Sliding Mode Control for Nonholonomic Mobile Robot

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Abstract

A new control scheme is presented for nonholonomic mobile robots. The main idea of this paper is to consider the natural algebraic structure of the chained form system together with ideas from sliding mode theory while designing the control law. At first, the system model is converted into a single-input time-varying linear system by setting one input as a time-varying function. We design the sliding mode control law by using pole placement based on pseudo-linearized model. The point stabilization and path-tracking problem for chained form are studied based on these ideas. Simulations for both unicycle car and car like robot showed this control algorithm can make the mobile robot stabilized at desired configuration and following the desired trajectory with a high precision.

1 Introduction

It is now well known that nonholonomic mobile robot (such as unicycle-type and car-like vehicles) cannot be stabilized about a desired configuration via continuous pure state feedback from a theorem due to Brockett[1]. Various approaches have been proposed for the control and stabilization problems of such system, focusing mainly on using either non-smooth feedback or smooth time-varying feedback. Smooth time-varying feedback, which explicitly depend on the time variable, had previously been received little attention in control theory, but now is the main control method for nonholonomic system. Pioneer work on nonlinear time-varying feedback for the stabilization of mobile robot was presented by Samson [3] [4]. Coron [5] established most controllable systems that can be asymptotically stabilized with time varying feedback. In order to quicken the convergence rate, M'Closkey and R. Murray[6] used the properties associated with homogeneous system. The use of discontinuous feedback has been proposed in [7] for mobile robots with unicycle kinematics. More general results were obtained in [8], where the coordinate transformation of system equation in chained form is used to simplify the analysis.

In this paper, we consider a sliding mode approach to the stabilization and tracking problem for the so-called chained form nonhonolomic system. Sliding mode is a nonlinear feedback control with variable structure with respect to the system states. The main advantage of sliding mode control is that the system is insensitive to extraneous disturbance and internal parameter variations while the trajectories are on the switching surface. Contrast to general control laws, sliding mode control is more robust and is easy to be implemented. Related recent work includes the following. A. Bloch[9] discussed stabilization and tracking of nonholonomic system using sliding mode. Utkin[10] et al presented a sliding mode controller for holonomic mobile robot navigation.

The basic idea of this paper is to use the natural algebraic structure of the system together with ideas from sliding mode theory. In Section 2, a sliding mode control for point stabilization is presented based on the pseudo-linearized model, the system is converted into a single-input time-varying linear system by setting input as a variable function. The switching function is selected response to pole placement. The path tracking problem of chained form mobile robot is studied in Section 3 by converting the system model into linear error equations and using the ideas developed in section 2. Simulations for both point stabilization and path tracking of car-like and unicycle mobile robot are given in Section 4.

2 Sliding Mode Control of Chained Form System

2.1 Chained Form System

The possibility of modelling the kinematics equation of a wheeled mobile robot by so called canonical chained form equation has been pointed in many different papers when treating the case of car-like mobile robots. It was known that the equations of unicycle-type vehicles (a simple case) could be written in this form, but this had not been used explicitly at the control design level. Recently, it has been shown that the equation of vehicles with trailers could also be locally converted into a chained form. The chained form considered here is given as follows:

$$\begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = u_1 x_3 \\ \dot{x}_3 = u_1 x_4 \\ \vdots \\ \dot{x}_{n-1} = u_1 x_n \\ \dot{x}_n = u_2 \end{cases} \tag{1}$$

or, equivalently:

$$\dot{\mathbf{X}} = \boldsymbol{h}_1(\mathbf{X})u_1 + \boldsymbol{h}_2(\mathbf{X})u_2 \tag{2}$$

here,

$$\begin{cases} \boldsymbol{h}_1(\mathbf{X}) = [1 \quad x_3 \quad \cdots \quad x_n]^T \\ \boldsymbol{h}_2(\mathbf{X}) = [0 \quad 0 \quad \cdots \quad 1]^T \end{cases}$$

