A Conceptual Space Logic

Jørgen Fischer Nilsson

Department of Information Technology
Technical University of Denmark

Abstract: Conceptual spaces have been proposed as topological or geometric means for establishing conceptual structures and models. This paper, after briefly reviewing conceptual spaces, focusses on the relationship between conceptual spaces and logical concept languages with operations for combining concepts to form concepts. Specifically is introduced an algebraic concept logic, for which conceptual spaces are installed as semantic domain as replacement for, or enrichment of, the traditional extensional set-theoretic semantics. The more pragmatic concern is to make the conceptual space paradigm appealing and useful for formal conceptual modeling and knowledge base systems.

Keywords: Conceptual spaces, conceptual models, reasoning with concepts, algebraic logic, topology.

1 Introduction

Conventional approaches to conceptual modeling and knowledge representation in the computational setting make extensive use of formal, symbolic languages. Typically in the symbol paradigm is adapted an appropriate logic formalised by axioms or inference rules providing computational reasoning abilities. Such languages are usually supported by a formal semantics rooted in set-theoretical notions.

By contrast, the conceptual space paradigm discards formal symbol languages as representation media in favour of topological or metric structures. In a number of papers, e.g. [7, 8, 9, 10], Gärdenfors has presented and advocated the conceptual space paradigm as an alternative to the symbol paradigm. The conceptual space paradigm relates to perception and sense qualities and therefore is biased towards a conceptualistic stance, in contrast to the concept realism inherent in traditional set theory and semantics. In this view [10, 12] propose a cognitive semantics in which conceptual spaces function as semantic domains.

Following a brief review of conceptual spaces in the next section, this paper in subsequent sections attempts to connect systematically the symbol paradigm and the conceptual space paradigm. This liason is established in the vein of [10, 11] by installing conceptual spaces as semantical target domains for an appropriate algebraic logic of concepts and properties as an alternative to the set-based model-theoretical underpinning for logic. In this way conceptual spaces become equipped with a formal language for introducing and describing the concept inhabitancy of the spaces and for carrying out operations on the concepts. This is done so that

> *the semantically significant regionalisations in the concept space are directly reflected as symbolic operations in the concept language.*

As mediation between the usual set models and the conceptual spaces is drawn on elementary topological notions in sect. 4.

The final sections summarise and discuss perspectives in this attempted reconciliation of the two paradigms also in relation to conceptual domain modeling.

2 Conceptual Spaces

Formal accounts of the semantics of logics and natural languages are dominated by a tradition in which linguistic phrases are systematically assigned mathematical meaning structures composed as sets of discrete atomic abstract entities. This meaning assignment serves to provide truth conditions and in turn entailment notions for sentences.

By contrast, the conceptual space approach takes as basis physical measures or sensory data such as colours and pitches, and organises these in domains and dimensions (quality dimensions). These domains and dimensions, which may be thought of as continuous (analog) or discrete, in turn span multi-dimensional concept spaces. Conceptual spaces often can be conveniently visualized in Euclidean space, though they should be understood as conceptualizations existing independently of the geometrical space concept.

It is a key idea that concepts occupy or are identified with regions within the space according to a meaning assignment mapping, which is to be elaborated in the present context:

$$\textit{concept expression} \quad \longrightarrow \quad \textit{region in conceptual space}$$

Moreover, concepts corresponding to natural kinds (typically lexicalized concepts) are conjectured to be associated with convex or at least connected (contiguous) regions, cf. [16] for an extended discussion of this aspect. The convexity hypothesis seeks justification by considerations concerning acceptance of concepts formed by induction, thereby affecting the overall occupancy of concept regions in the spaces.

2.1 Sample Concept Structure with Quality Dimensions

We consider as instructive example of a concept structure the prominent perceptional-physiological colour-circle model of colours, following [8, 10].

