Superfield Effective Action in the Noncommutative Wess-Zumino Model

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Abstract

We introduce the concept of superfield effective action in noncommutative $\mathcal{N}=1$ supersymmetric field theories containing chiral superfields. One and two loops low-energy contributions to the effective action are found for the noncommutative Wess-Zumino model. The one loop Kählerian effective potential coincides with its commutative counterpart. We show that the two loops nonplanar contributions to the Kählerian effective potential are leading in the case of small noncommutativity. The structure of the leading chiral corrections to the effective action and the behaviour of the chiral effective potential in the limit of large noncommutativity are also investigated.

Nowadays, an enormous effort is being done to understand the properties of noncommutative field theories. There are two main reasons for that. By one side, they are the field theory limit of open strings in the presence of a constant B-field [1]. On the other side, although being nonlocal field theories, they are still tractable giving rise to new and interesting phenomena [2]. In particular, in spite of their nonlocality, noncommutative models allow the construction of causal quantum field theories.

The main characteristic of noncommutative field theories is the mixture of ultraviolet and infrared divergences which may turn the ordinary (commutative) renormalizable theories into nonrenormalizable ones [3]. Supersymmetry seems to be needed to recover renormalizability at least for the case of non-gauge theories [4]. The complex scalar field theory with interaction $\phi^* \star \phi^* \star \phi \star \phi$ is one loop nonrenormalizable. However, its supersymmetric extension, the noncommutative Wess-Zumino model is renormalizable to all loop orders [5]. In $2+1$ dimensions the dynamical mass generation in the Gross-Neveu model is spoiled by noncommutativity. Also, the noncommutative nonlinear sigma model turns out to be nonrenormalizable due to the mixture of ultraviolet and infrared divergences. However, the noncommutative supersymmetric nonlinear sigma model, which includes both models above, is one loop renormalizable [6]. For supersymmetric gauge theories the situation is more involved since the effective action has quantum corrections for nonplanar graphs which require the introduction of generalized Moyal products [7].

An essential ingredient in quantum field theory is the effective action. It allows the study of several aspects of quantum field models including the structure of ultraviolet divergences, the infrared behaviour and quantum symmetries. Therefore, the effective action is a valuable tool which will provide the necessary means to investigate the problem of ultraviolet/infrared mixing in noncommutative supersymmetric theories and clarify how the noncommutativity can influence the known properties of standard supersymmetric field models.

In this paper we calculate the leading chiral correction to the superfield effective action in the massless noncommutative Wess-Zumino model. We consider the massless case because there are no chiral corrections for the massive theory as in the commutative case [8]. The first nonvanishing correction appears at the two loops level, also as in the commutative case, and presents neither ultraviolet nor infrared divergences. We also calculate the one and two loops contributions to the Kählerian effective potential. For the one loop case there is no dependence on the noncommutativity parameter and the result coincides with the commutative one. At two loops the Kälerian effective potential has a nonplanar part which strongly depends on the noncommutativity.

The most natural way to study the effective action makes use of superspace concepts. The formulation of noncommutative supersymmetric field theories in superspace has already been performed [9]. Noncommutativity is only introduced for bosonic coordinates, the Grassmannian coordinates still being taken as anticommuting (see nevertheless the attempts to construct a superspace with non-anticommuting Grassmann coordinates [10]). In the commutative case the effective action in superspace was developed in [11] (see also [12]). Its application to the low-energy leading contributions to the effective action were found for several superfield theories [8, 13, 14, 15, 16]. In the noncommutative case, one loop quantum corrections to the effective action in superfield form were investigated for the Wess-Zumino model [17] and for gauge theories [18]. However, a systematic development of the concept of superfield effective action still remains to be done in the noncommutative case. So, this paper is also devoted to carry out such a generalization for any order in perturbation theory. We will obtain the noncommutative analogs of [11, 14], i.e., the Kählerian and chiral effective potentials for the noncommutative Wess-Zumino model.

The noncommutative massless Wess-Zumino model in superspace has the action

$$S[\bar{\Phi}, \Phi] = \int d^8 z \bar{\Phi}\Phi + (\lambda \int d^6 z \Phi^{*3} + h.c.), \tag{1}$$

where $\Phi(z)$ and $\bar{\Phi}(z)$ are chiral and antichiral superfields respectively, λ is a real coupling constant and $\Phi^{*3} = \Phi * \Phi * \Phi$. The interaction term has the following expression

$$\int d^6 z \Phi^{*3}(z) = \int d^2\theta \int d^4x \int d^4k_1 d^4k_2 d^4k_3 e^{i(k_1+k_2+k_3)x} e^{-i\sum_{i<j}^{3} k_i \times k_j} \times$$
$$\times \int d^4x_1 d^4x_2 d^4x_3 e^{-k_1x_1 - k_2x_2 - k_3x_3} \Phi(x_1,\theta)\Phi(x_2,\theta)\Phi(x_3,\theta), \tag{2}$$

where $k_i \times k_j = k_i^\mu \theta_{\mu\nu} k_j^\nu$ and $\theta_{\mu\nu}$ is the noncommutativity parameter. The propagator is (we use the conventions of [12])

$$< \Phi(z_1)\bar{\Phi}(z_2) >= \frac{\bar{D}^2 D^2}{16\Box}\delta^8(z_1 - z_2), \tag{3}$$

and has the same expression an in the commutative case. The vertex is, however, modified. It reads [17]

$$\lambda(2\pi)^4\delta(k+l+p)\cos(k\times l), \tag{4}$$

where k, l, p are the momenta of the superfields associated to the vertex.