Although it is nonlinear, above chained system has a strong underlying linear structure. If we set a variable function to the input u_1 of the chained form of above equation, the system can be described as fellows:

$$\begin{cases} \dot{\mathbf{X}}_1 = u_1 \\ \dot{\mathbf{X}}_2 = \mathbf{A}\mathbf{X}_2 + \mathbf{B}u_2 \end{cases} \tag{3}$$

with,

$$\mathbf{X}_1 = x_1$$

$$\mathbf{X}_2 = [x_2 \quad \cdots \quad x_{n-1} \quad x_n]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 0 & u_1(t) & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ \cdots & \cdots & \cdots & u_1(t) \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

This clearly appears the second part of the system (3) becomes a single-input time-varying linear system when u_1 is taken as a function of time and no longer as a control variable. At the mean time, when the input u_1 is taken as a function of time, the system is clearly no longer always controllable due to the first equation in (3). However, if a suitable input of u_1 is chosen, the second part of system involving $\mathbf{X}_2$ remains controllable. The second part of above equation is a state equation of linear system. We can apply the sliding mode control scheme to this part based on the linear system theory. This makes the design of control law very easy.

This property is very important. For instance, it is used further in the study of the derivation of smooth time-varying feedback which asymptotically stabilize the original point. It can also be utilized to solve the open loop steering problem.

For above model, the design of control law consists of two steps:

- Choose an integerable function $u_1(t)$ which ensures the controllability of the second part of the system, and determines a control $u_2(t)$ which drive $\mathbf{X}_2$ to its desired value in finite time.
- After $\mathbf{X}_2$ reaches its desired position, keep $u_2(t)$ equal to zero, so as to make $\mathbf{X}_2$ unchanged, and determine $u_1(t)$ to drive x_1 to its desired value in finite time.

Certainly, we can adopt the process presented in [11]. At first, select a suitable input to drive x_1 to its desired value; Then, select $u_1(t)$ as a suitable time periodic function and determinate $u_2(t)$ to drive $\mathbf{X}_2$ to its desired position. Due to system property, x_1 will obviously unchanged after one or several periods.

2.2 Sliding Mode Controller

From control theory, if the control input $u_1(t)$ is not equal to zero, the second part of equation (3) is a generalized normal form. The basic idea of designing a sliding mode controller by utilizing the generalized normal form will essentially be the same as the ones used for the generalized controller canonical form. In this section, we suppose that the control input $u_1(t)$ is selected to assure the second part of equation (3) is controllable. We will consider the control of the second part mainly. Rewrite the second part of equation (3) in following form:

$$\dot{\mathbf{X}}_2 = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ 0 & 0 \end{bmatrix} \mathbf{X}_2 + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} u_2 \tag{4}$$

Without lose of generality, suppose our goal is to drive the system states to its origin. Defined a switching function

$$\boldsymbol{S} = \mathbf{C} \cdot \mathbf{X}_2 \tag{5}$$

For sliding mode control, one of the most important things is to select a suitable matrix $\mathbf{C}$, which is mostly related to some stabilization problem. Those selected parameters, which involved in the shape of switching surface, must make sure that the sliding motion achieves convergence or stabilization in sliding regime. There are two ways to select matrix $\mathbf{C}$ [12]: one is the pole placement and another is the optimization method. Several papers extend these two methods, but the basic idea is almost the same. In this paper, the first method is adopted. Different from conventional sliding mode, where $\mathbf{C}$ is a changeable matrix respect to time-varying input $u_1(t)$, the matrix $\mathbf{C}$ is a variable matrix in out control laws. It forms a dynamic sliding mode.

