Linear Dilaton Background
and
Fully Localized Intersecting Five-branes

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Abstract

We investigate a near-horizon geometry of NS5-branes wrapping on a Riemann surface, which asymptotically approaches to linear dilaton backgrounds. We concretely find a fully localized solution of the near-horizon geometry of intersecting NS5-branes. We also discuss a relation to a description of Landau-Ginzburg theories.

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1   Introduction

Non-gravitational theories in six dimensions having string-like properties, which are called "little string theories" (LSTs), have been attracted much interests. (See for review [1].) There is no consistent Lagrangian and these theories are considered as non-local field theories.

To understand the LSTs we have various approaches from superstring theories, even though there is no field theoretical treatment. The first construction of the LSTs [2] was obtained by using discrete light-cone quantization (DLCQ) of M(atrix) theory [3]. More intuitive definition is world-volume effective theory of N parallel NS5-branes in the limit taking the string coupling to zero ($g_s \to 0$) and fixing the string scale. The LSTs can be holographically understood from the near-horizon geometry of parallel NS5-branes [4]. This approach is T-dual equivalent to Type II string theory on an A_{k-1} singularity with $g_s \to 0$ [5, 6].

Originally, LSTs are six dimensional non-gravitational theories, but we expect that there exist lower dimensional LSTs by compactifying the 6d LSTs. For example, if we consider NS5-branes whose world-volume is $\mathbf{R}^{1,3} \times \Sigma$, where Σ is a compact two-dimensional Riemann surface, we obtain a four dimensional LST. When we especially choose the Riemann surface as a Seiberg-Witten curve and lift the NS5-brane to M5-brane wrapping on Σ in M-theory [7], we expect a relation between some phase of four dimensional QCD and the LST.

Holographic analysis for this four dimensional LST needs a near-horizon of SUGRA solution of the NS5-branes wrapping on the Σ. Some partially and fully localized solutions were constructed by [8, 9, 10]. The fully localized near-horizon geometry wrapping on the Σ, which is the warped AdS geometry, was found by [11]. However, we now need some linear dilaton backgrounds rather than the AdS in order to investigate the four dimensional LSTs which are not conformal field theory. In this paper, we search for the near-horizon SUGRA solution of the NS5-brane wrapping on the Σ as a kind of Fayyazuddin-Smith geometry [9]. Especially, we find the solution in the case of highly degenerate Riemann surface, which describes intersecting NS5-branes with four dimensional common world-volume.

Our solution have very similar structure to the parallel NS5-branes solution by using a suitable singular coordinate transformation. Superstring theory on the near-horizon geometry of the N parallel NS5-branes has a dual description to a Landau-Ginzburg (LG) theory with a suitable superpotential, which have the same form as a curve for ALE space with A_{N-1} singularity [5, 6]. If we apply these results to our solution, we can

expect that dual theory on the NS5-branes intersection relates to the LG theory with a superpotential of a curve for a singular Calabi-Yau three fold. This result agrees with [12].

This paper is organized as follows. We start with a brief review of the Fayyazuddin-Smith geometry and discuss some decoupling limits of this geometry in a subsequent section. In section 4 we construct a fully localized solution of the intersecting NS5-branes which enjoys a linear dilaton form. We assume the linear dilation form at first in the approach to the solution in this section. Alternative general construction, in which we first make a intersection M5-branes solution in M-theory, is given in section 5. These two approaches are consistent with each other. In section 6 we will discuss a relation to the LG model. The final section is devoted to conclusion and discussion.

2 A brief review of the Fayyazuddin-Smith geometry

In this section, we review some results on the M5-brane solution wrapping on a Riemann surface [9].

The general solution for M5-brane configurations which preserve at least 1/4 supersymmetries is studied in [9]. Such M5-branes have world-volume directions along x^0, x^1, x^2, x^3 and wrap on a Riemann surface holomorphically embedded in $\mathbf{C}^2 \ni (v, s)$, where we define $v \equiv x^4 + ix^5$ and $s = x^6 + ix^7$. The overall transverse directions to the M5-branes are x^8, x^9, x^{10}. The metric is given in [9],

$$ds^2 = H^{-\frac{1}{3}}dx_{3+1}^2 + H^{-\frac{1}{3}}g_{m\bar{n}}dz^m dz^{\bar{n}} + H^{\frac{2}{3}}\delta_{\alpha\beta}dx^\alpha dx^\beta, \tag{2.1}$$

where $\alpha, \beta = 8, 9, 10$, $m, n = v, s$ and $g_{m\bar{n}}$ is required to be a Kähler metric. Here H is the determinant of $g_{m\bar{n}}$,

$$H = g_{v\bar{v}}g_{s\bar{s}} - g_{v\bar{s}}g_{s\bar{v}}. \tag{2.2}$$

The source equation is

$$dF = J_{m\bar{n}}dz^m \wedge dz^{\bar{n}} \wedge dx^8 \wedge dx^9 \wedge dx^{10} \tag{2.3}$$

where F is 4-form field strength and

$$J_{m\bar{n}} = \partial_\alpha\partial_\alpha g_{m\bar{n}} + 4\partial_m\partial_{\bar{n}}H. \tag{2.4}$$

