# Modern B-Tree Techniques

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Abstract

Invented about 40 years ago and called ubiquitous less than 10 years later, B-tree indexes have been used in a wide variety of computing systems from handheld devices to mainframes and server farms. Over the years, many techniques have been added to the basic design in order to improve efficiency or to add functionality. Examples include separation of updates to structure or contents, utility operations such as non-logged yet transactional index creation, and robust query processing such as graceful degradation during index-to-index navigation.

This survey reviews the basics of B-trees and of B-tree indexes in databases, transactional techniques and query processing techniques related to B-trees, B-tree utilities essential for database operations, and many optimizations and improvements. It is intended both as a survey and as a reference, enabling researchers to compare index innovations with advanced B-tree techniques and enabling professionals to select features, functions, and tradeoffs most appropriate for their data management challenges.
1

Introduction

Less than 10 years after Bayer and McCreight [7] introduced B-trees, and now more than a quarter century ago, Comer called B-tree indexes ubiquitous [27]. Gray and Reuter asserted that “B-trees are by far the most important access path structure in database and file systems” [59]. B-trees in various forms and variants are used in databases, information retrieval, and file systems. It could be said that the world’s information is at our fingertips because of B-trees.

1.1 Perspectives on B-trees

Figure 1.1 shows a very simple B-tree with a root node and four leaf nodes. Individual records and keys within the nodes are not shown. The leaf nodes contain records with keys in disjoint key ranges. The root node contains pointers to the leaf nodes and separator keys that divide the key ranges in the leaves. If the number of leaf nodes exceeds the number of pointers and separator keys that fit in the root node, an intermediate layer of “branch” nodes is introduced. The separator keys in the root node divide key ranges covered by the branch nodes (also known as internal, intermediate, or interior nodes), and separator
1.1 Perspectives on B-trees

Fig. 1.1 A simple B-tree with root node and four leaf nodes.

keys in the branch nodes divide key ranges in the leaves. For very large data collections, B-trees with multiple layers of branch nodes are used. One or two branch levels are common in B-trees used as database indexes.

Complementing this “data structures perspective” on B-trees is the following “algorithms perspective.” Binary search in a sorted array permits efficient search with robust performance characteristics. For example, a search among $10^9$ or $2^{30}$ items can be accomplished with only 30 comparisons. If the array of data items is larger than memory, however, some form of paging is required, typically relying on virtual memory or on a buffer pool. It is fairly inefficient with respect to I/O, however, because for all but the last few comparisons, entire pages containing tens or hundreds of keys are fetched but only a single key is inspected. Thus, a cache might be introduced that contains the keys most frequently used during binary searches in the large array. These are the median key in the sorted array, the median of each resulting half array, the median of each resulting quarter array, etc., until the cache reaches the size of a page. In effect, the root of a B-tree is this cache, with some flexibility added in order to enable array sizes that are not powers of two as well as efficient insertions and deletions. If the keys in the root page cannot divide the original large array into sub-arrays smaller than a single page, keys of each sub-array are cached, forming branch levels between the root page and page-sized sub-arrays.

B-tree indexes perform very well for a wide variety of operations that are required in information retrieval and database management, even if some other index structure is faster for some individual index operations. Perhaps the “B” in their name “B-trees” should stand for their balanced performance across queries, updates, and utilities. Queries include exact-match queries (“=” and “in” predicates), range queries (“<” and “between” predicates), and full scans, with sorted output if
required. Updates include insertion, deletion, modifications of existing data associated with a specific key value, and “bulk” variants of those operations, for example bulk loading new information and purging out-of-date records. Utilities include creation and removal of entire indexes, defragmentation, and consistency checks. For all of those operations, including incremental and online variants of the utilities, B-trees also enable efficient concurrency control and recovery.

1.2 Purpose and Scope

Many students, researchers, and professionals know the basic facts about B-tree indexes. Basic knowledge includes their organization in nodes including one root and many leaves, the uniform distance between root and leaves, their logarithmic height and logarithmic search effort, and their efficiency during insertions and deletions. This survey briefly reviews the basics of B-tree indexes but assumes that the reader is interested in more detailed and more complete information about modern B-tree techniques.

Commonly held knowledge often falls short when it comes to deeper topics such as concurrency control and recovery or to practical topics such as incremental bulk loading and structural consistency checking. The same is true about the many ways in which B-trees assist in query processing, e.g., in relational databases. The goal here is to make such knowledge readily available as a survey and as a reference for the advanced student or professional.

The present survey goes beyond the “classic” B-tree references [7, 8, 27, 59] in multiple ways. First, more recent techniques are covered, both research ideas and proven implementation techniques. Whereas the first twenty years of B-tree improvements are covered in those references, the last twenty years are not. Second, in addition to core data structure and algorithms, the present survey also discusses their usage, for example in query processing and in efficient update plans. Finally, auxiliary algorithms are covered, for example defragmentation and consistency checks.

During the time since their invention, the basic design of B-trees has been improved upon in many ways. These improvements pertain
to additional levels in the memory hierarchy such as CPU caches, to multi-dimensional data and multi-dimensional queries, to concurrency control techniques such as multi-level locking and key range locking, to utilities such as online index creation, and to many more aspects of B-trees. Another goal here is to gather many of these improvements and techniques in a single document.

The focus and primary context of this survey are B-tree indexes in database management systems, primarily in relational databases. This is reflected in many specific explanations, examples, and arguments. Nonetheless, many of the techniques are readily applicable or at least transferable to other possible application domains of B-trees, in particular to information retrieval [83], file systems [71], and “No SQL” databases and key-value stores recently popularized for web services and cloud computing [21, 29].

A survey of techniques cannot provide a comprehensive performance evaluation or immediate implementation guidance. The reader still must choose what techniques are required or appropriate for specific environments and requirements. Issues to consider include the expected data size and workload, the anticipated hardware and its memory hierarchy, expected reliability requirements, degree of parallelism and needs for concurrency control, the supported data model and query patterns, etc.

### 1.3 New Hardware

Flash memory, flash devices, and other solid state storage technology are about to change the memory hierarchy in computer systems in general and in data management in particular. For example, most current software assumes two levels in the memory hierarchy, namely RAM and disk, whereas any further levels such as CPU caches and disk caches are hidden by hardware and its embedded control software. Flash memory might also remain hidden, perhaps as large and fast virtual memory or as fast disk storage. The more likely design for databases, however, seems to be explicit modeling of a memory hierarchy with three or even more levels. Not only algorithms such as external merge sort but
also storage structures such as B-tree indexes will need a re-design and perhaps a re-implementation.

Among other effects, flash devices with their very fast access latency are about to change database query processing. They likely will shift the break-even point toward query execution plans based on index-to-index navigation, away from large scans and large set operations such as sort and hash join. With more index-to-index navigation, tuning the set of indexes including automatic incremental index creation, growth, optimization, etc. will come more into focus in future database engines.

As much as solid state storage will change tradeoffs and optimizations for data structures and access algorithms, many-core processors will change tradeoffs and optimizations for concurrency control and recovery. High degrees of concurrency can be enabled only by appropriate definitions of consistent states and of transaction boundaries, and recovery techniques for individual transactions and for the system state must support them. These consistent intermediate states must be defined for each kind of index and data structure, and B-trees will likely be first index structure for which such techniques are implemented in production-ready database systems, file systems, and key-value stores.

In spite of future changes for databases and indexes on flash devices and other solid state storage technology, the present survey often mentions tradeoffs or design choices appropriate for traditional disk drives, because much of the presently known and implemented techniques have been invented and designed in this context. The goal is to provide comprehensive background knowledge about B-trees for those researching and implementing techniques appropriate for the new types of storage.

1.4 Overview

The next section (Section 2) sets out the basics as they may be found in a college level text book. The following sections cover implementation techniques for mature database management products. Their topics are implementation techniques for data structures and algorithms
(Section 3), transactional techniques (Section 4), query processing using B-trees (Section 5), utility operations specific to B-tree indexes (Section 6), and B-trees with advanced key structures (Section 7). These sections might be more suitable for an advanced course on data management implementation techniques and for a professional developer desiring in-depth knowledge about B-tree indexes.
B-trees enable efficient retrieval of records in the native sort order of the index because, in a certain sense, B-trees capture and preserve the result of a sort operation. Moreover, they preserve the sort effort in a representation that can accommodate insertions, deletions, and updates. The relationship between B-trees and sorting can be exploited in many ways; the most common ones are that a sort operation can be avoided if an appropriate B-tree exists and that the most efficient algorithm for B-tree creation eschews random “insert” operations and instead pays the cost of an initial sort for the benefit of efficient “append” operations.

Figure 2.1 illustrates how a B-tree index can preserve or cache the sort effort. With the output of a sort operation, the B-tree with root, leaf nodes, etc. can be created very efficiently. A subsequent scan can retrieve data sorted without additional sort effort. In addition
to preserving the sort effort over an arbitrary length of time, B-trees also permit efficient insertions and deletions, retaining their native sort order and enabling efficient scans in sorted order at any time.

Ordered retrieval aids many database operations, in particular subsequent join and grouping operations. This is true if the list of sort keys required in the subsequent operation is precisely equal to or a prefix of that in the B-tree. It turns out, however, that B-trees can save a lot of sort effort in many more cases. A later section will consider the relationship between B-tree indexes and database query operations in detail.

B-trees share many of their characteristics with binary trees, raising the question why binary trees are commonly used for in-memory data structures and B-trees for on-disk data. The reason is quite simple: disk drives have always been block-access devices, with a high overhead per access. B-trees exploit disk pages by matching the node size to the page size, e.g., 4 KB. In fact, B-trees on today’s high-bandwidth disks perform best with nodes of multiple pages, e.g., 64 KB or 256 KB. Inasmuch as main memory should be treated as a block-access device when accessed through CPU caches and their cache lines, B-trees in memory also make sense. Later sections will resume this discussion of memory hierarchies and their effect on optimal data structures and algorithm for indexes and specifically B-trees.

B-trees are more similar to 2-3-trees, in particular as both data structures have a variable number of keys and child pointers in a node. In fact, B-trees can be seen as a generalization of 2-3-trees. Some books treat them both as special cases of \((a, b)\)-trees with \(a \geq 2\) and \(b \geq 2a - 1\) [92]. The number of child pointers in a canonical B-tree node varies between \(N\) and \(2N - 1\). For a small page size and a particularly large key size, this might indeed be the range between 2 and 3. The representation of a single node in a 2-3-tree by linking two binary nodes also has a parallel in B-trees, discussed later as B\(^{\text{link}}\)-trees.

Figure 2.2 shows a ternary node in a 2-3-tree represented by two binary nodes, one pointing to the other half of the ternary node rather than a child. There is only one pointer to this ternary node from a parent node, and the node has three child pointers.

In a perfectly balanced tree such as a B-tree, it makes sense to count the levels of nodes not from the root but from the leaves. Thus, leaves
are sometimes called level-0 nodes, which are children of level-1 nodes, etc. In addition to the notion of child pointers, many family terms are used in connection with B-trees: parent, grandparent, ancestor, descendant, sibling, and cousin. Siblings are children of the same parent node. Cousins are nodes in the same B-tree level with different parent nodes but the same grandparent node. If the first common ancestor is a great-grandparent, the nodes are second cousins, etc. Family analogies are not used throughout, however. Two siblings or cousins with adjoining key ranges are called neighbors, because there is no commonly used term for such siblings in families. The two neighbor nodes are called left and right neighbors; their key ranges are called the adjacent lower and upper key ranges.

In most relational database management systems, the B-tree code is part of the access methods module within the storage layer, which also includes buffer pool management, lock manager, log manager, and more. The relational layer relies on the storage layer and implements query optimization, query execution, catalogs, and more. Sorting and index maintenance span those two layers. For example, large updates may use an update execution plan similar to a query execution plan to maintain each B-tree index as efficiently as possible, but individual B-tree modifications as well as read-ahead and write-behind may remain within the storage layer. Details of such advanced update and prefetch strategies will be discussed later.

In summary:

- B-trees are indexes optimized for paged environments, i.e., storage not supporting byte access. A B-tree node occupies a page or a set of contiguous pages. Access to individual records requires a buffer pool in byte-addressable storage such as RAM.
• B-trees are ordered; they effectively preserve the effort spent on sorting during index creation. Differently than sorted arrays, B-trees permit efficient insertions and deletions.
• Nodes are leaves or branch nodes. One node is distinguished as root node.
• Other terms to know: parent, grandparent, ancestor, child, descendent, sibling, cousin, neighbor.
• Most implementations maintain the sort order within each node, both leaf nodes and branch nodes, in order to enable efficient binary search.
• B-trees are balanced, with a uniform path length in root-to-leaf searches. This guarantees uniformly efficient search.

2.1 Data Structures

In general, a B-tree has three kinds of nodes: a single root, a lot of leaf nodes, and as many branch nodes as required to connect the root and the leaves. The root contains at least one key and at least two child pointers; all other nodes are at least half full at all times. Usually all nodes have the same size, but this is not truly required.

The original design for B-trees has user data in all nodes. The design used much more commonly today holds user data only in the leaf nodes. The root node and the branch nodes contain only separator keys that guide the search algorithm to the correct leaf node. These separator keys may be equal to keys of current or former data, but the only requirement is that they can guide the search algorithm.

This design has been called B+-tree but it is nowadays the default design when B-trees are discussed. The value of this design is that deletion can affect only leaf nodes, not branch nodes and that separator keys in branch nodes can be freely chosen within the appropriate key range. If variable-length records are supported as discussed later, the separator keys can often be very short. Short separator keys increase the node fan-out, i.e., the number of child pointers per node, and decrease the B-tree height, i.e., the number of nodes visited in a root-to-leaf search.

The records in leaf nodes contain a search key plus some associated information. This information can be all the columns associated with
a table in a database, it can be a pointer to a record with all those columns, or it can be anything else. In most parts of this survey, the nature, contents, and semantics of this information are not important and not discussed further.

In both branch nodes and leaves, the entries are kept in sorted order. The purpose is to enable fast search within each node, typically using binary search. A branch node with $N$ separator keys contains $N + 1$ child pointers, one for each key range between neighboring separator keys, one for the key, range below the smallest separator key, and one for the key range above the largest separator key.

Figure 2.3 illustrates a B-tree more complex than the one in Figure 1.1, including one level of branch nodes between the leaves and the root. In the diagram, the root and all branch nodes have fan-outs of 2 or 3. In a B-tree index stored on disk, the fan-out is determined by the sizes of disk pages, child pointers, and separator keys. Keys are omitted in Figure 2.3 and in many of the following figures unless they are required for the discussion at hand.

Among all possible node-to-node pointers, only the child pointers are truly required. Many implementations also maintain neighbor pointers, sometimes only between leaf nodes and sometimes only in one direction. Some rare implementations have used parent pointers, too, e.g., a Siemens product [80]. The problem with parent pointers is that they force updates in many child nodes when a parent node is moved or split. In a disk-based B-tree, all these pointers are represented as page identifiers.

B-tree nodes may include many additional fields, typically in a page header. For consistency checking, there are table or index identifier plus the B-tree level, starting with 0 for leaf pages; for space
management, there is a record count; for space management with variable-size records, there are slot count, byte count, and lowest record offset; for data compression, there may be a shared key prefix including its size plus information as required for other compression techniques; for write-ahead logging and recovery, there usually is a Page LSN (log sequence number) [95]; for concurrency control, in particular in shared-memory systems, there may be information about current locks; and for efficient key range locking, consistency checking, and page movement as in defragmentation, there may be fence keys, i.e., copies of separator keys in ancestor pages. Each field, its purpose, and its use are discussed in a subsequent section.

- Leaf nodes contain key values and some associated information. In most B-trees, branch nodes including the root node contain separator keys and child pointers but no associated information.
- Child pointers are essential. Sibling pointers are often implemented but not truly required. Parent pointers are hardly ever employed.
- B-tree nodes usually contain a fixed-format page header, a variable-size array of fixed-size slots, and a variable-size data area. The header contains a slot counter, information pertaining to compression and recovery, and more. The slots serve space management for variable-size records.

### 2.2 Sizes, Tree Height, etc.

In traditional database designs, the typical size of a B-tree node is 4–8 KB. Larger B-tree nodes might seem more efficient for today’s disk drives based on multiple analyses [57, 86] but nonetheless are rarely used in practice. The size of a separator key can be as large as a record but it can also be much smaller, as discussed later in the section on prefix B-trees. Thus, the typical fan-out, i.e., the number of children or of child pointers, is sometimes only in the tens, typically in the hundreds, and sometimes in the thousands.

If a B-tree contains $N$ records and $L$ records per leaf, the B-tree requires $N/L$ leaf nodes. If the average number of children per parent
is $F$, the number of branch levels is $\log_F(N/L)$. For example, the B-tree in Figure 2.3 has 9 leaf nodes, a fan-out $F = 3$, and thus $\log_3 9 = 2$ branch levels. Depending on the convention, the height of this B-tree is 2 (levels above the leaves) or 3 (levels including the leaves). In order to reflect the fact that the root node usually has a different fan-out, this expression is rounded up. In fact, after some random insertions and deletions, space utilization in the nodes will vary among nodes. The average space utilization in B-trees is usually given as about 70% [75], but various policies used in practice and discussed later may result in higher space utilization. Our goal here is not to be precise but to show crucial effects, basic calculations, and the orders of magnitude of various choices and parameters.

If a single branch node can point to hundreds of children, then the distance between root and leaves is usually very small and 99% or more of all B-tree nodes are leaves. In other words, great-grandparents and even more distant ancestors are rare in practice. Thus, for the performance of random searches based on root-to-leaf B-tree traversals, treatment of only 1% of a B-tree index and thus perhaps only 1% of a database determine much of the performance. For example, keeping the root of a frequently used B-tree index in memory benefits many searches with little cost in memory or cache space.

- The B-tree depth (nodes along a root-to-leaf path) is logarithmic in the number of records. It is usually small.
- Often more than 99% of all nodes in a B-tree are leaf nodes.
- B-tree pages are filled between 50% and 100%, permitting insertions and deletions as well as splitting and merging nodes. Average utilization after random updates is about 70%.

### 2.3 Algorithms

The most basic, and also the most crucial, algorithm for B-trees is search. Given a specific value for the search key of a B-tree or for a prefix thereof, the goal is to find, correctly and as efficiently as possible, all entries in the B-tree matching the search key. For range queries, the search finds the lowest key satisfying the predicate.
A search requires one root-to-leaf pass. In each branch node, the search finds the pair of neighboring separator keys smaller and larger than the search key, and then continues by following the child pointer between those two separator keys.

The number of comparisons during binary search among $L$ records in a leaf is $\log_2(L)$, ignoring rounding effects. Similarly, binary search among $F$ child pointers in a branch node requires $\log_2(F)$ comparisons. The number of leaf nodes in a B-tree with $N$ records and $L$ records per leaf is $N/L$. The depth of a B-tree is $\log_F(N/L)$, which is also the number of branch nodes visited in a root-to-leaf search. Together, the number of comparisons in a search inspecting both branch nodes and a leaf node is $\log_F(N/L) \times \log_2(F) + \log_2(L)$. By elementary rules for algebra with logarithms, the product term simplifies to $\log_2(N/L)$ and then the entire expression simplifies to $\log_2(N)$. In other words, node size and record size may produce secondary rounding effects in this calculation but the record count is the only primary influence on the number of comparisons in a root-to-leaf search in a B-tree.