The desired controller is to force the system states sliding along the surface and drives system states to their desired states. In sliding regime, the dimension of dynamic system is reduced by the number of component defined in the sliding surface. It must be noted, the sliding regime is uneffected by perturbations satisfying the well-known matching conditions. In sliding regime, the following condition is desired:

$$\boldsymbol{S}(t) = \mathbf{C}\mathbf{X}_2 = [\mathbf{C}_1 \quad \mathbf{C}_2] \begin{bmatrix} \mathbf{X}_{21} \\ \mathbf{X}_{22} \end{bmatrix} = 0 \tag{6}$$

Once the above condition is reached, it is always accompanied with condition

$$\boldsymbol{S}(t) = \mathbf{C}\mathbf{X}_2 = [\mathbf{C}_1 \quad \mathbf{C}_2] \begin{bmatrix} \mathbf{X}_{21} \\ \mathbf{X}_{22} \end{bmatrix} = 0 \tag{7}$$

Suppose the matrix $\mathbf{C}$ is so designed that matrix $\mathbf{CB}$ is nonsingular. Therefore, the equivalent control in sliding surface can be derived from those two conditions:

$$u_{eq}(t) = -(\mathbf{CB})^{-1}\mathbf{CAX}_2 \tag{8}$$

***Remark 1**: The above equivalent control is only a mathematically derived tool for the analysis of a sliding surface rather than a real control law. It represents the control action which is required to maintain the states on the sliding mode. In fact, the equivalent control is not* *realizable through a nonlinear controller even the system is nominal, or the system is in the absence of uncertainties.*

Substituting equivalent control into equation (4) yields the equivalent dynamics system with nonlinear input in the sliding mode as

$$\dot{\mathbf{X}}_{21} = (\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{C}_2^{-1}\mathbf{C}_1)\mathbf{X}_{21} \tag{9}$$

Before determine the sliding surface parameter C, we first introduce the following lemma.

***Lemma** 1 For system (3), if (*$\mathbf{A}$, $\mathbf{B}$*) is controllable*,($\mathbf{A}_{11}$, $\mathbf{A}_{12}$) *is controllable.*

Proof:($\mathbf{A}$,$\mathbf{B}$) is controllable, this is to say:

$$Rank\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

$$= Rank\left[\begin{bmatrix} 0 \\ B_2 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ B_2 \end{bmatrix} \cdots \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}^{n-1}\begin{bmatrix} 0 \\ B_2 \end{bmatrix}\right]$$

$$= Rank\begin{bmatrix} 0 & A_{11}B_2 & A_{11}A_{12}B_2 & \cdots & A^{n-2}{}_{11}A_{12}B_2 \\ B_2 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$= Rank\begin{bmatrix} A_{11}B_2 & A_{11}A_{12}B_2 & \cdots & A^{n-2}{}_{11}A_{12}B_2 \end{bmatrix} + Rank B_2$$

$$= n$$

Because $\mathbf{B}_2$=1, this implies Rank$\mathbf{B}_2$=1, it is easy to see:

$$Rank\begin{bmatrix} \mathbf{A}_{11}\mathbf{B}_2 & \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{B}_2 & \cdots & \mathbf{A}^{n-2}{}_{11}\mathbf{A}_{12}\mathbf{B}_2 \end{bmatrix} = n-1$$

$\mathbf{A}_{11}$, $\mathbf{A}_{12}$ is controllable.

Theorem 1:*If* $\mathbf{A}_{11}+\mathbf{K}\mathbf{A}_{12}=L$, $\mathbf{C}=[\mathbf{K},1]$ *is one implement of switching function, which can keep the pole point of system (9) at* L.

***Proof**:* Because (A_{11}, A_{12}) is controllable, this implies there exists matrix K:

$$s\,(\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{K}) = L$$

Compare above equation with equation (9), we get:

$$\mathbf{C}_2^{-1}\mathbf{C}_1 = \mathbf{K}$$

Set $\mathbf{C}_2$= 1, there exists

$$\mathbf{C} = [\mathbf{C}_1 \quad 1] = [\mathbf{C}_2\mathbf{K} \quad \mathbf{C}_2] = [\mathbf{K} \quad 1]$$