This model is set up as follows:

1. The colours of the spectrum, *hues*, are in the idealization represented as a colour circle, where complement colour pairs red/green, and yellow(orange)/blue form opposites at the circumference. The colour circle comes about in that the visible extremes of the wavelength spectrum, purple and violet, meet and overlap in the perception of colours as indicated in fig. 1. Thus the hue dimension is a polar coordinate.

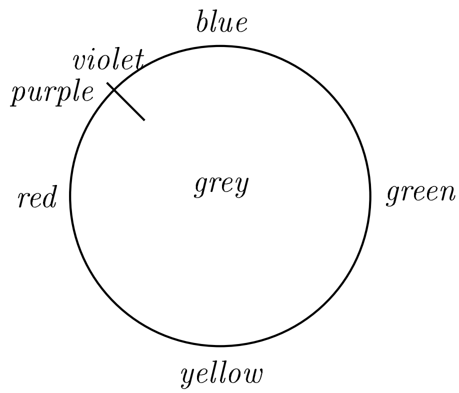

Fig.1 Colour circle and disk

2. The colour disk arises by introducing the *saturation* dimension as a radial coordinate stretching from the center to a maximum colour circle periphery. As indicated in the figure this means that the saturation decreases in the direction from the periphery to the center of the disk, where neutral grey is obtained.

3. The colour spindle appears by introducing a *brightness* dimension perpendicular to the colour disk as an interval with endpoints pure black and pure white, and with various shades of grey along the interval.

Figure 2 below shows these three dimensions of the colour space.

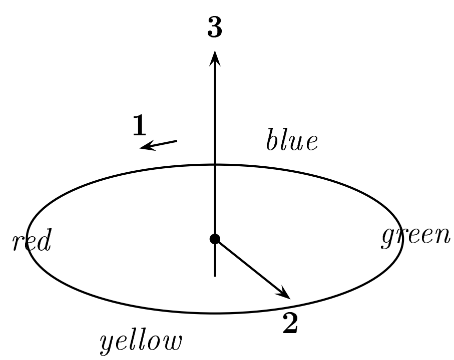

Fig.2 Colour coordinates

It is seen that, say, red colours correspond to a convex region with coherent intervals of hue, saturation and brightness. A specific red colour defined exactly by appropriate scales with hue together with saturation and brightness corresponds to a point in the colour space. A specific red hue corresponds to a vertical plane through the brightness axis in the space. The red region borders or overlaps regions with similar colours, e.g. orange-yellow and redish brown. The colour brown covers a region with yellow/red orientation and low saturation close to black (i.e. low brightness).

In general in the conceptual space approach quality dimensions may also be discrete, either inherently or by design decision. For instance there is the biological classification of living beings into species and sexes, and conventional social classification of humans into age groups.

3 Formal Concept Languages

This section discusses languages for concepts to be used subsequently for describing and formalising conceptual space structures. The section focusses on formal logical algebras with reasoning abilities. The expressions of these languages are to denote regions in conceptual spaces.

3.1 Concept Logic

It is a conventional approach in the context of knowledge representation to describe concepts, classes and properties within a selected target domain by adopting an appropriate version of predicate logic. Usually concepts are then identified with 1-ary predicates, so that, say, *dog* is the class of dogs. A problem with this approach is that the logical language constructs of first order predicate logic denote either truth values as in the formula *dog(fido)* or individuals as in the term *fido*, but not concepts as such.

Thus concept inclusion, such as say, inclusion of dogs in the class of animals, cannot be expressed directly by an operator applied to the pertinent concepts, but is formulated as a requirement on extensions of concepts as for instance $\forall x(dog(x) \rightarrow animal(x))$.

A logical language with constructs and operations devoted to concepts can be obtained with constructions from higher-order logic or logical type theory as applied in Montague semantics for natural language, see e.g. [14]. In such an approach the above universal quantifier-with-implication construct can be abstracted as a higher-order predicate taking concept classes as arguments. A more selective approach amounts to introducing a number of combinators on unary and binary predicates as done in decription logic, see e.g. [2] for an introduction and for further references.

3.2 Concept Algebraic Logic

As a variant of the above-mentioned logical approaches is considered a dedicated algebraic logic akin to the well-known Boolean algebras of classes offering operations on concepts.