Figure 2.4 shows parts of a B-tree including some key values. A search for the value 31 starts at the root. The pointer between the key values 7 and 89 is followed to the appropriate branch node. As the search key is larger than all keys in that node, the right-most pointer is followed to a leaf. A search within that node determines that the key value 31 does not exist in the B-tree. A search for key value 23 would lead to the center node in Figure 2.4, assuming a convention that a separator key serves as inclusive upper bound for a key range. The search cannot terminate when the value 23 is found at the branch

![Fig. 2.4 B-tree with root-to-leaf search.](image-url)
level. This is because the purpose of most B-tree searches is retrieving the information attached to each key and information contents exists only in leaves in most B-tree implementations. Moreover, as can be seen for key value 15 in Figure 2.4, a key that might have existed at some time in a valid leaf entry may continue to serve as separator key in a nonleaf node even after the leaf entry has been removed.

An exact-match query is complete after the search, but a range query must scan leaf nodes from the low end of the range to the high end. The scan can employ neighbor pointers if they exist in the B-tree. Otherwise, parent and grandparent nodes and their child pointers must be employed. In order to exploit multiple asynchronous requests, e.g., for a B-tree index stored in a disk array or in network-attached storage, parent and grandparent nodes are needed. Range scans relying on neighbor pointers are limited to one asynchronous prefetch at-a-time and therefore unsuitable for arrays of storage devices or for virtualized storage.

Insertions start with a search for the correct leaf to place the new record. If that leaf has the required free space, the insertion is complete. Otherwise, a case called “overflow,” the leaf needs to be split into two leaves and a new separator key must be inserted into the parent. If the parent is already full, the parent is split and a separator key is inserted into the appropriate grandparent node. If the root node needs to be split, the B-tree grows by one more level, i.e., a new root node with two children and only one separator key. In other words, whereas the depth of many tree data structures grows at the leaves, the depth of B-trees grows at the root. This is what guarantees perfect balance in a B-tree. At the leaf level, B-trees grow only in width, enabled by the variable number of child nodes in each parent node. In some implementations of B-trees, the old root page becomes the new root page and the old root contents are distributed into the two newly allocated nodes. This is a valuable technique if modifying the page identifier of the root node in the database catalogs is expensive or if the page identifier is cached in compiled query execution plans.

Figure 2.5 shows the B-tree of Figure 2.4 after insertion of the key 22 and a resulting leaf split. Note that the separator key propagated to the parent node can be chosen freely; any key value that separates the
two nodes resulting from the split is acceptable. This is particularly useful for variable-length keys: the shortest possible separator key can be employed in order to reduce space requirements in the parent node.

Some implementations of B-trees delay splits as much as possible, for example by load balancing among siblings such that a full node can make space for an insertion. This design raises the code complexity but also the space utilization. High space utilization enables high scan rates if data transfer from storage devices is the bottleneck. Moreover, splits and new page allocations may force additional seek operations during a scan that are expensive in disk-based B-trees.

Deletions also start with a search for the correct leaf that contains the appropriate record. If that leaf ends up less than half full, a case called “underflow,” either load balancing or merging with a sibling node can ensure the traditional B-tree invariant that all nodes other than the root be at least half full. Merging two sibling nodes may result in underflow in their parent node. If the only two children of the root node merge, the resulting node becomes the root and the old root is removed. In other words, the depth of B-trees both grows and shrinks at the root. If the page identifier of the root node is cached as mentioned earlier, it might be practical to move all contents to the root node and de-allocate the two children of the root node.

Figure 2.6 shows the B-tree from Figure 2.5 after deletion of key value 23. Due to underflow, two leaves were merged. Note that the separator key 23 was not removed because it still serves the required function.

Many implementations, however, avoid the complexities of load balancing and of merging and simply let underflows persist. A subsequent insertion or defragmentation will presumably resolve
it later. A recent study of worst case and average case behaviors of B-trees concludes that “adding periodic rebuilding of the tree, . . . the data structure . . . is theoretically superior to standard B+-trees in many ways [and] . . . rebalancing on deletion can be considered harmful” [116].

Updates of key fields in B-tree records often require deletion in one place and insertion in another place in the B-tree. Updates of nonkey fixed-length fields happen in place. If records contain variable-length fields, a change in the record size might force overflow or underflow similar to insertion or deletion.

The final basic B-tree algorithm is B-tree creation. Actually, there are two algorithms, characterized by random insertions and by prior sorting. Some database products used random insertions in their initial releases but their customers found creation of large indexes very slow. Sorting the future index entries prior to B-tree creation permits many efficiency gains, from massive I/O savings to various techniques saving CPU effort. As the future index grows larger than the available buffer pool, more and more insertions require reading, updating, and writing a page. A database system might also require logging each such change in the recovery log, whereas most systems nowadays employ non-logged index creation, which is discussed later. Finally, a stream of append operations also encourages a B-tree layout on disk that permits efficient scans with a minimal number of disk seeks. Efficient sort algorithms for database systems have been discussed elsewhere [46].

- If binary search is employed in each node, the number of comparisons in a search is independent of record and node sizes except for rounding effects.
• B-trees support both equality (exact-match) predicates and range predicate. Ordered scans can exploit neighbor pointers or ancestor nodes for deep (multi-page) read-ahead.
• Insertions use existing free space or split a full node into two half-full nodes. A split requires adding a separator key and a child pointer to the parent node. If the root node splits, a new root node is required and the B-tree grows by one level.
• Deletions may merge half-full nodes. Many implementations ignore this case and rely on subsequent insertions or defragmentation (reorganization) of the B-tree.
• Loading B-trees by repeated random insertion is very slow; sorting future B-tree entries permits efficient index creation.

2.4 B-trees in Databases

Having reviewed the basics of B-trees as a data structure, it is also required to review the basics of B-trees as indexes, for example in database systems, where B-tree indexes have been essential and ubiquitous for decades. Recent developments in database query processing have focused on improvements of large scans, e.g., by sharing scans among concurrent queries [33, 132], by a columnar data layout that reduces the scan volume in many queries [17, 121], or by predicate evaluation by special hardware, such as FPGAs. The advent of flash devices in database servers will likely result in more index usage in database query processing — their fast access times encourage small random accesses whereas traditional disk drives with high capacity and high bandwidth favor large sequential accesses. With the B-tree index the default choice in most systems, the various roles and usage patterns of B-tree indexes in databases deserve attention. We focus here on relational databases because their conceptual model is fairly close to the records and fields used in the storage layer of all database systems as well as other storage services.

In a relational database, all data is logically organized in tables with columns identified by name and rows identified by unique values in columns forming the table’s primary key. Relationships among tables are captured in foreign key constraints. Relationships among rows are
expressed in foreign key columns, which contain copies of primary key values elsewhere. Other forms of integrity constraints include uniqueness of one or more columns; uniqueness constraints are often enforced using a B-tree index.

The simplest representation for a database table is a heap, a collection of pages holding records in no particular order, although often in the order of insertion. Individual records are identified and located by means of page identifier and slot number (see below in Section 3.3), where the page identifier may include a device identifier. When a record grows due to an update, it might need to move to a new page. In that case, either the original location retains “forwarding” information or all references to the old location, e.g., in indexes, must be updated. In the former case, all future accesses incur additional overhead, possibly the cost of a disk read; in the latter case, a seemingly simple change may incur a substantial unforeseen and unpredictable cost.

Figure 2.7 shows records (solid lines) and pages (dashed lines) within a heap file. Records are of variable size. Modifications of two records have changed their sizes and forced moving the record contents to another page with forwarding pointers (dotted lines) left behind in the original locations. If an index points to records in this file, forwarding does not affect the index contents but does affect the access times in subsequent queries.

If a B-tree structure rather than a heap is employed to store all columns in a table, it is called a primary index here. Other commonly used names include clustered index or index-organized table. In a secondary index, also commonly called a non-clustered index, each entry must contain a reference to a row or a record in the primary index. This reference can be a search key in a primary index or it can be a record identifier including a page identifier. The term “reference” will often be used below to refer to either one. References to records in a primary index are also called bookmarks in some contexts.

Fig. 2.7 Heap file with variable-length records and forwarding pointers.
Both designs, reference by search key and reference by record identifier, have advantages and disadvantages [68]; there is no perfect design. The former design requires a root-to-leaf search in the primary index after each search in the secondary index. Figure 2.8 illustrates the double index search when the primary data structure for a table is a primary index and references in secondary indexes use search keys in the primary index. The search key extracted from the query requires an index search and root-to-leaf traversal in the secondary index. The information associated with a key in the secondary index is a search key for the primary index. Thus, after a successful search in the secondary index, another B-tree search is required in the primary index including a root-to-leaf traversal there.

The latter design permits faster access to records in the primary index after a search in the secondary index. When a leaf in the primary index splits, however, this design requires many updates in all relevant secondary indexes. These updates are expensive with many I/O operations and B-tree searches, they are infrequent enough to be always surprising, they are frequent enough to be disruptive, and they impose a substantial penalty due to concurrency control and logging for these updates.

A combination is also possible, with the page identifier as a hint and the search key as the fall-back. This design has some intrinsic difficulties, e.g., when a referenced page is de-allocated and later re-allocated for a different data structure. Finally, some systems employ clustering indexes over heap files; their goal is to keep the heap file sorted if possible but nonetheless enable fast record access via record identifiers.
Basic Techniques

In databases, all B-tree keys must be unique, even if the user-defined B-tree key columns are not. In a primary index, unique keys are required for correct retrieval. For example, each reference found in a secondary index must guide a query to exactly one record in the primary index — therefore, the search keys in the primary index must be unambiguous in the reference-by-search-key design. If the user-defined search key for a primary index is a person’s last name, values such as “Smith” are unlikely to safely identify individual records in the primary index.

In a secondary index, unique keys are required for correct deletion. Otherwise, deletion of a logical row and its record in a primary index might be followed by deletion of the wrong entry in a non-unique secondary index. For example, if the user-defined search in a secondary index is a person’s first name, deletion of a record in the primary index containing “Bob Smith” must delete only the correct matching record in the secondary index, not all records with search key “Bob” or a random such record.

If the search key specified during index creation is unique due to uniqueness constraints, the user-defined search key is sufficient. Of course, once a logical integrity constraint is relied upon in an index structure, dropping the integrity constraint must be prevented or followed by index reorganization. Otherwise, some artificial field must be added to the user-defined index key. For primary indexes, some systems employ a globally unique “database key” such as a microsecond timestamp, some use an integer value unique within the table, and some use an integer value unique among B-tree entries with the same value in the user-defined index key. For secondary indexes, most systems simply add the reference to the search key of the primary index.

B-tree entries are kept sorted on their entire unique key. In a primary index, this aids efficient retrieval; in a secondary index, it aids efficient deletion. Moreover, the sorted lists of references for each unique search key enable efficient list intersection and union. For example, for a query predicate “A = 5 and B = 15,” sorted lists of references can be obtained from indexes on columns A and B and their intersection computed by a simple merge algorithm.

The relationships between tables and indexes need not be as tight and simple as discussed so far. A table may have calculated columns
that are not stored at all, e.g., the difference (interval) between two date (timestamp) columns. On the other hand, a secondary index might be organized on such a column, and in this case necessarily store a copy of the column. An index might even include calculated columns that effectively copy values from another table, e.g., the table of order details might include a customer identifier (from the table of orders) or a customer name (from the table of customers), if the appropriate functional dependencies and foreign key constraints are in place.

Another relationship that is usually fixed, but need not be, is the relationship between uniqueness constraints and indexes. Many systems automatically create an index when a uniqueness constraint is defined and drop the index when the constraint is dropped. Older systems did not support uniqueness constraints at all but only unique indexes. The index is created even if an index on the same column set already exists, and the index is dropped even if it would be useful in future queries. An alternative design merely requires that some index with the appropriate column set exists while a uniqueness constraint is active. For instant definition of a uniqueness constraint with existing useful index, a possible design counts the number of unique keys during each insertion and deletion in any index. In a sorted index such as a B-tree, a count should be maintained for each key prefix, i.e., for the first key field only, the first and second key fields together, etc. The required comparisons are practically free as they are a necessary part of searching for the correct insertion or deletion point. A new uniqueness constraint is instantly verified if the count of unique key values is equal to the record count in the index.

Finally, tables and indexes might be partitioned horizontally (into sets of rows) or vertically (into sets of columns), as will be discussed later. Partitions usually are disjoint but this is not truly required. Horizontal partitioning can be applied to a table such that all indexes of that table follow the same partitioning rule, sometimes called “local indexes.” Alternatively, partitioning can be applied to each index individually, with secondary indexes partitioned with their own partitioning rule different from the primary index, which is sometimes called “global indexes.” In general, physical database design or the separation
Basic Techniques

of logical tables and physical indexes remains an area of opportunity and innovation.

- B-trees are ubiquitous in databases and information retrieval.
- If multiple B-trees are related, e.g., the primary index and the secondary index of a database table, pointers can be physical addresses (record identifiers) or logical references (search keys in the primary index). Neither choice is perfect, both choices have been used.
- B-tree entries must be unique in order to ensure correct updates and deletions. Various mechanisms exist to force uniqueness by adding an artificial key value.
- Traditional database design rigidly connects tables and B-trees, much more rigidly than truly required.

2.5 B-trees Versus Hash Indexes

It might seem surprising that B-tree indexes have become ubiquitous whereas hash indexes have not, at least not in database systems. Two arguments seem to strongly favor hash indexes. First, hash indexes should save I/O costs due to a single I/O per look-up, whereas B-trees require a complete root-to-leaf traversal for each search. Second, hash indexes and hash values should also save CPU effort due to efficient comparisons and address calculations. Both of these arguments have only very limited validity, however, as explained in the following paragraphs. Moreover, B-trees have substantial advantages over hash indexes with respect to index creation, range predicates, sorted retrieval, phantom protection in concurrency control, and more. These advantages, too, are discussed in the following paragraphs. All techniques mentioned here are explained in more depth in subsequent sections.

With respect to I/O savings, it turns out that fairly simple implementation techniques can render B-tree indexes competitive with hash indexes in this regard. Most B-trees have a fan-out of 100s or 1,000s. For example, for node of 8 KB and records of 20 bytes, 70% utilization means 140 child nodes per parent node. For larger node sizes
(say 64 KB), good defragmentation (enabling run-length encoding of child pointers, say 2 bytes on average), key compression using prefix and suffix truncation (say 4 bytes on average per entry), 70% utilization means 5,600 child nodes per parent node. Thus, root-to-leaf paths are short and more than 99% or even 99.9% of pages in a B-tree are leaf nodes. These considerations must be combined with the traditional rule that many database servers run with memory size equal to 1–3% of storage size. Today and in the future, the percentage might be higher, up to 100% for in-memory databases. In other words, for any B-tree index that is “warm” in the buffer pool, all branch nodes will be present in the buffer pool. Thus, each B-tree search only requires a single I/O, the leaf page. Moreover, the branch nodes could be fetched into the buffer pool in preparation of repeated look-up operations, perhaps even pinned in the buffer pool. If they are pinned, further optimizations could be applied, e.g., spreading separator keys into a separate array such that interpolation search is most effective, replacing or augmenting child pointers in form of page identifiers with child pointers in form of memory pointers, etc.

Figure 2.9 illustrates the argument. All B-tree levels but the leaf nodes easily fit into the buffer pool in RAM memory. For leaf pages, a buffer pool might employ the least-recently-used (LRU) replacement policy. Thus, for searches with random search keys, only a single I/O operation is required, similar to a hash index if one is available in a database system.

With respect to CPU savings, B-tree indexes can compete with hash indexes using a few simple implementation techniques. B-tree indexes support a wide variety of search keys, but they also support very simple
ones such as hash values. Where hash indexes can be used, a B-tree on hash values will also provide sufficient functionality. In other cases, a “poor man’s normalized key” can be employed and even be sufficient, rendering all additional comparison effort unnecessary. Later sections discuss normalized keys, poor man’s normalized keys, and caching poor man’s normalized keys in the “indirection vector” that is required for variable-size records. In sum, poor man’s normalized keys and the indirection vector can behave similarly to hash values and hash buckets.

B-trees also permit direct address calculation. Specifically, interpolation search may guide the search faster than binary search. A later section discusses interpolation search including avoiding worst-case behavior of pure interpolation by switching to binary search after two interpolation steps, and more.

While B-tree indexes can be competitive with hash indexes based on a few implementation techniques, B-trees also have distinct advantages over hash indexes. For example, space management in B-trees is very straightforward. In the simplest implementations, full nodes are split into two halves and empty nodes are removed. Multiple schemes have been invented for hash indexes to grow gracefully, but none seems quite as simple and robust. Algorithms for graceful shrinking of hash indexes are not widely known.

Probably the strongest arguments for B-trees over hash indexes pertain to multi-field indexes and to nonuniform distributions of key values. A hash index on multiple fields requires search keys for all those fields such that a hash value can be calculated. A B-tree index, on the other hand, can efficiently support exact-match queries for a prefix of the index key, i.e., any number of leading index fields. In this way, a B-tree with \( N \) search keys can be as useful as \( N \) hash indexes. In fact, B-tree indexes can support many other forms of queries; it is not even required that the restricted fields are leading fields in the B-tree’s sort order [82].

With respect to nonuniform (“skewed”) distributions of key values, imagine a table with \( 10^9 \) rows that needs a secondary index on a column with the same value in 10% of the rows. A hash index requires introduction of overflow pages, with additional code for index creation, insertion, search, concurrency control, recovery, consistency checks, etc.
For example, when a row in the table is deleted, an expensive search is required before the correct entry in the secondary index can be found and removed, whereupon overflow pages might need to be merged. In a B-tree, entries are always unique, if necessary by appending a field to the search key as discussed earlier. In hash indexes, the additional code requires additional execution time as well as additional effort for testing and maintenance. Due to the well-defined sort order in B-trees, neither special code nor extra time is required in any of the index functions.

Another strong argument in favor of B-trees is index creation. After extracting future index entries and sorting them, B-tree creation is simple and very efficient, even for the largest data collections. An efficient, general-purpose sorting algorithm is readily available in most systems managing large data. Equally efficient index creation for hash indexes would require a special-purpose algorithm, if it is possible at all. Index creation by repeated random insertions is extremely inefficient for both B-trees and hash indexes. Techniques for online index creation (with concurrent database updates) are well known and widely implemented for B-trees but not for hash indexes.

An obvious advantage of B-trees over hash indexes is the support for ordered scans and for range predicates. Ordered scans are important for key columns and set operations such as merge join and grouping; range predicates are usually more important for nonkey columns. In other words, B-trees are superior to hash indexes for both key columns and nonkey columns in relational databases, also known as dimensions and measures in online analytical processing. Ordering also has advantages for concurrency control, in particular phantom protection by means of key range locking (covered in detail later) rather than locking key values only.

Taken together, these arguments favor B-trees over hash indexes as a general indexing technique for databases and many other data collections. Where hash indexes seem to have an advantage, appropriate B-tree implementation techniques minimize it. Thus, very few database implementation teams find hash indexes among the opportunities or features with a high ratio of benefit and effort, in particular if B-tree indexes are required in any case in order to support range queries and ordered scans.
While nodes of 10 KB likely result in B-trees with multiple levels of branch nodes, nodes of 1 MB probably do not. In other words, the considerations above may apply to B-tree indexes on flash storage but probably not on disks. For disks, it is probably best to cache all branch nodes in memory and to employ fairly small leaf nodes such that neither transfer bandwidth nor buffer space is wasted on unwanted records.