The source J specifies the position of M5-branes. For example, the source for the intersecting N M5-branes whose world-volume are the $x^0, x^1, x^2, x^3, x^4, x^5$ directions and N′

M5′-branes whose world-volume are the $x^0, x^1, x^2, x^3, x^6, x^7$ directions is given by

$$
\begin{align}
J_{s\bar{s}} &= N \, l_p^3 \, \delta^{(3)}(x^\alpha) \, \delta^{(2)}(s), \tag{2.5} \\
J_{v\bar{v}} &= N' \, l_p^3 \, \delta^{(3)}(x^\alpha) \, \delta^{(2)}(v), \tag{2.6} \\
J_{v\bar{s}} \quad (&= \ J_{s\bar{v}}) = 0. \tag{2.7}
\end{align}
$$

Given a M5-brane configuration which determine the source equation (2.4), solving it and Kählar condition on $g_{m\bar{n}}$, we have the SUGRA solution for the M5-brane configuration. In other words, if we have the metric which satisfies the Kähler condition, then we find the M5-brane configuration by using the source equation (2.4).

3 Decoupling limit

The world-volume theory on NS5-branes was studied in the decoupling limit $g_s \to 0$ with l_s fixed [2], where g_s is the string coupling constant and l_s is the string length. In this limit, the modes in the bulk of space-time decouple, while the dynamics of modes on the NS5-branes remains.

Since NS5-branes are dual to the M5-branes which are localized on the eleventh (compactified) direction, we consider the decoupling limit in the M-theory. The parallel 5-branes case is studied in [4] and the six-dimensional decoupled theory appears. In the configurations which we will consider in this paper, there are common four-directions on the 5-branes. Hence we will have the four-dimensional decoupled theory. The string length l_s of Type IIA theory is given by $l_s{}^2 = l_p^3/R$, where R is the radius of the compactified direction. We take the limit in which R and l_p go to zero with l_s fixed.

We choose the natural coordinates in this limit as follows. We would like to keep the excitations which remain interacting on the M5-branes in this limit. The M2-branes may end on the M5-branes. On the M5-brane world-volume theory, two types of BPS objects appear. We would like to fix the their mass. The first is that mass of the M2-branes which streched between the M5-branes along the overall transverse directions x^α appears as x^α/l_p^3. The other is M2-branes which have a disc topology bounded on a cycle of the Riemann surface in (v, s)-space. The mass of the M2-branes is given by the area of the disk and appears as vs/l_p^3. Thus we choose the natural coordinates as

$$
\begin{align}
y^\alpha &= \frac{x^\alpha}{l_p^3}, \tag{3.1} \\
w\,t &= \frac{v\,s}{l_p^3}. \tag{3.2}
\end{align}
$$

In terms of y, w, t, the M5-branes metric (2.1) becomes

$$\frac{ds^2}{l_p^2} = H^{-\frac{1}{3}} dx_{3+1}^2 + H^{-\frac{1}{3}} g_{m\bar{n}} dz^m dz^{\bar{n}} + H^{\frac{2}{3}} \delta_{\alpha\beta} dy^\alpha dy^\beta, \tag{3.3}$$

where m, n represent w, t and we redefine as $H = g_{w\bar{w}} g_{t\bar{t}} - g_{w\bar{t}} g_{t\bar{w}}$. The l_p^3 factors in the source equations disappear. For example, the source equations for the intersecting M5-branes (2.4)-(2.7) become

$$\partial_\alpha \partial_\alpha g_{t\bar{t}} + 4\partial_t \partial_{\bar{t}} H = N \, \delta^{(3)}(y^\alpha) \, \delta^{(2)}(t), \tag{3.4}$$

$$\partial_\alpha \partial_\alpha g_{w\bar{w}} + 4\partial_w \partial_{\bar{w}} H = N' \, \delta^{(3)}(y^\alpha) \, \delta^{(2)}(w), \tag{3.5}$$

$$\partial_\alpha \partial_\alpha g_{w\bar{t}} + 4\partial_w \partial_{\bar{t}} H = 0. \tag{3.6}$$

Since the source equations do not have l_p factor, solving these equations, the solutions of $g_{m\bar{n}}$ do not include l_p factor. Therefore the metric (3.3) does not depend on l_p except for the overall l_p factor. This l_p factor will not effect on any physical computations as in the case of a similar factor in [13].

The assignment of the l_p factor between v and s does not change above results. But we would like to treat v and s equally, we choose as

$$w = \frac{v}{l_p^{\frac{3}{2}}}, \tag{3.7}$$

$$t = \frac{s}{l_p^{\frac{3}{2}}}. \tag{3.8}$$

In the next section, we will search for the 5-brane solution in the decoupling limit.