The effective action $\Gamma[\bar{\Phi},\Phi]$ can be presented as a series in supercovariant derivatives $D_A = (\partial_a, D_\alpha, \bar{D}_{\dot{\alpha}})$ in the form

$$\begin{aligned}\Gamma[\bar{\Phi},\Phi] &= \int d^8z \mathcal{L}_{eff}(\Phi, D_A\Phi, D_AD_B\Phi, \ldots, \bar{\Phi}, D_A\bar{\Phi}, D_AD_B\bar{\Phi}, \ldots) + \\ &+ (\int d^6z \mathcal{L}^{(c)}_{eff}(\Phi, \partial_a\Phi, \partial_a\partial_b\Phi, \ldots) + h.c.),\end{aligned} \tag{5}$$

where $\mathcal{L}_{eff}$ is the general effective Lagrangian and $\mathcal{L}^{(c)}_{eff}$ is the chiral effective Lagrangian. It is clear that these effective Lagrangians contain the effects induced by noncommutativity. Our purpose is to find the leading low-energy contributions to the effective Lagrangians. We assume that they have the structure

$$\mathcal{L}_{eff} = K_{eff}(\bar{\Phi},\Phi)_* + \ldots = \bar{\Phi}\Phi + \sum_{n=1}^{\infty} K^{(n)}_{eff}(\bar{\Phi},\Phi) + \ldots, \tag{6}$$

$$\mathcal{L}^{(c)}_{eff} = W_{eff}(\Phi)_* = \lambda\Phi^{*3} + \sum_{n=1}^{\infty} W^{(n)}_{eff}(\Phi) + \ldots, \tag{7}$$

where dots in Eq.(6) mean space-time derivatives of the superfields $\bar{\Phi}$ and Φ, and dots in Eq.(7) mean space-time derivatives of Φ. From these derivative dependent terms we will keep only the leading ones in momentum and in the noncommutativity parameter $\theta_{\mu\nu}$. In Eq.(6) we call $K_{eff}(\bar{\Phi},\Phi,...)$ the Kählerian effective potential in noncommutative theory and $W_{eff}(\Phi,...)$, in Eq.(7), the chiral (or holomorphic) effective potential. Here $K^{(n)}_{eff}(\Phi,\bar{\Phi})$ is the n-th correction to the Kählerian potential and $W^{(n)}_{eff}(\Phi)$ is the n-th correction to the chiral potential.

To consider further the effective Lagrangians $\mathcal{L}_{eff}$ and $\mathcal{L}^{(c)}_{eff}$ we use the path integral representation of the effective action [12, 19]

$$\begin{aligned}&\exp(\frac{i}{\hbar}\Gamma[\bar{\Phi},\Phi]) = \\ &\int \mathcal{D}\phi\mathcal{D}\bar{\phi}\exp\left(\frac{i}{\hbar}S[\bar{\Phi}+\sqrt{\hbar}\bar{\phi}, \Phi+\sqrt{\hbar}\phi] - \frac{1}{\sqrt{\hbar}}(\int d^6z\frac{\delta\Gamma[\bar{\Phi},\Phi]}{\delta\Phi(z)}\phi(z) + h.c.)\right),\end{aligned} \tag{8}$$

where Φ and $\bar{\Phi}$ are the background superfields and ϕ and $\bar{\phi}$ are the quantum ones. The effective action can be written as $\Gamma[\bar{\Phi},\Phi] = S[\bar{\Phi},\Phi] + \tilde{\Gamma}[\bar{\Phi},\Phi]$, where $\tilde{\Gamma}[\bar{\Phi},\Phi]$ is the quantum correction to the classical action. Then Eq.(8) allows us to obtain $\tilde{\Gamma}[\bar{\Phi},\Phi]$ in the form of a loop expansion $\tilde{\Gamma}[\bar{\Phi},\Phi] = \sum_{n=1}^{\infty}\hbar^n\Gamma^{(n)}[\bar{\Phi},\Phi]$ and hence we get the loop expansion for the effective Lagrangians $\mathcal{L}_{eff}$ and $\mathcal{L}^{(c)}_{eff}$.

To find the loop corrections $\Gamma^{(n)}[\bar{\Phi},\Phi]$ in explicit form we expand the right-hand side of Eq.(8) in a power series in the quantum superfields ϕ, $\bar{\phi}$. For slowly varying background fields in space-time, the quadratic part of the expansion of $\frac{1}{\hbar}S[\bar{\Phi}+\sqrt{\hbar}\bar{\phi},\Phi+\sqrt{\hbar}\phi]$ in quantum superfields $\phi,\bar{\phi}$ is given by

$$S_2=\frac{1}{2}\int d^8z\begin{pmatrix}\phi & \bar{\phi}\end{pmatrix}\begin{pmatrix}\lambda\Phi & (-\frac{1}{4})D^2\\ (-\frac{1}{4})\bar{D}^2 & \lambda\bar{\Phi}\end{pmatrix}\begin{pmatrix}\phi\\ \bar{\phi}\end{pmatrix}. \tag{9}$$

No Moyal product is present because Φ is a slowly varying superfield. It means that the full low-energy one loop effective action in the noncommutative theory will be the same as in the corresponding commutative one [20, 21, 22]. We can expect non-trivial corrections due to the noncommutativity only in the two loops approximation.