- B-tree indexes are ubiquitous, whereas hash indexes are not, even though hash indexes promise exact-match look-up with direct address calculation in the hash directory and a single I/O.
- B-tree software can provide similar benefits if desired. In addition, B-trees support efficient index creation based on sorting, support for exact match predicates and for partial predicates, graceful degradation in case of duplicate or distribution skew among the key values, and ordered scans.

2.6 Summary

In summary of this section on the basic data structure, B-trees are ordered, balanced search trees optimized for block-access devices such as disks. They guarantee good performance for various types of searches well as for insertions, deletions, and updates. Thus, they are particularly suitable to databases and in fact have been ubiquitous in databases for decades.

Over time, many techniques have been invented and implemented beyond the basic algorithms and data structures. These practical improvements are covered in the next few sections.
The present section focuses on data structures and algorithms found in mature data management systems but usually not in college-level text books; the subsequent sections cover transactional techniques, B-trees and their usage in database query processing, and B-tree utilities.

While only a single sub-section below is named “data compression,” almost all sub-sections pertain to compression in some form: storing fewer bytes per record, describing multiple records together, comparing fewer bytes in each search, modifying fewer bytes in each update, and avoiding fragmentation and wasted space. Efficiency in space and time is the theme of this section.

The following sub-sections are organized such that the first group pertains to the size and internal structure of nodes, the next group to compression specific to B-trees, and the last group to management of free space. Most of the techniques in the individual sub-sections are independent of others, although certain combinations may ease their implementation. For example, prefix- and suffix-truncation require detailed and perhaps excessive record keeping unless key values are normalized into binary strings.
3.1 Node Size

Even the earliest papers on B-trees discussed the optimal node size for B-trees on disk [7]. It is governed primarily by access latency and transfer bandwidth as well as the record size. High latency and high bandwidth both increase the optimal node size; therefore, the optimal node size for modern disks approaches 1 MB and the optimal on flash devices is just a few KB [50]. A node size with equal access latency and transfer time is a promising heuristic — it guarantees a sustained transfer bandwidth at least half of the theoretical optimum as well as an I/O rate at least half of the theoretical optimum. It is calculated by multiplying access latency and transfer bandwidth. For example, for a disk with 5 ms access latency and 200 MB/s transfer bandwidth, this leads to 1 MB. An estimated access latency of 0.1 ms and a transfer bandwidth of 100 MB/s lead to 10 KB as a promising node size for B-trees on flash devices.

For a more precise optimization, the goal is maximize the number of comparisons per unit of I/O time. Examples for this calculation can already be found in the original B-tree papers [7]. This optimization assumes that the goal is to optimize root-to-leaf searches and not large range scans, I/O time and not CPU effort is the bottleneck, binary search is used within nodes, and a fixed total number of comparisons in a root-to-leaf B-tree search independent of the node size as discussed above.

Figure 3.1 shows a calculation similar to those in [57]. It assumes pages filled to 70% with records of 20 bytes, typical in secondary indexes. For example, in a page of 4 KB holding 143 records, binary

<table>
<thead>
<tr>
<th>Page size [KB]</th>
<th>Records / page</th>
<th>Node utility</th>
<th>I/O time [ms]</th>
<th>Utility / time</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>143</td>
<td>7.163</td>
<td>5.020</td>
<td>1.427</td>
</tr>
<tr>
<td>16</td>
<td>573</td>
<td>9.163</td>
<td>5.080</td>
<td>1.804</td>
</tr>
<tr>
<td>64</td>
<td>2,294</td>
<td>11.163</td>
<td>5.320</td>
<td>2.098</td>
</tr>
<tr>
<td>128</td>
<td>4,588</td>
<td>12.163</td>
<td>5.640</td>
<td>2.157</td>
</tr>
<tr>
<td>256</td>
<td>9,175</td>
<td>13.163</td>
<td>6.280</td>
<td>2.096</td>
</tr>
<tr>
<td>1,024</td>
<td>36,700</td>
<td>15.163</td>
<td>10.120</td>
<td>1.498</td>
</tr>
<tr>
<td>4,096</td>
<td>146,801</td>
<td>17.163</td>
<td>25.480</td>
<td>0.674</td>
</tr>
</tbody>
</table>

Fig. 3.1 Utility for pages sizes one a traditional disk.
search performs a little over 7 comparisons on average. The number of comparisons is termed the utility of the node with respect to searching the index. I/O times in Figure 3.1 are calculated assuming 5 ms access time and 200 MB/s (burst) transfer bandwidth. The heuristic above would suggest a page size of 5 ms × 200 MB/s = 1,000 KB. B-tree nodes of 128 KB enable the most comparisons (in binary search) relative to the disk device time. Historically common disk pages of 4 KB are far from optimal for B-tree indexes on traditional disk drives. Different record sizes and different devices will result in different optimal page sizes for B-tree indexes. Most importantly, devices based on flash devices may achieve 100 times faster access times without substantially different transfer bandwidth. Optimal B-tree node sizes will be much smaller, e.g., 2 KB [50].

- The node size should be optimized based on latency and bandwidth of the underlying storage. For example, the optimal page size differs for traditional disks and semiconductor storage.

### 3.2 Interpolation Search

Like binary search, interpolation search employs the concept of a remaining search interval, initially comprising the entire page. Instead of inspecting the key in the center of the remaining interval like binary search, interpolation search estimates the position of the sought key value, typically using a linear interpolation based on the lowest and highest key value in the remaining interval. For some keys, e.g., artificial identifier values generated by a sequential process such as invoice numbers in a business operation, interpolation search works extremely well.

In the best case, interpolation search is practically unbeatable. Consider an index on the column Order-Number in the table Orders given that order numbers and invoice numbers are assigned sequentially. Since each order number exists precisely once, interpolation among hundreds

\(^1\)Much of this section is derived from [45].
or even thousands of records within a B-tree node instantly guides the search to the correct record.

In the worst case, however, the performance of pure interpolation search equals that of linear search due to a nonuniform distribution of key values. The theoretical complexity is $O(\log \log N)$ for search among $N$ keys [36, 107], or 2 to 4 steps for practical page sizes. Thus, if the sought key has not yet been found after 3 or 4 steps, the actual key distribution is not uniform and it might be best to perform the remaining search using binary search.

Rather than switching from pure interpolation search to pure binary search, a gradual transition may pay off. If interpolation search has guided the search to one end of the remaining interval but not directly to the sought key value, the interval remaining for binary search may be very small or very large. Thus, it seems advisable to bias the last interpolation step in such a way to make it very likely that the sought key is in the smaller remaining interval.

The initial interpolation calculation might use the lowest and highest possible values in a page, the lowest and highest actual values, or a regression line based on all current values. The latter technique may be augmented with a correlation calculation that guides the initial search steps toward interpolation or binary search. Sums and counts required to quickly derive regression and correlation coefficients can easily be maintained incrementally during updates of individual records in a page.

Figure 3.2 shows two B-tree nodes and their key values. In the upper one, the correlation between slot numbers and key values is very high ($>0.998$). Slope and intercept are 3.1 and 5.9, respectively (slot numbers start with 0). An interpolation search for key value 12 immediately probes slot number $(12 - 5.9) \div 3.1 = 2$ (rounded), which is where key 5, 6, 15, 16, 43, 95, 96, 97, 128, 499

Fig. 3.2 Sample key values.
value 12 indeed can be found. In other words, if the correlation between position and key value is very strong, interpolation search is promising. In the lower B-tree node shown in Figure 3.2, slope and intercept are $-64$ and $31$, respectively. More importantly, the correlation coefficient is much lower ($<0.75$). Not surprisingly, interpolation search for key value 97 starts probing at slot $(97 - 64) \div 31 = 5$ whereas the correct slot number of key value 97 is 8. Thus, if the correlation between position and key value is weak, binary search is the more promising approach.

- If the key value distribution within a page is close to uniform, interpolation search requires fewer comparisons and incurs fewer cache faults than binary search. Artificial identifiers such as order numbers are ideal cases for interpolation search.
- For cases on non-uniform key value distributions, various techniques can prevent repeated erroneous interpolation.

3.3 Variable-length Records

While B-trees are usually explained for fixed-length records in the leaves and fixed-length separator keys in the branch nodes, B-trees in practically all database systems support variable-length records and variable-length separator keys. Thus, space management within B-tree nodes is not trivial.

The standard design for variable-length records in fixed-length pages, both in B-trees and in heap files, employs an indirection vector (also known as slot array) with entries of fixed size. Each entry represents one record. An entry must contain the byte offset of the record and may contain additional information, e.g., the size of the record.

Figure 3.3 shows the most important parts of a disk page in a database. The page header, shown far left within the page, contains index identifier, B-tree level (for consistency checks), record count, etc. This is followed in Figure 3.3 by the indirection vector. In heap files, slots remain unused after a record deletion in order to ensure that the remaining valid records retain their record identifier. In B-trees,
insertions or deletions require shifting some slot entries in order to ensure that binary search can work correctly. (Figure 4.7 in Section 4.2 shows an alternative to this traditional design with less shifting due to intentional gaps in the sequence of records.) Each used slot contains a pointer (in form of a byte offset within the page) to a record. In the diagram, the indirection vector grows from left to right and the set of records grows from right to left. The opposite design is also possible. Letting two data structures grow toward each other enables equally well many small records or fewer large records.

For efficient binary search, the entries in the indirection vector are sorted on their search key. It is not required that the entries be sorted on their offsets, i.e., the placement of records. For example, the sequence of slots in the left half of Figure 3.3 differs from the sequence of records in the right half. A sort order on offsets is needed only temporarily for consistency checks and for compaction or free space consolidation, which may be invoked by a record insertion, by a size-changing record update, or by a defragmentation utility.

Record insertion requires free space both for the record and for the entry in the indirection vector. In the standard design, the indirection vector grows from one end of the page and the data space occupied by records grows from the opposite end. Free space for the record is usually found very quickly by growing the data space into the free space in the middle. Free space for the entry requires finding the correct placement in the sorted indirection vector and then shifting entries as appropriate. On average, half of the indirection must shift by one position.

Record deletion is fast as it typically just leaves a gap in the data space. However, it must keep the indirection vector dense and sorted, and thus requires shifting just like insertion. Some recent designs require less shifting [12]. Some designs also separate separator keys and child pointers in branch nodes in order to achieve more effective
compression as well as more efficient search within each branch node. Those techniques are also discussed below.

- Variable-size records can be supported efficiently by a level of indirection within a page.
- Shift operations in the indirection vector can be minimized by gaps (invalid entries).

### 3.4 Normalized Keys

In order to reduce the cost of comparisons, many implementations of B-trees transform keys into a binary string such that simply binary comparisons suffice to sort the records during index creation and to guide a search in the B-tree to the correct record. The key sequence for the sort order of the original key and for the binary string are the same, and all comparisons equivalent. This binary string may encode multiple columns, their sort direction (e.g., descending) and collation including local characters (e.g., case-insensitive German), string length or string termination, etc.

Key normalization is a very old technique. It is already mentioned by Singleton [118] without citation, presumably because it seemed a well-known or trivial concept: “integer comparisons were used to order normalized floating-point numbers.”

Figure 3.4 illustrates the idea based on an integer column followed by two string columns. The initial single bit (shown underlined) indicates whether the leading key column contains a valid value. Using 0 for null values and 1 for other values ensures that a null value “sorts lower” than all other values. If the integer column value is not null, it is stored in the next 32 bits. Signed integers require reversing some bits to ensure the proper sort order, just like floating point values.

<table>
<thead>
<tr>
<th>Integer</th>
<th>First string</th>
<th>Second string</th>
<th>Normalized key</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>“flow”</td>
<td>“error”</td>
<td>1 0...0 0000 0000 0010 1_flow\0 1_error\0</td>
</tr>
<tr>
<td>3</td>
<td>“flower”</td>
<td>“rare”</td>
<td>1 0...0 0000 0000 0011 1_flower\0 1_rare\0</td>
</tr>
<tr>
<td>1024</td>
<td>Null</td>
<td>“brush”</td>
<td>1 0...0 0010 0000 0000 0 brush\0</td>
</tr>
<tr>
<td>Null</td>
<td>””</td>
<td>Null</td>
<td>0 1 0 0</td>
</tr>
</tbody>
</table>

Fig. 3.4 Normalized keys.
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require proper treatment of exponent, mantissa, and the two sign bits. Figure 3.4 assumes that the first column is unsigned. The following single bit (also shown underlined) indicates whether the first string column contains a valid value. This value is shown here as text but really ought to be stored in a binary format as appropriate for the desired international collation sequence. A string termination symbol (shown as \0) marks the end of the string. A termination symbol is required to ensure the proper sort order. A length indicator, for example, would destroy the main value of normalized keys, namely sorting with simple binary comparisons. If the string termination symbol can occur as a valid character in some strings, the binary representation must offer one more symbol than the alphabet contains. Notice the difference in representations between a missing value in a string column (in the third row) and an empty string (in the fourth row).

For some collation sequences, “normalized keys” lose information. A typical example is a language with lower and upper case letters sorted and indexed in a case-insensitive order. In that case, two different original strings might map to the same normalized key, and it is impossible from the normalized key to decide which original style was used. One solution for this problem is to store both the normalized key and the original string value. A second solution is to append to the normalized key the minimal information that enables a precise recovery of the original writing style. A third solution, specific to B-tree indexes, is to employ normalized keys only in branch nodes; recall that key values in branch nodes merely guide the search to the correct child but do not contain user data.

In many operating systems, appropriate functions are provided to compute a normalized key from a localized string value, date value, or time value. This functionality is used, for example, to list files in a directory as appropriate for the local language. Adding normalization for numeric data types is relatively straightforward, as is concatenation of multiple normalized values. Database code must not rely on such operating system code, however. The problem with relying on operating systems support for database indexes is the update frequency. An operating system might update its normalization code due to an error or extension in the code or in the definition of a local sort order; it is
unacceptable, however, if such an update silently renders existing large database indexes incorrect.

Another issue with normalized keys is that they tend to be longer than the original string values, in particular for some languages and their complex rules for order, sorting, and index look-up. Compression of normalized keys seems quite possible but a detailed description seems to be missing yet in the literature. Thus, normalized keys are currently used primarily in internal B-tree nodes, where they simplify the implementation of prefix and suffix truncation but never require recovery of original key values.

- Normalized keys enable comparisons by traditional hardware instructions, much faster than column-by-column interpolation of metadata about international sort order, ascending versus descending sort order, etc.
- Normalized keys can be longer than a traditional representation but are amenable to compression.
- Some systems employ normalized keys in branch nodes but not in leaf nodes.

3.5 Prefix B-trees

Once keys have been normalized into a simple binary string, another B-tree optimization becomes much easier to implement, namely prefix and suffix truncation or compression [10]. Without key normalization, these techniques would require a fair bit of bookkeeping, even if they were applied only to entire key fields rather than to individual bytes; with key normalization, their implementation is relatively straightforward.

Prefix truncation analyzes the keys in a B-tree node and stores the common prefix only once, truncating it from all keys stored in the node. Saving storage space permits increasing the number of records per leaf and increasing the fan-out of branch nodes. In addition, the truncated key bytes do not need to be considered in comparisons during a search.

Figure 3.5 shows the same records within a B-tree node represented without and with prefix truncation. It is immediately obvious that the latter representation is more efficient. It is possible to combine prefix
truncation with some additional compression technique, e.g., to eliminate symbols from the birthdates given. Of course, it is always required to weigh gains in run-time performance and storage efficiency against implementation complexity including testing effort.

Prefix truncation can be applied to entire nodes or to some subset of the keys within a node. Code simplicity argues for truncating the same prefix from all entries in a B-tree node. Moreover, one can apply prefix truncation based on the actual keys currently held in a node or based on the possible key range as defined by the separator keys in parent (and possibly other ancestor) pages. Code simplicity, in particular for insertions, argues for prefix truncation based on the maximal possible key range, even if prefix truncation based on actual keys might produce better compression [87]. If prefix truncation is based on actual keys, insertion of a new key might force reformatting all existing keys. In an extreme case, a new record might be much smaller than the free space in a B-tree page yet its insertion might force a page split.

The maximal possible key range for a B-tree page can be captured by retaining two fence keys in each node, i.e., copies of separator keys posted in parent nodes while splitting nodes. Figure 4.11 (in Section 4.4) illustrates fence keys in multiple nodes in a B-tree index. Fence keys have multiple benefits in B-tree implementations, e.g., for key range locking. With respect to prefix truncation, the leading bytes shared by the two fence keys of a page define the bytes by all current and future key values in the page. At the same time, prefix truncation reduces the overhead imposed by fence keys, and suffix truncation...
(applied when leaf nodes are split) ensures that separator keys and thus fence keys are always as short as possible.

Prefix truncation interacts with interpolation search. In particular, if the interpolation calculation uses fixed and limited precision, truncating common prefixes enables more accurate interpolation. Thus, normalized keys, prefix truncation, suffix truncation, and interpolation search are a likely combination in an implementation.

A very different approach to prefix truncation technique is offset-value coding [28]. It is used in high-performance implementations of sorting, in particular in sorted runs and in the merge logic [72]. In this representation, each string is compared to its immediate predecessor in the sorted sequence and the shared prefix is replaced by an indication of its length. The sign bit is reserved to make the indicator order-preserving, i.e., a short shared prefix sorts later than a long shared prefix. The result is combined with the data at this offset such that a single machine instruction can compare both offset and value. This representation saves more space than prefix truncation applied uniformly to an entire page. It is very suitable to sequential scans and merging but not to binary search or interpolation search. Instead, a trie representation could attempt to combine the advantages of prefix truncation and binary search, but it is used in very few database systems. The probable reasons are code complexity and update overhead.

Even if prefix truncation is not implemented in a B-tree and its page format, it can be exploited for faster comparisons and thus faster search. The following technique might be called dynamic prefix truncation. While searching for the correct child pointer in a parent node, the two keys flanking the child pointer will be inspected. If they agree on some leading bytes, all keys found by following the child pointer must agree on the same bytes, which can be skipped in all subsequent comparisons. It is not necessary to actually compare the two neighboring separator keys with each other, because the required information is readily available from the necessary comparisons of these separator keys with the search key. In other words, dynamic prefix truncation can be exploited without adding comparison steps to a root-to-leaf search in a B-tree.

For example, assume a binary search within the B-tree node shown on the left side of Figure 3.5, with the remaining search interval from
“Smith, Jack” to “Smith, Jason.” Thus, the search argument must be in that range and also start with “Smith, Ja.” For all remaining comparisons, this prefix may be assumed and thus skipped in all remaining comparisons within this search. Note that dynamic prefix truncation also applies to B-tree nodes stored with prefix truncation. In this example, the string “a” beyond the truncated prefix “Smith, J” may be skipped in all remaining comparisons.

While prefix truncation can be employed to all nodes in a B-tree, suffix truncation pertains specifically to separator keys in branch nodes [10]. Prefix truncation is most effective in leaf nodes whereas suffix truncation primarily affects branch nodes and the root node. When a leaf is split into two neighbor leaves, a new separator key is required. Rather than taking the highest key from the left neighbor or the lowest key from the right neighbor, the separator is chosen as the shortest string that separates those two keys in the leaves.

For example, assume the key values shown in Figure 3.6 are in the middle of a node that needs to be split. The precise center is near the long arrow. The minimal key splitting the node there requires at least 9 letters, including the first letter of the given name. If, on the other hand, a split point anywhere between the short arrows is acceptable, a single letter suffices. A single comparison of the two keys defining the range of acceptable split points can determine the shortest possible separator key. For example, in Figure 3.6, a comparison between “Johnson, Lucy” and “Smith, Eric” shows their first difference in the first letter, indicating that a separator key with a single letter suffices. Any letter

![Fig. 3.6 Finding a separator key during a leaf split.](image-url)
can be chosen that is larger than \( J \) and not larger than \( S \). It is not required that the letter actually occurs in the current key values.

