SYNTHETIC ATTITUDE AND HEADING REFERENCE FOR SAAB GRIPEN

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Abstract

In future versions of Saab Gripen, the mechanical artificial horizon will be replaced by a computer calculated attitude and heading, independent of the inertial navigation system (INS). The system uses data from sensors already existing in the aircraft, which are easily available in a highly integrated, 4th generation combat aircraft such as the Gripen. The sensor information used is a three-axis magnetic detector, true airspeed, angle of attack, barometric altitude, flight control rate gyros and load factor. The sensor data is fused together in an Extended Kalman filter (EKF).

Each sensor by itself is of relatively poor quality. For instance, the accuracy of the rate gyros is in the order of degrees per second, rather than degrees per hour as is the case in gyros dedicated for navigation use.
However, when all data are combined, they provide an attitude and heading estimate with sufficient quality for its purpose; to cross-monitor the INS, and to serve as a backup in case the INS fails or data can not be displayed. The system is called Synthetic Attitude and Heading Reference System (SAHRS), and is a Saab patent. A similar system is developed and operational in the Saab Viggen.

Nomenclature

α, β	Angle of attack and sideslip
ϕ,θ,ψ	Euler angles; roll, pitch and heading
$\boldsymbol{x}$, $\boldsymbol{C}$	Bold font indicates vectors and matrices
ω	Angular rate vector; roll, pitch and yaw
$\dot{h}$	Time derivative of barometric altitude
$\hat{\boldsymbol{x}}_{k+1\|k}$	State estimate for *k+1*, at time *k*

1. Introduction

Most passenger and military aircraft of today are equipped with high-accuracy inertial navigation systems (INS), that among other things provide aircraft attitude and heading, essential information for safe all-weather flight. In case of an INS failure, or a failure in the data bus/display chain, there has to be redundancy in the system so that the risk of losing attitude information is very small. Therefore, most all-weather aircraft have a mechanical-electrical artificial horizon as a back-up. The use of this can be divided into three general tasks; 1) to cross-monitor the INS data during normal operation, 2) to complete or abort an advanced maneuver in case of a failure, and 3) to navigate with reasonable accuracy during calm maneuvering, and land safely. These three tasks impose different requirements on the back-up system, regarding accuracy, availability and integrity. In future versions of Saab Gripen, the mechanical horizon gyro will be replaced by computer calculations. The calculations are based on data from sensors already existing in the aircraft. In combination with a redundant hardware architecture, the so-called Synthetic Attitude and Heading Reference System (SAHRS) fulfils the requirements, with equal or bigger margins than the mechanical gyro. It also allows optimization of the man-machine interface; the INS monitoring can be done automatically, and the displays can easily be reconfigured.

There is a SAHRS system already operational in the Saab Viggen aircraft, used by the Swedish Air Force. Although the algorithms and architecture differ from those in the Gripen, the system is based on the same concept, as it is described so far. The SAHRS concept is patented [1].

The SAHRS in the Gripen consists of four major components; 1) A global model of the geomagnetic field, 2) algorithms for calibrating the magnetic field detector, 3) a routine that propagates the angular rates forward in time, and 4) a Kalman filter that weighs together all available information to obtain an optimal attitude estimate.

This paper gives a brief description of the Gripen SAHRS algorithms.

2. Available Sensors

The big advantage of the SAHRS is that it utilizes information from sensors that already exist in the aircraft, and they are easily available due to the high integration level in a modern military aircraft. A constraint is that the information must be independent of the INS, so that the calculated attitude and heading is truly redundant. That must also be kept in mind when analyzing the total hardware architecture.

The data below are used in the SAHRS algorithms:

- Angular rates from the Electrical Flight Control System (EFCS). These are triple redundant and very reliable, but the accuracy is far from what is usually required for navigation purposes. The SAHRS algorithms identify and compensate for bias errors, however.
- A Strap-Down Magnetic Detector Unit (SMDU). This sensor is also inaccurate, due to magnetic interference from the aircraft and individual variations. The SMDU is also calibrated, see below. Since the measurement is three-dimensional, and is compared to a three-dimensional geomagnetic field model, it gives information about attitude as well as heading.
- Barometric altitude from the Air Data Computer (ADC).
- True airspeed, angles of attack and sideslip. The velocity vector, in combination with rate of change in altitude provides information about pitch attitude.
- Vertical load factor from the EFCS. Together with angular rates and airspeed, a roll angle measurement can be achieved, see below.