An algebra of concepts is to comprise expressions (terms), which are to represent concepts composed from concept identifiers by operators. The operators represent functions of various number of arguments from concepts to concepts.

In the formal universal algebraic framework employed here the operators are to be defined (axiomatised) through equational axioms. Initially are considered purely classificatory language expressions; however, this is as preparation for the notion of conceptual space with multiple quality dimensions being addressed in sect. 4.

3.3 Concept Lattices

A most fundamental notion for concepts is the notion of inclusion of one concept c_1 in another one c_2. Strictly, we have to distinguish extensional and intensional concept inclusion, cf. e.g. [5] and references therein. The former is set-based inclusion of the set of individuals falling under c_1 in the set of individuals falling under c_2. The latter is inclusion of all the properties and features of the c_2 concept in the properties and features possessed by the c_1 concept. The intensional inclusion relation prescribes the extensional one, but not the other way round. The following is concerned also with the intensional concept relationship.

This relationship, which is often called the *isa* relationship, is conjectured to be a partial ordering (i.e., reflexive, antisymmetric, and transitive) with some additional properties calling for use of lattice algebras [3].

This partial ordering, $\leq$, algebraically in lattices is manifest as presence of two operators known as lattice join and meet, conceived here as the following two dyadic concept operations

1. Concept *sum*, $+$, which in an extensional set understanding is union $\cup$
2. Concept *crux*, $\times$, which in an extensional understanding is intersection $\cap$.

Thus terms φ in the lattice algebra are either
1) simple terms in the form of (concept) identifiers, or
2) compound terms consisting of an operator with arguments composed recursively as terms.

In addition, for use in the axioms there are simple terms in the form of variables.

The concept operations sum, $+$, and crux, $\times$, are axiomatised by the equational axioms (laws) of idempotency, commutativity, associativity, and absorption, see e.g. [3, 4]. Thus, for instance, there is the law of idempotency for sum: $x + x = x$. One recalls that these laws hold for sets when $+$ is conceived as set union, and $\times$ as set intersection, see further next subsection. Since we are interested in such set-oriented models it is further assumed that the lattices be distributive. Accordingly, the distributive law, holding for sets, is included among the equational axioms, making the algebra a distributive lattice.

Lattice theory establishes the following fundamental connections between the ordering relation and the operations:

$$\varphi \leq \psi \quad \text{iff} \quad \varphi = \varphi \times \psi \quad \text{iff} \quad \psi = \varphi + \psi$$

so that concept inclusion can be expressed in terms of the two algebraic operations.

3.4 Denotations of Concept Terms

In general, an interpretation of a language comes about by assigning meaning objects (denotations) to the linguistic constructs in the form of components from a semantical domain. Let $[\![\varphi]\!]$ be the denotation of the term φ.

The semantical domain for distributive lattices may be taken to be subsets of a given universe of elements. This is because a distributive lattice algebra is isomorphic to a collection of sets closed under the set operations of union $\cup$ and intersection $\cap$.

Thus $[\![\varphi]\!]$ is a subset of a universe $\mathcal{U}$ consisting of not further specified abstract entities, and the interpretation is stipulated as follows:

$$[\![\varphi + \psi]\!] = [\![\varphi]\!] \cup [\![\psi]\!]$$
$$[\![\varphi \times \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$$
$$[\![\varphi \leq \psi]\!] = [\![\varphi]\!] \subseteq [\![\psi]\!]$$

In the language, besides concept identifiers according to needs, is assumed presence of two distinguished concept constants: $\bot$, which is the empty or null concept, and $\top$, which is the universal concept.

These two concepts form respectively the bottom and the top in the lattice, as expressed by axioms corresponding to $\bot \leq \varphi \leq \top$, for any term φ, and in keeping with the following denotations

$$[\![\bot]\!] = \{\}$$
$$[\![\top]\!] = \mathcal{U}$$

A conceptual structure is specified by algebraic equations as to be exemplified in the next subsection. A specification of the form $\varphi \times \psi = \bot$ has the set interpretation $[\![\varphi]\!] \cap [\![\psi]\!] = \{\}$, hence stipulating φ and ψ as non-overlapping (disjoint) concepts.