4 Linear dilaton solution

It was proposed [4] that any vacua of string theories which have asymptotic linear dilaton backgrounds are holographic. The lower dimensional decoupled theories, which are in general not local QFT, are described by string theory on the backgrounds,

$$ds^2 = dx_{1,d-1}^2 + d\phi^2 + ds^2(\mathcal{M}), \tag{4.1}$$

$$g_s^2 = e^{-Q\phi}, \tag{4.2}$$

where Q is a constant and ϕ relates to the dilaton linearly. We call this ϕ the dilation in the following. The metric on the $9 - d$ dimensional manifold $\mathcal{M}$ is independent of x and ϕ.

Authors in [4] commented that NS5-branes wrapping on a Riemann surface which is considered in this paper is a possible candidate for the proposition, but the linear dilaton background for such configuration has been not known.

Another approach to the decoupled theory was discussed in [12, 14]. String theory on $\mathbf{R}^{1,d-1} \times X^{2n}$ was studied in the decoupling limit. Here $2n = 10 - d$ and X^{2n} is a singular (non-compact) Calabi-Yau $n-$fold (CY_n). The decoupled theory arises near the singularity on X^{2n}, and is dual to string theory on the linear background which is

$$\mathbf{R}^{1,d-1} \times \mathbf{R}_{\phi} \times \mathcal{N} \tag{4.3}$$

where $\mathcal{N}$ is the manifold at a fixed distance from the singular point on X^{2n}.

For $n = 3$ case, non-compact CY_3 is considered as T-dual to the NS5-branes wrapping a Riemann surface [15]. Since string theory on (CY_n) is dual to string theory on the linear dilaton background, the metric for NS5-branes wrapping a Riemann surface expect to be also the linear dilaton form. Thus we look for the NS5-brane solutions which have the linear dilaton background.

We have the type IIA NS5-brane solution by the dimensional reduction along one of the overall transverse directions to the M5-brane solution (3.3). The standard relation between 11-dimensional supergravity and type IIA supergravity is given in [16, 17]. We compactify along the eleventh direction. The NS5-brane metric becomes

$$ds^2 = dx_{3+1}^2 + g_{m\bar{n}}dz^m d\bar{z}^{\bar{n}} + Hdzd\bar{z}, \tag{4.4}$$

where we define $z = y^8 + iy^9$. The dilaton ϕ is determined by the compactification as $e^{\phi/\sqrt{N}l_s} = H^{1/2}$ where N is a dimensionless real number. Thus, if the metric has the linear dilaton form (4.1), it might become

$$ds^2 = dx_{3+1}^2 + Nl_s{}^2\frac{dH^2}{4H^2} + ds^2(\mathcal{M}). \tag{4.5}$$

We first seek the solutions such that the general solution (4.4) has the linear dilaton form (4.5). Then, we check if the solutions satisfy the Kähler condition and the source equations.

Now we have three dimensionful complex coordinates w, t and z whose dimensions in length are $-1/2, -1/2$ and -2. It is convenient to choose only one dimensionful (radial) coordinate. We define that

$$w = \tilde{w}^a, \tag{4.6}$$

$$t = \tilde{t}^a, \tag{4.7}$$

$$z = \tilde{z}^b, \tag{4.8}$$

where a, b are some rational parameters. Since we have the fixed string length l_s as another dimensionful parameter, we select the dimensionful radial coordinate as

$$\rho^2 = \frac{|\tilde{v}|^2 + |\tilde{s}|^2}{l_s^{2c}} + \frac{|\tilde{z}|^2}{l_s^{2d}}, \tag{4.9}$$

where c, d are also rational parameters. We adopt the radial coordinate in terms of $\tilde{w}, \tilde{t}, \tilde{z}$, not w, t, z, because the powers of original coordinates in the radial coordinate often deviate from usual ones in the case of localized solutions [8, 9, 11, 18]. The first term and second term in (4.9) must have the same dimension, so that

$$\frac{1}{2a} + c = \frac{2}{b} + d. \tag{4.10}$$

The radial coordinate ρ is related to the dilaton ϕ, and $\mathbf{R}^{1,3}$ and the manifold $\mathcal{M}$ does not depend on ϕ. Therefore, since the manifold $\mathcal{M}$ does not depend on ρ either, all elements related to ρ in (4.4) come from the second term in (4.5) which have the l_s^2 factor. Those in (4.4) must also have the l_s^2 factor, so that

$$ac = bd = 1. \tag{4.11}$$

From these equations, we find that

$$b = 2a, \quad c = 1/a, \quad d = 1/2a. \tag{4.12}$$

So we parameterize w, t, z as like follows,

$$w = \tilde{w}^a, \tag{4.13}$$

$$t = \tilde{t}^a, \tag{4.14}$$

$$z = \tilde{z}^{2a}, \tag{4.15}$$

$$\rho^2 = \frac{|\tilde{w}|^2 + |\tilde{t}|^2}{l_s^{2/a}} + \frac{|\tilde{z}|^2}{l_s^{1/a}}, \tag{4.16}$$

$$\tilde{w} = l_s^{\frac{1}{a}} \rho \sin\theta_1 \cos\theta_2 e^{i\varphi_1}, \tag{4.17}$$

$$\tilde{t} = l_s^{\frac{1}{a}} \rho \sin\theta_1 \sin\theta_2 e^{i\varphi_2}, \tag{4.18}$$

$$\tilde{z} = l_s^{\frac{1}{2a}} \rho \cos\theta_1 e^{i\varphi_3}. \tag{4.19}$$

Here θ_i $(i = 1, 2)$ and φ_j $(j = 1, 2, 3)$ are dimensionless angular variables.