3. Kalman filter representation

The heart of the algorithms is an Extended Kalman filter (EKF) [2] with nine states and five measurements.

3.1. State vector

To minimize computational load with maintained accuracy, the algorithms represent aircraft attitude in two ways; one in the time propagation, and another in the measure update. The time propagation is performed on the transformation matrix from aircraft body axes to navigation axes (North, East, Down), denoted $\hat{\boldsymbol{C}}_B^N$. The state derivative then becomes [3]

$$\dot{\hat{\boldsymbol{C}}}_B^N = \hat{\boldsymbol{C}}_B^N \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}, \text{ where } \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \omega .$$

The attitude matrix is updated with 30 Hz, using second-order Runge-Kutta integration [4].

The Euler angles are computed as [3]

$$\hat{\varphi} = \operatorname{atan2}(\hat{\boldsymbol{C}}_B^N(3,2), \hat{\boldsymbol{C}}_B^N(3,3)),$$

$$\hat{\theta} = -\operatorname{asin}(\hat{\boldsymbol{C}}_B^N(3,1)),$$

$$\hat{\psi} = \operatorname{atan2}(\hat{\boldsymbol{C}}_B^N(2,1), \hat{\boldsymbol{C}}_B^N(1,1)).$$

In the measure update, the attitude is represented by three states $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$, describing a small correction in roll, pitch and yaw (body axes) of the attitude matrix from the current estimate. The correction is done as

$$\hat{\boldsymbol{C}}_{B_{new}}^N = \hat{\boldsymbol{C}}_B^N \begin{bmatrix} 1 & x_3 & -x_2 \\ -x_3 & 1 & x_1 \\ x_2 & -x_1 & 1 \end{bmatrix}.$$

After each measure update, the state matrix is updated as above, and the three states $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ are zeroed.

In the state vector, there are also three states, $\begin{bmatrix} x_4 & x_5 & x_6 \end{bmatrix}^T$, that estimate a bias error $\Delta\omega_m$ in the angular rate measurement. Finally, the states $\begin{bmatrix} x_7 & x_8 & x_9 \end{bmatrix}^T$ are estimates of biases errors in the magnetic detector, $\Delta\boldsymbol{b}_m$ (errors remaining after calibration, see below).

3.2. Innovation vector

The sensor data described above are formulated in an innovation vector, based on measurements and state estimate.

The first three elements are the magnetic measurement $\boldsymbol{b}_m^B$, transformed to earth axes:

$$\boldsymbol{b}_m^N = \hat{\boldsymbol{C}}_B^N \boldsymbol{b}_m^B$$

This vector is to be compared with the geomagnetic field, $\boldsymbol{b}_{IGRF}^N$.

Then we create a θ measurement from air data:

$\theta_m = \operatorname{asin}\dfrac{\dot{h}}{|\boldsymbol{V}|} + \alpha\cos\hat{\varphi} + \beta\sin\hat{\varphi}$, where $\boldsymbol{V}$ is the velocity vector in body axes. Note that estimate of roll angle is used in the expression.

The last measurement is a roll angle estimate. Consider the following equation of motion for an aircraft (or any mass in an inertial frame of reference), [3]:

$$\dot{\boldsymbol{V}} = \boldsymbol{C}_N^B \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} - \boldsymbol{n}g - \omega \times \boldsymbol{V},$$

where $\boldsymbol{C}_N^B = (\boldsymbol{C}_B^N)^T$, $\boldsymbol{n}$ is body axes load factor (net total of forces divided by mass and gravity, with reversed sign), and g is earth gravity.

We make two assumptions about the flight state of the aircraft:
1) It is flying at a relatively constant airspeed and angle of

attack, i.e. $\dot{V}_z$ is negligible,
2) It is flying 'coordinated' with low sideslip and lateral load factor, i.e. V_y, $\dot{V}_y$ and n_y can be neglected. Since the a/c is inherently stable in yaw, and the flight control system auto-trims sideslip to zero, this is usually a good assumption.