It is noticed that these conventional set interpretations comply with an understanding of terms as denoting point sets in regions of a conceptual space. An individual concept corresponds to a singleton set.

3.5 Example Concept Structure: Colour Circle Revisited

Let us consider formalisation in the concept algebra of the 1-dimensional colour circle from sect. 2.1, postponing the complete 3-dimensional colour spindle for consideration in a subsequent section.

A possible logico-algebraic specification is

orange = yellow × red
violet = purple = blue × red
yellowishgreen = yellow × green
blueishgreen = green × blue

accompanied by the following equations expressing disjointness of complementary colours

$\perp$ = green × red
$\perp$ = yellow × blue

specifiying complementary colours as non-overlapping sets (regions).

This specification is to reflect the conceptual model expressed pictorially as follows:

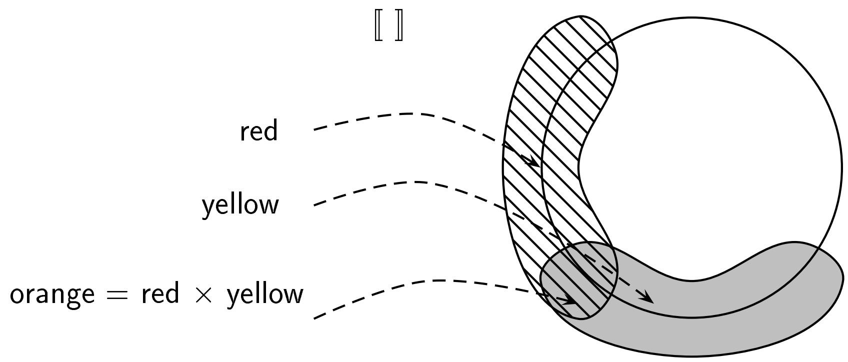

Fig 3. Denotational mapping of terms into regions (arcs) of colour circle

The specification can be further detailed, e.g. by definitional equations for "pure green":

green = yellowishgreen + propergreen+ blueish green
$\perp$ = propergreen × blue = propergreen × yellow

and similarly for yellow, red, and blue.

This specification, augmented with the lattice axioms, enables the carrying out of inferences as term rewriting. For instance one can infer propergreen $\leq$ green and green × orange = $\perp$.

3.6 Complementation

It is possible to extend the concept lattice algebra with a unary operation of complementation of a concept (negation). By introducing proper complementation the lattice becomes a Boolean algebra of classes. By introducing pseudo-complementation defined as the sum of all concepts being disjoint with the argument concept, is obtained a Heyting algebra. In Boolean algebra, in contrast to Heyting algebra, the complementation of the complementation of a concept yields the original concept.

Logical complementation (not to be confused with complementation of colours) tends to introduce non-convex regions. As an example in the above specification the full complement of green is red + blue (less bluish green) + yellow (less yellowish green). This compound concept has a non-convex region and is therefore to be rejected in the conceptual space approach to inductive formation of concepts. Therefore complementation is here disregarded in the following.

4 Dimensionality by Attribution of Properties

The above considered algebraic logic falls short in accounting for the various dimensions of conceptual spaces as presented in sect. 2. We now address the crucial problem of establishing a systematic relationship between a formal concept language and the multiple dimensions of the conceptual spaces.

As starting point we notice that the various dimensions treat aspects of concepts independently, similarly to dimensions in Euclidean spaces. This might suggest shaping of regions in the dimensional space through a Cartesian product construction.

However, it is a key point in the present proposal that the languge for coping with the dimensional space be devised as a natural extension of the already proposed formal logico-algebraic framework. To this end is considered an extended version of binary relation algebras [18, 2]. These universal algebras comprise besides the already introduced sort of concepts (with extension sets) a distinct sort of binary relations.