It is tempting to apply suffix truncation not only when splitting leaf nodes but also when splitting branch nodes. The problem with this idea, however, is that a separator key in a grandparent node must guide the search not only to the correct parent but also to the correct leaf. In other words, applying suffix truncation again might guide a search to the highest node in the left sub-tree rather than to the lowest node in the right sub-tree, or vice versa. Fortunately, if 99\% of all B-tree nodes are leaves and 99\% of the remaining nodes are immediate parents of leaves, additional truncation could benefit at most 1\% of 1\% of all nodes. Thus, this problematic idea, even if it worked flawlessly, would probably never have a substantial effect on B-tree size or search performance.

Figure 3.7 illustrates the problem. The set of separator keys in the upper B-tree is split by the shortened key “g,” but the set of leaf entries is not. Thus, a root-to-leaf search for the key “gh” will be guided to the right sub-tree and thus fail, obviously incorrectly. The correct solution is to guide searches based on the original separator key “gp.” In other words, when the branch node is split, no further suffix truncation must be applied. The only choice when splitting a branch node is the split point and the key found there.

- A simple technique for compression, particularly effective in leaf pages, is to identify the prefix shared by all key values and to store the prefix only once.

![Diagram](image-url)
• Alternatively, or in addition, binary search and interpolation search can ignore key bytes shared by the lower and upper bounds of the remaining search interval. In a root-to-leaf search, such dynamic prefix truncation carries from parent to child.

• Key values in branch pages need not be actual key values. They merely need to guide root-to-leaf searching. When posting a separator key while splitting a leaf page, a good choice is the shortest value that splits near the middle.

• Offset-value coding compares each key value with its immediate neighbor and truncates the shared prefix. It achieves better compression than page-wide prefix truncation but disables efficient binary search and interpolation search.

• Normalized keys significantly reduce the implementation complexity of prefix and suffix truncation as well as of offset-value coding.

3.6 CPU Caches

Cache faults contribute a substantial fraction to the cost of searching within a B-tree page. If a B-tree needs to be searched with many keys and the sequence of search operations may be modified, temporal locality may be exploited [128]. Otherwise, optimization of data structures is required. Cache faults for instructions can be reduced by use of normalized keys — comparisons of individual fields with international sort order, collation sequence, etc., plus interpretation of schema information, can require a large amount of code whereas two normalized keys can be compared by a single hardware instruction. Moreover, normalized keys simplify the implementation not only of prefix and suffix truncation but also of optimizations targeted at reducing cache faults for data accesses. In fact, many optimizations seem practical only if normalized keys are used.

After prefix truncation has been applied, many comparisons in a binary search are decided by the first few bytes. Even where normalized keys are not used in the records, e.g., in B-tree leaves, storing a few bytes of the normalized key can speed up comparisons. If only those few bytes are
stored, not the entire normalized key, such that they can decide many but not all comparisons, they are called “poor man’s normalized keys” [41].

In order to enable key comparisons and search without cache faults for data records, poor man’s normalized keys can be an additional field in the elements of the indirection vector. This design has been employed successfully in the implementation of AlphaSort [101] and can be equally beneficial in B-tree pages [87].

On the other hand, it is desirable to keep each element in the indirection vector small. While traditional designs often include the record size in the elements of the indirection vector as was mentioned in the discussion of Figure 3.3, the record length is hardly ever accessed without access to the related record. Thus, the field indicating the record length might as well be placed with the main record rather than in the indirection vector.

Figure 3.8 illustrates such a B-tree page with keys indicating three European countries. On the left are page header and indirection vector, on the right are the variable-size records. The poor man’s normalized key, indicated here by a single letter, is kept in the indirection vector. The main record contains the total record size and the remaining bytes of the key. A search for “Denmark” can eliminate all records by the poor man’s normalized keys without incurring cache faults for the main records. A search for “Finland,” on the other hand, can rely on the poor man’s normalized key for the binary search but eventually must access the main record for “France.” While the poor man’s normalized key in Figure 3.8 comprises only a single letter, 2 or 4 bytes seem more appropriate, depending on the page size. For example, in a small database page optimized for flash storage and its fast access latency, 2 bytes might be optimal; whereas in large database pages optimized for traditional disks and their fast transfer bandwidth, 4 bytes might be optimal.

An alternative design organizes the indirection vector not as a linear array but as a B-tree of cache lines. The size of each node in this

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Fig. 3.8 Poor man’s normalized keys in the indirection vector.
B-tree is equal to a single cache line or a small number of them [64]. Root-to-leaf navigation in this B-tree might employ pointers or address calculations [110, 87]. Search time and cache faults within a B-tree page may be cut in half compared to node formats not optimized for CPU caches [24]. A complementary, more theoretical design of cache-efficient B-tree formats is even more complex but achieves optimal asymptotic performance independently of the sizes of disk page and cache line [11]. Both organizations of B-tree nodes, i.e., linear arrays as shown in Figure 3.8 and B-trees within B-tree nodes, can benefit from ghost slots, i.e., entries with valid key values but marked invalid, which will be discussed shortly.

- A cache fault may waste 100s of CPU cycles. B-tree pages can be optimized to reduce cache faults just like B-trees are optimized (compared to binary trees) to reduce page faults.

### 3.7 Duplicate Key Values

Duplicate values in search keys are fairly common. Duplicate records are less common but do occur in some databases, namely if there is confusion between relation and table and if a primary key has not been defined. For duplicate records, the standard representations are either multiple copies or a single copy with a counter. The former method might seem simpler to implement as the latter method requires maintenance of counters during query operations, e.g., a multiplication in the calculation of sums and averages, setting the counter to one during duplicate elimination, and a multiplication of two counters in joins.

Duplicate key values in B-tree indexes are not desirable because they may lead to ambiguities, for example during navigation from a secondary index to a primary index or during deletion of B-tree entries pertaining to a specific row in a table. Therefore, all B-tree entries must be made unique as discussed earlier. Nonetheless, duplicate values in the leading fields of a search key can be exploited to reduce storage space as well as search effort.

The most obvious way to store non-unique keys and their associated information combines each key value with an array representing the information. In non-unique secondary indexes with record identifiers as
3.7 Duplicate Key Values

the information associated with a key, this is a traditional format. For efficient search, for example during deletion, the list of record identifiers is kept sorted. Some simple forms of compression might be employed. One such scheme stores differences between neighboring values using the minimal number of bytes instead of storing full record identifiers. A similar scheme has been discussed above as an alternative scheme for prefix B-trees. For efficient sequential search, offset-value coding [28] can be adapted.

A more sophisticated variant of this scheme permits explicit control over the key prefix stored only once and the record remainder stored in an array. If, for example, the leading key fields are large with few distinct values, and the final key field is small with very many distinct values, then storing values of those leading fields once can save storage space.

An alternative representation of non-unique secondary index employs bitmaps. There are various forms and variants. These will be discussed below.

The rows in Figure 3.9 show alternative representations of the same information: (a) shows individual records repeating the duplicate key value for each distinct record identifier associated with the key value, which is a simple scheme that requires the most space. Example (b) shows a list of record identifiers with each unique key value, and (c) shows a combination of these two techniques suitable for breaking

<table>
<thead>
<tr>
<th></th>
<th>Smith, 4711</th>
<th>Smith, 4712</th>
<th>Smith, 4723</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Smith, 4711, 4712, 4723</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Smith, 4711, 4712</td>
<td>Smith, 4723</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Smith, 4711</td>
<td>(9) 2</td>
<td>(8) 23</td>
</tr>
<tr>
<td>e</td>
<td>Smith, 4711(2), 4723</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Smith, 4708, 00011000'00000001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3.9 Alternative representations of duplicates.
up extremely long lists, e.g., those spanning multiple pages. Example (d) shows a simple compression based on truncation of shared prefixes. For example, “(9)2” indicates that this entry is equal to the preceding one in its first 9 letters or “Smith, 471,” followed by the string “2.” Note that this is different from prefix B-trees, which truncate the same prefix from all records in a page or B-tree nodes. Example (e) shows another simple compression schemes based on run-length encoding. The encoding “4711(2)” indicates a contiguous series with 2 entries starting with 4711. Example (f) shows a bitmap as might be used in a bitmap index. The leading value 4708 indicates the integer represented by the first bit in the bitmap; the “1” bits in the bitmap represent the values 4711, 4712, and 4723. Bitmaps themselves are often compressed using some variant of run-length encoding. Without doubt, many readers could design additional variations and combinations.

Each of these schemes has its own strengths and weaknesses. For example, (d) seems to combine the simplicity of (a) with space efficiency comparable to that of (b), but it might require special considerations for efficient search, whether binary or interpolation search is employed. In other words, there does not seem to be a perfect scheme. Perhaps the reason is that compression techniques focus on sequential access rather than random access within the compressed data structure.

These schemes can be extended for multi-column B-tree keys. For example, each distinct value in the first field may be paired with a list of values of the second field, and each of those has a list of detail information. In a relational database about students and courses, as a specific example, an index for a many-to-many relationship may have many distinct values for the first foreign key (e.g., student identifier), each with a list of values for the second foreign key (e.g., course number), and additional attributes about the relationship between pair of key values (e.g., the semester when the student took the course). For information retrieval, a full-text index might have many distinct keywords, each with a list of documents containing a given keyword, each document entry having a list of occurrences of keywords with documents. Ignoring compression, this is the basic format of many text indexes.

Duplicate key values pertain not only to representation choices but also to integrity constraints in relational databases. B-trees are
often used to prevent violations of unique constraints by insertion of a duplicate key value. Another technique, not commonly used, employs existing B-tree indexes for instant creation and verification of newly defined uniqueness constraints. During insertion of new key values, the search for the appropriate insertion location could indicate the longest shared prefix with either of the future neighbor keys. The required logic is similar to the logic in dynamic prefix truncation. Based on the lengths of such shared prefixes, the metadata of a B-tree index may include a counter of distinct values. In multi-column B-trees, multiple counters can be maintained. When a uniqueness constraint is declared, these counters immediately indicate whether the candidate constraint is already violated.

- Even if each B-tree entry is unique, keys might be divided into prefix and suffix such that there are many suffix values for each prefix value. This enables many compression techniques.
- Long lists may need to be broken up into segments, with each segment smaller than a page.

### 3.8 Bitmap Indexes

The term bitmap index is commonly used, but it is quite ambiguous without explanation of the index structure. Bitmaps can be used in B-trees just as well as in hash indexes and other forms of indexes. As seen in Figure 3.9, bitmaps are one or many representation techniques for a set of integers. Wherever a set of integers is associated with each index key, the index can be a bitmap index. In the following, however, a non-unique secondary B-tree index is assumed.

Bitmaps in database indexes are a fairly old idea [65, 103] that gained importance with the rise of relational data warehousing. The only requirement is a one-to-one mapping between information associated with index keys and integers, i.e., the positions of bits in a bitmap. For example, record identifiers consisting of device number, page number, and slot number can be interpreted as a single large integer and thus can be encoded in bitmaps and bitmap indexes.
In addition, bitmaps can be segmented and compressed. For segmentation, the domain of possible bit positions is divided into ranges. These ranges are numbered and a separate bitmap is created for each non-empty range. The search key is repeated for each segment and extended by the range number. An example for breaking lists into segments is shown in Figure 3.9, albeit with lists of references rather than with bitmaps.

A segment size with $2^{15}$ bit positions ensures that the bitmap for any segment easily fits into a database page; a segment size with $2^{30}$ bit positions ensures that standard integer values can be used in compression by run-length encoding. Dividing bitmaps into segments of $2^{15}$ or $2^{30}$ bit positions also enables reasonably efficient updates. For example, insertion of a single record require decompression and re-compression of only a single bitmap segment, and space management very similar to changing the length of a traditional B-tree record.

For bitmap compression, most schemes rely primarily on run-length encoding. For example, WHA [126] divides a bitmap into sections of 31 bits and replaces multiple neighboring sections with a count. In the compressed image, a 32-bit word contains an indicator bit plus either a literal bitmap of 31 bits or a run of constant values. In each run, a 30-bit count leaves one bit to indicate whether the replaced sections contain “0” bits or “1” bits. Bitmap compression schemes based on bytes rather than words tend to achieve tighter compression but require more expensive operations [126]. This is true in particular if run lengths are encoded in variable-length integers.

Figure 3.10 illustrates this compression technique. Example (a) shows a bitmap similar to the ones in Figure 3.10 although with a different third value. Example (b) shows WAH compression. Commas indicate word boundaries in the compressed representation, underlined bit values indicate the word usage. The bitmap starts with 151 groups

\begin{verbatim}
a Smith, 4711 Smith, 4712 Smith, 4923
b Smith, 10...151, 00...01, 010...0, 10...5, 00...010...0
\end{verbatim}

Fig. 3.10 A WAH-compressed bitmap.
of 31 “0” bits. The following two words show literal bitmaps; two are required because bit positions 4711 and 4712 fall into different groups of 31 bit positions. Five more groups of 31 “0” bits then skip forward toward bit position 4923, which is shown as a single “1” bit in the final literal group of 31 bits.

Without compression, bitmap indexes are space-efficient only for very few distinct key values in the index. With effective compression, the size of bitmap indexes is about equal to that of traditional indexes with lists of references broken into segments, as shown in Figure 3.10. For example, with WAH compression, each reference requires at most one run of “0” sections plus a bitmap of 31 bits. A traditional representation with record identifiers might also require 64 bits per reference. Thus, bitmap indexes are useful for both sparse and dense bitmaps, i.e., for both low- and high-cardinality attributes [125, 126].

Bitmaps are used primarily for read-only or read-mostly data, not for update-intensive databases and indexes. This is due to the perceived difficulty of updating compressed bitmaps, e.g., insertion of a new value in run-length encoding schemes such as WAH. On the other hand, lists of record identifiers compressed using numeric differences are very similar to the counters in run-length encoding. Update costs should be very similar in these two compressed storage formats.

The primary operations on bitmaps are creation, intersection, union, difference, and scanning. Bitmap creation occurs during index creation, and, when bitmaps are used to represent intermediate query results, during query execution. Bitmap intersection aids in conjunctive (“and”) query predicates, union in disjunctive (“or”) predicates. Note that range queries on integer keys can often be translated into disjunctions, e.g., “... between 3 and 5” is equivalent to “... = 3 or ... = 4 or ... = 5.” Thus, even if most query predicates are written as conjunctions rather than disjunctions, union operations are important for bitmaps and lists of references.

Using a bitmap representation for an intermediate query result implicitly sorts the data. This is particularly useful when retrieving an unpredictable number of rows from a table using references obtained from a secondary index. Gathering references in a bitmap and then fetching the required database rows in sorted order is often more
efficient then fetching the rows without sorting. A traditional sort operation might require more memory and more effort than a bitmap.

In theory, bitmaps can be employed for any Boolean property. In other words, a bit in a bitmap indicates whether or not a certain record has the property of interest. The discussion above and the example in Figure 3.9 implicitly assume that this property is equality with a certain key value. Thus, there is a bitmap for each key value in an index indicating the records with those key values. Another scheme is based on modulo operations [112]. For example, if a column to be indexed is a 32-bit integer, there are 32 bitmaps. The bitmap for bit position $k$ indicates the records in which the key value modulo $2^k$ is nonzero. Queries need to perform intersection and union operations. Many other schemes, e.g., based on range predicates, could also be designed. O’Neil et al. [102] survey many of the design choices.

Usually, bitmap indexes represent one-to-many relationships, e.g., between key values and references. In these cases, a specific bit position is set to “1” in precisely one of the bitmaps in the index (assuming there is a row corresponding to the bit position). In some cases, however, a bitmap index may represent a many-to-many relationship. In those cases, the same bit position may be set to “1” in multiple bitmaps. For example, if a table contains two foreign keys to capture a many-to-many relationship, one of the foreign key columns might provide the key values in a secondary index and the other foreign key column is represented by bitmaps. As a more specific example, the many-to-many relationship enrollment between students and courses might be represented by a B-tree on student identifier. A student’s courses can be captured in a bitmap. The same bit position representing a specific course is set to “1” in many bitmaps, namely in the bitmaps of all students enrolled in that course.

- Bitmaps require a one-to-one relationship between values and bit positions.
- Bitmaps and compressed bitmaps are just another format to represent duplicate (prefix) values.
- Bitmaps can be useful to represent all suffix values associated with distinct prefix values.
• Run-length encoding as a compression technique for bitmaps is similar to compressing a list of integer values by sorting the list and storing the differences between neighbors. Based on this similarity, both techniques for representing duplicate values can be similarly space efficient.

3.9 Data Compression

Data compression reduces the expense of purchasing storage devices. It also reduces the cost to house, connect, power, and cool these devices. Moreover, it can improve the effective scan bandwidth as well as the bandwidths of utilities such as defragmentation, consistency checks, backup, and restore. Flash devices, due to their high cost per unit of storage space, are likely to increase the interest in data compression for file systems, databases, etc.

B-trees are the primary data structure in databases, justifying compression techniques tuned specifically for B-tree indexes. Compression in B-tree indexes can be divided into compression of key values, compression of node references (primarily child pointers), and representation of duplicates. Duplicates have been discussed above; the other two topics are surveyed here.

For key values, prefix and suffix truncation have already been mentioned, as has single storage of non-unique key values. Compression of normalized keys has also been mentioned, albeit as a problem without published techniques. Another desirable form of compression is truncation of zeroes and spaces, with careful attention to order-preserving truncation in keys [2].

Other order-preserving compression methods seem largely ignored in database systems, for example order-preserving Huffman coding or arithmetic coding. Order-preserving dictionary codes received initial attention [127]. Their potential usage in sorting, in particular sorting in database query processing, is surveyed elsewhere [46]; many of the considerations there also apply to B-tree indexes.

For both compression and de-compression, order-preserving Huffman codes rely on binary trees. For static codes, the tree is similar to the tree for nonorder-preserving techniques. Construction of a
Huffman code starts with each individual symbol forming a singleton set and then repeatedly merges two sets of symbols. For a standard Huffman code, the two sets with the lowest frequencies are merged. For an order-preserving Huffman code, the pair of immediate neighbors with the lowest combined frequency is chosen. Both techniques support static and adaptive codes. Adaptive methods start with a tree created as for a static method but modify it according to the actual, observed frequency of symbols in the uncompressed stream. Each such modification rotates nodes in the binary tree.

Figure 3.11, copied from [46], shows a rotation in the binary tree central to encoding and decoding in order-preserving Huffman compression. The leaf nodes represent symbols and the root-to-leaf paths represent the encodings. With a left branch encoded by a 0 and a right branch by a 1, the symbols “A,” “B,” and “C” have encodings “0,” “10,” and “11,” respectively. The branch nodes of the tree contain separator keys, very similar to separator keys in B-trees. The left tree in Figure 3.11 is designed for relatively frequent “A” symbols. If the symbol “C” is particularly frequent, the encoding tree can be rotated into the right tree, such that the symbols “A,” “B,” and “C” have encodings “00,” “01,” and “1,” respectively. The rotation from the left tree in Figure 3.11 to the right tree is worthwhile if the accumulated weight in leaf node C is higher than that in leaf node A, i.e., if effective compression is more important for leaf node C than for leaf node A. Note that the frequency of leaf node B is not relevant and the size of its encoding is not affected by the rotation, and that this tree transformation is not suitable to minimize the path to node B or the representation of B.

