Design of the Rectifying Inspection Plans and An Optimization Scheme

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Abstract. Two optimization algorithms for the rectifying inspection are presented. They minimize either *ATI* (Average Total Inspection) or *AOQ* (Average Outgoing Quality) and use another as a constraint. The definitions for *ATI* and *AOQ* are also generalized in order to handle the varying fraction defective of the incoming lots. A comprehensive study is conducted for comparing the performance characteristics of several different rectifying inspection plans. The results show that the optimum algorithms developed in this article can usually produce the best outputs and always satisfy the design constraint. Finally, a manufacturing example is used to show the applications of the optimum rectifying inspection plans.

1. Introduction

The rectifying inspection is a sampling program that is able to improve the outgoing quality of the products. In the rectifying inspection plan, a random sample of size n is picked from a lot of size N and inspected. If the number d of observed defectives in the sample is less than or equal to a predetermined acceptance number c, the lot is accepted immediately and no further action will be taken. However, if d is greater than c, the lot is rejected and a 100% inspection will be carried out on the remaining units of the lot. All of the found defectives are replaced by good units. The rectifying inspection plans can be used in receiving inspection, process inspection, and also the final inspection.

A rectifying inspection plan is characterized by four input parameters and two output ones:

Input parameters:

- p_0 the incoming fraction defective.
- N the lot size, i.e., the number of units in a lot.
- n the sample size, i.e., the number of units in a sample.
- c the acceptance number.

Output parameters:

- *ATI* the Average Total Inspection, i.e., the average number of inspected units over the accepted and rejected lots.

AOQ the Average Outgoing Quality. i.e., the average outgoing fraction defective over the accepted and rejected lots.

If a process has a fixed incoming fraction defective p_0, an optimum rectifying inspection plan can be determined by optimizing *n* and *c*, so that *ATI* is minimized and *AOQ* is held equal to or smaller than a specified value. However, in reality, many unavoidable factors (such as the wear of cutting tools and the fluctuation of power supply) result in nonhomogeneous lots, that is, different incoming lots may have different p_0 values. Several more realistic design algorithms are proposed, in which *ATI* is minimized at the *process average* $\overline{p}_0$ (the average of p_0) subject to different constraints. Typical constraints are related to AOQL (Average Outgoing Quality Limit), LTPD (Lot Tolerance Percent Defective) and OQ (Outgoing Quality) (Guenther 1984). The AOQL method proposed by Dodge and Roming (1995) is most widely used. Some researchers (Bebbington and Govindaraju 1998) have developed sampling schemes similar to the rectifying inspections, but the procedure is much more sophisticated. Others (Kleijnen *et al*. 1992) have studied the applications of the rectifying inspections in financial auditing. However, none of these rectifying inspection plans takes the probability distribution of p_0 into consideration and no algorithm that minimizes *AOQ* and uses *ATI* as a constraint has been presented in the open literature.

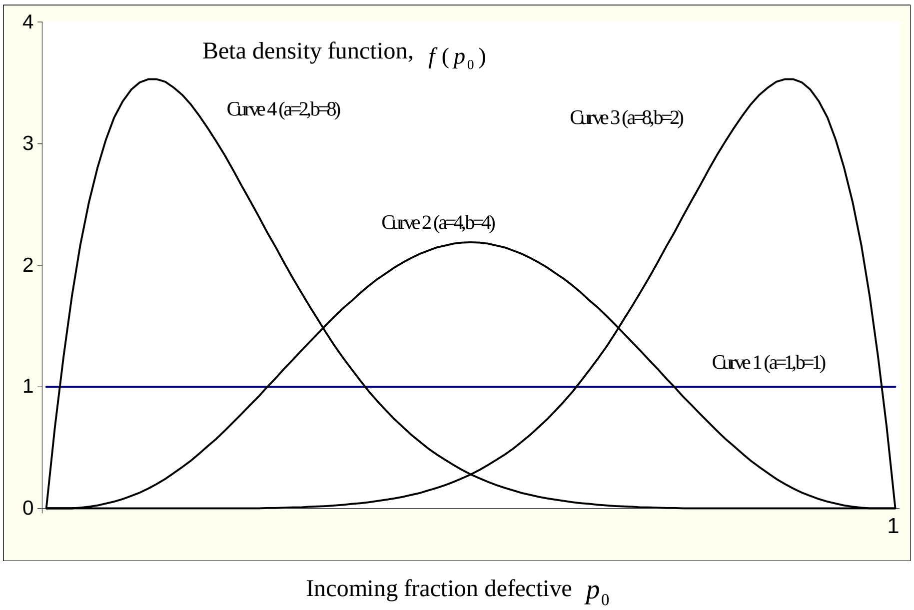


Figure 1 Beta distribution

A review by Lauer (1978) suggests that, the random variable p_0 can be better represented by the beta distribution than by any other distributions. The probability density function of the beta distribution is as follows,