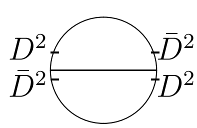
To find the two loops correction to the Kählerian potential we need to calculate the superpropagator associated to Eq.(9). It is given by the solution of

$$\begin{pmatrix}\lambda\Phi & (-\frac{1}{4})D^2\\ (-\frac{1}{4})\bar{D}^2 & \lambda\bar{\Phi}\end{pmatrix}\begin{pmatrix}G_{++} & G_{+-}\\ G_{-+} & G_{--}\end{pmatrix}=-\begin{pmatrix}\delta_+ & 0\\ 0 & \delta_-\end{pmatrix}, \tag{10}$$

where $\delta_+=-\frac{1}{4}\bar{D}^2\delta^8(z_1-z_2)$ and $\delta_-=-\frac{1}{4}D^2\delta^8(z_1-z_2)$. The components of the matrix superpropagator for the case of constant superfields is given by

$$\begin{aligned}G_{++}&=\frac{\lambda\bar{\Phi}}{\Box+\lambda^2|\Phi|^2}\frac{\bar{D}_1^2}{4}\delta_{12},\quad G_{+-}=\frac{1}{\Box+\lambda^2|\Phi|^2}\frac{\bar{D}_1^2D_2^2}{16}\delta_{12},\\ G_{-+}&=\frac{1}{\Box+\lambda^2|\Phi|^2}\frac{D_1^2\bar{D}_2^2}{16}\delta_{12},\quad G_{--}=\frac{\lambda\Phi}{\Box+\lambda^2|\Phi|^2}\frac{D_1^2}{4}\delta_{12}.\end{aligned} \tag{11}$$

Since the Kählerian effective potential depends only on the superfields Φ and $\bar{\Phi}$ but not on their derivatives, supergraphs contributing to it must include an equal number of D^2 and $\bar{D}^2$ factors with all vertices rewritten in the form of an integral over the whole superspace. The only supergraph with equal number of D^2 and $\bar{D}^2$ factors is given by:



The contribution of the supergraph, after evident D-algebra manipulations, takes the form

$$K^{(2)}=\frac{\lambda^2}{6}\int\frac{d^4kd^4l}{(2\pi)^8}\cos^2(k\times l)\frac{1}{(k^2+m^2)(l^2+m^2)((k+l)^2+m^2)}, \tag{12}$$

where $m^2\equiv\lambda^2|\Phi|^2$.

This can be split into a planar and a nonplanar part. The planar part is given by the integral

$$K^{(2)}_{pl}=\frac{\lambda^2}{12}\int\frac{d^4kd^4l}{(2\pi)^8}\frac{1}{(k^2+m^2)(l^2+m^2)((k+l)^2+m^2)}, \tag{13}$$

whose contribution is analogous to the contribution for the commutative Wess-Zumino model. The only difference is a multiplicative factor of $\frac{1}{2}$. After calculating the integrals and subtracting divergences we get (cf. [8])

$$K_{pl}^{(2)} = \frac{\lambda^2}{2(4\pi)^4} m^2 \Big(-\frac{1}{4}\log^2\frac{m^2}{\mu^2} + \frac{3-\gamma}{2}\log\frac{m^2}{\mu^2} + \frac{3}{2}(\gamma - 1) + \frac{1}{4}(\gamma^2 + \zeta(2)) - b \Big), \quad (14)$$

where b is a finite constant whose origin is due to the choice of a non-minimal subtraction scheme. Its value has to be fixed by proper normalization conditions. We renormalized only the planar part since, as we shall show, the nonplanar part is finite.

To evaluate the nonplanar part of Eq.(12) let us consider the integral

$$\int \frac{d^4k}{(2\pi)^4} \frac{e^{2i(k\times l)}}{(k^2+m^2)((k+l)^2+m^2)}. \quad (15)$$

Using the Feynman representation and the α-representation for the denominator we can perform the integration over the momenta arriving at

$$\frac{1}{16\pi^2}\int_0^1 dx \int_0^\infty \frac{d\alpha}{\alpha} e^{-\alpha(m^2 + l^2 x(1-x)) - \frac{l\circ l}{\alpha}}, \quad (16)$$

where $l \circ l \equiv l^a (\theta^2)_{ab} l^b$ and $(\theta^2)_{ab} = \theta_{ac}\theta^c{}_b$. We note that if we set $\theta = 0$ the integral becomes divergent due to the absence of the factor $e^{-l\circ l/\alpha}$. Then, the nonplanar contribution to Eq.(12) is

$$K_{np}^{(2)} = \frac{\lambda^2}{24}\frac{1}{16\pi^2}\int_0^1 dx \int_0^\infty \frac{d\alpha}{\alpha}\int \frac{d^4 l}{(2\pi)^4}\frac{1}{l^2+m^2} e^{-\alpha(m^2 + l^2 x(1-x)) - \frac{l\circ l}{\alpha}}. \quad (17)$$