Compression of B-tree child pointers may exploit the fact that neighboring nodes are likely to have been allocated in neighboring locations while a B-tree is created from a sorted stream of future index
entries. In this case, child pointers in a parent page can be compressed by storing not the absolute values of pointers but their numeric differences, and by storing those in the fewest words possible [131]. In the extreme case, a form of run-length encoding can be employed that simply indicates a starting node location and the number of neighbor nodes allocated contiguously. Since careful layout of B-tree nodes can improve scan performance, such allocation of B-tree nodes is often created and maintained using appropriate space management techniques. Thus, this compression technique often applies and it is used in products. In addition to child pointers within B-tree indexes, a variant can also be applied to a list of references associated with a key value in a non-unique secondary index.

Compression using numeric differences is also a mainstay technique in document retrieval, where “an inverted index... records, for each distinct word or term, the list of documents that contain the term, and depending on the query modalities that are being supported, may also incorporate the frequencies and impacts of each term in each document, plus a list of the positions in each document at which that word appears. For effective compression, the lists of document and position numbers are usually sorted and transformed to the corresponding sequence of differences (or gaps) between adjacent values.” [1]. Research continues to optimize compression effectiveness, i.e., the bits required for values and length indicators for the values, and decompression bandwidth. For example, Anh and Moffat [1] evaluate schemes in which a single length indicator applies to all differences encoded in a single machine word. Many more ideas and techniques can be found in dedicated books and surveys, e.g., [124, 129].

- Various data compression schemes exist for separator keys and child pointers in branch nodes and for key values and their associated information in leaf nodes.
- Standard techniques are truncation of blank spaces and zeroes, representing values by their difference from a base value, and representing a sorted list of numbers by their differences. Offset-value coding is particularly effective for sorted runs in a merge sort but can also be used in B-trees.
• Order-preserving, dynamic variants exist for Huffman 
  compression, dictionary compression, and arithmetic com-

3.10 Space Management

It is sometimes said that, in contrast to heap files, B-trees have space 
management for records built-in. On the other hand, one could also say 
that record placement in B-trees offers no choice even if multiple pages 
have some free space; instead, a new record must be placed where its 
key belongs and cannot be placed anywhere else.

There are some opportunities for good space management, however. 
First, when an insertion fails due to insufficient space in the appropri-
ate node, a choice is required among compaction (reclamation of free 
space within the page), compression (re-coding keys and their associ-
ated information), load balancing (among sibling nodes), and splitting. 
As simple and local operations are preferable, the sequence given indi-
cates the best approach. Load balancing among two neighbors is rarely 
implemented; load balancing among more than two neighbors hardly 
ever. Some defragmentation utilities, however, might be invoked for 
specific key ranges only rather than for an entire B-tree.

Second, when splitting and thus page allocation are required, the 
location of the new page offers some opportunities for optimization. If 
large range scans and index-order scans are frequent, and if the B-tree 
is stored on disks with expensive seek operations, it is important to 
allocate the new page near the existing page.

Third, during deletion, similar choices exist. Load balancing among 
two neighbors can be required during deletion in order to avoid under-
flow, whereas it is an optional optimization for insertion. A commonly 
used alternative to the “text book” design for deletion in B-trees ignores 
underflows and, in the extreme cases, permits even empty pages in a 
B-tree. Space reclamation is left to future insertions or to a defragmen-
tation utility.

In order to avoid or at least delay node splits, many database 
systems permit leaving some free space in every page during index 
creation, bulk loading, and defragmentation. For example, leaving 10%
free space in all branch nodes hardly affects their fan-out or the height of the tree, but it reduces the overhead of node splits during transaction processing. In addition, some systems permit leaving free pages on disk. For example, if the unit of I/O in large scans contains multiple B-tree nodes, it can be advantageous to leave a few pages unallocated in each such unit. If a node splits, a nearby page is readily available for allocation. Until many nodes in the B-tree have been split due to many insertions, scan performance is not affected.

An interesting approach to free space management on disk relies on the core logic of B-trees. O’Neil’s SB-trees [104] allocate disk space in large contiguous extents of many pages, leaving some free pages in each extent during index creation and defragmentation. When a node splits, a new node is allocated within the same extent. If that is not possible because the entire extent is allocated, the extent is split into two extents, each half full. This split is quite similar to a node split in a B-tree. While simple and promising, this idea has not been widely adopted. This pattern of “self-similar” data structures and algorithms can be applied at multiple levels of the memory hierarchy.

Figure 3.12 shows the two kinds of nodes in an SB-tree. Both extents and pages are nodes in the sense that they may overflow and then are split in half. The child pointers in page 75.2 contain very similar values for page identifiers and thus are amenable to compression. When, for example, page 93.4 must be split in response to an insertion, the entire extent 93 is split and multiple pages, e.g., 93.3–93.5, moved to a new extent.

- B-trees rigidly place a new record according to its sort key but handle space management gracefully, e.g., by load balancing among neighbor nodes.
• B-tree concepts apply not only to placement of records in pages but also to placement of pages in contiguous clusters of pages on the storage media.

3.11 Splitting Nodes

After a leaf node is split into two, a new separator key must be posted in the parent node. This might cause an overflow in the parent, whereupon the parent node must be split into two and a separator key must be posted in the grandparent node. In the extreme case, nodes from a leaf to the root must be split and a new root must be added to the B-tree.

The original B-tree algorithms called for leaf-to-root splitting as just described. If, however, multiple threads or transactions share a B-tree, then the bottom-up (leaf-to-root) splits in one thread might conflict with a top-down (root-to-leaf) search of the other thread. The earliest design relied on the concept of a “safe” node, i.e., one with space for on more insertion, and retained locks from the last safe node during a root-to-leaf search [9]. A more drastic approach restricts each B-tree to only one structural change at a time [93]. Three other, less restrictive solutions have been used for this problem.

First, since only few insertions require split operations, one can force such an insertion to perform an additional root-to-leaf traversal. The first traversal determines the level at which a split is required. The second traversal performs a node split at the appropriate level. If it is unable to post the separator key as required, it stops and instead invokes another root-to-leaf pass that performs a split at the next higher level. This additional root-to-leaf traversal can be optimized. For example, if the upper B-tree nodes have not been changed in the meantime, there is no need to repeat binary search with known outcomes.

Second, the initial root-to-leaf search of an insertion operation may verify that all visited nodes have sufficient free space for one more separator key. A branch node without sufficient free space is split preventively [99]. Thus, a single root-to-leaf search promises to perform all insertions and node splits. If each node can hold hundreds of separator keys, splitting a little earlier than truly required does not materially affect B-tree space utilization, node fan-out, or tree height.
Unfortunately, variable-length separator keys present a problem; either the splitting decision must be extremely conservative or there may be rare cases in which a second root-to-leaf pass is required as in the first solution described in the preceding paragraph. In other words, an implementation of the first solution might be required in any case. If node splits are rare, adding a heuristic code path with its own test cases, regression tests, etc. might not provide a worthwhile or even measurable performance gain.

Third, splitting a B-tree node and posting a new separator key in the node’s parent is divided into two steps [81]. During the intermediate state, which may last a long time but ideally does not, the B-tree node looks similar to the ternary node in a 2-3-tree as shown in Figure 2.2. In other words, two separate steps split a full node in two and post a separator key in the parent node. For a short time, the new node is linked to the old neighbor, not its parent, giving rise to the name B<sub>link</sub>-trees. As soon as convenient, e.g., during the next root-to-leaf traversal, the separator key and the pointer are copied from the formerly overflowing sibling node to the parent node.

- Some variations of the original B-tree structure enable high concurrency and efficient concurrency control. B<sub>link</sub>-trees seem particularly promising although they seem to have been overlooked in products.

### 3.12 Summary

In summary, the basic B-tree design, both data structure and algorithms, have been refined in many ways in decades of research and implementation efforts. Many industrial implementations employ many of the techniques reviewed so far. Research that ignores or even contradicts these techniques may be perceived as irrelevant to commercial database management products.
The previous section surveys optimizations for B-tree data structures and algorithms; the current section focuses on concurrency control and recovery techniques. A large part of the development and testing effort for real systems is spent on concurrency control and recovery of on-disk data structures, meaning primarily B-trees. Transaction support, query processing, and a full suite of utilities, i.e., the topics of the present section and the following sections, differentiate traditional database management systems from key-value stores now employed in various web services and their implementations [21, 29].

Implicit in this section is that B-tree structures can support not only read-only searches but also — concurrently — updates including insertions, deletions, and modifications of existing records, both of key and nonkey fields. The focus here is on immediate updates rather than deferred updates using techniques such as differential files [117]. A later section covers update plans used in mature database management systems for maintenance of multiple related indexes, materialized views, integrity constraints, etc.

Since the complexity and expense of setting up a database can usually be justified only by sharing the data among many users,
many applications, etc., database access by concurrent transactions and execution threads has been at the forefront of database research and development right from the beginning, as have high availability and fast, reliable recovery from failures of software or hardware. More recently, many-core processors have increased the focus on high degrees of concurrency for in-memory data structures. Transactional memory may be part of the solution but requires understanding of appropriate transaction boundaries and thus requires choices of consistent intermediate states.

In addition to concurrent users, there is also a trend toward asynchronous, parallel, online, and incremental utilities. These perform optional or mandatory tasks on permanent storage. A typical mandatory asynchronous task is consolidation of free space, e.g., after a table or a B-tree index has been removed from the database by simply marking it obsolete without adding its pages to the free space. A typical optional asynchronous task is defragmentation, e.g., load balancing among B-tree leaves and optimization of the on-disk layout for efficient range queries and index-order scans. There are many other asynchronous tasks that do not pertain specifically to B-trees, e.g., gathering or updating statistics for use in compile-time query optimization.

Figure 4.1 lists the four “ACID” properties of transactions together with brief explanations. These properties are discussed in more detail in any database text book. The word “logical” in the explanation of atomicity deserves further clarification by means of a concrete example. Consider a user transaction that attempts to insert a new row into a database table, splits a B-tree node in order to create sufficient free space, but then fails. During transaction rollback, the row insertion must be undone, but rolling back the node split is not strictly required in order to ensure correct database contents. If the effects of the node split remain in the database, there is no logical database change after

<table>
<thead>
<tr>
<th>Atomicity</th>
<th>“All or nothing:” full success or no (logical) database change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consistency</td>
<td>A consistent database state is transformed into a new consistent database state</td>
</tr>
<tr>
<td>Isolation</td>
<td>Transaction output and database changes as if no other transaction were active</td>
</tr>
<tr>
<td>Durability</td>
<td>Once commited, database changes persist “through fire, flood, or insurrection”</td>
</tr>
</tbody>
</table>

Fig. 4.1 The ACID properties of transactions.
transaction rollback even if there is a physical change. “Logical” might be defined here by “query results” and “physical” by the database representation such as bits on a disk.

This distinction of logical and physical database, or database contents versus database representation, permeates the following discussion. One particularly useful implementation technique is the notion of “system transactions,” i.e., transactions that modify, log, and commit changes in the database representation but have no effect on database contents. System transactions are extremely useful for node splits in B-trees, space allocation and consolidation, etc. In the example above, the user transaction invokes a system transaction that performs, logs, and commits the node split; when the user transaction rolls back, the committed node split remains in place. System transactions are usually quite simple and run in a single thread, typically the thread of the user transaction such that the user transaction waits for the system transaction to complete. If the user transaction runs in multiple threads, each thread may invoke its own system transactions.

If large tables and indexes are partitioned and partitions assigned to nodes in a distributed system, the usual implementation permits each node to perform local concurrency control and recovery coordinated by two-phase commit when required. Similar techniques are required if a single site employs multiple recovery logs. Distributed transactions, two-phase commit, etc. are beyond the scope of this survey on B-tree indexes.

Locks are the usual mechanism for concurrency control. Figure 4.2 shows a basic lock compatibility matrix with no lock (N), shared (S), exclusive (X), and update (U) modes. The left column indicates the lock currently held and the top row indicates the lock requested. An empty space in the matrix indicates that the requested lock cannot be

<table>
<thead>
<tr>
<th>Lock held</th>
<th>S</th>
<th>U</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>S</td>
<td>U</td>
<td>X</td>
</tr>
<tr>
<td>S</td>
<td>S</td>
<td>U</td>
<td>?</td>
</tr>
<tr>
<td>U</td>
<td>U?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4.2 Basic lock compatibility matrix.
granted. If no lock is currently active, any lock can be granted. Two shared locks are compatible, which of course is the essence of sharing, whereas exclusive locks are not compatible with any other locks. Shared locks are also known as read locks, exclusive locks as write locks.

For permissible lock requests, the matrix indicates the aggregate lock mode. Its purpose is to speed up processing of new lock requests. Even if many transactions hold a lock on a specific resource, the new lock request must be tested only against the aggregate lock mode. There is no need to verify compatibility of the new lock request with each lock already granted. In other words, the existing lock mode in the left column of Figure 4.2 is the existing aggregate lock mode. In Figure 4.2, aggregation of lock modes is trivial in most cases. Later examples with additional lock modes include cases in which the new aggregate lock mode differs from both the old aggregate lock mode and the requested lock mode. A special case not reflected in Figure 4.2 is downgrading a lock from update to shared modes. Since only one transaction can hold an update lock, the aggregate lock mode after the downgrade from update mode to shared mode is a shared lock.

Update locks are designed for applications that first test a predicate before updating a data record. Taking an exclusive lock from the start prevents other transactions from processing the same logic; taking merely a shared lock permits two locks to acquire a shared lock on the same data items but then enter a deadlock if both attempt to upgrade to an exclusive lock. An update lock permits only one transaction at a time in a state of indecision about its future actions. After predicate evaluation, either the update lock is indeed upgraded to an exclusive lock or downgraded to a shared lock. Note that the update lock bestows no right beyond those of a shared lock; their difference is in scheduling and deadlock prevention rather than in concurrency control or permitted data accesses. Thus, downgrading to a shared lock is permissible.

Update locks are also known as upgrade locks. Given that an update lock gives priority to upgrade a lock rather than permission to update a data item, upgrade is a more accurate name. Update lock seems to have become more commonly used, however. Korth [79] explores in depth the relationships between derived locks such as the upgrade lock and basic locks such as shared and exclusive locks.
One field in Figure 4.2 shows a question mark. Some systems permit new shared locks while one transaction already holds an update lock, some do not. The former group stops additional shared locks only when a transaction requests an exclusive lock. Most likely, this is the transaction holding the update lock, but not necessarily. Thus, the latter design is more effective at preventing deadlocks [59] even if it introduces an asymmetry in the lock matrix. Subsequent examples of lock matrices assume the asymmetric design.

The primary means for providing failure atomicity and durability is write-ahead logging, which requires that the recovery log describes changes before any in-place updates of the database take place. Each type of update requires a “do” method invoked during initial processing, a “redo” method to ensure the database reflects an update even after a failure or crash, and an “undo” method to bring the database back to its prior state. The “do” method also creates log records with sufficient information for “redo” and “undo” invocations and instructs the buffer pool to retain dirty data pages until those log records have arrived safely on “stable storage.” Recovery is reliable inasmuch as the stable storage is. Mirroring the log device is a common technique. Log pages, once written, must never be modified or overwritten.

In early recovery techniques, “redo” and “undo” actions must be idempotent [56], i.e., repeated application of the same action results in the same state as a single application. An underlying assumption is that the recovery process keeps the recovery log in read-only mode, i.e., no logging during recovery from a failure. Later techniques, notably ARIES [95], reliably apply “redo” and “undo” actions exactly once by logging “undo” operations and by keeping a “Page LSN” (log sequence number) in each data page, which indicates the most recent log record already applied. Moreover, they “compensate” updates logically rather than physically. For example, a deletion compensates an insertion, yet after a leaf split in a B-tree index, the deletion may occur in a different leaf page than the insertion. Aborting a transaction applies compensating updates and then commits normally, except that there is no need to immediately force the commit record to stable storage.

- The ACID properties atomicity, consistency, isolation, and durability define transactions. Write-ahead logging and the
4.1 Latching and Locking

“do-redo-undo” triple are the cornerstones of recovery and reliability. Latching and locking are the cornerstones of concurrency control.

- Record-level locking in B-trees is key value locking and key range locking. A granularity of locking (e.g., record, key) smaller than the granularity of recovery (e.g., page) requires logging “undo” actions and logical compensation rather than strict physical recovery invoking unlogged, idempotent actions.

- Physical data independence separates logical database contents and their physical representation. In the relational layer of a database system, it creates freedom in physical database design and forces the need for automatic query optimization. In the storage layer of a database system, it enables many optimizations in the implementation of concurrency control and recovery.

- An important optimization is the separation of user transactions that query or modify logical database contents and system transactions that affect only the physical representation of contents. The prototypical example for the advantages of a system transaction is splitting a node in a B-tree.

4.1 Latching and Locking

B-tree locking, or locking in B-tree indexes, means two things. First, it means concurrency control among concurrent database transactions querying or modifying database contents. The primary concern in this context is the logical database contents, independent of its representation in data structures such as B-tree indexes. Second, it means concurrency control among concurrent threads modifying data structures in memory, including in particular images of disk-based B-tree nodes in the buffer pool.

These two aspects have not always been separated cleanly. Their difference becomes very apparent when a single database request is

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1 Most of this section is copied from [51].
processed by multiple parallel threads. Specifically, two threads within
the same transaction must “see” the same database contents, the same
count of rows in a table, etc. This includes one thread “seeing” updates
applied on behalf of the same transaction by another thread. However,
while one thread splits a B-tree node, i.e., modifies representation of
database contents in specific data structures, the other thread must not
observe intermediate and incomplete data structures. The difference
also becomes apparent in the opposite case when a single execution
thread serves multiple transactions.

These two purposes are usually accomplished by two different mech-
nisms, locks and latches. Unfortunately, the literature on operating
systems and programming environments usually uses the term locks
for the mechanisms that in database systems are called latches, which
can be confusing.

Figure 4.3 summarizes their differences. Locks separate transactions
using read and write locks on pages, on B-tree keys, or even on gaps
(open intervals) between keys. The latter two methods are called key
value locking and key range locking. Key range locking is a form of
predicate locking that uses actual key values in the B-tree and the
B-tree’s sort order to define predicates. By default, locks participate

<table>
<thead>
<tr>
<th></th>
<th>Locks</th>
<th>Latches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separate …</td>
<td>User transactions</td>
<td>Threads</td>
</tr>
<tr>
<td>Protect …</td>
<td>Database contents</td>
<td>In-memory data structures</td>
</tr>
<tr>
<td>During …</td>
<td>Entire transactions(^2)</td>
<td>Critical sections</td>
</tr>
<tr>
<td>Modes …</td>
<td>Shared, exclusive, update,</td>
<td>Read, writes,</td>
</tr>
<tr>
<td></td>
<td>intention, escrow, schema, etc.</td>
<td>(perhaps) update</td>
</tr>
<tr>
<td>Deadlock …</td>
<td>Analysis of the waits-for graph,</td>
<td>Coding discipline,</td>
</tr>
<tr>
<td>… by …</td>
<td>timeout, transaction abort,</td>
<td>instant-timeout requests,</td>
</tr>
<tr>
<td></td>
<td>partial rollback, lock de-escalation</td>
<td>Avoidance</td>
</tr>
<tr>
<td>Kept in …</td>
<td>Lock manager’s hash table</td>
<td>Protected data structure</td>
</tr>
</tbody>
</table>

\(^2\)Transactions must retain locks to transaction commit in order to equivalence to serial execution, also known as transaction isolation level “serializable.” Weaker transaction isolation permits shorter lock durations. In many database systems, weak transaction isolation is the default, thus achieving higher concurrency at the expense of correct and complete isolation of concurrent transactions.