4.1 Peirce Product

The crucial operation in the two-sorted concept relational algebra is the so called Peirce product [2] introduced by the 19th century logician and philosopher Charles S. Peirce. This is a binary operation $a : \varphi$, whose first operand a is a binary relation, and whose second operand φ is a concept term. It is here assumed that the relation a is functional from its first to its second argument in order that it behaves as an attribution, as to be explained.

As augmentation of the above introduced set interpretation, sect. 3.4, the Peirce product is defined denotationally as follows:

$$[\![a : \varphi]\!] = \{x \mid \exists y((x, y) \in [\![a]\!] \wedge y \in [\![\varphi]\!])\}$$

where $[\![a]\!]$ is the (functional) binary relation associated with the symbol a.

4.2 Attribution

The Peirce product may be imagined as an attribute value pair, where a is the attribute and φ the attributed value in the form of a concept. Therefore in the following for $(a : \varphi)$ is used $a(\varphi)$, with a being re-conceived as a unary operator in the algebra lifting a concept into a property. Such an attribution term $a(\varphi)$ may attach as a qualifier in connection with the crux operation as in

$$\varphi \times a(\psi)$$

which is the sub-concept of φ possessing the a-attributed property concept ψ.

More generally a product term of the form

$$c \times a_1(\varphi_1) \times a_2(\varphi_2) \times ... \times a_m(\varphi_m)$$

called a *frame term* serves as a feature structure, with a concept c qualified with what may be understood as a map of attribute names into associated (concept) values. The concept c may be the universal concept $\top$.

Intuitively $a_1(\varphi_1) \times a_2(\varphi_2) \times ... \times a_m(\varphi_m)$ is imagined as an unordered bundle of attribute/concept-value pairs: a_i is a "slot" with the "filler" φ_i. Therefore, is introduced the suggestive shorthand term forms

$$\prod_i^m a_i(\varphi_i) \qquad \text{or} \qquad \begin{bmatrix} a_1 : \varphi_1 \\ ... \\ a_m : \varphi_m \end{bmatrix}$$

One may observe that frames are formed algebraically *ab initio* by "lattice inheritance" of constituents, $\times$ being lattice meet. The operations $\times$ and $+$ are defined on frame terms, giving frame terms as result by way of the axioms. It is essential to keep in mind that crux in this way remains being interpreted as intersection rather than Cartesian product. Nevertheless the frame terms serve to span axes in the dimensional conceptual space as to be elucidated in the next subsections.

In addition to the lattice axioms of idempotency, commutativity, associativity, absorption and distribution for $+$ and $\times$, the following axioms are devised for applied attributes $a_1, a_2, ...$ in the concept algebra, cf. [4].

Annihilation or strictness:

$$a_i(\bot) = \bot$$

Distribution of $+$ over attribution:

$$a_i(x) + a_i(y) = a_i(x + y)$$

Distribution of $\times$ over attribution:

$$a_i(x) \times a_i(y) = a_i(x \times y)$$

4.3 Topological Interpretation of Attribution

The set interpretation for concept algebra of sect. 3.4 is now extended to handle interpretation of attribution, drawing on introductory notions from general topology, cf. e.g. [1, 13].

In these interpretations each attribute, a, is first associated with some function $f_a()$ from values to values (points to points in the space): $\mathcal{U} \rightarrow \mathcal{U}$. Then the denotation of a in the term $a(\phi)$ is stipulated as a set function $2^{\mathcal{U}} \rightarrow 2^{\mathcal{U}}$, namely *the inverse image* defined as usual as

$$f^{-1}(Y) = \{x | f(x) \in Y\}$$

where the range of Y are the sets in $2^{\mathcal{U}}$.