Finally, we can exponentiate $\frac{1}{l^2+m^2}$ to arrive at

$$K_{np}^{(2)} = \frac{\lambda^2}{24}\frac{1}{16\pi^2}\int_0^1 dx \int_0^\infty \frac{d\alpha}{\alpha}\int_0^\infty dz e^{-(\alpha+z)m^2}\int \frac{d^4 l}{(2\pi)^4}\exp[-l^m A_{mn} l^n], \quad (18)$$

where A_{mn} is a matrix of the form

$$A_{mn} = \eta_{mn}(\alpha x(1-x) + z) + \frac{1}{\alpha}(\theta^2)_{mn}. \quad (19)$$

Notice that all integrals are convergent. After a Wick rotation we can perform the integration over the momenta obtaining

$$K_{np}^{(2)} = \frac{\lambda^2}{24}\frac{1}{(16\pi^2)^2}\int_0^1 dx \int_0^\infty \frac{d\alpha}{\alpha}\int_0^\infty dz e^{-(\alpha+z)m^2}\det{}^{-1/2}[A_{mn}]. \quad (20)$$

Carrying out the remaining integrations is quite complicated. Therefore we specialize the matrix $\theta_{\mu\nu}$ to its canonical form with diagonal blocks. Furthermore, to avoid troubles with causality we allow only space-space noncommutativity. Then, the nonvanishing components are $\theta_{23} = -\theta_{32} = a$, with a having mass dimension -2. Hence, the nonvanishing components of θ^2 are $(\theta^2)_{22} = (\theta^2)_{33} = -a^2$ and

$$\det{}^{-1/2} A = \frac{1}{(\alpha x(1-x) + z + \frac{a^2}{\alpha})(\alpha x(1-x) + z)}. \quad (21)$$

Now we can rescale $\alpha m^2 \rightarrow \alpha$ and $zm^2 \rightarrow z$ so that the new variables are dimensionless. We can also introduce a new dimensionless noncommutativity parameter $\tilde{a}^2 = m^4 a^2$. Then the nonplanar correction takes the form

$$K_{np}^{(2)} = m^2 \frac{\lambda^2}{24} \frac{1}{(16\pi^2)^2} \int_0^1 dx \int_0^\infty d\alpha \int_0^\infty dz \frac{e^{-(\alpha+z)}}{(\alpha^2 x(1-x) + \alpha z + \tilde{a}^2)(\alpha x(1-x) + z)}. \quad (22)$$

Eq.(22) is the exact two loops nonplanar correction to the Kählerian effective potential.

The integral in the right hand side of Eq.(22) is still complicated. However, there are two limits of $\tilde{a}^2$ for which the integral can be performed. Let us first consider the case $\tilde{a}^2 >> 1$. We can expand Eq.(22) in a power series in $\frac{1}{a}$ and arrive at

$$\begin{aligned} K_{np}^{(2)} &= m^2 \frac{\lambda^2}{24} \frac{1}{(16\pi^2)^2} \int_0^1 dx \int_0^\infty d\alpha \int_0^\infty dz \frac{\alpha}{\tilde{a}^2} \frac{e^{-(\alpha+z)}}{\alpha x(1-x) + z} (1 - \frac{\alpha^2 x(1-x) + \alpha z}{\tilde{a}^2}) + \\ &+ O(\frac{1}{\tilde{a}^6}). \end{aligned} \quad (23)$$

After integration over x, α and z, and restoring the manifest Φ dependence, the two loops nonplanar correction can be expressed as

$$K_{np}^{(2)} = \frac{\lambda^4}{24} |\Phi|^2 \frac{1}{(16\pi^2)^2} (\frac{c_1}{\tilde{a}^2} + \frac{c_2}{\tilde{a}^4}) + O(\frac{1}{\tilde{a}^6}), \quad (24)$$

where c_1 and c_2 are real numbers. Therefore the nonplanar contribution is suppressed at large value of the noncommutativity parameter $\tilde{a}$. Its leading term is proportional to $\lambda^4 |\Phi|^2 \frac{1}{\tilde{a}^2}$. Note that this correction is finite and does not contain any singularity coming from the UV/IR mixing.