\(^3\)In this technique, a level is assigned to any latch. A thread may request only latches with a higher level than the highest latch already held.
in deadlock detection and are held until end-of-transaction. Locks also support sophisticated scheduling, e.g., using queues for pending lock requests and delaying new lock acquisitions in favor of lock conversions, e.g., an existing shared lock to an exclusive lock. This level of sophistication makes lock acquisition and release fairly expensive, often hundreds of CPU instructions and thousands of CPU cycles, some of those due to cache faults in the lock manager’s hash table.

Latches separate threads accessing B-tree pages, the buffer pool’s management tables, and all other in-memory data structures shared among multiple threads. Since the lock manager’s hash table is one of the data structures shared by many threads, latches are required while inspecting or modifying a database system’s lock information. With respect to shared data structures, even threads of the same user transaction conflict if one thread requires a write latch. Latches are held only during a critical section, i.e., while a data structure is read or updated. Deadlocks are avoided by appropriate coding disciplines, e.g., requesting multiple latches in carefully designed sequences. Deadlock resolution requires a facility to roll back prior actions, whereas deadlock avoidance does not. Thus, deadlock avoidance is more appropriate for latches, which are designed for minimal overhead and maximal performance and scalability. Latch acquisition and release may require tens of instructions only, usually with no additional cache faults since a latch can be embedded in the data structure it protects. For images of disk pages in the buffer pool, the latch can be embedded in the descriptor structure that also contains the page identifier etc.

Since locks pertain to database contents but not their representation, a B-tree with all contents in the leaf nodes does not require locks for the nonleaf levels of the B-tree. Latches, on the other hand, are required for all pages, independent of their role in the database. The difference between locking and latching also becomes apparent in concurrency control for secondary indexes, i.e., redundant indexes that point into non-redundant storage structures. For example, in data-only locking of ARIES/IM [97], a single lock covers all records and B-tree entries pertaining to a logical row. The secondary indexes and their keys are not used to separate transactions at finer granularity and to permit more concurrency. Latches, on the other hand, are required for
any in-memory data structure touched by multiple concurrent threads, including of course the pages and nodes of secondary indexes.

- Latching coordinates threads to protect in-memory data structures including page images in the buffer pool. Locking coordinates transactions to protect database contents.
- Deadlock detection and resolution is usually provided for transactions and locks but not for threads and latches. Deadlock avoidance for latches requires coding discipline and latch acquisition requests that fail rather than wait.
- Latching is closely related to critical sections and could be supported by hardware, e.g., hardware transactional memory.

4.2 Ghost Records

If a transaction deletes a record in a B-tree, it must retain the ability to roll back until the transaction commits. Thus, the transaction must ensure that space allocation cannot fail during rollback and that another transaction cannot insert a new record with the same unique B-tree key. A simple technique to satisfy both these requirements is to retain the record and its key in the B-tree, merely marking it invalid. The record and its key remain locked until the deleting transaction commits. A side benefit is that the user transaction also delays or even avoids some effort for space management, e.g., shifting entries in the indirection array of the page. Moreover, the user transaction merely needs to lock the record being deleted, not the entire key range between the record’s two immediate neighbor keys.

The resulting record is called a pseudo-deleted record or a ghost record. A single bit in the record header suffices to indicate the ghost status of a record. Thus, a deletion turns into a modification of the ghost bit. If concurrency control relies on key range locking (discussed below), only the key itself needs to be locked and all gaps between keys may remain unlocked.

Figure 4.4 illustrates a B-tree page with a ghost record, i.e., the intermediate state immediately after deletion of the record with key 27. Obviously, this is prior to ghost removal and space reclamation within
the page. The valid records contain some information associated with their keys, indicated with ellipses, whereas the information fields in the ghost record are probably retained but not meaningful. A first step toward space reclamation could to shorten those fields as much as possible, although complete removal of the ghost record is probably the method of choice.

Queries must ignore (skip over) ghost records; thus, a scan in a system with ghost records always has a hidden predicate, although evaluation of this predicate is compiled into the B-tree code without any need for a predicate interpreter. Space reclamation is left to subsequent transactions, which might be an insertion that requires more free space than is readily available in the page, an explicitly invoked page compaction, or a B-tree defragmentation utility.

A ghost record cannot be removed while it is locked. In other words, the ghost record remains in place at least until the transaction commits that turned the valid record into a ghost record. Subsequently, another transaction might lock a ghost record, e.g., to ensure continued absence of key values. Locking absence is essential for serializability; without it, repeated “select count (*)” queries within the same transaction might return different results.

Multiple ghost records may exist at the same time within the same page and a single system transaction may remove all of them. Merging the log record for ghost removal with the log record for transaction commit eliminates the need to log the contents of the deleted record. Merging these log records renders it impossible that the transaction might fail between ghost removal and commit. Therefore, there never
can be a need to roll back the ghost removal and thus for the record contents in the recovery log. In other words, ghost records not only ensure successful transaction rollback if required but also often reduce the overall log volume associated with a deletion.

Figure 4.5 illustrates the log records for record removal without and with ghost records. On the left, the user transaction removes the record and logs its entire contents. If needed, the record can be re-inserted using information in the recovery log. On the right, the user transaction merely modifies the ghost bit. At some later time, a system transaction creates a single log record with transaction start, ghost removal, and transaction commit. There is no way to re-insert the removed ghost record from the recovery log, but there is no need to do so because the removal is committed as it is logged.

If a new row is inserted with the same key as a ghost record in a B-tree, the old record can be reused. Thus, an insertion may turn into a modification of the ghost bit and, in most cases, some other non-key fields in the record. As during deletion, key range locking needs to lock only the key value, not the key range into which a new key is inserted.

While ghost records are usually associated with record deletion in B-trees, they also can aid insertion of new keys. Splitting the insertion into two steps reduces the locks required for a transaction. First, a ghost record is created with the desired key under the protection of a latch. A lock is not required for this step. Second, the new record is locked and modified as appropriate by the user transaction. If the user transaction fails and rolls back, the ghost record remains. This second step requires a lock on the key value but not the key range into which the new key is inserted.
An additional refinement of this idea is creation of multiple ghost records with likely future keys. This is particularly useful if future key insertions are entirely predictable as in the case of order numbers and invoice numbers. Even if the precise values of future keys are not predictable, such ghost records may help separate future insertions and thus enable more concurrency among future insertion transactions. For example, a key may consist of multiple fields but values are predictable only for the leading field, for example, order numbers and line numbers within each order.

Figure 4.6 illustrates insertion of multiple ghost records. After valid records with key values 11, 12, 13, and 14 have been inserted, it is likely that the next operations will be insertions of records with key values 15, 16, 17, etc. Performance of these insertions can be improved by pre-allocation of appropriate space on the space with these keys already filled in. User transactions save the allocation effort and lock merely the key values, neither the gaps between keys nor the gap between the highest existing key value and infinity, which is often a bottleneck in transaction sequences with such insertions.

Finally, it can be beneficial to insert very short ghost records that merely contain the key without any of the remaining fields in valid B-tree records. Sprinkling such “ghost slots” into the ordered sequence of records (or slots in the indirection array) enables efficient insertions
into a page. In a page without such ghost slots, an insertion needs to shift half of all entries, for example slots in the indirection array. In a page with ghost slots, the complexity of insertions is not \( O(N) \) but \( O(\log N) \) [12]. For example, in a secondary index with thousands of small records per page, an insertion needs to shift perhaps ten rather than thousands of entries in the indirection vector, a removal shifts none at all and simply leaves behind a ghost slot, and a page reorganization leaves perhaps 10% or 20% of slots as ghost slots.

Figure 4.7 is a refinement of Figure 3.3, showing two differences. First, entries in the indirection vector contain keys or actually key prefixes. The diagram shows letters but an actual implementation would use poor man’s normalized keys and interpret them as integer values. Second, one of the slots is a ghost slot as it contains a key (“d”) but no reference to a record. This slot can participate in binary search and in key range locking. It might have been put there during page reorganization or, just as likely, it might be the result of a fast record deletion without shifting the two slots with keys “g” and “k.” Once it exists, it can speed up insertions. For example, insertion of a new record with key “e” can simply modify the slot currently containing “d.” Of course, this requires that the key “d” is currently not locked or that the lock manager permits appropriate adjustments.

- Ghost records (also known as pseudo-deleted records) are commonly used to reduce the locking requirements during deletion and to simplify “undo” of a deletion.
- Ghost records do not contribute to query results but participate in key range locking.
- A ghost record or its space may be reclaimed during an insertion or during asynchronous clean-up, but only if it is not locked.
- Ghost records could speed up and simplify insertions as well.
4.3 Key Range Locking

The terms key value locking and key range locking are often used interchangeably. The purpose of locking key ranges instead of key values only is to protect one transaction from insertions by another transaction. For example, if a transaction executes an SQL query of the type “select count (*) from... where... between... and...,” i.e., a query with a range predicate for an indexed column, and if that query runs in serializable transaction isolation, then a second execution of the same query ought to produce the same count. In other words, in addition to protecting the existing B-tree entries within the query range from deletion, locks obtained and held by that transaction must also protect the gaps between existing key values against insertion of new B-tree entries with new key values. In other words, key range locking ensures continued absence of key values by locking the gaps between existing key values. Transaction isolation levels weaker than serializability do not offer this guarantee but many application developers fail to grasp their precise semantics and their detrimental implications for application correctness.

Key range locking is a special form of predicate locking [31]. Neither general predicate locking nor the more pragmatic precision locking [76] has been adopted in major products. In key range locking, the predicates are defined by intervals in the sort order of the B-tree. Interval boundaries are the key values currently existing in the B-tree. The usual form are half-open intervals including the gap between two neighboring keys and one of the end points, with “next-key locking” perhaps more common than “prior-key locking.” Next-key locking requires the ability to lock an artificial key value “+∞.” Prior-key locking can get by with locking the NULL value, assuming this is the lowest possible value in the B-tree’s sort order.

In the simplest form of key range locking, a key and the gap to the neighbor are locked as a single unit. An exclusive lock is required for any form of update of the B-tree entry, its key, or the gap to its neighbor, including modifying non-key fields of the record, deletion of the key,

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4 Much of this section is copied from [51], which covers neither update locks nor the notion of an aggregate lock mode.
insertion of a new key into the gap, etc. Removal of a key requires a lock on both the old key and its neighbor; the latter is required to ensure the ability to re-insert the key in case of transaction rollback.

Figure 4.8 illustrates alternatives of what a key range lock on a single key value might protect, using as example a B-tree leaf containing three records with key values between 1170 to 1180. A lock on key value 1174 might have any of the ranges indicated by the arrows. The first arrow illustrates traditional next-key locking, i.e., the lock covers the gap between two key values and the following record and its key value. The second arrow indicates prior-key locking. The third arrow shows a lock limited to the key value only, without coverage of either one of the neighboring gaps. Thus, this lock cannot guarantee absence of a key for a transaction’s duration, e.g., key value 1176, and it therefore cannot guarantee serializability.

The fourth arrow shows a lock, to be discussed shortly, that complements the key value lock: it can guarantee absence of a key without locking an existing key. While one transaction holds a lock on key value 1174 as shown in the fourth arrow, a second transaction may update the record with key value 1174. More specifically, the second transaction can modify nonkey fields in the record but not the record’s key value. Thus, the second transaction cannot remove the record or the key until the first transaction releases its lock. On the other hand, the second transaction may update the record’s ghost bit. For example, if it finds that the record with key value 1174 is a valid record, it can turn it into a ghost record, thus excluding the key value from future query results. Inversely, it could turn a ghost record into a valid record and update all non-key fields in the record, thus applying a logical insertion into the B-tree. Figure 4.8 could also show a fifth lock scope that covers the gap preceding the locked key; this arrow is omitted because it might confuse the discussion below.
4.3 Key Range Locking

Key range locking is commonly used in commercial systems. Both ARIES/KVL (“key value locking”) [93] and ARIES/IM (“index management”) [97] are forms of key range locking. Neither technique locks individual entries in individual indexes. ARIES/KVL locks unique key values within individual indexes. In a non-unique secondary index, a single lock covers the entire list of records with the same key value, plus the open interval to the prior unique key value. Insertion into such an open interval in serializable transaction isolation requires such a lock, even if only with instant duration. If a concurrent transaction holds a conflicting lock, e.g., due to a read access to one of the records in the list, the insertion fails or is delayed. There is no actual conflict between reading a record with one key value and insertion of a record with a different key value; the choice of lock scopes in the design only make it appear as if there is. Perhaps it is due to such artificial conflicts that many database installations run with transaction isolation levels weaker than serializability and that the software of many vendors ships with such weaker isolation levels as default.

A lock in ARIES/IM covers a row in a table including all its index entries, plus the prior open interval in each index (“data-only locking,” later in DB2 “type-1 indexes”). In non-unique indexes, this open interval may be bounded by another entry with the same index key but a different record identifier. In ARIES/IM page locking, a lock covers all the rows in a data page, their index entries, and the appropriate open intervals between keys in the appropriate indexes. “Structure modification operations” acquire an X latch specific to the index tree, which read-only operations do not acquire, not even in S mode, unless such an operation encounters a page with its “structure modification bit” set. Since both ARIES methods are fairly complex due to a myriad of details, readers are encouraged to read the original papers instead of relying on a secondary source such as this survey. Hopefully, reading this survey first will enable reading the original ARIES papers with less effort.

Microsoft’s SQL Server product employs key range locking based on Lomet’s design [85], which builds on ARIES/IM and ARIES/KVL [93, 97]. Like ARIES, this design requires “instant locks,” i.e., locks with extremely short duration. In addition, it requires “insert locks,”
i.e., a new lock mode that applies only to the open intervals between two key values in a B-tree index. In the lock matrix published for SQL Server, however, insert locks are so similar to exclusive locks that it is not clear why this distinction is even required or useful. The most recent design requires neither instant locks nor insert locks yet it permits more concurrency than Lomet’s design [48].

In order to enable a variety of locking scopes, key range locking is based on hierarchical locking [58]. Before one or more items of a small granularity of locking are locked, an appropriate intention lock on the large granularity of locking is required first. A typical use case is searching a file locked in S mode and updating a few individual records in X mode, which requires the IX mode (intent to acquire exclusive locks) for the file in addition to locking individual records in X mode. Conflicts are detected on the file level, specifically by the conflict between S and IX locks.

Figure 4.9 shows the lock compatibility matrix for hierarchical locking. The combinations marked “a” indicate where locks are not compatible due to the asymmetry of the update lock mode. This matrix goes beyond a traditional lock compatibility matrix by the addition of aggregate lock modes. For example, if a resource is already locked in IS mode by multiple transactions (and no other modes), the aggregate lock mode is also IS. A request for an IX lock can be granted based on the aggregate lock mode without inspecting the individual locks held by the prior transactions, and the new aggregate lock mode becomes IX.

As is readily visible in Figure 4.9, two intention locks are always compatible with one another because any actual conflict will be detected at a smaller granularity of locking. Otherwise, intention locks and absolute locks are compatible precisely like traditional absolute

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Fig. 4.9 Lock compatibility in hierarchical locking.
locks. The combined mode $S + IX$ is compatible with those lock modes that are compatible with both the $S$ mode and the $IX$ mode.

Figure 4.9 shows update (U) locks but not intention-to-update (IU and SIU) locks, following Gray and Reuter [59]. Intention-to-write (IX and SIX) locks should be acquired instead. An IX lock at a large granularity of locking covers a U lock at a small granularity of locking.

In key range locking based on hierarchical locking, the large granularity of locking is the half-open interval; the small granularities of locking are either the key value or the open interval. This simple hierarchy permits very precise locks appropriate for each transaction’s needs. The disadvantage of this design is that locking a key (or an open interval) requires two invocations of the lock manager, one for the intention lock on the half-open interval and one for the absolute lock on the key value.

Given that all three locks (key value, open interval, and their combination in a half-open interval) are identified by the key value, a tradeoff is possible between the number of lock modes and the number of lock manager invocations. Additional, artificial lock modes can describe combinations of locks on the half-open interval, the key value, and the open interval. Thus, a system that employs hierarchical locking for half-open interval, key value, and open interval requires no more lock management effort than one that locks only half-open intervals. Without additional run-time effort, such a system permits additional concurrency between transactions that lock a key value and an open interval separately, e.g., to ensure absence of key values in the open interval and to update a record’s nonkey attributes. A record’s nonkey attributes include the property whether the record is a valid record or a ghost record; thus, even logical insertion and deletion are possible while another transaction locks a neighboring open interval.

Specifically, the half-open interval can be locked in $S$, $X$, $IS$, $IX$ modes. The SIX mode is not required because with precisely two resources, more exact lock modes are easily possible. The key value and the open interval each can be locked in $S$ or $X$ modes. The new lock modes must cover all possible combinations of $S$, $X$, or $N$ (no lock) modes of precisely two resources, the key value and the open interval. The intention locks $IS$ and $IX$ can remain implied. For example, if the
key value is locked in X mode, the half-open interval is implicitly locked in IX mode; if the key value is locked in S mode and the open interval in X mode, the implied lock on the half-open interval containing both is the IX mode. Locks can readily be identified using two lock modes, one for the key value and one for the open interval. Assuming previous-key locking, a SN lock protects a key value in S mode and leaves the following open interval unlocked. A NS lock leaves the key unlocked but locks the open interval. This lock mode can be used for phantom protection as required for true serializability.

Figure 4.10 shows the lock compatibility matrix. It can be derived simply by checking for compatibility of both the first and the second components. For example, XS is compatible with NS because X is compatible with N and S is compatible with S. Single-letter locks are equivalent to using the same letter twice, but there is no benefit in introducing more lock modes than absolutely necessary.

Figure 4.10 includes examples in which the new aggregate lock mode differs from both the prior aggregate lock mode and the requested lock mode. SN and NS combine to S, but more interestingly, SN and NX are not only compatible but also combine to a lock mode derived and explained entirely within the scheme, without need for a new lock mode specific to aggregate lock modes.

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Fig. 4.10 Lock table with combined lock modes.
4.3 Key Range Locking

If entries in a secondary index are not unique, multiple row identifiers may be associated with each value of the search key. Even thousands of record identifiers per key value are possible due to a single frequent key value or due to attributes with few distinct values. In non-unique indexes, key value locking may lock each value (and its entire cluster of row identifiers) or it may lock each unique pair of value and row identifier. The former saves lock requests in search queries, while the latter may permit higher concurrency during updates. For high concurrency in the former design, intention locks may be applied to values. Depending on the details of the design, it may not be required to lock individual row identifiers if those are already locked in the table to which the secondary index belongs.

In addition to the traditional read and write locks, or shared and exclusive locks, other lock modes have been investigated. Most notable is the “increment” lock. Increment locks enable concurrent transactions to increment and decrement sums and counts. This is rarely required in detail tables but can be a concurrency bottleneck in summary views. In B-tree indexes defined for such materialized views, the combination of ghost records, key range locking, and increment locks enables high concurrency even when insertion and deletions in the detail table affect entire groups and thus the existence of summary records and their index entries. Key range locking can readily be extended to include increment locks including increment locks on intervals between existing key values in a B-tree. More details can be found elsewhere [51, 55, 79] O’Neil 1987.