$$f(p_0 \mid a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p_0^{a-1}(1-p_0)^{b-1} \qquad (1)$$

where, Γ(.) is the gamma function. By varying the two parameters *a* and *b*, the beta distribution can be used to describe a wide variety of p_0 distributions as shown in Figure 1. Some researchers (Hald 1960, Wu 1997) have developed algorithms to estimate the parameters *a* and *b* based on the observed sample values. Furthermore, some researchers find that the knowledge of the probability distribution does not need to be very precise in sampling plans provided that the form of the distribution chosen is reasonable (Wetherill and Chiu 1975).

This article studies two dual optimization algorithms for rectifying inspections. One is to minimize *ATI* using *AOQ* as a constraint; another is to minimize *AOQ* using *ATI* as the constraint. The definitions for *ATI* and *AOQ* will be generalized as the overall means over the entire range of the random variable p_0. p_0 is assumed to follow a beta distribution with known parameters *a* and *b* (in practice, they are estimated from the observed data). It is also assumed that the number *d* of observed defectives in a sample has a hypergeometric distribution. This distribution produces more accurate results for large sample size than the binomial distribution. Finally, the performance characteristics of six different types of rectifying inspection plans will be studied and compared. The findings from this study will provide the QA (Quality Assurance) practitioners with useful guidelines in applying the rectifying inspections.

2. Two optimization algorithms (models)

In this article, $ati(p_0)$ and $aoq(p_0)$ denote the Average Total Inspection and Average Outgoing Quality at a fixed value of the incoming fraction defective p_0. They can be calculated as follows (Montgomery 1997):

$$ati(p_0) = n + (1 - P_a)(N - n) \qquad (2)$$

$$aoq(p_0) = p_0 P_a (N - n)/N \qquad (3)$$

where, P_a is the probability of accepting a lot. It is dependent on p_0. If hypergeometric distribution is used,

$$P_a = \sum_{d=0}^{c} \frac{\binom{p_0 N}{d}\binom{N - p_0 N}{n-d}}{\binom{N}{n}} \qquad (4)$$

The first optimization algorithm (model 1) (Guenther 1984) has been used widely in industry. For a given value of p_0, this algorithm minimizes $ati(p_0)$ on the condition that the resultant $aoq(p_0)$ is smaller than or equal to a specified value AOQ_{max}.

Optimization model 1:

Objective function:	$ati(p_0)$ = minimum	(5)
Constraints:	$aoq(p_0) \leq AOQ_{max}$	(6)

Design variables: n and c

The second optimization algorithm (model 2) is proposed in this article. It minimizes $aoq(p_0)$ on the condition that the resultant $ati(p_0)$ is smaller than or equal to a specified value ATI_{max}.

Optimization model 2:

Objective function:	$aoq(p_0)$ = minimum	(7)
Constraints:	$ati(p_0) \leq ATI_{max}$	(8)

Design variables: n and c

The procedure for optimization model 1 has been described in many papers e.g. Guenther 1984) and will not be repeated here. For optimization model 2, the most straightforward search procedure is to check every possible combination of n and c ($1 \leq n \leq N$, $0 \leq c \leq n-1$), and identify the optimum pair of n and c that produces the minimum $aoq(p_0)$ while holding $ati(p_0) \leq ATI_{max}$. However, the search region can be substantially reduced by noting that:

(1) It is clear from equation (2) that $ati(p_0)$ is always larger than n. So, n must be smaller than ATI_{max}, otherwise, $ati(p_0)$ is bound to be greater than ATI_{max} and the design is infeasible.

(2) For a given n value, the probability of acceptance, P_a, is an increasing function of c (from equation (4)). That is, $ati(p_0)$ is a decreasing function of c (from equation (2)) and $aoq(p_0)$ is an increasing function of c (from equation (3)). Consequently, for a given n, the optimum acceptance number is the first (or smallest possible) c value that makes the inequality constraint (8) stand.