In the case of $\tilde{a}^2 << 1$ we redefine the variables α and z by $\alpha' = \alpha a$ and $z' = za$, respectively. As a result, Eq.(22) takes the form

$$K_{np}^{(2)} = m^2 \frac{\lambda^2}{24} \frac{1}{a} \frac{1}{(16\pi^2)^2} \int_0^1 dx \int_0^\infty d\alpha' \int_0^\infty dz' \frac{e^{-a(\alpha'+z')}}{((\alpha')^2 x(1-x) + \alpha' z' + 1)(\alpha' x(1-x) + z')}, \quad (25)$$

resulting in

$$K_{np}^{(2)} = \frac{\lambda^4}{24} |\Phi|^2 (\frac{d}{a} + O(a^0)). \quad (26)$$

where

$$d = \frac{1}{(16\pi^2)^2} \int_0^1 dx \int_0^\infty d\alpha' \int_0^\infty dz' \frac{e^{-a(\alpha'+z')}}{((\alpha')^2 x(1-x) + \alpha' z' + 1)(\alpha' x(1-x) + z')}, \quad (27)$$

is a constant. In other words, in the case of small noncommutativity the nonplanar correction becomes the leading one. This result agrees with the predictions given in [23] for the non-supersymmetric case.

Now let us turn to the evaluation of the corrections to the chiral effective potential. First we set the background antichiral superfield $\bar{\Phi}$ to zero. Then the expansion of $\frac{1}{\hbar}S[\bar{\Phi}+\sqrt{\hbar}\bar{\phi}, \Phi+\sqrt{\hbar}\phi]$ in quantum superfields yields

$$S = \int d^8z\phi\bar{\phi} + \lambda\int d^6z(3\Phi * \phi * \phi + \phi^{*3}) + \lambda\int d^6\bar{z}\bar{\phi}^{*3}. \tag{28}$$

As we will show, chiral loop contributions begin at two loops. Therefore we retain in Eq.(28) only the terms of second and third orders in quantum superfields (note that vertices of fourth order in quantum superfields, which in general are essential for calculating the two loops effective action, are absent in this theory).

The chiral action can be written as [17]

$$S_c = \int d^6z\phi^{*3} = \int d^2\theta\int\frac{d^4p_1d^4p_2}{(2\pi)^8}e^{-p_1\times p_2}\phi(p_1,\theta)\phi(p_2,\theta)\phi(-(p_1+p_2),\theta). \tag{29}$$

We see that the quantum ϕ^{*3} corrections have the same structure as the original commutative interaction Lagrangian. The only difference is in the presence of an additional factor $S(p_1,p_2)$ which arises after integration over internal momenta. Then performing the inverse Fourier transformation we arrive at the possible form for the quantum correction

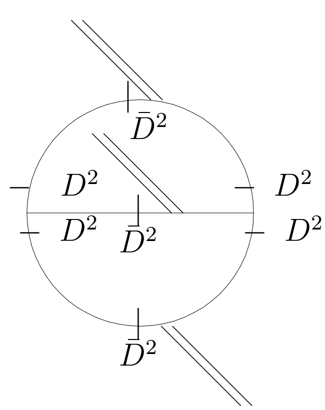
$$\begin{aligned}\Delta S_c &= \int d^2\theta\int d^4x_1d^4x_2d^4x_3\frac{d^4p_1d^4p_2}{(2\pi)^8}e^{-p_1\times p_2}\phi(x_1,\theta)\phi(x_2,\theta)\phi(x_3,\theta)\times\\ &\times e^{ix_1p_1+ix_2p_2+ix_3(-p_1-p_2)}S(p_1,p_2).\end{aligned} \tag{30}$$

We see that all quantum corrections are included in the single function $S(p_1,p_2)$. Assuming that the superfields under consideration are slowly varying in space-time we can integrate over x_2 and x_3 and over the momenta k_1 and k_2 getting

$$L_c = \int d^2\theta\int d^4x_1\phi^3(x_1,\theta)S(p_1,p_2)|_{p_1,p_2=0}. \tag{31}$$

This correction has precisely the same form as that in the commutative case. Thus, we showed that for slowly varying superfields their Moyal product coincides with their standard product.

The structure of the vertices and propagators are similar to those of the commutative Wess-Zumino model and allow us to show that there is only one supergraph contributing to the chiral effective potential at two loops:



The double external lines denote the background superfield Φ. The superpropagator is given by Eq.(3). The contribution of this supergraph is then

$$\frac{\lambda^5}{12}\int\frac{d^4p_1d^4p_2}{(2\pi)^8}\frac{d^4kd^4l}{(2\pi)^8}\int d^4\theta_1d^4\theta_2d^4\theta_3d^4\theta_4d^4\theta_5\Phi(-p_1,\theta_3)\Phi(-p_2,\theta_4)\Phi(p_1+p_2,\theta_5)\times$$
$$\times\quad\frac{\cos(k\times l)\cos[(k+p_1)\times(l+p_2)]\cos(k\times p_1)\cos(l\times p_2)\cos[(k+l)\times(p_1+p_2)]}{k^2l^2(k+p_1)^2(l+p_2)^2(l+k)^2(l+k+p_1+p_2)^2}\times$$
$$\times\quad\delta_{13}\frac{\bar{D}_3^2}{4}\delta_{32}\frac{D_1^2\bar{D}_4^2}{16}\delta_{14}\delta_{42}\frac{D_1^2\bar{D}_5^2}{16}\delta_{15}\delta_{52}. \tag{32}$$