High rates of insertion can create a hotspot at the “right edge” of a B-tree index on an attribute correlated with time. With next-key locking, one solution verifies the ability to acquire a lock on +∞ (infinity) but does not actually retain it. Such “instant locks” violate two-phase locking but work correctly if a single acquisition of the page latch protects both verification of the lock and creation of the new key on the page. Another solution relies on system transactions to insert ghost records, letting user transactions turn them into valid records without interfering with each other. The system transaction does not require any locks as it does not modify the logical database contents and the subsequent user transactions require only key value locks for the affected B-tree entries. If the key values of future B-tree entries are
predictable, e.g., order numbers, a single system transaction can insert multiple ghost records and thus prepare for multiple user transactions.

- Key range locking locks key values and the gaps between key values. It is a special and practical form of predicate locking. Designs differ in their simplicity and in the concurrency they enable.
- Locking a gap between existing key values, i.e., locking the absence of new keys, is required for serializability, i.e., true isolation of concurrent transactions equivalent to their serial execution.

4.4 Key Range Locking at Leaf Boundaries

In traditional key range locking, another source of complexity and inefficiency are range locks that span boundaries between neighboring leaf nodes. For example, in order to insert a new B-tree entry with a key higher than all existing key values in the leaf, next-key locking needs to find the lowest key value in the next leaf. Prior-key locking has the same problem during insertion of a new low key value in a leaf. For efficient access of the next leaf, many systems include a next-neighbor pointer in each B-tree node, at least in the leaf nodes. An alternative solution avoids the neighbor pointers and instead employs two fence keys in each B-tree node. They define the range of keys that may be inserted in the future into that node. One of the fences is an inclusive bound, the other an exclusive bound, depending on the decision to be taken when a separator key in a parent node is precisely equal to a search key.

In the initial, empty B-tree with one node that is both root and leaf, negative and positive infinity are represented with special fence values. All other fence key values are exact copies of separator keys established while splitting leaf nodes. When a B-tree node (a leaf or a branch node) overflows and is split, the key that is installed in the parent node is also retained in the two pages resulting from the split as upper and lower fences.

A fence may be a valid B-tree record but it does not have to be. Specifically, the fence key that is an inclusive bound can be a valid
4.4 Key Range Locking at Leaf Boundaries

Figure 4.11 shows a B-tree with fence keys in both leaf nodes and nonleaf nodes (the root). As the fence keys define the possible key range within a page, there is never a need to lock a key value in a neighboring leaf. When a fence is turned from a ghost record into a valid record, i.e., during insertion of a new B-tree entry with a key value precisely equal to a fence key, there is no need to lock a key range. Only the key value must be locked because the insertion is performed by modifying an existing B-tree entry rather than by creating a new one.

- Traditional designs lock the gap between two key values stored on neighboring B-tree leaves by accessing a neighbor node, even if that neighbor node cannot otherwise contribute to the query or update.
- Fence keys are copies of the separator keys posted while splitting leaf nodes. In each leaf, one fence key (e.g., the upper fence) is always a ghost record and one fence key can be valid or a ghost. Fence keys participate in key range locking and thus avoid the need to access neighboring leaf nodes for concurrency control.
4.5 Key Range Locking of Separator Keys

In most commercial database systems, the granularities of locking are an entire index, an index leaf (page), or an individual key (with the sub-hierarchy of key value and open interval between keys, as discussed above). Locking both physical pages and logical key ranges can be confusing, in particular when page splits, defragmentation, etc. must be considered. An alternative model relies on key range locking for separator keys in the B-tree level immediately above the leaves [48]. This is different from locking fence keys at the level of B-tree leaves, even if the same key values are used. The scope of each such lock is similar to a page lock, but locks on separator keys are predicate locks in the same way as key range locks in B-tree leaves. Lock management during splits of leaf pages can rely on the intermediate states of B-link-trees or by copying the locks from one separator key to a newly posted separator.

Very large database tables and their indexes, however, may require millions of leaf pages, forcing many transactions to acquire many thousands of locks or lock much more data than they access. Lock hierarchies with intermediate levels between an index lock and a page lock have been proposed, although not yet used in commercial systems.

One such proposal [48] employs the B-tree structure, adding key range locking on separator keys in upper B-tree levels to key range locking on leaf keys. In this proposal, the lock identifier includes not only a key value but also the level in the B-tree index (e.g., level 0 are leaves). This technique promises to adapt naturally to skewed key distributions just like the set of separator keys also adapts to the actual key distribution.

Another proposal [48] focuses on the B-tree keys, deriving granularities of locking from compound (i.e., multi-column) keys such as “last name, first name.” The advantage of this method is that it promises to match predicates in queries and database applications, such that it may minimize the number of locks required. Tandem’s “generic locking” is a rigid form of this, using a fixed number of leading bytes in the key to define ranges for key range locking.\(^5\)

\(^5\) Saracco and Bontempo [113] describe Tandem’s generic locking as follows: “In addition to the ability to lock a row or a table’s partition, NonStop SQL/MP supports the notion of
Both proposals for locks on large key ranges need many details worked out, many of which will become apparent only during a first industrial-strength implementation. A variant of this method [4] has been employed in XML storage where node identifiers follow a hierarchical scheme such that an ancestor’s identifier is always a prefix of its descendents.

- Large indexes require an intermediate granularity of locking between locking a key value and locking an entire index.
- Traditional designs include locks on leaf pages in addition to (or instead of) locking key values. The number of pages in an index and thus the number of page locks in a query may far exceed the threshold at which the lock manager escalates to a larger granularity of locking, which is usually a few thousand locks.
- Alternatively, key range locking can be applied to separator keys in some or all branch nodes in a B-tree. This design adapts traditional hierarchical locking to B-trees and their organization in levels.

4.6 $B^{\text{link}}$-trees

In the original design for B-trees, splitting an overflowing node updates at least three nodes: the overflowing node, the newly allocated node, and their parent node. In the worst case, multiple ancestors must be split. Preventing other threads or transactions from reading or even updating a data structure with incomplete updates requires latches on all affected nodes. A single thread holding latches on many B-tree

generic locks for key-sequenced tables. Generic locks typically affect multiple rows within a certain key range. The number of affected rows might be less than, equal to, or more than a single page. When creating a table, a database designer can specify a “lock length” parameter to be applied to the primary key. This parameter determines the table’s finest level of lock granularity. Imagine an insurance policy table with a 10-character ID column as its primary key. If a value of “3” was set for the lock length parameter, the system would lock all rows whose first three bytes of the ID column matched the user-defined search argument in the query.” Note that Gray and Reuter [1993] explain key-range locking as locking a key prefix, not necessarily entire keys.

6 Most of this section is copied from [51].
nodes obviously restricts concurrency, scalability, and thus system performance. Rather than weakening the separation of threads and thus risking inconsistent B-trees, the definition of correct B-trees requires some relaxation. One such design divides a node split into two independent steps, i.e., splitting the nodes and posting a new separator key in the parent node. After the first step, the overflowing node requires a separator key and a pointer to its right neighbor, thus the name B\textsuperscript{link}-trees [81].

Until the second step, the right neighbor is not yet referenced in the node’s parent. In other words, a single key range in the parent node and its associated child pointer really refer to two child nodes. A root-to-leaf search, upon following this pointer, must first compare the sought key with the child node’s high fence and proceed to the right neighbor if the sought key is higher. In order to ensure efficient, logarithmic search behavior, this state is only transient and ends at the first opportunity.

The first step of splitting a node defines the separator key, creates a new right neighbor node, ensures correct fence keys in both nodes, and retains the high fence key of the new node also in the old node. The last action is not required for correct searching in the B-tree but it enables efficient consistency checks of a B-tree even with some nodes in this transient state. In this transient state, the old node could be called a “foster parent” of the new node.

The second, independent step posts the separator key in the parent. The second step can be made a side effect of any future root-to-leaf traversal, should happen as soon as possible, yet may be delayed beyond a system reboot or even a crash and its recovery without data loss or inconsistency of the on-disk data structures (see Figure 6.11 for more details on the permissible states and invariants).

The advantage of B\textsuperscript{link}-trees is that allocation of a new node and its initial introduction into the B-tree is a local step, affecting only one preexisting node and requiring a latch only on the overflowing node. The disadvantages are that search may be a bit less efficient during the transient state, a solution is needed to prevent long lists of neighbors nodes during periods of high insertion rates, and verification of a B-tree’s structural consistency is more complex and perhaps less efficient.
Figure 4.12 illustrates a state that is not possible in a standard B-tree but is a correct intermediate state in a B$^{\text{link}}$-tree. “Correct” here means that search and update algorithms must cope with this state and that a database utility that verifies correct on-disk data structures must not report an error. In the original state, the parent node has three children. Note that these children might be leaves or branch nodes, and the parent might be the B-tree root or a branch node. The first step is to split a child, resulting in the intermediate state shown in Figure 4.12. The second step later places a fourth child pointer into the parent and abandons the neighbor pointer, unless neighbor pointers are required in a specific implementation of B-trees. Note the similarity to a ternary node in a 2-3-tree as shown in Figure 2.2.

In most cases, posting the separator key in the parent node (the second step above) can be a very fast system transaction invoked by the next root-to-leaf traversal. It is not required that this thread be part of an update transaction, because any changes in the B-tree structure will be part of the system transaction, not the user transaction. When a thread holds latches on both parent and child node, it can check for the presence of a separator key not yet posted. If so, it upgrades its latches to exclusive latches, allocates a new entry in the parent node, and moves the separator key from the child to the parent. If another thread holds a shared latch, the operation is abandoned and left to a subsequent root-to-leaf search. If the parent node cannot accommodate another separator key, a new overflow node is allocated, populated, and linked into the parent. Splitting the parent node should be a separate system transaction. If a root-to-leaf search finds that the root node has a linked overflow node, the tree should grow by another level. If any of the required latches cannot be acquired instantaneously, the system transaction may abort and leave it to a later B-tree traversal to post the separator key in the parent node.
In the unlikely event that a node must be split again before a separator key is posted in the parent node, multiple overflow nodes can form a linked list. Long linked lists due to multiple splits can be prevented by restricting the split operation to nodes pointed to by the appropriate parent node. These and further details of B\textsuperscript{link}-trees have recently been described in a detailed paper [73].

The split process of B\textsuperscript{link}-trees can be reversed in order to enable removal of B-tree nodes [88]. The first step creates a neighbor pointer and removes the child pointer from the parent node, whereupon the second step merges the removal victim with its neighbor node. The transient state of B\textsuperscript{link}-trees might even be useful for load balancing among sibling nodes and for defragmentation of B-trees, although this idea has not been tried in research prototypes or industrial implementations.

- B\textsuperscript{link}-trees relax the strict B-tree structure in order to enable more concurrency. Splitting a node and posting a new separator key in the parent are two separate steps.
- Each step can be a system transaction that commits to make its changes visible to other threads and other transactions.
- In the transient state between these two steps, the old node is a “foster parent” to the new node. The transient state should be short-lived but may persist if the second step is delayed, e.g., due to concurrency conflicts.
- B\textsuperscript{link}-trees and their transient state may be useful for other structural changes in B-trees, e.g., removal of a node (merging the key ranges of two nodes) and load balancing among two nodes (replacing the separator key).

4.7 Latches During Lock Acquisition

If locks are defined by actual key values in a B-tree, latches must be managed carefully. Specifically, while a transaction attempts to acquire a key range lock, its thread must hold a latch on the data structure in the buffer pool such that the key value cannot be removed by another thread. On the other hand, if the lock cannot be granted immediately, the thread should not hold a latch while the transaction waits. In fact,
the statement could be more general: a thread must never wait while holding a latch. Otherwise, multiple threads may deadlock each other. Recall that deadlock detection and resolution is usually provided only for locks but not for latches.

There are several designs to address this potential problem. In one solution, the lock manager invocation, upon detecting a conflict, only queues a lock request and then returns. This lets the thread release appropriate latches prior to invoking the lock manager a second time and waiting for the lock to become available. It is not sufficient to merely fail the lock request in the first invocation. Until the lock request is inserted into the lock manager’s data structures, the latch on the data structure in the buffer pool is required to ensure the existence of the key value.

Another solution passes a function and appropriate function arguments to the lock manager to be called prior to waiting. This call-back function may release the latch on the data structure in the buffer pool, to be re-acquired after the wait and the lock is acquired. In either case, the sequence of actions during the lock request needs to indicate not only success versus failure but also instant versus delayed success.

While a transaction waits for a key value lock without holding the latch on the data structure in the buffer pool, other transactions might change the B-tree structure with splits, merges, load balancing, or page movements, e.g., during B-tree defragmentation or in a write-optimized B-tree on RAID or flash storage [44]. Thus, after waiting for a key value lock, a transaction must repeat its root-to-leaf search for the key. In order to minimize the cost for this repeated search, log sequence numbers of pages along the root-to-leaf path might be retained prior to the wait and verified after the wait. Alternatively, counters (of structure modifications) might be employed to decide quickly whether or not the B-tree structure might have changed [88]. These counters can be part of the system state, i.e., not part of the database state, and there is no need to recover their prior values after a system crash.

- A data page must remain latched while a key value lock is acquired in order to protect the key value from removal, but the latch on the data page must not be retained while waiting for a lock.
Transactional Techniques

• Solutions require a call-back or repeated lock manager calls, one to insert the lock into the wait queue and one to wait for lock acquisition.

4.8 Latch Coupling

When a root-to-leaf traversal advances from one B-tree node to one of its children, there is a brief window of vulnerability between reading a pointer value (the page identifier of the child node) and accessing the child node. In the worst case, another thread deletes the child page from the B-tree during that time and perhaps even starts using the page in another B-tree. The probability is low if the child page is present in the buffer pool, but it cannot be ignored. If ignored or not implemented correctly, identifying this vulnerability as the cause for a corrupted database is very difficult. Additional considerations apply if the child page is not present in the buffer pool and I/O is required, which is discussed in the next sub-section.

A technique called latch coupling avoids this problem. The root-to-leaf search retains the latch on the parent page, thus protecting the page from updates, until it has acquired a latch on the child page. Once the child page is located, pinned, and latched in the buffer pool, the latch on the parent page is released. If the child page is readily available in the buffer pool, latches on parent and child pages overlap only for a very short period of time.

Latch coupling was invented fairly early in the history of B-trees [9]. For read-only queries, at most two nodes need to be locked (latched) at a time, both in shared mode. In the original design for insertions, exclusive locks are retained on all nodes along a root-to-leaf path until a node is found with sufficient free space to permit splitting a child and posting a separator key. Unfortunately, variable-size records and keys may force very conservative decisions. Instead, newer designs rely on B_{link}-trees (temporary neighbor pointers until a separator key can be posted) or on repeated root-to-leaf passes. The initial root-to-leaf pass employs shared latches on root and branch nodes even if the intended operation will modify the leaf node with an insertion or deletion.
Systems that rely on neighbor pointers for efficient cursors, scans, and key range locking, i.e., implementations not exploiting fence keys, employ latch coupling also among neighbor nodes. In those systems, multiple threads may attempt to latch a leaf page from different directions, which could lead to deadlocks. Recall that latches usually do not support deadlock avoidance or detection. Therefore, latch acquisition must include a fail-fast no-wait mode and the B-tree code must cope with failed latch acquisitions.

Most root-to-leaf traversals hold latches on at most two B-tree nodes at a time, i.e., a parent and a child. A split operation needs to hold three B-tree latches, including one for the newly allocated node. In addition, it needs to latch the free space information. In $B^{\text{link}}$-trees, split operations require only two latches at a time. Even the final operation that moves separator key and pointer from the child to the parent requires only two latches; there is no need to latch the new node. On the other hand, a complete split sequence in a $B^{\text{link}}$-trees requires two periods with exclusive latches, even if the final operation can be delayed until the appropriate latches are readily available.

- During navigation from one B-tree node to another, the pointer must remain valid. The usual implementation keeps the source latched until the destination has been latched.
- If I/O is required, the latch ought to be released. The B-tree navigation might need to be repeated, possibly starting from the root node.
- $B^{\text{link}}$-trees latch at most two nodes at a time, even while splitting a node and while posting a separator key.

4.9 Physiological Logging

In addition to locking, the other fundamental technique required to support transactions is logging, i.e., writing sufficient redundant information about database changes to cope with transaction failure, media failure, and system failure [56, 59]. A log record describing an update action must be written to reliable storage before the modified data page may be written from the buffer pool to its place in the database,
motivating the name “write-ahead logging.” The principal optimization for logging is reduction of the log volume.

Each change in a database page must be recoverable, both in “undo” and “redo” modes in cases of transaction failure or media failure. In a traditional physical logging scheme, these operations must be logged in detail. If only a single record is affected by the change, it is sufficient to copy “before-image” and “after-image” of that record to the recovery log. In the earliest schemes, both images of the entire page were logged. In other words, changing 20 bytes in a page of 8 KB required writing 16 KB to the recovery log, plus appropriate record headers, which are fairly large for log records [59].

In a logical logging scheme, merely insertion and deletion are logged including the appropriate record contents, without reference to the specific physical location of the change. The problem with this scheme is that modifications of free space and of data structures are not logged. For example, splitting a B-tree node leaves no trace in the recovery log. Thus, some recovery cases become rather complex and slow. For example, if a single B-tree is stored on multiple disks and one of these disks fails, all of them must be recovered by restoring an earlier backup copy and “replaying” the logged history for the entire set of devices. Recovery after a system crash is even more expensive unless checkpoints force a quiescent system and force all dirty pages to permanent storage, which contradicts today’s high-performance checkpoint techniques such as second-chance checkpoints, fuzzy checkpoints, and checkpoint intervals.

A third alternative combines physical logging and logical logging by logging each contents change for a page yet referring to records within a page only by their slot number, not their byte position [59]. In this “physiological” logging, recovery of individual media and even of individual pages is possible and efficient, but logging copies of entire pages can often be avoided and the remaining log records can be simplified or shortened. In particular, changes in space management within a page need not be logged in detail.

\[7\] The name is a combination of “physical” and “logical;” it is not a reference to the medical term “physiology,” which might be confusing.
Figure 4.13 summarizes physical, logical, and physiological logging for two operations in B-tree nodes. Physical logging is simple but expensive with respect to log volume. Logical logging implies complex recovery. Physiological logging is designed to strike a good balance and is commonly used in modern databases.

Gray and Reuter [59] describe physiological logging as “physical to a page, logical within a page.” The logical aspect of physiological logging is sometimes confused with the difference between physical “undo” operations and logical compensation of an operation [95]. For example, insertion of a new record into a B-tree leaf might require logical compensation in a different page, specifically if the newly inserted B-tree entry has moved after the relevant leaf node was split due to another insertion by the same or another transaction. In other words, physiological logging tolerates representation changes within a page, e.g., due to compression or free space compaction, but it does not by itself enable logical “undo” by compensation as required by a fine granularity of locking and modern recovery schemes.

Insertions, deletions, and updates that increase record sizes can require that a B-tree node be reorganized. Such compaction operations include removal of ghost records, consolidation of free space, and perhaps improvements in compression. In a traditional physical logging scheme, these operations must be logged in detail, typically by copying both the before-image and the after-image of the entire page into the recovery log. In physiological logging, consolidation of free space and the required movement of records within a page are not logged in detail; in fact, it is not required that such movements be logged at all. Removal of ghost records, on the other hand, must be logged, because ghost records occupy slots in the indirection vector and their removal thus affects the slot numbers of other valid records. If ghost removal were not logged, a subsequent log record referring to specific page and
slot numbers might be applied to the wrong record during recovery. Recall, however, that removal of one or more ghost records can be logged with a short log record that merely mentions their slot numbers, omitting the record contents, if this log record also includes the commit of the removal transaction.