The complete search procedure for optimization model 2 is outlined below:

(1) Input p_0, N, ATI_{max} and make AOQ_{min} equal to one (the largest possible value of AOQ).

(2) Start the optimization search by setting n as one.

(3) For a given n, first set c equal to zero.

(4) For a particular design combination (n, c),
   - (4.1) Calculate P_a by equation (4).
   - (4.2) Calculate $ati(p_0)$ by equation (2).
   - (4.3) If $ati(p_0)$ is larger than ATI_{max}, increase c by one and go back to the

beginning of step (4); otherwise, go to the next step (the current *c* is the optimum acceptance number for the given *n*).

(5) For the current design pair (*n*, *c*) identified in step (4),

(5.1) Calculate $aoq(p_0)$ by equation (3).

(5.2) If $aoq(p_0)$ is smaller than the current value of AOQ_{min}, store these pairs (*n*, *c*) as the temporarily optimum sample size n_{optim} and acceptance number c_{optim}. Otherwise, do nothing.

(6) If current *n* is smaller than (ATI_{max} – 1), increase *n* by one and go back to step (3); Otherwise the whole optimization search is completed, and the (n_{optim}, c_{optim}) is finalized.

Above two optimization models can be applied to different circumstances. For example, when a company is conducting an initial design of a rectifying inspection plan, it may use optimization model 1. The purpose is to ensure the outgoing quality and minimize the required resources (e.g., manpower and instrument). On the other hand, for an existing rectifying inspection plan, further reducing *ATI* may be impossible or meaningless (for example, the number of operators cannot be reduced from one to 0.8). Under such a situation, optimization model 2 can be employed to optimize the outgoing quality by making full use of the available resources.

If the variation of p_0 has to be taken into consideration, two worst-case approaches can be employed. The first one is actually the AOQL (Dodge and Roming 1995) method and corresponds to optimization model 1.

Objective function: $ati(\overline{p}_0)$ = minimum (9)

Constraints: $\underset{p_L \le p_0 \le p_U}{MAX}\left(aoq(p_0)\right) \le AOQ_{max}$ (10)

Design variables: *n* and *c*

where, $\overline{p}_0$ is called the process average. p_L and p_U are the lower and upper bounds of p_0.

$$\overline{p}_0 = 0.5 \times (p_L + p_U) \tag{11}$$

The second worst-case approach corresponds to optimization model 2. It is derived with reference to the AOQL method.

Objective function: $aoq(\overline{p}_0)$ = minimum (12)

Constraints: $\underset{p_L \le p_0 \le p_U}{MAX}\left(ati(p_0)\right) \le ATI_{max}$ (13)

Design variables: *n* and *c*

From above, it can be seen that the worst-case approaches only take into account the range of p_0, rather than its probability distribution.

3. Generalization of *ATI* and *AOQ*

Equation (2) and (3) are only valid for a fixed p_0 value. If p_0 is taken as a random variable with a certain probability distribution, it is more rational to define the Average Total Inspection and Average Outgoing Quality as the overall means over the whole range of p_0. In this article, *ATI* and *AOQ* are generalized as follows:

$$ATI = \int_{p_L}^{p_U} ati(p_0) \cdot f(p_0) dp_0 = \int_{p_L}^{p_U} [n + (1 - P_a)(N - n)] \cdot f(p_0) dp_0 \quad (14)$$

$$AOQ = \int_{p_L}^{p_U} aoq(p_0) \cdot f(p_0) dp_0 = \int_{p_L}^{p_U} \frac{p_0 (N - n)}{N} P_a \cdot f(p_0) dp_0 \quad (15)$$

where, $f(p_0)$ is the probability density function. It is noted that, P_a is also a function of p_0 (equation (4)).

If the generalized *ATI* and *AOQ* are adopted, minor modifications are needed for the two optimization models. Namely, $ati(p_0)$ and $aoq(p_0)$ should be replaced by *ATI* (equation (14)) and *AOQ* (equation (15)), respectively. Optimization model 1 is rewritten as,

Objective function: *ATI* = minimum (16)
Constraints: $AOQ \leq AOQ_{max}$ (17)
Design variables: *n* and *c*

Optimization model 2 becomes

Objective function: *AOQ* = minimum (18)
Constraints: $ATI \leq ATI_{max}$ (19)
Design variables: *n* and *c*

A computer program **rectify.c** in C language has been developed by the authors. It is able to design the optimum rectifying inspection plan for both optimization models. The operation of the optimum rectifying inspection plan is as easy as that for any ordinary rectifying inspection plans.