In accordance with the definition of Peirce product in sect. 4.1 the interpretation of attribution terms then becomes

$$[\![a(\phi)]\!] = f_a^{-1}([\![\phi]\!]) = \{x \mid f_a(x) \in [\![\phi]\!]\}$$

As mentioned, it is a key point in the set up that attributes span independent quality dimensions in a conceptual space. The dimensions may be understood to come about through the inverse image formation as exemplified by the term

$$a_1(\varphi_1) \times a_2(\varphi_2)$$

visualised in 2-dimensional space:

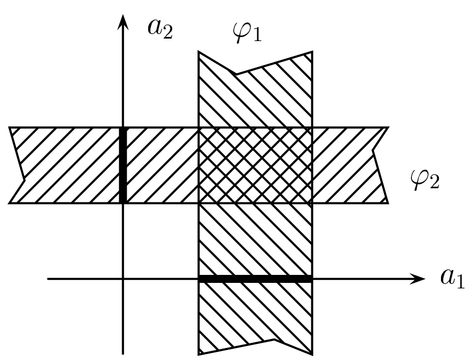

Fig. 4 Crux, $\times$, as intersection in 2-dimensional space

Thus in the conceptual space understanding a in $a(\varphi)$ maps φ (exemplified by an interval) in the quality dimension a into the corresponding *cylinder* in the dimensional space.

The three axioms for attribution in the concept algebra stated in sect. 4.2 reflect the following laws for inverse images [13] [1] :

[1]One may observe that the direct image of the function fails, since it has only the weaker $f(X \cap Y) \subseteq f(Y) \cap f(X)$.

1. $f^{-1}(\{\}) = \{x | f(x) \in \{\}\} = \{\}$
2. $f^{-1}(X \cup Y)\} =$
   $$\{x | f(x) \in (X \cup Y)\} =$$
   $$\{x | f(x) \in X\} \cup \{x | f(x) \in Y\} =$$
   $f^{-1}(X) \cup f^{-1}(Y)$
3. $f^{-1}(X \cap Y) =$
   $$\{x | f(x) \in (X \cap Y)\} =$$
   $$\{x | f(x) \in X\} \cap \{x | f(x) \in Y\} =$$
   $f^{-1}(Y) \cap f^{-1}(X)$

These equations tell that the inverse images in topological interpretation act as structure preserving mappings in the institution of attributes as independent quality dimensions.

4.4 Example Revisited: Colours in Dimensional Space

The above construction endorsing dimensional conceptual spaces is now to be exemplified with the colour space. The following specification steps reflect the construction in sect. 2.1.

1. The colour circle is made an independent circular or polar dimension in space by introducing the attribute hue. Concepts may now be defined algebraically, for instance as in $\mathsf{red} = \mathsf{hue}(0.65\mu\ ..\ 0.75\mu)$, assuming availability of a scalar domain.
2. The color disk is obtained by introducing the radial coordinate of saturation, say, with the attribute *saturation*. For the sake of the example let us assume a discrete saturation domain comprising the sequential intervals zero (point), weak, medium, strong, where strong is (close to) the circumference of the disk. medium may be defined as an overlapping interval: $\mathsf{medium} = \mathsf{weak} \times \mathsf{strong}$.

   Regions at the disk can now be described by sample terms such as
   $\mathsf{red} \times \mathsf{saturation}(\mathsf{strong})$ (i.e., a strong red) and
   $(\mathsf{red} + \mathsf{yellow} + \mathsf{yellowishgreen}) \times \mathsf{saturation}(\mathsf{weak} + \mathsf{strong})$ (i.e., a warm colour)
3. Finally, the fully-fledged colour spindle is achieved by adding the third orthogonal dimension of brightness through an attribute bright, say with intervals min and low (below the disk) medium, and high and max above the disk.

