, London, 1997.

[228] W. Schwabhäuser, W. Szmielew, and A. Tarski. *Metamathematische Methoden in der Geometrie.* Hochschul text. Springer–Verlag, Berlin, 1983.

[229] I.R. Shafarevich. *Basic Notions of Algebra*, volume 11 of *Encyclopedia of Mathematical Sciences.* Springer–Verlag, Berlin, 1997.

[230] S. Shelah. *Around classification theory of models.* Lecture Notes in Mathematics 1182, Springer-Verlag, Berlin, 1986.

[231] W.F. Stinespring. Integration theorems for gauges and duality for unimodular groups. *Trans. Amer. Math. Soc.*, 90:15–56, 1959.

[232] P. Suppes. The axiomatic method in the empirical sciences. In L. Henkin, J. Addison, C.C. Chang, W. Craig, D. Scott, and R. Vaught, editors, *Proceedings of the Tarski Symposium*, Proceedings of Symposia in Pure Mathematics Volume XXV, pages 465–479, Providence, Rhode Island, 1974. American Mathematical Society.

[233] L.E. Szabó. The problem of open future, determinism in the light of relativity and quantum theory. Manuscript, Dept. of Theor. Physics, Eötvös Loránd University, Budapest, 1998.

[234] L.W. Szczerba and A. Tarski. Metamathematical properties of some affine geometries. In Y. Bar-Hiller, editor, *Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science*, Studies in Logic and the Foundations of Mathematics, pages 166–178, 1965.

[235] W. Szmielew. The role of the Pasch axiom in the foundations of Euclidean geometry. In *Proc. of the Tarski Symp. held in Berkeley in 1971*, pages 123–132. Amer. Math. Soc., Providence, RI, 1974.

[236] W. Szmielew. *From Affine to Euclidean Geometry.* Reidel Publ. Co., 1983.

[237] A. Tarski. A problem concerning the notion of definability. In [241], Vol. 3, pp. 163-170.

[238] A. Tarski. Some methodological investigations on the definability of concepts. (in German), in [241], Voll 1, pp.637-639. The Polish version contains more detail: in Revue Philosophique, Vol.37 (1934).

[239] A. Tarski. Fundamentale begriffe der methodologie der deduktiven wissenchaften, i. *Monatsh. Math. Phys.*, 37:361–404, 1930. Also in "Logic, Semantics, Metamathematics. Papers from 1923 to 1938. Clarendon Press, Oxford, 1956. pp.60–109.

[240] A. Tarski. What is elementary geometry? In Henkin et al. [126], pages 16–29.

[241] A. Tarski. *Collected Papers, Volumes 1–4.* Birkhäuser (Basel–Boston–Stuttgart), 1986. Editors: Steven R. Givant and Ralph N. McKenzie.

[242] A. Tarski and S. Givant. *A Formalization of Set Theory Without Variables*, volume 41 of *AMS Colloquium Publications.* Providence, Rhode Island, 1987.

[243] A. Tarski and S. Givant. Tarski's system of geometry. *Bulletin of Symbolic Logic*, 5,2:175–214, 1999.

[244] A. Tarski, A. Mostowski and R.M. Robinson. *Undecidable theories.* North-Holland, Amsterdam, 1953. xii+98pp.

[245] E.F. Taylor and J.A. Wheeler. *Spacetime Physics, second edition, fourth printing.* W. H. Freeman and Company, New York, 1997.

[246] R.J. Thompson. Complete description of substitutions in cylindric algebras and other algebraic logics. In *Algebraic Methods in Logic and in Computer*

Science (Proceedings of the '91 Banach Semester, Banach Center), pages 327–342, Warsaw, 1993. Polish Academic Publishers, Banach Center Publications, Volume 28.

[247] K.S. Thorne. *Black holes and time warps*. W. W. Norton and Company, 1994.

[248] Cs. Tőke. Some problems in the analysis of relativity theory in first-order logic. Master's thesis, Eötvös Loránd University, Budapest, 2000. 58pp.

[249] A. Urquhart. Decision problems for distributive lattice-ordered semigroups. *Algebra Universalis*, 34:399–418, 1995.

[250] F. Valentine. *Convex sets*. McGraw Hill, 1964.

[251] S. Vályi. Modal version of basic axiom system for special relativity. Manuscript, Institute of Mathematics, Kossuth Lajos University Debrecen, 1998.

[252] J. van Benthem. *Modal logic and classical logic*. Bibliopolis, Napoli, 1983.

[253] J. van Benthem and A. ter Meulen (eds). *Handbook of Logic and Language*. North-Holland, Amsterdam, 1997.

[254] J.A.F.K. van Benthem. *The Logic of Time*. D. Reidel, 1983.

[255] J.A.F.K. van Benthem. *Language in Action*. Elsevier Science Publishers, Amsterdam, 1991.

[256] J.A.F.K. van Benthem. Content versus wrapping: an essay in semantic complexity. In M. Marx, L. Pólos, and M. Masuch, editors, *Arrow Logic and Multimodal Logic*, pages 203–219. CSLI Publications, Stanford, 1996.

[257] J.A.F.K. van Benthem. *Exploring logical dynamics*. Studies in Logic, Language and Information. CSLI Publications, Stanford, 1996.

[258] D. van Dalen. *Logic and Structure*. Springer Universitext, 1997. Third Edition, Second printing.

[259] V.S. Varadarajan. *Geometry of Quantum Theory*, volume I. D. Van Nostrand Co. Inc., Princeton, New Jersey, 1968.

[260] J. von Neumann. *Continuous Geometry*, volume 25 of *Princeton Mathematical Series (I. Halperin, ed.)*. Princeton University Press, Princeton NJ, 1960. xi+299 pp.

[261] K.G.C. von Staudt. *Beiträge zur Geometrie der Lage.* F. Korn, Nürenberg, 1857.

[262] R.M. Wald. *General relativity.* The University of Chicago Press, Chicago, 1984.

[263] J.A. Winnie. Special relativity without one-way velocity assumptions. *Philosophy of Science*, 37:81–99; 223–238, 1970.

[264] E.C. Zeeman. Causality implies the Lorentz group. *Journal of Mathematical Physics*, 5:490–493, 1964.

[265] P. Zlatos. On conceptual completeness of syntactical-semantical systems. *Periodica Math. Hungar.*, 16(3):145–174, 1985.

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