We now define that

$$g_{w\bar{w}} = \rho^{-2a} \tilde{g}_{w\bar{w}}, \tag{4.20}$$

$$g_{t\bar{t}} = \rho^{-2a} \tilde{g}_{t\bar{t}}, \tag{4.21}$$

$$g_{w\bar{t}} = \rho^{-2a} e^{-ai(\varphi_1 - \varphi_2)} \tilde{g}_{w\bar{t}}, \tag{4.22}$$

$$g_{t\bar{w}} = \rho^{-2a} e^{ai(\varphi_1 - \varphi_2)} \tilde{g}_{w\bar{t}}, \tag{4.23}$$

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and

$$\tilde{H} \equiv \tilde{g}_{w\bar{w}}\tilde{g}_{t\bar{t}} - \tilde{g}_{w\bar{t}}^2 \left(= \rho^{4a}H\right). \tag{4.24}$$

Here we assume that $\tilde{g}_{w\bar{w}}, \tilde{g}_{t\bar{t}}$ and $\tilde{g}_{w\bar{t}}$ do not depend on the dimensionless combination $\rho l_s^{\frac{3}{2a}}$, therefore $\tilde{g}_{w\bar{w}}, \tilde{g}_{t\bar{t}}$ and $\tilde{g}_{w\bar{t}}$ are real functions which depend only on θ_1 and θ_2. [3] By the definition of the parameterization (4.13)-(4.19), we have the overall l_s^2 factor in the general solution (4.4) except for dx_{3+1}^2 part. From the dimensional analysis, the metric elements $\tilde{g}_{kl}, k, l = \theta_i, \varphi_j$ do not depend on ρ because of above assumption. Since the manifold $\mathcal{M}$ does not depend on the dilaton (and ρ), all elements in (4.4) which relate to ρ come from the second term in the RHS of (4.5).

In these parameterization, the elements related to ρ in the linear dilaton metric (4.5) are

$$g_{\rho\rho} = \frac{4a^2Nl_s^2}{\rho^2}, \tag{4.25}$$

$$g_{\rho\theta_1} = -\frac{2aNl_s^2\partial_{\theta_1}\tilde{H}}{\rho}, \tag{4.26}$$

$$g_{\rho\theta_2} = -\frac{2aNl_s^2\partial_{\theta_2}\tilde{H}}{\rho}, \tag{4.27}$$

$$g_{\rho\varphi_j} = 0 \quad \text{for} \quad j = 1, 2, 3. \tag{4.28}$$

On the other hand, those in the general solution (4.4)

$$g_{\rho\rho} = \frac{a^2l_s^2}{\rho^2}\left(4\cos^{4a}\theta_1\tilde{H} + \sin^{2a}\theta_1 A\right), \tag{4.29}$$

$$g_{\rho\theta_1} = -\frac{2a^2l_s^2}{\rho}\left(4\cos^{4a-1}\theta_1\sin\theta_1\tilde{H} - \cos\theta_1\sin^{2a-1}\theta_1 A\right), \tag{4.30}$$

$$g_{\rho\theta_2} = -\frac{2a^2l_s^2}{\rho}\sin^{2a}\theta_1\left(\tilde{g}_{w\bar{w}}\cos^{2a-1}\theta_2\sin\theta_2 - \tilde{g}_{t\bar{t}}\cos\theta_2\sin^{2a-1}\theta_2\right.$$
$$\left. +\tilde{g}_{w\bar{t}}\left(\cos^{a-1}\theta_2\sin^{a+1}\theta_2 - \cos^{a+1}\theta_2\sin^{a-1}\theta_2\right)\right), \tag{4.31}$$

$$g_{\rho\varphi_j} = 0 \quad \text{for} \quad j = 1, 2, 3. \tag{4.32}$$

Here we define

$$A \equiv \left(\tilde{g}_{w\bar{w}}\cos^{2a}\theta_2 + 2\tilde{g}_{w\bar{t}}\cos^a\theta_2\sin^a\theta_2 + \tilde{g}_{t\bar{t}}\sin^{2a}\theta_2\right). \tag{4.33}$$

If the general solution become the linear dilaton form, the correspond elements in each metric must be equal. Solving these equations (4.25)-(4.32), we find

$$\tilde{H} = \frac{N}{\cos^{4a}\theta_1 + f(\theta_2)\sin^{4a}\theta_1}, \tag{4.34}$$