4. Performance studies

Two experiments are conducted to study and compare the performance characteristics of several different types of rectifying inspection plans under two optimization models.

4.1 Experiment one: using optimization model 1

In this experiment, the Average Total Inspection is minimized with the Average Outgoing Quality specified. The two responses for the experiment are $T1$ and $Q1$. $T1$ is used to measure the final Average Total Inspection ATI_{min} with reference to the lot size.

$$T1 = ATI_{\min} / N \tag{20}$$

$Q1$ is used to check the feasibility of the plan.

$$Q1 = \begin{cases} 0 & if \quad AOQ \leq AOQ_{\max} \\ 1 & otherwise \end{cases} \tag{21}$$

4.2 Experiment two: using optimization model 2

In this model, the Average Outgoing Quality is minimized with the Average Total Inspection specified. There are also two responses, $T2$ and $Q2$. Where, $Q2$ is used to assess the final Average Outgoing Quality AOQ_{min} with reference to μ_{p0} (the mean value of p_0).

$$Q2 = AOQ_{\min} / \mu_{p0} \tag{22}$$

$T2$ is then used to check the feasibility of the plan.

$$T2 = \begin{cases} 0 & if \quad ATI \leq ATI_{\max} \\ 1 & otherwise \end{cases} \tag{23}$$

In each experiment six different types of rectifying sampling plan will be studied. For the first experiment, they are:

(1) Optimum plan, i.e., optimization model 1 with variable p_0 (equations (16) and (17)).

(2) Mean-point plan, i.e., optimization model 1 with p_0 fixed at the mean value μ_{p0} (equations (5) and (6)).

(3) Center-point plan, i.e., optimization model 1 with p_0 fixed at the process average $\bar{p}_0$.

(4) Lower-end plan, i.e., optimization model 1 with p_0 fixed at the lower end p_L.

(5) Upper-end plan, i.e., optimization model 1 with p_0 fixed at the upper end p_U.

(6) Worst-case plan for model 1 (equations (9) and (10)).

The mean-point and center-point plans are sometimes used in the designs of the rectifying inspection plans due to their simplicity. Lower- and upper- end plans are included here for the purpose of studying the consequence of having poorly under-estimate or over-estimate p_0.

Six similar plans are studied in experiment two. However, formulae for optimization model 2 are used. Specifically, equations (18) and (19) are used by plan (1); equations (7) and (8) by plans (2) to (5), and equations (12) and (13) by plan (6).
For both experiments, the effects of the following five factors are investigated:

(1) **Factor A: probability distribution** (four levels)

Four beta distributions (equation (1)) with different values for the parameters *a* and *b* are used to describe the variation of p_0 (Figure 1).

[1] Distribution 1 ($a = b = 1$) is actually an uniform one.

[2] Distribution 2 ($a = b = 4$) is very close to a normal distribution (with the same mean and standard deviation).

[3] Distribution 3 ($a = 8$, $b = 2$) is skewed to left.

[4] Distribution 1 ($a = 2$, $b = 8$) is skewed to right.

(2) **Factor B: lower bound p_L of the incoming fraction defective** (two levels)

[1] $p_L = 0.001$; [2] $p_L = 0.01$

Factor B represents the general level of the fraction defective.

(3) **Factor C: upper bound p_U of the incoming fraction defective** (two levels)

[1] $p_U = 2p_L$; [2] $p_U = 6p_L$

Factor C, together with factor B, decides the range of the fraction defective.