After D-algebra transformations, which can be carried out in the same manner as in the commutative Wess-Zumino model, this expression can be written as

$$\frac{\lambda^5}{12}\int\frac{d^4p_1d^4p_2}{(2\pi)^8}\frac{d^4kd^4l}{(2\pi)^8}\int d^2\theta\Phi(-p_1,\theta)\Phi(-p_2,\theta)\Phi(p_1+p_2,\theta)\times$$
$$\times\quad\frac{k^2p_1^2+l^2p_2^2+2(kl)(p_1p_2)}{k^2l^2(k+p_1)^2(l+p_2)^2(l+k)^2(l+k+p_1+p_2)^2}\times \tag{33}$$
$$\times\quad\cos(k\times l)\cos((k+p_1)\times(l+p_2))\cos(k\times p_1)\cos(l\times p_2)\cos((k+l)\times(p_1+p_2)),$$

where $kl=k^\mu l_\mu$. It has the same form as Eq.(30), as expected. We then find

$$S(p_1,p_2)=\frac{\lambda^5}{12}\int\frac{d^4kd^4l}{(2\pi)^8}\frac{k^2p_2^2+l^2p_1^2+2(kl)(p_1p_2)}{k^2l^2(k+p_1)^2(l+p_2)^2(l+k)^2(l+k+p_1+p_2)^2}\times \tag{34}$$
$$\times\quad\cos(k\times l)\cos[(k+p_1)\times(l+p_2)]\cos(k\times p_1)\cos(l\times p_2)\cos[(k+l)\times(p_1+p_2)],$$

where p_1,p_2 are regarded as external momenta. Therefore we need to analyze the behavior of $S(p_1,p_2)$ Eq.(34) in the limit $p_1,p_2\to 0$. It is natural to consider this limit in the following way. We must first set one of these external momenta (e.g. p_2) to zero, and then consider the limit of the expression as $p_1\to 0$. If we set $p_2=0$, multiply the cosine factors and make several changes of variables we arrive at

$$\begin{aligned}S(p)\ &=\frac{\lambda^5}{12}\frac{p^2}{8}\int\frac{d^4kd^4l}{(2\pi)^8}\Big[\frac{1}{k^2(k+p)^2l^2(l+k)^2(l+k+p)^2}+\\ &+\quad\frac{3\cos(2p\times l)}{k^2(k+p)^2l^2(l+k)^2(l+k+p)^2}+\frac{2\cos(2k\times l)}{k^2(k+p)^2l^2(l+k)^2(l+k+p)^2}\\ &+\quad\frac{2\cos(2k\times l)}{k^2(k-p)^2(l+p)^2(l+k)^2(l+k+p)^2}\Big],\end{aligned} \tag{35}$$

where $S(p)=S(p_1,p_2)|_{p_1=0}$ and $p=p_1$. Let us analyze the limit $p\to 0$. Since the numerator has a p^2 and the denominator is proportional to at most $1/p^2$, Eq.(35) has zeroth leading order in p, and $S(p)|_{p\to 0}\equiv S$ is constant. Another reason for this is the following one. If we omit all noncommutative factors the result is not singular at $p=0$ since it is of zeroth order in $1/p$. If we introduce noncommutativity, additional infrared singularities can arise if and only if the supergraph is divergent [3]. However, this supergraph is evidently ultraviolet finite, hence there is no infrared singularity in it (notice that the external momentum p plays the role of an infrared cutoff).

The constant S can be written as $S = S_{pl} + S_{np}$ where S_{pl} is a planar contribution to S given by the first two terms in Eq.(35), and S_{np} is a nonplanar contribution given by the two last terms. It is evident that all p-dependent cosine factors cannot decrease the power of p. Therefore if we can set $p = 0$ in all cosines (but not in denominator!) it will not change the infrared behavior. Let us find the leading contributions at $p \to 0$ to effective action from the second term of Eq.(35). After using the Feynman representation and integration over l we arrive at

$$\frac{\lambda^5}{8}\frac{p^2}{(4\pi)^2}\int\frac{d^4k}{(2\pi)^4}\int_0^1 dxdy\frac{1}{k^2(k+p)^2}e^{-2ik\times px}\times \tag{36}$$
$$\times\quad \frac{\sqrt{(p^2x+k^2y+(k+p)^2(1-x-y)-(px+ky+(k+p)(1-x-y))^2)p\circ p}}{p^2x+k^2y+(k+p)^2(1-x-y)-(px+ky+(k+p)(1-x-y))^2}\times$$
$$\times\quad K_{-1}\Big(\sqrt{(p^2x+k^2y+(k+p)^2(1-x-y)-(px+ky+(k+p)(1-x-y))^2)p\circ p}\Big).$$

Here $K_{-1}(z)$ is the modified Bessel function of order -1. Let us consider this expression in limit $p \to 0$. Remind that $K_{-1}(x) \sim \frac{1}{4x} + O(x)$ for $x \to 0$ (we do not use the explicit form of $O(x)$ since it corresponds to terms proportional to p^4 which are not essential for our purposes). We find that this expression has the same $p \to 0$ limit as

$$\frac{\lambda^5}{32}\frac{p^2}{(4\pi)^2}\int\frac{d^4k}{(2\pi)^4}\int_0^1 dxdy\frac{1}{k^2(k+p)^2}\times \tag{37}$$
$$\frac{1}{p^2x+k^2y+(k+p)^2(1-x-y)-(px+ky+(k+p)(1-x-y))^2}+O(p^4).$$