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[3] If we assume a Kähler potential $K = K(|w|, |t|, |z|)$, since $g_{m\bar{n}} = \partial_m\partial_{\bar{n}}K$ for $m, n = w, t$, $\tilde{g}_{w\bar{w}}, \tilde{g}_{t\bar{t}}$ and $\tilde{g}_{w\bar{t}}$ do not depend on φ_j.

$$\tilde{g}_{w\bar{w}} = \frac{1}{B\cos^{2a}\theta_2 \sin^{2a}\theta_1}\left(64a^2 Nf(\theta_2)^2\cos^4\theta_2 \sin^{4a}\theta_1 + 4\,a^2\,\cos^{4a}\theta_1\,\cos^{2a}\theta_2\,\sin^{2a}\theta_2\right.$$

$$\left. +Nf'(\theta_2)^2\sin^{4a}\theta_1\,\sin^2 2\theta_2 + 4\,a\,f(\theta_2)\sin^{4a}\theta_1\left(a\,\cos^{2a}\theta_2\,\sin^{2a}\theta_2 - 8Nf'(\theta_2)\,\cos^3\theta_2\,\sin\theta_2\right)\right) \quad (4.35)$$

$$\tilde{g}_{t\bar{t}} = \frac{1}{B\sin^{2a}\theta_1 \sin^{2a}\theta_2}\left(64a^2 Nf(\theta_2)^2\,\sin^{4a}\theta_1\,\sin\theta_2{}^4 + 4\,a^2\,\cos^{4a}\theta_1\,\cos^{2a}\theta_2\,\sin^{2a}\theta_2\right.$$

$$\left. +Nf'(\theta_2)^2\sin^{4a}\theta_1\,\sin^2 2\theta_2 + 4\,a\,f(\theta_2)\sin^{4a}\theta_1\left(a\,\cos^{2a}\theta_2\,\sin^{2a}\theta_2 + 8Nf'(\theta_2)\,\cos\theta_2\,\sin^3\theta_2\right)\right) \quad (4.36)$$

$$\tilde{g}_{w\bar{t}} = \frac{-1}{B\cos^{a}\theta_2 \sin^{2a}\theta_1 \sin^{a}\theta_2}\left(4\,a^2\,\cos^{4a}\theta_1\,\cos^{2a}\theta_2\,\sin^{2a}\theta_2 - 16a^2 Nf(\theta_2)^2\,\sin^{4a}\theta_1\,\sin^2 2\theta_2\right.$$

$$\left. +Nf'(\theta_2)^2\sin^{4a}\theta_1\,\sin^2 2\theta_2 + 4\,a\,f(\theta_2)\sin^{4a}\theta_1\left(a\,\cos^{2a}\theta_2\,\sin^{2a}\theta_2 - Nf'(\theta_2)\sin 4\,\theta_2\right)\right), \quad (4.37)$$

where $f(\theta_2)$ is an arbitrary function depending only on θ_2, and

$$B \equiv 16\,a^2\,f(\theta_2)\left(\cos^{4a}\theta_1 + f(\theta_2)\sin^{4a}\theta_1\right). \quad (4.38)$$

Moreover, these solutions must satisfy the Kähler condition, $\partial_l g_{m\bar{n}} = \partial_m g_{l\bar{n}}$ for $l, m, n = w, t$. The Kähler condition is satisfied if:

$$f(\theta_2) = k\cos^{2a-n}\theta_2\,\sin^{2a+n}\theta_2, \quad (4.39)$$

where k, n are arbitrary number. In terms of w, t and z, these solutions become

$$g_{m\bar{n}} = \partial_m G\partial_{\bar{n}}\bar{G} + H\partial_m F\partial_{\bar{n}}\bar{F}, \quad (4.40)$$

$$H = \frac{Nl_s{}^2}{|z|^2 + |F|^2}, \quad (4.41)$$

where

$$F = \sqrt{k}w^{1-p}t^{1+p}/l_s, \quad (4.42)$$

$$G = \begin{cases} \frac{l_s}{2\sqrt{k}}\log\frac{t}{w} & \text{for } p = 0, \\ l_s\frac{w^p t^{-p}}{2p\sqrt{k}} & \text{for } p \neq 0. \end{cases} \quad (4.43)$$

Here we define $p \equiv \frac{n}{2a}$. The dilaton ϕ is $e^{\phi/\sqrt{N}l_s} = H^{\frac{1}{2}}$.

The source equations for these solutions are

$$\partial_z\partial_{\bar{z}}g_{w\bar{w}} + \partial_w\partial_{\bar{w}}H = N(1-p)l_s{}^2\delta^{(2)}(z)\delta^{(2)}(w), \quad (4.44)$$

$$\partial_z\partial_{\bar{z}}g_{t\bar{t}} + \partial_t\partial_{\bar{t}}H = N(1+p)l_s{}^2\delta^{(2)}(z)\delta^{(2)}(t), \quad (4.45)$$

$$\partial_z\partial_{\bar{z}}g_{w\bar{t}} + \partial_w\partial_{\bar{t}}H = 0. \quad (4.46)$$

These equations imply that the solutions describe intersecting $N(1+p)$ NS5-branes and $N(1-p)$ NS5′-branes. Here $N > 0$ and $|p| < 1$ since both charges of NS5-branes and NS5′-branes are positive. We have fully localized intersecting NS5-branes solutions in the near horizon limit which have linear dilaton background (4.1).