(4) **Factor D: lot size *N*** (two levels)

[1] $N = 1000$; [2] $N = 10000$

(5) **Factor E: allowable AOQ_{max} or ATI_{max}** (two levels)

For experiment one, the constraint is AOQ_{max}:

[1] $AOQ_{max} = 0.8\,\overline{p}_0$; [2] $AOQ_{max} = 0.98\,\overline{p}_0$

For experiment two, the constraint is ATI_{max}:

[1] $ATI_{max} = 0.02N$; [2] $ATI_{max} = 0.2N$

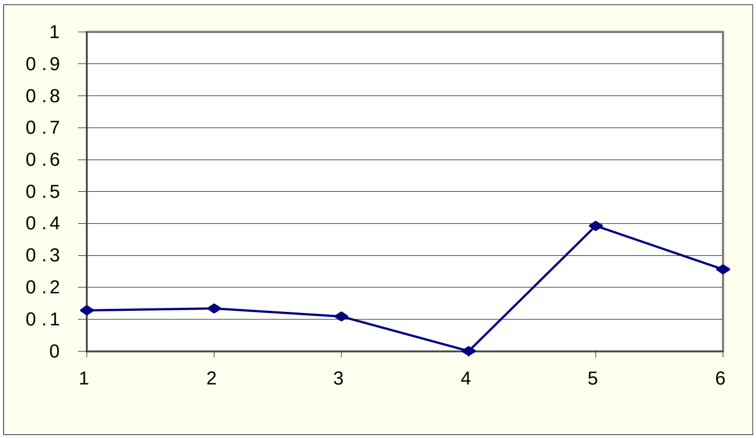
The total number of runs (combinations of the levels of all five factors) for each plan in each experiment is equal to 64 (=4×2×2×2×2). For a particular run, we first use a plan (e.g. the mean-point plan) to decide the optimum sample size n_{optim} and acceptance number $c_{optim.}$ Then the variation of p_0 is brought into play (the p_0 distribution is the beta distribution associated with this run). We substitute the n_{optim} and c_{optim} for *n* and *c* in formulae (14) and (15) to find out *ATI* and *AOQ*. Finally, *T1* and *Q1* (or *T2* and *Q2*) are calculated from *ATI* and *AOQ* (equations (20) to (23)).

4.3 Discussion on the results of experiment one

Figure 2 displays the average values, $\overline{T1}$ and $\overline{Q1}$, for the two responses in experiment one. $\overline{T1}$ and $\overline{Q1}$ are the averages over the 64 runs for each rectifying sampling plan. Actually, $\overline{Q1}$ is the probability that the constraint on *AOQ* is violated. Follow interesting points are discovered as follows:

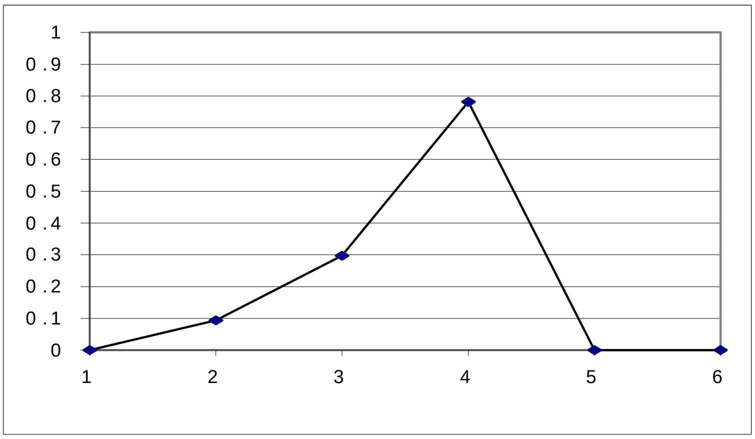
(1) In Figure 2 (b), the $\overline{Q1}$ values resulting from the mean-point, center-point and lower-end plans are larger than zero. It means that it is infeasible to use these three plans to minimize *ATI* due to the severe violation of the

constraint on *AOQ*. Especially, if the lower-end plan is used, 78% of the designed plans will result in an *AOQ* larger than the specification. Among



a) $\overline{T1}$ = *ATImin/N* (objective)

b) $\overline{Q1}$ (constraint)



Plans:

1. optimum	2. mean-point
3. center-point	4. lower-end
5. upper-end	6. worst-case

Figure 2. Results of experiment 1

the remaining three plans (the optimum, upper-end and worst-case plans), the optimum plan has a minimum value of $\overline{T1}$ (shown in Figure 2 (a)), and therefore, excel the other two plans. It can be concluded that the optimum plan can produce the lowest *ATI*, on average, when the probability distribution of p_o is taken into consideration. Moreover, it always keeps the resultant *AOQ* equal to or smaller than the specification.