If we take rows two and three (components *y* and *z*), and express them in terms of Euler, attack and sideslip angles, they can be written as

$$\begin{bmatrix} g(\sin\varphi\cos\theta) \\ g(\cos\varphi\cos\theta) \end{bmatrix} = \begin{bmatrix} |\boldsymbol{V}|(r\cos\alpha - p\sin\alpha) \\ gn_z - q|\boldsymbol{V}|\cos\alpha \end{bmatrix}$$

Dividing the first row by the latter gives us the tangent of the roll angle. Therefore, we can formulate a roll angle measurement as

$$\varphi_m = \operatorname{atan2}(|\boldsymbol{V}|(r\cos\alpha - p\sin\alpha), gn_z - |\boldsymbol{V}|\cos\alpha)$$

In the Kalman filter, a check is done that the assumptions above are relevant; if not, the measurement is not used in the update.
The final expression for the innovation is

$$\boldsymbol{z} = \begin{bmatrix} \hat{\boldsymbol{C}}_B^N \boldsymbol{b}_m^B - \boldsymbol{b}_{IGRF}^N \\ \theta_m - \hat{\theta} \\ \varphi_m - \hat{\varphi} \end{bmatrix}$$

3.3. Kalman filter equations

Given the system dynamics
$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$,
the state time update is usually done as

$$\hat{\boldsymbol{x}}_{k+1|k} = \hat{\boldsymbol{x}}_{k|k} + \int_0^T \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})dt \quad .$$

But since the attitude states are updated as above, there are no dynamic states left to update in this case.

The time update of the probability matrix $\boldsymbol{P}$ is done as

$\boldsymbol{P}_{k+1|k} = \boldsymbol{F}_k \boldsymbol{P}_{k|k} \boldsymbol{F}_k^T + \boldsymbol{Q}_k$, where $\boldsymbol{F} = \frac{\partial}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$ and $\boldsymbol{Q}$ is the so-called process noise matrix.
Finally, measure update is done by calculating the Kalman gain $\boldsymbol{K}$:

$\boldsymbol{K} = \boldsymbol{P}_{k+1|k} \boldsymbol{H}^T [\boldsymbol{H} \boldsymbol{P}_{k+1|k} \boldsymbol{H}^T + \boldsymbol{R}_k]^{-1}$, where

$\boldsymbol{H} = \frac{\partial}{\partial \boldsymbol{x}} \boldsymbol{z}(\boldsymbol{x}, \boldsymbol{u})$ and $\boldsymbol{R}$ is measurement noise.
The state and probability matrix are then updated as

$$\hat{\boldsymbol{x}}_{k+1|k+1} = \hat{\boldsymbol{x}}_{k+1|k} + \boldsymbol{K} \boldsymbol{z}_{k+1}$$

$$\boldsymbol{P}_{k+1|k+1} = [\boldsymbol{I} - \boldsymbol{K}\boldsymbol{H}] \boldsymbol{P}_{k+1|k}$$

$\boldsymbol{I}$ is the identity matrix.

4. Geomagnetic Model

As a reference model of the geomagnetic field, the so-called International Geomagnetic Reference Field (IGRF) is used [5]. The geomagnetic potential is defined as a spherical-harmonic expansion series, written as

$$V(r, \theta, \varphi) =$$

$$R_E \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+1} \sum_{m=0}^{n} \{g_{nm}\cos(m\varphi) + h_{nm}\sin(m\varphi)\} P_n^m(\theta)$$

where (r, θ, φ) is position in spherical coordinates ($\theta = 90^o$ - latitude), R_E is earth radius, and P_n^m is a Schmidt-normalized associated Legendre polynomial of spherical-harmonic degree *n* and order *m* [6]. *g* and *h* are coefficient data, which are slowly varying with time. New data are released every five years, including time derivatives that can be used for extrapolation at least another five years.
The magnetic field is the gradient of the potential;

$$\boldsymbol{b}_{IGRF}^N(r, \theta, \varphi) = -\nabla V(r, \theta, \varphi) .$$

The field vector is transformed to cartesian North-East-Down coordinates in the Kalman filter.