8

5 M5-brane solutions

In the previous section, we have assumed the linear dilaton form metric at first, and then looked for the solutions. In this section, we consider M5-brane solutions for the equations (2.1)-(2.4) in eleven dimensions and see how the solutions relate to the linear dilaton metric in Type IIA theory. The solutions in the near horizon limit are first given in [19] and we revisit them.

We now consider the solutions in terms of v, s, x^α for $\alpha = 8, 9, 10$ not fixed coordinates w, t, y^α in the decoupling limit. We define that

$$r^2 \equiv \sum_{\alpha=8}^{10} (x^\alpha)^2. \tag{5.1}$$

We assume that the determinant H of $g_{m\bar{n}}$ for v, s as

$$H = \frac{N l_p{}^3}{(r^2 + |F|^2)^{3/2}}, \tag{5.2}$$

where N is a (dimensionless) constant and F is an arbitrary holomorphic function which depends only on v and s, $F = F(v, s)$. The determinant H has diverges on the palace where r and F vanish simultaneously. These singularities specify the position of the M5-branes as we will see, therefore, the M5-branes localize at the origin in the overall transverse directions and wrap on the Riemann surface in the (v, s)-space.

The metric elements $g_{m\bar{n}}$ must be a Kähler for the M5-brane solution. So we take a Kähler potential as $K = K(r, |v|, |s|)$ where we assume that the solution has rotational symmetry in the overall transverse directions. The sources (2.4) vanish at the place where M5-branes do not localize. Since $g_{m\bar{n}} = \partial_m \partial_{\bar{n}} K$, the source equations (2.4) away from the M5-branes become

$$\partial_m \partial_{\bar{n}} \left(\frac{1}{r^2} \partial_r (r^2 \partial_r K) + 4H \right) = 0. \tag{5.3}$$

We find the general solution up to some Kähler transformation which contain functions depending only on r,

$$K = \frac{4N l_p{}^3}{r} \log \left(\sqrt{r^2 + |F|^2} + r \right) + \frac{1}{r} K^{(1)} + K^{(2)}, \tag{5.4}$$

where $K^{(i)}$ for $i = 1, 2$ are the real functions which do not depend on r, so $K^{(i)} = K^{(i)}(|v|, |s|)$. For simplicity, we take [4]

$$K^{(1)} = 0, \tag{5.5}$$

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[4] The first term in (5.4) is the same as the solution (12) in [19] up to a Kähler transformation. To do this, we choose $K^{(1)} = -2N \log |F|^2$.

and

$$K^{(2)} = |G|^2, \tag{5.6}$$

where G is a holomorphic function, $G = G(v, s)$. The function G is determined by the condition that the determinant of $g_{m\bar{n}}$ obtained from the Kähler potential equals to H again, so

$$(\det g_{m\bar{n}} =)\partial_v \partial_{\bar{v}} K \, \partial_s \partial_{\bar{s}} K - \partial_v \partial_{\bar{s}} K \, \partial_s \partial_{\bar{v}} K = H. \tag{5.7}$$

This equation gives the condition on G that

$$\partial_v F \, \partial_s G - \partial_s F \, \partial_v G = 1. \tag{5.8}$$

Thus, we find the general solution for the M5-branes, which is formally

$$ds^2 = H^{-\frac{1}{3}} \left(dx_{3+1}^2 + |dG|^2 + H(|dF|^2 + dr^2 + r^2 d\Omega_2{}^2) \right), \tag{5.9}$$

where $d\Omega_2$ is the standard S^2 in the overall transverse directions. This metric is naively the same as the parallel M5-branes solution in terms of F and G, however, the coordinate map from (v, s) to (F, G) is generally not well-defined on the M5-branes as we saw in the previous section. Therefore, the metric is defined in terms of v and s.

The source equations (2.4) become

$$
\begin{aligned}
&\partial_\alpha \partial_\alpha g_{m\bar{n}} + 4\partial_m \partial_{\bar{n}} H \\
&= \partial_m F \partial_n \bar{F} \left(\partial_\alpha \partial_\alpha + 4\partial_F \partial_{\bar{F}} \right) H \\
&= N \, l_p{}^3 \delta^{(3)}(r)\delta^{(2)}(F)\partial_m F \partial_{\bar{n}} \bar{F}.
\end{aligned}
\tag{5.10}
$$

So N represents a number of the M5-branes which wrap on the holomorphic curve $F = 0$.