(2) The lower-end plan always produces an infeasible plan unless the probability distribution is skewed to the right. Many factors, such as B (fraction defective level), C (fraction defective range) and E (allowable AOQ_{max}), have strong influence on the feasibility of the mean-point plans which are usually infeasible at the lower level of these factors.

(3) For the three feasible plans, i.e., the optimum, upper-end and worst-case plans, the *ATI* resulting from the optimum and worst-case plans is strongly influenced by the beta probability distribution; and the *ATI* from the upper-end plan is affected by the fraction defective range. Furthermore, when the constraint on *AOQ* is relaxed (i.e., AOQ_{max} = 0.98 $\overline{p}_0$), the *ATI* resulting from all plans are reduced.

4.4 Discussion on the results of experiment two

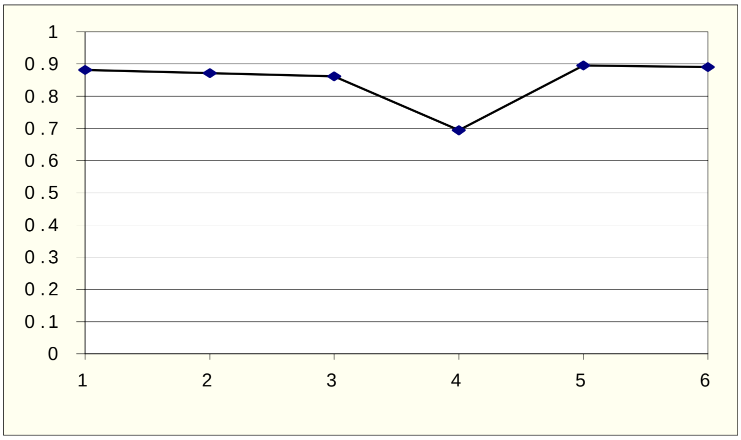
Figure 3 displays the average values $\overline{T2}$ and $\overline{Q2}$ for the two responses in experiment two. Again, many interesting points can be noted. The finding in the experiment can be summarized as follows:

(1) From Figure 3(b), the $\overline{T2}$ values (the probability of violating the *ATI* constraint) resulting from the mean-point, center-point and lower-end plans are much larger than zero. It means that these plans are again infeasible when used to design the rectifying inspection plans with minimum *AOQ*. For the remaining three plans (the optimum, upper-end and worst-case plans), the *AOQ* (indicated by $\overline{Q2}$) resulting from the optimum plan is slightly smaller than that from the other two plans (see Figure 3(a)). It may be stated that the optimum plan is again the overally best rectifying inspection plan with minimum *AOQ*.

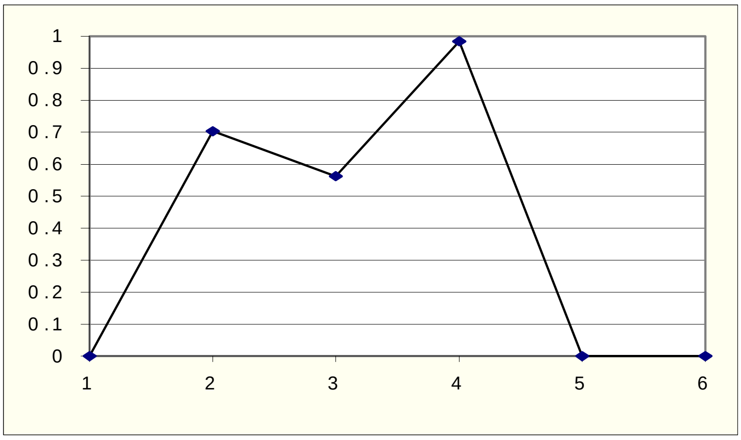
(2) It is found that the lower-end plan violates the *ATI* constraint almost by 100%. The feasibility of the mean-point plan is strongly influenced by the fraction defective level, the lot size and the allowed ATI_{max}; and the feasibility of the center-point plan is strongly influenced by the fraction defective level and the lot size.

(3) It is also found that the *AOQ* resulting from the three feasible plans, (i.e., the

a) $\overline{Q2} = AOQmin / \mu_{p_0}$ (objective)



b) $\overline{T2}$ (constraint)



Plans:

1. optimum	2. mean-point
3. center-point	4. lower-end
5. upper-end	6. worst-case

Figure 3 Results of experiment 2

optimum, upper-end and worst-case plans,) is almost insensitive to the different values of the factors. Only when the constraint of ATI_{max} is increased (i.e., AOQ_{max} = 0.2N), is the AOQ reduced significantly for all of the plans. It is becomes that more units are inspected and more defective items are replaced by conforming ones.