5. SMDU Calibration

To achieve required accuracy on the attitude and heading estimate, the magnetic detector cannot be used without some sort of calibration, due to 1) magnetic interference from the aircraft, and 2) calibration errors in the detector itself.
To calibrate the unit, a Kalman filter models a sensor error with 12 degrees of freedom. The calibrated field in body axes, $\boldsymbol{b}_m^B$, is calculated from the raw measurement $\boldsymbol{b}_r^B$ as

$$\boldsymbol{b}_m^B = \boldsymbol{b}_r^B - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_4 & x_7 & x_8 \\ x_{10} & x_5 & x_9 \\ x_{11} & x_{12} & x_6 \end{bmatrix} \boldsymbol{b}_r^B$$

In other words, $x_{1\text{-}3}$ are bias errors, $x_{4\text{-}6}$ are scale errors, and $x_{7\text{-}12}$ can be seen as misalignment errors.

The innovation vector in the Kalman filter is defined as

$\boldsymbol{z} = \boldsymbol{b}_m - \boldsymbol{C}_N^B \boldsymbol{b}_{IGRF}^N$,

where the attitude matrix $\boldsymbol{C}_N^B$ is computed from INS angles. In the beginning of the paper, we said that the cal-

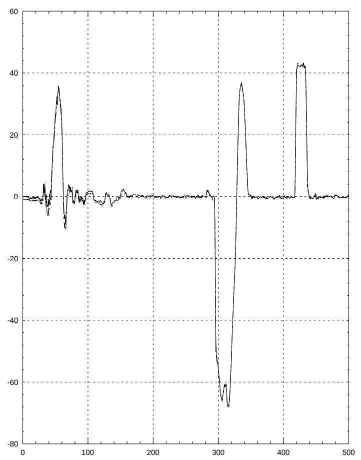
culations have to be independent of the INS. This is still true, since the calibration is performed throughout an entire flight, but it is stored and not used until the subsequent flight. If there is a failure, or if the flight is too short to achieve good accuracy, the calibration will not be stored.
Before first flight, a ground calibration is performed. The ground calibration has only four degrees of freedom, since twelve cannot be identified by ground maneuvering.

6. Results

The accuracy requirements on the SAHRS attitude and heading estimations were defined as;

- During calm maneuvering: rms (random mean square) error values of maximum 2^o in roll and pitch, 5^o in heading. The errors shall not exceed 10^o in roll and heading, or 5^o in pitch.
- During hard maneuvering, errors shall not exceed 30^o in roll, or 20^o in pitch.

During calm maneuvering, the estimates are very stable, and fulfil the accuracy requirements with good margin. Figure 1-3 below show SAHRS angles from the first 500 seconds of a flight test, with INS angles as comparison.



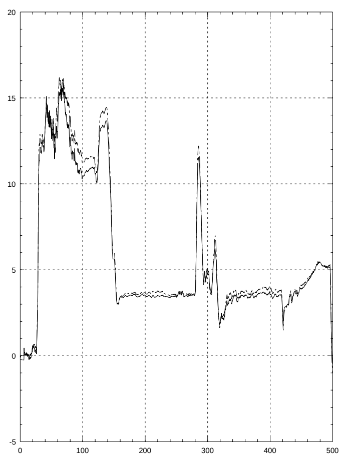


Figure 1. Roll and pitch angle estimates (INS dashed)

During hard maneuvering, the performance is also good, with no mentionable drift. The “gimbal limit” problem associated with conventional gyros is non-existent. After the maneuvers, the estimates are just as good as before.

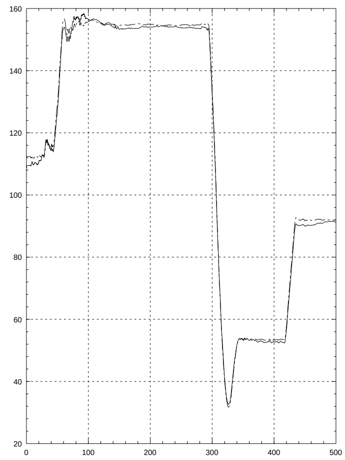


Figure 2. Heading estimate (INS dashed)

7. Conclusions

The SAHRS algorithms deliver estimates of attitude and heading, with an accuracy that is good enough for the purpose. The function requires nothing more than software changes, since the sensors already exist, and are available in a highly integrated 4th generation combat aircraft.

In combination with redundancies in data links and digital displays, the SAHRS provides a back-up reference that is both reliable and independent of the INS.

A digital representation of attitude and heading also gives more flexibility to optimize the man-machine interface than analog instruments.

References

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