It is easy to see ($\mathbf{A}$, $\mathbf{B}$) in (3) is controllable. We can get the switching surface parameter according to theorem 1. In order to force the system states sliding in the switch surface, impose the following discontinuous dynamics on sliding manifold:

$$\dot{S} = -W Sign S \tag{10}$$

where $W>0$ and

$$Sign(S) = \begin{cases} 1 & S > 0 \\ 0 & S = 0 \\ -1 & S < 0 \end{cases}$$

Substituting this equation into equation (7), yields dynamics system with nonlinear input as:

$$u(t) = (\mathbf{CB})^{-1}(-\mathbf{CAX}_2 - W sign S) \tag{11}$$

***Remark** 2: The above control law is based on generalized states. The control input is also generalized input. In practice, we can convert the generalized input into physical control input by using inverse transformation of the input.*

***Remark** 3: In practice, following discontinuous function is often used to replace equation (10):*

$$\dot{S} = \begin{cases} -W sign\, S & \|S\| > d \\ -W\, S/d & \|S\| < d \end{cases}$$

to reduce the chattering. However, the precision of the control system will be reduced due to this function.

3 Path Tracking Using Sliding Mode Control

In this section, we consider the trajectory tracking problem. For the chained form representation (1), denote the system states and inputs as x_i and u_i. And the desired state as x_{ri} and u_{ri}. Denote the states and inputs errors respectively as

$$\tilde{x}_i = x_{ri} - x_i \qquad i=1,n$$

$$\tilde{u}_j = u_{rj} - u_j \qquad j=1,2$$

The nonlinear error equations are

$$\begin{cases} \dot{\tilde{x}}_1 = \tilde{u}_1 \\ \dot{\tilde{x}}_1 = u_{r1}x_{r3} - u_1 x_3 \\ \cdots \\ \dot{\tilde{x}}_{n-1} = u_{r1}x_{rn} - u_1 x_n \\ \dot{\tilde{x}}_n = \tilde{u}_2 \end{cases} \tag{12}$$

For the above system, during the control periods, we can keep the longitude velocity invariable. This means $\tilde{u}_1 = 0$. In this case, rewrite the above equations:

$$\begin{cases} \dot{\tilde{x}}_1 = 0 \\ \dot{\tilde{x}}_1 = u_1\tilde{x}_3 \\ \cdots \\ \dot{\tilde{x}}_{n-1} = u_1\tilde{x}_n \\ \dot{\tilde{x}}_n = \tilde{u}_2 \end{cases} \tag{13}$$

The above equations can be divided into two sections as what we have done in Section II:

$$\begin{cases} \dot{\tilde{x}}_1 = 0 \\ \dot{\tilde{\mathbf{X}}}_2 = \mathbf{A}\tilde{\mathbf{X}}_2 + \mathbf{B}\tilde{u}_2 \end{cases} \tag{14}$$

The second section of the system is a general normal form:

$$\dot{\tilde{\mathbf{X}}}_2 = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \tilde{\mathbf{X}}_{21} \\ \tilde{\mathbf{X}}_{22} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} u_2 \tag{15}$$

with, $\tilde{\mathbf{X}}_{21} = [\tilde{x}_2 \quad \tilde{x}_3 \quad \cdots \quad \tilde{x}_{n-1}]$ and $\tilde{\mathbf{X}}_{22} = \tilde{x}_n$

We can use the same algorithm stated in section II:

$$u(t) = (\mathbf{CB})^{-1}(-\mathbf{CA}\tilde{\mathbf{X}}_2 - W Sign\tilde{S}) \tag{16}$$

Here, $\tilde{S} = \mathbf{C}\tilde{\mathbf{X}}$

For such system, if a suitable switching function is gotten, the system states will follow the desired states, i.e. $t \to \infty$, $\tilde{x}_i \to 0$. The desired states in equation (12), can be derived from the reference trajectory and longitude position.