   Here min yields the colour black, while max is pure white. As an auxiliary definition covering non-extreme brightness there is, say,

   $$\mathsf{nonextreme} = \mathsf{low} + \mathsf{medium} + \mathsf{high}$$

The region of red hues of medium saturation and of unspecified brightness is now formed as $\mathsf{red} \times \mathsf{saturation}(\mathsf{medium})$ giving the overlap of a circle and a sector:

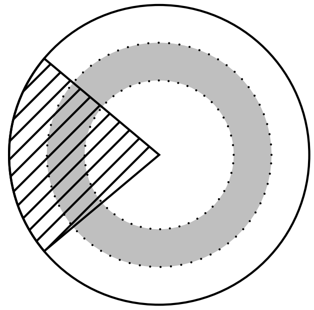

Fig. 5 The overlapping regions of red hues and of medium saturation

Derived colours can now be defined as for instance

$$\mathsf{pink} = \mathsf{red} \times \mathsf{saturation}(\mathsf{weak} + \mathsf{strong}) \times \mathsf{bright}(\mathsf{high})$$

with further, say

$$\mathsf{shockingpink} \;=\; \mathsf{red} \times \mathsf{saturation}(\mathsf{strong}) \times \mathsf{bright}(\mathsf{high})$$

which makes shocking pink a distinguished pink in that $\mathsf{shockingpink} \leq \mathsf{pink}$ can be inferred by rewritings using the axioms and the colour definitions. This conforms with the region $\mathsf{shockingpink}$ being included in the pink region.

5 Conceptual Space Algebraic Logic

The below table summarises the above proposal for logical algebraisation of conceptual spaces.

Symbol System Concept algebra	**Conceptualization System** Conceptual space
Concept ident	given (contiguous) region
$\bot$, null	empty region $\{\}$
$\top$, univ	whole space
$+$	union of regions $\cup$
$\times$	intersection of regions $\cap$
$a_i(\,)$	inverse image map of quality dimension a_i
$\leq$	spatial inclusion $\subseteq$

In order to elucidate the spanning of quality dimensions in a 3-dimensional conceptual space as extension of the 2-dimensional case in fig. 4 and 5, consider the reduction (term rewriting) of the crux of the two frame terms

$$a_1(\varphi') \times a_2(\psi) \quad \text{and} \quad a_1(\varphi'') \times a_3(\xi)$$

yielding the three-entry term

$$a_1(\varphi' \times \varphi'') \times a_2(\psi) \times a_3(\xi)$$

using, among other axioms, $a(x) \times a(y) = a(x \times y)$. In the sugared notation this can be expressed as the equality

$$\left[\begin{array}{l} a_1 : \phi' \\ a_2 : \psi \end{array} \right] \times \left[\begin{array}{l} a_1 : \phi'' \\ a_3 : \xi \end{array} \right] = \left[\begin{array}{l} a_1 : \phi' \times \phi'' \\ a_2 : \psi \\ a_3 : \xi \end{array} \right]$$

This calculation may be visualised as 3-dimensional intersection of cylinders as extension of the two dimensional intersections of fig. 4 and 5. Cylindrification is applied to a geometrical interpretation of composition of binary relations in the Calculus of Relations in [18], and is also known from cylindrical algebra.

These pictorial explanations of crux, $\times$, also explains the combining of multiple (partially) conflicting properties through intersection. At the symbolic language level this may be compared with multiple inheritance of attribute values in feature structures and objects.

5.1 Intensionality

In the pure extensional approach to semantics the meaning of a descriptive phrase reduces to the set of entities fulfilling the specified property, in particular a singleton set for individual concepts. This extensional view faces difficulties with intensional aspects of concepts: What is formally distinction between two concepts, which happen to be coextensional, that is to say, fulfilled by the same set of individuals ? In particular, how is distinguished formally between two concepts which happen to be extensionally empty ? The traditional solution is to invent or posit an abstract collection of possible worlds (acquired from the gray zone between respectable mathematics with Platonic orientation and metaphysics proper). Coextensionality then emerges as a coincidental phenomenon in some of the worlds, including the "actual" world. Thus the denotation of a concept in principle in this approach becomes a mapping of possible world names into ordinary extensions.

By contrast the intensional nature of concepts and properties is intended to be accounted for in the conceptual space paradigm through the organisational structure and locational aspect of domains, dimensions and regions as reflected in the described algebraic logic.