Let us slightly generalize above solutions. We take H, instead of (5.2), as

$$H = 1 + \frac{N \, l_p{}^3}{(r^2 + |F|^2)^{3/2}} \tag{5.11}$$

The source equation away from M5-branes and the solution of Kähler potential for it are the same as (5.3) and (5.4). We take $K^{(1)} = 0$ again and

$$K^{(2)} = |F|^2 + |G|^2, \tag{5.12}$$

where G is an arbitrary holomorphic function as like as before. It is easy to check that the determinant of $g_{m\bar{n}}$ also gives the same condition as (5.8) on G. So the metric and the source equation are described by the same equations (5.9) and (5.10) again in which H is given by (5.11).

The holomorphic function F must have one dimension of the length. If we take $F = v$ in particular, G becomes $G = s$. So the solution become the standard parallel N M5-branes solution.

For the general F, in the limit $r \to \infty$, the metric becomes asymptotically flat in $\mathbf{R}^{1,3}$ and the overall transverse space, but, does not in (v, s)-space. So we expect that there are more general solutions which become asymptotically flat far away from the M5-branes.

5.1 Dimensional reduction to linear dilaton background

In this subsection, we consider the conditions that the solutions (5.9) have the linear dilaton form background in the decoupling limit.

In the near horizon limit r and $F \to 0$, the first term in H (5.11) drops out,

$$H = \frac{N\, l_p{}^3}{(r^2 + |F|^2)^{3/2}} \tag{5.13}$$

In the decoupling limit, this becomes

$$H = \frac{1}{l_p{}^6} \tilde{H}, \tag{5.14}$$

where we define

$$\tilde{H} \equiv \frac{N}{(|y^\alpha|^2 + |\tilde{F}|^2)^{\frac{3}{2}}}, \tag{5.15}$$

$$\tilde{F} \equiv F/l_p{}^3. \tag{5.16}$$

Then the metric becomes

$$ds^2 = l_p{}^2 \tilde{H}^{-\frac{1}{3}} \left(dx_{3+1}^2 + |dG|^2 + \tilde{H}(|d\tilde{F}|^2 + (dy^\alpha)^2) \right). \tag{5.17}$$

The condition on G (5.8) becomes

$$\partial_w \tilde{F} \partial_t G - \partial_t \tilde{F} \partial_w G = 1. \tag{5.18}$$

Compactifying the metric along the eleventh direction, the metric becomes, in type IIA theory,

$$ds^2 = dx_{3+1}^2 + |dG|^2 + \tilde{H}(|d\tilde{F}|^2 + |z|^2). \tag{5.19}$$

Here

$$\tilde{H} = \frac{N\, l_s{}^2}{|z|^2 + |\tilde{F}|^2}. \tag{5.20}$$

The dilaton ϕ is given by

$$e^{\phi/\sqrt{N}l_s} = \tilde{H}^{1/2}. \tag{5.21}$$

From the dimensional analysis, $\tilde{F}$ must have -2 dimensions of the length . We take [5]

$$\tilde{F} = k\, w^n t^m l_s^{\frac{n+m}{2}-2}. \tag{5.23}$$

So we consider the intersecting NS5-branes case. We choose the "radial" coordinate as

$$\rho^2 = |z|^2 + |\tilde{F}|^2 \tag{5.24}$$

The dilaton ϕ depends only on ρ in the six dimensional transverse space w, t, z, therefore, G does not depend on ρ. From dimensional analysis again, G must have a form as [6]

$$G = l_s \tilde{G}(w/t), \tag{5.25}$$

where $\tilde{G}(w/t)$ does not include l_s. Since the right hand side in (5.18) is the constant which does not depend on l_s, F must depend only on l_s^{-1}. So we have the condition on n and m that

$$n + m = 2. \tag{5.26}$$

Here we take

$$n = 1 - p, \tag{5.27}$$
$$m = 1 + p. \tag{5.28}$$

The holomorphic function G which determined by (5.18) is the same as (4.43). Thus we find that the solution in section 5 reduces to the same linear dilaton backgrounds in section 4. There might be more general solutions to M5-branes as we mentioned, however, if the solutions reduce to the linear dilaton backgrounds they might become the form in section 4.

In the parallel NS5-branes case, there is only one (dimensionless) parameter N which is number of the NS5-branes. This corresponds to the size of S^3 surrounding the NS5-branes. On the other hand, in the intersecting NS5-branes case, there are three parameter

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[5] In general, we may take

$$\tilde{F} = \sum_i a_i\, w^{n_i} t^{m_i} l_s^{\frac{n_i+m_i}{2}-2}, \tag{5.22}$$

where n_i and m_i are arbitrary real number and a_i are arbitrary complex number. However, it seems that there is generally no solution for G to the equation (5.18) except for the case of the monomial F and the special cases where F reduces to a monomial by a one-to-one coordinate transformation. This supports the result of section 4 which is general solution for the linear dilaton background.

[6] G does not depend on z becouse of the condition (5.18) on G.