As a conclusion drawn from above two experiments, the optimum plan always produces the feasible and best rectifying inspection plan, especially, if the design objective is to minimize ATI (optimization model 1). The lower-end plan is the poorest one, almost always generates infeasible plans. Noteworthily, the mean-point and center-point are also unsatisfactory, resulting in infeasible designs for many cases. The upper-end plan is the only feasible plan with fixed p_0 value, but final object function values are usually inferior. The worst-case plan can guarantee the feasible designs, but the objective function values are still larger than that generated by the optimum plans.

5. Example

A manufacturing company decides to design a rectifying inspection plan to inspect the final products in a production line. The following information is available

(1) The fraction defective p_0 has a probability distribution that is approximately between 0.01 and 0.02, and can be very well described by a beta distribution with the shape as curve 2 ($a = b = 4$) in Figure 1.

(2) The lot size is N = 10000.

The rectifying inspection plan should make ATI as small as possible in order to save the required resources. The resultant AOQ must be no larger than 0.012.
This is a typical design using optimization model one (equations (16) and (17)). The inputs can be summarized as follows:

E(p_0)	= 0.015, mean value of p_0.
R(p_0)	= 0.01, the range of p_0. The lower and upper bounds of p_0 are 0.01 and 0.02 respectively.
N	= 10000, lot size.
AOQ_{max}	= 0.012, allowed maximum AOQ.

Using the computer program **rectify.c**, the optimization design is completed in 4.18 second of CPU time (in a Pentium II 300 PC). The results are listed below:

n	= 506, sample size.
c	= 10, acceptance number.

AOQ = 0.011998, resultant Average Outgoing Quality.
ATI = 1912, minimum Average Total Inspection.

The design is fairly desired, because *ATI* is minimized and *AOQ* is just smaller than the specification (satisfactory convergence).

Since each inspector is able to inspect 1500 units per lot, the manager decides to allocate two inspectors for this job. Thus, the total inspection capacity per lot is 3000 units, larger than the optimum *ATI* (= 1912) obtained in the first design.
Now the manager is interested in conducting a second design to minimize *AOQ*, using ATI_{max} = 3000 as a constraint. The purpose is to make full use of the inspection capacity. This time, optimization model 2 will be employed. All of the parameters remain unchanged as in the first design, except that the constraint on *AOQ* should be replaced by that on *ATI*. That is
ATI_{max} = 3000, allowed maximum *ATI*.

The second design takes 10.22 seconds of CPU time and results in the following rectifying inspection plan.
n = 1194, sample size.
c = 21, acceptance number.
AOQ = 0.010304, minimum Average Outgoing Quality.
ATI = 2999, resultant Average Total Inspection.

The second design is also quite successful. *AOQ* is reduced by 14% compared to that in the first design, and the resultant *ATI* is just smaller than the specification of 3000 (within the capacity of two inspectors).
Before this rectifying inspection plan is released to the real production line, a simulation run (with n = 1194 and c=21) is carried out. The simulation results listed below are very close to the calculated ones.
AOQ_{simu} = 0.010296, simulated Average Outgoing Quality.
ATI_{simu} = 3016, simulated Average Total Inspection.

6. Conclusions

This article presents two optimization design algorithms (models) for the rectifying inspection plans. One is to minimize the Average Total Inspection (*ATI*), another minimizes the Average Outgoing Quality (*AOQ*). Each optimization model can be applied to certain circumstances, and can always produce the optimal and feasible plans.

The definitions for ATI and AOQ are generalized as the overall mean over the distribution of the random variable p_0 (the incoming fraction defective).
A comprehensive study is also carried out to compare the optimization algorithms with some traditional design plans. The general findings are that the fixed point plans

usually result in infeasible or inferior rectifying inspection plans (design constraint violated). The worst-case plan can guarantee a feasible inspection plans, but the resultant *ATI* or *AOQ* is always poorer than that from the optimization algorithms.
The optimization design algorithms can be easily computerized, and should be used exclusively as long as the distribution of p_0 is known or can be reasonably estimated.

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