6 Conclusion

We have introduced a logic in the form a concept algebra for specifying and symbolically reasoning with conceptual space structures, thereby opening for a reconciliation of the symbolic and the conceptual space paradigm.

The present concept algebraic approach is related to the so called Boolean semantics for natural language, cf. [14]. Moreover, the concept algebra has affinity to the feature structures favoured in computational natural language analysis. This makes it promising to attempt an elaboration of (computational) conceptual space semantics for natural language via the concept algebraic approach to conceptual spaces.

In [15] is further made an attempt to combine the concept algebra with "fuzzy" sets providing for "radial" categories with soft boundaries and graded overlaps with adjacent concepts.

For application of concept algebra to inductive formation of concepts is referred to [6], which complies with the conceptual space perspective on induction in [7, 8, 9].

It is also left as future work to clarify the possible connections between the present formalisation of conceptual spaces and the logical theories of mereotopology, [17], which seeks to apply topological means in investigating formally parts, boundaries, and interiors of wholes.

References

[1] C. Berge: *Topological Spaces*, Oliver & Boyd, 1963.

[2] C. Brink, K. Britz, and R.A. Schmidt: Peirce Algebras, *Formal Aspects of Computing*, Vol. 6, 1994, pp. 339-358.

[3] B.A. Davey & H.A. Priestley: *Introduction to Lattices and Order*, Cambridge University Press, 1990.

[4] J. Fischer Nilsson: An Algebraic Logic for Concept Structures, *Information Modelling and Knowledge Bases V* IOS Press, Amsterdam, 1994. pp. 75-84.

[5] J. Fischer Nilsson and J. Palomäki: Towards Computing with Extensions and Intensions of Concepts, P.-J. Charrel et al. (eds.): *Information Modelling and Knowledge Bases IX*, IOS Press, 1998.

[6] J. Fischer Nilsson and H.-M. Haav: Inducing queries from examples as concept formation, H, Jaakkola *et al.* (eds.), *Information Modelling and Knowledge Bases X*, IOS Press, 1999.

[7] P. Gärdenfors: Semantics, Conceptual Spaces and the Dimensions of Music, in V. Rantala *et. al.* (eds.): *Essays on the Philosophy of Music, Acta Philosophica Fennica*, Vol. 43, 1988. pp. 9-27.

[8] P. Gärdenfors: Induction, Conceptual Spaces and AI, *Philosophy of Science.* 57, 1990. pp. 78-95.

[9] P. Gärdenfors: A Geometric Model of Concept Formation, in S. Ohsuga *et al.* (eds.): *Information Modelling and Knowledge Bases III, Foundations, Theory and Aplications,* IOS Press, Amsterdam, 1992.

[10] P. Gärdenfors: Meaning as Conceptual Structures, Lund University Cognitive Studies - LUCS 40 1995, 1995.

[11] A. Hautamäki: A Conceptual Space Approach to Semantic Networks, *Computers Math. Applic.*, Vol. 23, No. 6-9, 1992.

[12] K. Holmqvist: Implementing Cognitive Semantics, Ph.D. dissertation, Dept. of Cognitive Science, Lund University, Sweden, 1993.

[13] S.-T. Hu: *Introduction to General Topology*, Holden-Day Inc., 1966.

[14] E.L. Keenan, & L.M. Faltz: *Boolean Semantics for Natural Language*, Reidel, 1986.

[15] H. Legind Larsen and J. Fischer Nilsson: Fuzzy Querying in a Concept Object Algebraic Datamodel, T. Andreasen *et al.* (eds.): *Flexible Query Answering Systems*, Kluwer, 1997.

[16] T. Mormann: Natural Predicates and Topological Structures of Conceptual Spaces, *Synthese*, vol. 95, 1993. pp. 219-240.

[17] B. Smith: Mereotopology: A Theory of Parts and Boundaries, *Data & Knowledge Engineering 20*, 1996. pp. 287-303.

[18] A. Tarski: On the Calculus of Relations, *J. of Symbolic Logic*, Vol. 6. No. 3., 1941. pp. 73-89.