4 Case Study: Mobile Robot in Smooth Plane

4.1 Point Stabilization of Unicycle Car

We first consider the problem of steering a unicycle. The control inputs are the driving speed and the steering speed. The kinematic equations for this system are:

$$\begin{cases} \dot{x} = \cos q \cdot u_1 \\ \dot{y} = \sin q \cdot u_1 \\ \dot{q} = u_2 \end{cases} \quad (17)$$

The above kinematic model can be converted into chained form by using following coordinate transformation.

$$\begin{cases} q_1 = x \\ q_2 = y \\ q_3 = \tan q \end{cases} \quad (18)$$

together with the input transformation

$$\begin{cases} u_1 = u_1 / \cos q \\ u_2 = u_2 / \cos^2 q \end{cases} \quad (19)$$

The system is in a chained form:

$$\begin{cases} \dot{q}_1 = u_1 \\ \dot{q}_2 = \dot{q}_3 u_1 \\ \dot{q}_3 = u_2 \end{cases} \quad (20)$$

As in Section II, divide the above system into two sections, For the second part, we build a sliding mode control. Using the results of section II, A control algorithm is proposed for the steering system of a unicycle car as follows:

1) Drive x_1 to its desired position using any input and disconsidering the evolution of other system state. Here, we choose,

   $u_1 = -2sign(x_1) \qquad u_2 = 0$

   By using equation (19), we can get the physical control inputs.

2) Design a sliding mode controller to drive the other states in equation (17) to their desired values.

   - Select a suitable periodical function u_1 to make sure that the system is controllable. Here, we choose

     $$u_1 = \begin{cases} k & if \quad t > t_0 + t_f / 2 \\ -k & if \quad t < t_0 + t_f / 2 \end{cases}$$

   - Calculate switching surface parameter
     Denote the system pole point in sliding phase is $L = -1$, According Theorem 1, we get:

     $$\mathbf{C} = [1/u_1 \quad 1]$$

     The switch function of the system is:

     $$\boldsymbol{S} = \mathbf{CX} = x_2 / u_1 + x_3$$

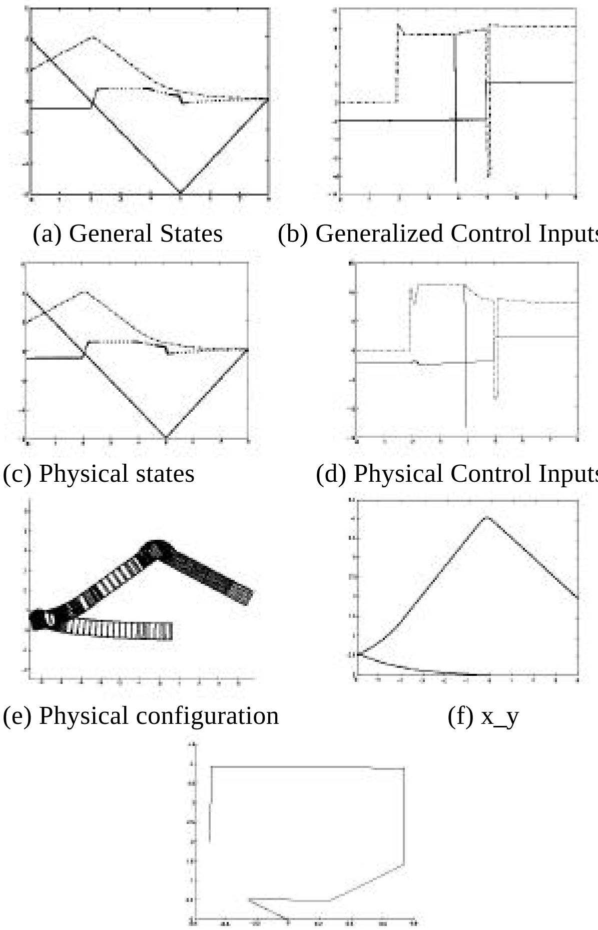
   - Calculate control input
     Following equation (11), the control input is

     $$u_2 = -x_3 - WsignS$$

     The physical control inputs are:

     $$\begin{cases} n_1 = u_1 \boldsymbol{cos(}A\boldsymbol{tan(}x_3\boldsymbol{))} \\ n_2 = -x_3 \boldsymbol{cos}^2\boldsymbol{(}A\boldsymbol{tan(}x_3\boldsymbol{))} - WSign\boldsymbol{S}\,\boldsymbol{cos}^2\boldsymbol{(}A\boldsymbol{tan(}x_3\boldsymbol{))} \end{cases}$$