N, p and k. Two of them N and p correspond to the number of NS5-branes. N represents the size of the base S^3 in $(z, F(w, t))$-space and p decide the fiber $G(w, t)$ structure in (w, t)-space. The other parameter k corresponds to the size of fiber. In the parallel case, we have the same background structure in the near horizon limit through any pass to close to the NS5-branes. In the intersecting case, however, the background structure depends on the pass in the near horizon limit, $F(w, t) \to 0$. If we are close to a parallel part of NS5-branes not the intersecting point, the background may be the same as the parallel NS5-branes case and the S^3 part will be dominant with the small size of fiber G. So if we would like to close to the intersecting point, we need to choose the appreciate pass and this seems to choose the appreciate k. In the CFT view, this may correspond to choose the consistent central charge for string theory.

6   Relation to LG models

The decoupled non-gravitational theory on the parallel NS5-branes background has studied by using CFT [4, 5, 6, 12, 14, 20] in detail. One approach to the theory is holographic description [4, 12, 14]. N parallel NS5-branes is given by the holomorphic curve $v^N = 0$ where v is a transverse complex coordinate. The NS5-branes are the dual to the ALE space [5, 6],

$$\mathcal{F}_2 \equiv v^N + z_1^2 + z_2^2 = 0, \tag{6.1}$$

where z_1 and z_2 are complex coordinates. The decoupled theory on the singularity $v = z_1 = z_2 = 0$ relates to string theory on

$$\mathbf{R}^{1,5} \times \mathbf{R}_\phi \times S^1 \times LG(W = \mathcal{F}_2), \tag{6.2}$$

where $\mathbf{R}_\phi$ is labeled by the dilaton ϕ.

The intersecting NS5-branes solutions (5.19) and (5.20) are formally the same forms as N parallel NS5-branes solutions in terms of F and G. F can be considered as transverse complex coordinate like as v in the parallel case. So we replace v in (6.1) by F and the decoupled theory formally relates to string theory on

$$\mathbf{R}^{1,5} \left(= \mathbf{R}^{1,3} \times \mathbf{C}_G \right) \times \mathbf{R}_\phi \times S^1 \times LG(W = \mathcal{F}_2). \tag{6.3}$$

Since, however, the coordinate map $w, t \to F, G$ is not well-defined at $w, t = 0$ in the intersecting NS5-branes case, F and G are not direct product through w and t. Therefore the world-volume theory on the intersecting NS5-branes must be described by string

theory on the backgrounds in terms of w, t rather than F, G. Replacing F, G with w, t, the CFT background (6.3) maps to

$$\mathbf{R}^{1,3} \times \mathbf{R}_\phi \times S^1 \times LG(W = \mathcal{F}_3) \tag{6.4}$$

where

$$\mathcal{F}_3 = w^{N(1-p)} t^{N(1+p)} + z_1^2 + z_2^2. \tag{6.5}$$

In [12], the authors discussed that, in general, string theory on the singular CY_n manifold which is described by $\mathcal{F}_n(z_a) = 0$ for $a = 0, \ldots, n$ relates to string theory on the background,

$$\mathbf{R}^{1,9-2n} \times \mathbf{R}_\phi \times S^1 \times LG(W = \mathcal{F}_n). \tag{6.6}$$

Explicit modular invariant partition functions on this background are constructed in [21, 22, 23].

The duality relation between the intersecting NS5-branes and the conifold are discussed from a viewpoint of the SUGRA solutions [24, 25]. The intersecting $N(1 - p)$ and $N(1 + p)$ NS5-branes are T-dual to the generalized conifold $\mathcal{F}_3 = 0$ [26]. Therefore the SUGRA solutions (4.40) for intersecting NS5-branes are consistent with the results of [12].

7 Conclusion and discussion

In this paper, we have found solutions of the near-horizon geometry of the intersecting NS5-branes, which is a kind of the linear dilation background. Closed string theory on this background holographically describes the LST on four dimensional common intersection of branes. We have been looking for a solution in two different ways. However, both approaches produce only the near-horizon geometry of a class of the intersecting branes if we restrict on the linear dilaton background. The second approach as we mentioned in section 5 seems to include more general solutions for the NS5-branes wrapping on the general Riemann surface. We are very interested in the case that the Riemann surface is other singular curves as like as Argyres-Douglas curves [27].

We have discussed the relation between our background and the LG theory. However, explicit connection is not so clear. In the case of parallel NS5-branes, near-horizon geometry is described by a product of compact coset spaces on which WZW model is dual to the LG theory. Our near-horizon solution except for the linear dilaton part might be non-compact and coset description of this theory is difficult to understand. So we need

the further investigation of the near-horizon geometry of the NS5-brane wrapping on the Σ.

Recently, an intersecting brane configuration on which standard model like theory appears is constructed [28]. To realize the Randall-Sundrum scenario on the intersection of branes in superstring theory, the SUGRA solution for these branes become more important. And also these solutions are indispensable to understand holographically the four dimensional QCD or standard model like theories. We hope that our solution give a hint to solve these problems.

Acknowledgments

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