The term containing the noncommutative factor vanishes. Hence this contribution in leading order is equal to $\frac{6}{(4\pi)^4}\zeta(3)$ which could have been obtained if we had set $\cos(p\times l) = 1$ from the very beginning. As a result, the sum of first two terms of Eq.(35), which corresponds to the planar correction in the limit $p \to 0$, has the contribution

$$\frac{\lambda^5}{24}p^2\int\frac{d^4kd^4l}{(2\pi)^8}\frac{1}{k^2(k+p)^2l^2(l+k)^2(l+k+p)^2} = \frac{\lambda^5}{2(4\pi)^4}\zeta(3). \tag{38}$$

We used the expression for this integral given in [14]. As pointed out before there is no noncommutative contribution to this result. Hence the total contribution to the chiral effective action from the planar sector is

$$\mathcal{L}^{(c)}_{pl} = \frac{\lambda^5}{4(4\pi)^4}\zeta(3)\int d^6z\Phi^3 + O(\Phi^2\Box^2\Phi). \tag{39}$$

It remains to find out the nonplanar contribution to the chiral effective potential given by the two last terms of Eq.(35). We can use the Feynman representation and then integrate over k with the help of the identity given in [26] to find

$$S_{np} = \frac{\lambda^5}{48}\frac{p^2}{32\pi^2}\int\frac{d^4l}{(2\pi)^4}\frac{1}{l^2(l+p)^2}G(l|p), \tag{40}$$

where

$$
\begin{aligned}
G(l|p) &= \int_0^1 dxdy e^{-2ip\times ly} K_{-1}\Big(\sqrt{[p^2x+(p-l)^2y-(px+(p-l)y)^2]l\circ l}\Big)\times \\
&\times \frac{\sqrt{4[p^2x+(p-l)^2y-(px+(p-l)y)^2]l\circ l}}{p^2x+(p-l)^2y-(px+(p-l)y)^2}+ \\
&+ \int_0^1 dxdy e^{-2ip\times l(x+y)} K_{-1}\Big(\sqrt{[l^2x+(p+l)^2y-(lx+(p+l)y)^2]l\circ l}\Big)\times \\
&\times \frac{\sqrt{4[l^2x+(p+l)^2y-(lx+(p+l)y)^2]l\circ l}}{l^2x+(p+l)^2y-(lx+(p+l)y)^2}. \qquad (41)
\end{aligned}
$$

This is the exact two loops result for the nonplanar contribution to the chiral effective potential.

The integral in the right hand side of Eq.(41) is very complicated. To estimate such an integral we use the following approximation. Let us rewrite the integral in the form

$$
S_{np} = \frac{\lambda^5}{48}\Big[\frac{p^2}{32\pi^2}\int_0^{\Lambda^2}\frac{d^4l}{(2\pi)^4}\frac{G(l|p)_{small}}{l^2(l+p)^2}+\frac{p^2}{32\pi^2}\int_{\Lambda^2}^{\infty}\frac{d^4l}{(2\pi)^4}\frac{G(l|p)_{large}}{l^2(l+p)^2}\Big], \qquad (42)
$$

where Λ is an arbitrary scale. We use the notation $G(l|p)_{small}$ and $G(l|p)_{large}$ to mean that for the corresponding interval we take the asymptotic form of the function $G(l|p)$ at small and large arguments, respectively. Since the modified Bessel function $K_{-1}(x)$ has the asymptotic behavior $K_{-1}(x) \sim \frac{1}{4x}+O(x)$ for small x and $K_{-1}(x) \sim (-\sqrt{\frac{\pi}{2x}}+O(\frac{1}{x}))e^{-x}$ for large x, we have for p small

$$
\begin{aligned}
G(l|p)_{small} &= \int_0^1 dxdy e^{-2ip\times ly}\frac{1}{p^2x+(p-l)^2y-(px+(p-l)y)^2}+ \\
&+ \int_0^1 dxdy e^{-2ip\times l(x+y)}\frac{1}{l^2x+(p-l)^2y-(lx+(p-l)y)^2}+\dots, \qquad (43)
\end{aligned}
$$

and

$$
\begin{aligned}
G(l|p)_{large} &= \int_0^1 dxdy\exp(-al^2)\frac{\sqrt[4]{[p^2x+(p-l)^2y-(px+(p-l)y)^2]l\circ l}}{p^2x+(p-l)^2y-(px+(p-l)y)^2}+ \\
&+ \int_0^1 dxdy\exp(-al^2)\frac{\sqrt[4]{[l^2x+(p+l)^2y-(lx+(p+l)y)^2]l\circ l}}{p^2x+(p-l)^2y-(px+(p-l)y)^2}. \qquad (44)
\end{aligned}
$$

Due to the asymptotics of $K_{-1}(x)$ at small values of the argument we can see that the next-to-leading term in its expansion (it is of first order in the argument) can lead only to contributions proportional to p^4.