Suppose the original position of the unicycle is (4, 2, -0.5) and the desired position is the origin. Using the above algorithm, we get the following simulation results. From Fig. 1, we can see that the system is stabilized at origin.



(a) General States (b) Generalized Control Inputs

(c) Physical states (d) Physical Control Inputs

(e) Physical configuration (f) x_y

(g) y-θ

Fig. 1 System States and Configuration Trajectory of Unicycle Point Stabilization

4.2 Point Stabilization of Car Like Robot

Similar to the previous example, let us now consider point stabilization of a car-like robot. As unicycle, The control inputs are also the driving speed and the steering speed. Its kinematic equations are:

$$\begin{cases} \dot{x} = \cos q \cdot u_1 \\ \dot{y} = \sin q \cdot u_1 \\ \dot{q} = \tan f \cdot u_1 / l \\ \dot{f} = u_2 \end{cases} \quad (21)$$

by letting

$$\begin{cases} q_1 = x \\ q_2 = y \\ q_3 = \boldsymbol{tan} q \\ q_4 = \boldsymbol{tan} f \,/\, l \,/\, \boldsymbol{cos}^3 q \end{cases} \quad (22)$$

and

$$\begin{cases} u_1 = u_1 / \boldsymbol{cos} q \\ u_2 = -3\boldsymbol{sin} q\, \boldsymbol{sin}^2 f u_1 / l \boldsymbol{cos}^2 q \\ + l \boldsymbol{cos}^3 q\, \boldsymbol{cos}^2 f u_2 \end{cases} \quad (23)$$

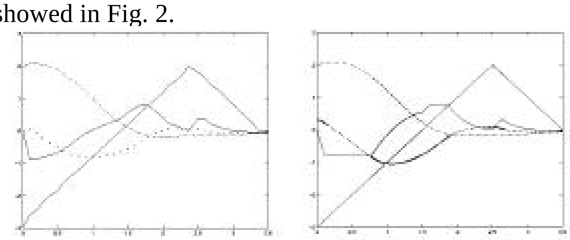
The system is in chained form:

$$\begin{cases}\dot{q}_1 = u_1\\ \dot{q}_2 = \dot{q}_3 u_1\\ \dot{q}_3 = \dot{q}_4 u_1\\ \dot{q}_4 = u_2\end{cases} \tag{24}$$

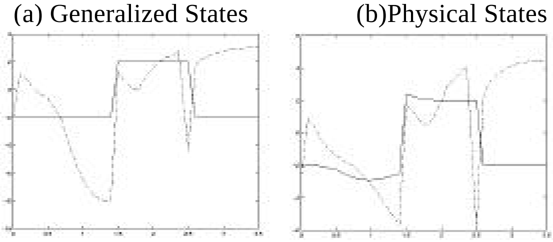
For such a system, we introduce the following algorithm to implement point stabilization problem:

1. Design a sliding mode controller
   - Selecting switching function
     Let the system pole points in sliding regime are Λ_1=-2 and Λ=-3, Based on theorem 1, there exists:
     $$\mathbf{C} = \left[6/u_1^2 \quad 5/u_1 \quad 1\right]$$
     The switching function of the system is:
     $$\boldsymbol{S}_2 = 6q_2/u^2{}_1 + 5q_3/u_1 + q_4$$
   - Design control law
     Following equation (11), the control input is
     $$u_2 = -6q_3/u_1 - 5q_4 - e_2 Sign(\boldsymbol{S}_2)$$
     Considering equation (23), we can get the physical control inputs easily.
2. Drive x_1 to its desired position using any input and control x_2, x_3, x_4 at the same time.
   $$\begin{cases}u_1 = -e_1 Sign(q_1)\\ u_2 = -6q_3/u_1 - 5q_4 - e_2 Sign(\boldsymbol{S}_2)\end{cases}$$
   From equation (23), we can get the physical control input.
3. Steer x_2, x_3, x_4 to their desired positions
   - Select a suitable perodical function u_1 to make sure that equation (21) is controllable. Here, we select:
     $$u_1 = \begin{cases} K & if \quad t > t_f/2\\ -K & if \quad t_f/2 > t > 0\end{cases}$$
   - Control law:
     $$\begin{cases}u_1 = \begin{cases} K & if \quad t > t_f/2\\ -K & if \quad t_f/2 > t > 0\end{cases}\\ u_2 = -6q_3/u_1 - 5q_4 - e_2 Sign(S_2)\end{cases}$$

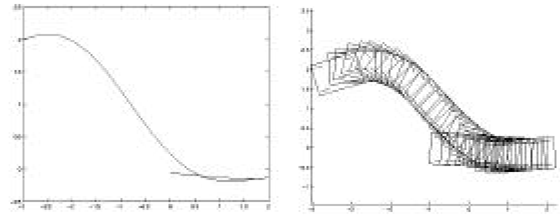
Suppose the original position is (-3, 2, 0.5, 0.0) and the desired position is the origin. The simulation results are showed in Fig. 2.



(a) Generalized States (b)Physical States



(c) generalized control inputs (d) physical control inputs



(e) X-Y (f) physical configuration

Fig 2 System States and Configuration Trajectory of Point Stabilization of Car-like Robot

4.3 Path Tracking of Car-like Robot

For path tracking problem, according to equation (13), the error equations are:

$$\begin{cases}\dot{\tilde{q}}_1 = 0\\ \dot{\tilde{q}}_2 = \tilde{q}_3 u_{r1}\\ \dot{\tilde{q}}_3 = \tilde{q}_4 u_{r1}\\ \dot{\tilde{q}}_4 = \tilde{u}_2\end{cases} \tag{25}$$

Suppose the ideal path is $\boldsymbol{y}=\boldsymbol{f}(\boldsymbol{x})$ and the generalized velocity is constant, denoted by u_{1c}, The following process is presented for control task:

1. Calculate the desired states of the system and the errors of system states
   - Desired system states:
     $$\begin{cases}x_r = x(t)\\ y_r = f(x_r)\\ q_r = A\boldsymbol{tan}\dfrac{\partial f(x_r)}{\partial x_r}\\ f_r = A\boldsymbol{tan}(l * \dfrac{\partial^2 f(x_r)}{\partial x^2{}_r}\boldsymbol{cos}^3 q_r)\end{cases}$$
   - Errors of the system:
     $$\begin{cases}\tilde{q}_1 = x_r - x = 0\\ \tilde{q}_2 = y_r - y\\ \tilde{q}_3 = \boldsymbol{tan}q_r - \boldsymbol{tan}q\\ \tilde{q}_4 = \boldsymbol{tan}f_r / l / \boldsymbol{cos}^3 q_r - \boldsymbol{tan}f / l / \boldsymbol{cos}^3 q\end{cases}$$
   - Ideal control inputs:
     $$\begin{cases}u_{1r} = u_{1c}\\ u_{2r} = \dfrac{1}{l * \boldsymbol{cos}^2 f_r \boldsymbol{cos}^3 q_r}\dot{f}_r + \dfrac{3\boldsymbol{tan}f_r\,\boldsymbol{sin}f_r}{l * \boldsymbol{cos}^4 f_r}\dot{f}_r\end{cases}$$
     here,
     $$\dot{q}_r = \frac{\partial f(x_r)}{\partial x_r}\boldsymbol{cos}q_r$$
     $$\dot{f}_r = \boldsymbol{cos}^2 f_r(-3l\frac{\partial^2 f(x_r)}{\partial x_r{}^2}\boldsymbol{cos}^2\dot{q}_r\,\boldsymbol{sin}q_r\dot{q}_r$$
     $$+ l\frac{\partial^3 f(x_r)}{\partial x_r{}^3}u_c\,\boldsymbol{cos}^3\dot{q}_r$$
2. Calculate the control input of the error equation
   - Select switching function
     Let the system pole points in sliding regime are Λ_1=-2 and Λ=-3. Based on theorem 1, there exists:
     $$\mathbf{C} = \left[6/u_1^2 \quad 5/u_1 \quad 1\right]$$

The switching function of the system is:

$$S_2 = 6q_2 / u^2{}_1 + 5q_3 / u_1 + q_4$$

- Design control law

  According to equation (16), the control input of the error equation is

  $$u_2 = -6q_3 / u_1 - 5q_4 - e_2 Sign(S_2)$$

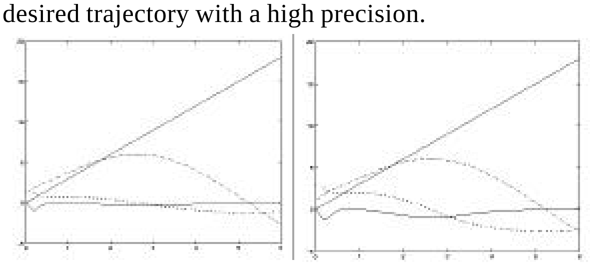
3. Get the physical control inputs:
   - Get the generalized control inputs:

     $$\begin{cases} u_1 = u_{1c} \\ u_2 = u_{2r} + \tilde{u}_2 \end{cases}$$

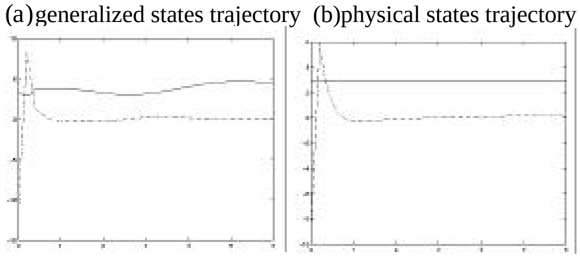
   - Convert the generalized control inputs into the physical control inputs:

     $$\begin{cases} v_1 = u_{1c} / \cos q \\ v_2 = \dfrac{-3\sin^2 f \sin q u_1}{l \cos^2 q} + l \cos^3 q \cos^2 f u_2 \end{cases}$$

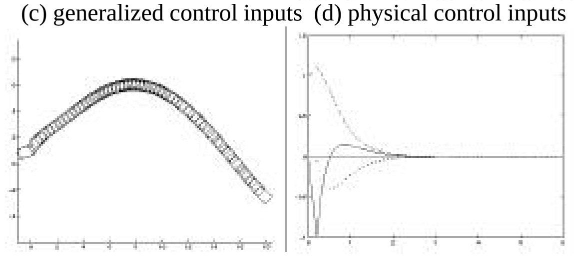
Suppose the desired trajectory of a car is $y=6sin(0.2x)$ and the original position of the car is (0.0, 1.0, 0.3, 0.0). The simulation results are showed in Fig. 3. From simulation results, we can see that the car can track the desired trajectory with a high precision.



(a) generalized states trajectory (b) physical states trajectory



(c) generalized control inputs (d) physical control inputs



(e) generalized states error (f) configuration trajectory

Fig. 3 Simulation Results of Path Tracking

5 Conclusion

In this paper, we proposed a method to control nonholonomic vehicle using sliding mode control. For the design of control law, we use the properties of the system structure together with ideas from sliding mode theory. This makes it very easy to design a controller. Point stabilization control and path tracking are studied using this algorithm. Finally, computer simulations for both unicycle and car-like mobile robots are presented.

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