We use the same choice for $\theta_{\mu\nu}$ as before (see the discussion which lead to Eq.(21)). For small p we then get

$$
\begin{aligned}
S_{np} &= \frac{\lambda^5}{48}\Big[\frac{p^2}{32\pi^2}\int_0^{\Lambda^2}\frac{d^4l}{(2\pi)^4}\frac{1}{l^2(l+p)^2(p^2x+(p-l)^2y+(px+(p-l)y))^2}+ \\
&+ \frac{p^2}{32\pi^2}\int_{\Lambda^2}^{\infty}\frac{d^4l}{(2\pi)^4}\exp(-al^2)\frac{1}{l^6}\Big]. \qquad (45)
\end{aligned}
$$

The first integral can be approximated as

$$\frac{\lambda^5}{8(4\pi^2)^2}\zeta(3) - \frac{\lambda^5}{96}\frac{p^2}{(4\pi)^4}\Lambda^2, \tag{46}$$

where we took into account that $\int_0^{\Lambda^2} = \int_0^\infty - \int_{\Lambda^2}^\infty$ and approximated $(l^2(l+p)^2(p^2x + (p-l)^2y + (px + (p-l)y))^2)^{-1}$, in the last interval, as l^{-6} for small p. The second term can be calculated straightforwardly. It is equal to $\frac{p^2 a}{2(16\pi^2)^2}\beta(a\Lambda^2)$ where $\beta(a\Lambda^2) = \int_{a\Lambda^2}^\infty \frac{dz}{z^{3/2}}e^{-z}$. Then the total nonplanar contribution is

$$S_{np} = \frac{\lambda^5}{4(16\pi^2)^2}\Big[\zeta(3) + p^2 a\beta(a\Lambda^2) - \frac{p^2}{24\Lambda^2}\Big]. \tag{47}$$

For $p << l$ the argument of K_{-1} in Eq.(40) is $l^2 a$. Since the border between the two asymptotic forms of the Bessel function is $l^2 a = 1$, then, it is natural to choose Λ satisfying $\Lambda^2 a = 1$, that is, $\Lambda = \frac{1}{\sqrt{a}}$. Hence we get

$$S_{np} = \frac{\lambda^5}{4(4\pi^2)^2}[\zeta(3) + (\beta - \frac{1}{24})p^2 a]. \tag{48}$$

where $\beta = \beta(a\Lambda^2)|_{\Lambda=\frac{1}{\sqrt{a}}} \simeq 0.178$. The corresponding contribution to the effective action is

$$\mathcal{L}^{(2)}_{np} = \frac{\lambda^5}{4(4\pi^2)^2}\int d^6z[\zeta(3)\Phi^3 + (\beta - \frac{1}{24})a\Phi^2\Box\Phi]. \tag{49}$$

The noncommutative effects arise in the terms proportional to $\int d^6z\Phi^2(a\Box)\Phi$. A natural interpretation is the following. Let us suppose that the external momentum p is very small but non-zero. This suggests that the noncommutativity parameter a may be very large. Then we find that at $ap^2 \sim 1$ (or as is the same $a\Box\Phi \sim \Phi$) we have sizable corrections to effective action which do not vanish at small energy. Therefore, the total contribution from the planar and nonplanar parts to the low-energy effective action is

$$\mathcal{L}^{(2)} = \frac{\lambda^5}{2(16\pi^2)^2}\zeta(3)\int d^6z\Phi^3 + \frac{\lambda^5}{4(16\pi^2)^2}(\beta - \frac{1}{24})a\int d^6z\Phi^2\Box\Phi + O(\Phi^2\Box^2\Phi). \tag{50}$$

This correction is finite and does not require any renormalization. It is evident that it reproduces the known results for Wess-Zumino model [8, 14, 24, 25] at the commutative limit $a \to 0$.

To conclude, we have calculated the leading chiral correction to the superfield effective action in the noncommutative Wess-Zumino model. It is finite and does not possess any singularity coming from the UV/IR mixing. We found that this correction contains a standard part which coincides with the two loops chiral effective potential in the commutative Wess-Zumino model and terms depending on $p^2 a$, where p plays the role of an energy scale and a is the noncommutativity parameter. In the standard case we set $p \to 0$, however, if we have very strong noncommutativity, that is $a \to \infty$, we obtain non-trivial corrections in Eq.(50) at $p^2 a \to const$. The presence of such a correction can be related to the quantum dynamics of the vacuum in which fluctuations of geometry are correlated with the energy of the particles created.

We have also calculated the one and two loops contributions to the Kählerian effective potential. This is the first calculation of higher loop contributions to the effective action in a noncommutative supersymmetric field theory carried out with the use of supergraph techniques. This approach allows us to preserve manifest supersymmetry at all steps of the calculation. In the one loop Kählerian effective potential all dependence on the noncommutativity parameter vanishes, and the result coincides with the commutative case [14]. It is natural to expect the same result for the one loop Kählerian effective potential in any noncommutative theory. The two loops Kählerian effective potential has a planar part which has the same form as in the commutative case [8], and a nonplanar part which is strongly dependent on the noncommutativity. It turns out that if the noncommutativity is large, the nonplanar contribution is suppressed by fast oscillations of the nonplanar term. Otherwise, if the noncommutativity is small, the nonplanar contribution becomes the leading one.

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