Figure 4.14 shows a possible sequence of log records that, eventually, put a new record into a B-tree node. A search in the B-tree determines the correct leaf, which is found to have sufficient unused space but it is not contiguous within the page. Thus, page compaction is invoked and generates very little log information, much less than two complete images (before and after) of the page. Even with compression, complete page images may be quite large. Run as a system transaction, page compaction requires no locks, only latches, and its effect remain valid even if the user transaction fails. Another system transaction creates a ghost record with the desired size and key value; this also determines the appropriate slot number within the page. This transaction also does not acquire locks but it needs to verify the absence of locks protecting serializability of another transaction. Finally, the user transaction turns the ghost record into a valid record and fills in the information associated with the key value. The two system transactions could be combined into one.

- Physiological logging has nothing to do with medicine or with physiology; it means logging “physical to the page, logical
within a page.” In other words, pages are referenced by their physical address (page identifier) and records within pages are referenced by their slot number or their key value but not their byte address.

- Space defragmentation within a page requires no log record. Ghost insertion or removal requires a log record if other log records refer to B-tree entries by slot number (rather than by key value).

### 4.10 Non-logged Page Operations

Another logging optimization pertains to structural B-tree operations, i.e., splitting a node, merging neighboring nodes, and balancing the load among neighboring nodes. As for in-page compaction, detailed logging can be avoided because those operations do not change the contents of the B-tree, only its representation. Differently from in-page compaction, however, there are multiple pages involved in these operations and the contents of individual pages indeed changes.

The operations considered are actually reflected in the recovery log; in that sense, the commonly used term “non-logged” is not literally accurate. A better descriptive name might be “allocation-only logging” instead. The savings are nonetheless substantial. For example, in strict physical logging, splitting a node of 8 KB might generate 24 KB of log volume plus log record headers, a few short log records for page allocation, and transaction commit, whereas an optimized implementation might required only these few short log records.

The key insight is that the old page contents, e.g., the full page prior to the split, can be employed to ensure recoverability of both pages after the split. Thus, the old contents must be protected against over-writing until the moved contents is safely written. For example, splitting a full page proceeds in multiple steps after the page is loaded into the buffer pool and found to require a split:

1. a new page is allocated on disk and this allocation is logged,
2. a new page frame for this new disk page is allocated in the buffer pool,
3. half the page contents is moved to the new page within the buffer pool; this movement is logged with a short log record that does not include the contents of the moved records but probably includes the record count,
4. the new page is written to the data store, and
5. the old page is written to the data store with only half its original content remaining, overwriting the old page contents and thus losing the half moved to the other page.

The careful write ordering in steps 4 and 5 is crucial. This list of actions does not include posting a new separator key in the parent node of the full node and its new sibling. Further optimizations are possible, in particular for B\textsuperscript{link}-trees [73]. The log records in the list above could be combined into a single log record in order to save space for record headers. The crucial aspect of the above list is that the last action must not be attempted until the prior one is complete. The delay between the first three actions and these last two actions can be arbitrarily long without putting reliability or recoverability in danger.

Variants of this technique also apply to other structural B-tree operations, in particular merging neighboring nodes, balancing the load among neighboring nodes, and moving entries in neighboring leaves or branch nodes in order to re-establish the desired fraction of free space for the optimal tradeoff between fast scans and fast future insertions. In all these cases, allocation-only logging as described above can save most of the log volume required in physical logging. More details on non-logged page operations are discussed in Section 6.6.

- “Non-logged” should be taken to mean “without logging page contents.” Another name is “allocation-only logging” or “minimal logging.”
- When moving records from one page to another (during split, load balancing, or defragmentation), the old page can serve as backup. It must be protected until the destination page is saved on storage.
4.11 Non-logged Index Creation

The term “non-logged index creation” seems to be commonly used although it is not entirely accurate. Changes in the database catalogs and in the free space management information are logged. The content of B-tree pages, however, is not logged. Thus, non-logged index creation saves perhaps 99% of the log volume compared to logged index creation.

All newly allocated B-tree pages, both leaves and branch nodes, are forced from the buffer pool to permanent storage in the database before committing the operation. Images of the B-tree nodes may, of course, remain in the buffer pool, depending on available space and the replacement policy in the buffer pool. Page allocation on disk is optimized to permit large sequential writes while writing the B-tree initially as well as large sequential reads during future index scans.

Figure 4.15 compares the log volume in logged and non-logged index creation. The voluminous operations, in particular individual record insertions or full B-tree pages, are not logged. For example, instead of millions of records, only thousands of page allocations are logged. Commit processing is slow if pages have been allowed to linger in the buffer pool during load processing. But just as table scans interact badly with LRU replacement in a buffer pool, pages filled during load processing should be ejected from the buffer pool as soon as possible.

Recovery of non-logged index creation requires precise repetition of the original index creation, in particular is space allocation operations, because subsequent user transactions and their log records may refer to specific keys in specific database pages, e.g., during row deletion. When those transactions are recovered, they must find those keys in those pages. Thus, node splits and allocation of database pages during recovery must precisely repeat the original execution.

<table>
<thead>
<tr>
<th>Action</th>
<th>Logged index creation</th>
<th>Non-logged index creation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Page allocation</td>
<td>Log change in allocation table</td>
<td>Same</td>
</tr>
<tr>
<td>Record insertion</td>
<td>1 log record per leaf page</td>
<td>Force leaf page at end</td>
</tr>
<tr>
<td>Leaf split</td>
<td>2-4 log records</td>
<td>Force branch nodes at end</td>
</tr>
<tr>
<td>Branch node split</td>
<td>2-4 log records</td>
<td>Force all nodes at end</td>
</tr>
</tbody>
</table>

Fig. 4.15 Logging details in logged and non-logged index creation.
• Since indexes can be very large, logging the entire contents of a new index can exceed the available log space. Most systems have facilities for non-logged creation of secondary indexes.
• Upon completion, the new index is forced to storage.
• A backup of the transaction log must include the new index; otherwise, subsequent updates to the new index cannot be guaranteed even if included in the transaction log and in a log backup.

4.12 Online Index Operations

The other important optimization for index creation is online index creation. Without online index creation, other transactions may be able to query the table based on pre-existing indexes; with online index creation, concurrent transactions may also update the table, including insertions and deletions, with the updates correctly applied to the index before index creation commits.

The traditional techniques described here are sufficient for small updates but it remains unadvisable to perform bulk insertions or deletion while concurrently modifying the physical database design with index creation and removal. There are two principal designs: either the concurrent updates are applied to the structure still being built or these updates are captured elsewhere and applied after the main index creation activity is complete. These designs have been called the “no side file” and “side file” [98]. The recovery log may serve as the “side file.”

Srinivasan and Carey [119] divide online algorithms for index creation further, specifically the “side file” approach. In their comparison study, all concurrent updates are captured in a list or in an index. They do not consider capturing updates in the recovery log or in the target index (the “no side file” approach). Their various algorithms permit concurrent updates throughout the index creation or only during its scan phase. Some of their algorithms sort the list of concurrent updates or even merge it with the candidate index entries scanned and sorted by the index builder. Their overall recommendation is to use a list of concurrent updates (a side file) and to merge it with the candidate index entries of the index builder.
4.12 Online Index Operations

Figure 4.16 illustrates the data flow for online index creation with and without “side file.” The upper operation starts with creating an empty side file (unless the recovery log serves as side file). Concurrent transactions buffer their updates there, and the entire contents of the side file is propagated after the scanning, sorting, and B-tree loading are complete. The lower operation starts with creating the new index, albeit entirely empty at this time. Concurrent transactions capture both insertions and deletions into the new index, even before and during B-tree loading.

The “side file” design lets the index creation proceed without regard for concurrent updates. This index creation process ought to build the initial index as fast as an offline index creation. The final “catch up” phase based on the “side file” requires either quiescent concurrent update activity or a race between capturing updates and applying them to the new index. Some systems perform a fixed number of catch-up phases, with the first catch-up phase applying updates captured during index creation, with the second catch-up phase applying updates captured during the first catch-up phase, and with the final catch-up phase applying the remaining updates and preventing new ones.

The “no side file” design requires that the future index be created empty at the start, concurrent updates modify the future index, and the index creation process work around records in the future index inserted by concurrent update transactions. One concern is that the index creation process may not achieve write bandwidth similar to an offline index creation. Another concern is that concurrent update
transactions may need to delete a key in a key range not yet inserted by the index creation process. For example, the index creation may still be sorting records to be inserted into the new index.

Such deletions can be represented by a negative or “anti-matter” record. When the index creation process encounters an anti-matter record, the corresponding record is suppressed and not inserted into the new index. At that time, the anti-matter record has served its function and is removed from the B-tree. When the index creation process has inserted all its records, all anti-matter records must have been removed from the B-tree index.

An anti-matter record is quite different from a ghost record. A ghost record represents a completed deletion, whereas an anti-matter record represents an incomplete deletion. Put in another way, an anti-matter record indicates that the index creation process must suppress a seemingly valid record. If one were to give weight to records, a valid record would have weight +1, a ghost record would have weight 0, and an anti-matter record would have weight −1.

It is possible that a second concurrent transaction inserts a key previously deleted by means of leaving an anti-matter record. In that case, a valid record with a suppression marker is required. The suppression marker indicates that the first transaction performed a deletion; the remainder of the valid record contains the information inserted by the second transaction into the database. A third concurrent transaction may delete this key again. Thus, the suppression marker and the ghost record are entirely orthogonal, except that a ghost record with a suppression marker must not be removed like other ghost records because that would lose the suppression information.

Figure 4.17 illustrates use of ghost bit and anti-matter bit during online index creation without side-file. Keys 47 and 11 are both updated in the index before the index creation process loads index entries with those key values. This bulk load is shown in the last two entries of Figure 4.17. The history of key value 47 starts with an insertion; thus, it never has the anti-matter bit set. The history of key value 11 starts with a deletion, which necessarily must refer to a future index entry to be loaded by the index creation process; thus, this key value retains its anti-matter bit until it is canceled against a record in the load stream.
4.12 Online Index Operations

<table>
<thead>
<tr>
<th>Action</th>
<th>Ghost</th>
<th>Anti-matter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insert key 47</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Delete key 11</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Delete key 47</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Insert key 11</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Delete key 11</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Load key 11</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Load key 47</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Fig. 4.17 Anti-matter during online index creation without side file.

The final result for key value 11 can be an invalid (ghost) record or no record at all.

In materialized summary (“group by”) views, however, ghost marker and suppression marker can be unified into a single counter that serves a role similar to a reference count [55]. In other words, if its reference count is zero, the summary record is a ghost record; if its reference count is negative, the summary record implies suppression semantics during an online index creation. In non-unique indexes with a list of references for each unique key value, an anti-matter bit is required for individual pairs of key value and reference. The count of references for a unique key value can be used similar to a ghost bit, i.e., a key value can be removed if and only if the count is zero.

Maintenance of indexes whose validity is in doubt applies not only to online index creation but also to index removal, i.e., dropping an index from a database, with two additional considerations. First, if index removal occurs within a larger transaction, concurrent transactions must continue standard index maintenance. This is required as long as the transaction including the index removal could still be aborted. Second, the actual de-allocation of database pages can be asynchronous. After the B-tee removal has been committed, updates by concurrent transaction must stop. At that time, an asynchronous utility may scan over the entire B-tree structure and inserts pages into the data structure with free space information. This process might be broken into multiple steps that may occur concurrently or with pauses in between steps.

Finally, online index creation and removal as described above can easily be perceived by database users as merely the first step. The
techniques above require one or two short quiescent periods of time at beginning and end. Exclusive locks are required on the appropriate database catalogs for the table or index. Depending on the application, its transaction sizes and its response time requirements, these quiescent periods may be painfully disruptive. An implementation of “fully online” index operations probably requires multi-value concurrency control for the database catalogs and the cache of pre-compiled query execution plans. No such implementation has been described in the literature.

- Online index operations permit updates by concurrent transactions while future index entries are extracted, sorted, and inserted into the new index. Most implementations lock the affected table and its schema while the new index is inserted in the database catalogs and during final transaction commit.
- Updates by concurrent transactions may be applied to the new index immediately (“no side file”) or after initial index creation is complete (“side file”). The former requires “anti-matter” records to reflect that the history of a key value in an index started with a deletion; the latter requires “catch-up” operations based on a log of the updates.

4.13 Transaction Isolation Levels

As is well established, transaction isolation levels weaker than serializability permit incorrect query results [59]. In update statements that run a query to compute the change set, weak transaction isolation levels can also result in wrong updates to a database. If concurrency control is applied to individual indexes, for example using key range locking in both primary and secondary B-trees, the possible results are generally not well understood. It does not help that commercial systems, their command set and their documentation differ in the definition of isolation levels, their names, and their semantics. For example, “repeatable read” in Microsoft SQL Server guarantees that records, once read, can be read again by the same transaction. Nonetheless, a transaction can insert records that satisfy another transaction’s query predicate, such that a second execution of the same query within the same transaction
produces additional result, so-called “phantom” rows. In IBM DB2
LUW, “repeatable read” guarantees full serializability, i.e., it protects
from phantoms. In terms of locking in B-trees, “repeatable read” in
SQL Server locks key values whereas “repeatable read” in DB2 also
locks the gaps between key values, i.e., it applies key range locking as
introduced in Section 4.3.

As transaction isolation levels are defined and explained elsewhere
[58, 59], let two examples suffice here. Both of these problems can arise
in the “read committed” (SQL Server) and “cursor stability” (DB2)
isolation level, which is the default isolation level in multiple products.
The first example shows a lost update, which is a standard textbook
example for the effects of weak transaction isolation levels. Two transac-
tions T1 and T2 read the same database record and the same attribute
value, say 10. Thereafter, transaction T1 increments the value by 1 to 11
and transaction T2 increments it by 2 to 12. After both transactions
commit, the final value is 11 or 12, depending on which transactions
writes the final value. Had both increment operations been applied seri-
ally, or had they been applied concurrently with serializable transaction
isolation, the final value would have been 13.

The second example illustrates inconsistency within a single result
row due to lock acquisition (and release) in individual indexes. If mul-
tiple indexes of a single table are used in an index intersection or an
index join, multiple predicate clauses may be evaluated on inconsistent
records. In an extreme case, a row might be included in (or excluded
from) a query result based on a row that never existed at all in the
database. For example, assume a query predicate “where \( x = 1 \) and
\( y = 2 \)” against a table with separate indexes on \( x \) and \( y \), and a query
execution plan that first probes the index on \( x \), then the index on \( y \),
and combines both using a hash join algorithm. One row might start
with values that satisfy the query predicate but its \( y \) value might have
changed before the scan of the index on \( y \); another row might end with
values that satisfy the query predicate but its \( x \) value was updated only
after the scan of the index on \( x \); etc.

Figure 4.18 illustrates this example. The database never contains a
valid, committed row satisfying the query predicate. Nonetheless, due
to the weak transaction isolation level and the interleaved execution
of transactions 1 and 2, the query returns one row. If the two indexes cannot cover the query and the required columns, the final operation in transaction 1 fetches all required columns from a primary index. In this case, the result row contains the value 2 for both columns, which does not satisfy the query predicate. If, on the other hand, the indexes cover the query, the result row contains values 1 and 2, which satisfy the predicate but never existed together in the database.

Also, an index scan may produce a reference for a row that no longer exists by the time index intersection is completed. In traditional query execution, the delay between index scan and fetching rows was small; if references are sorted or lists of references are combined from multiple secondary indexes, the delay may be substantial and more easily enable concurrent update transactions to interfere with a query execution.

Where the problem is understood, a common approach is to re-evaluate all predicate clauses for a table once entire rows from the table have been assembled. Unfortunately, this only seems to solve the problem, but it does not truly solve it. The technique ensures that no single result row obviously violates the query predicate but it does not ensure that the produced query result is complete. It also does not extend any guarantees across multiple tables in the database. For example, query optimization might remove a redundant join based on a foreign key constraint; if a query execution plan runs with a weak
4.13 Transaction Isolation Levels

transaction isolation level, however, omission or inclusion of the seemingly redundant join can lead to different query results.

The same problem exists if query optimization may freely choose between a materialized view (and its indexes) and the base table. The problem is worse if the query execution plan includes a join between a materialized view and its underlying tables. For example, the materialized view and its indexes may start a selection followed by fetching detail information from the base tables in the database.

An alternative approach promises more predictable results and execution semantics but it is more complex and restrictive. It relies on temporary serializability for the duration of one query execution. Once execution of one plan is complete, locks may be downgraded or released as appropriate. The concurrency control problems listed above for index intersection, semi-join removal based on foreign key constraints, “back-joins” from materialized view to base tables, etc. can be addressed in this way. This approach only works, however, if developers never employ program state or temporary database tables to carry information from one query to the next. For example, if query optimization fails to reliably choose a good plans, database application developers often resort to breaking a complex query into multiple statements using temporary tables. If one statement computes a set of primary key values, for example, the next statement may rely on all of them being present in the database. This is, of course, a variant of the earlier join and back-join problems.

Perhaps the most solid solution would be a recommendation to users to avoid the weak isolation levels or support for nested transactions with stronger transaction isolation levels. Nested transactions could be used implicitly for individual query execution plans or explicitly by users for individual statements or appropriate groups of statements in their scripts. Setting serializability as the default isolation level when a new database is created or deployed would be a good first step, because it would ensure that users would suffer wrong query results and wrong updates only after they actively “opt in” for this complex problem.

- Weak transaction isolation levels (repeatable read, read committed, etc.) eschew correct and complete isolation of
concurrent transactions in order to gain concurrency, performance, and scalability.

- Many database systems use a weak transaction isolation level as default. Many users and application developers probably do not completely understand the effect on application correctness.

### 4.14 Summary

In summary, necessity has been spawning many inventions that improve concurrency control, logging, and recovery performance for databases based on B-tree indexes. The separation of locking and latching, of database contents and in-memory data structures, is as important as key range locking aided by ghost records during deletion and possibly also insertion. Reducing log volume during large index utilities, in particular non-logged (or allocation-only logged) index creation, prevents the need for log space almost as large as the database but it introduces the need to force dirty pages from the buffer pool. Finally, weak transaction isolation levels might seem like a good idea for increased concurrency but they can introduce wrong query results and, when used in updates that compute the change from the database, wrong updates to the database.

Perhaps the most urgently needed future direction is simplification. Functionality and code for concurrency control and recovery are too complex to design, implement, test, debug, tune, explain, and maintain. Elimination of special cases without a severe drop in performance or scalability would be welcome to all database development and test teams.
B-tree indexes are only as useful as they are complemented by query execution techniques that exploit them and query optimization techniques that consider these query execution plans. Therefore, this section summarizes query execution techniques commonly used in conjunction with B-tree indexes. Preceding the individual sub-sections on B-tree techniques for query processing is a brief introduction of query processing and its two principal components, query optimization and query execution. The need and opportunity for automatic query optimization arises from a user interface based on a non-procedural database language such as SQL and from physical data independence.

The term physical data independence describes the separation of the logical data organization in tables and (materialized) views from the physical data organization in heap files and indexes. Tables contain columns identified by a name and rows identified by a unique primary key; rows contain column values. Files and indexes contain records identified by a record identifier in a heap or by a unique search key in an index; records contain fields. There are, of course, relationships between rows and records and between columns and fields, but physical data independence permits this relationship to be fairly loose.
Many optimization opportunities arise from exploiting these loose relationships. In the terms used above, there may be many-to-many relationships between tables and indexes as well as between logical rows and physical records. Interestingly, entities and relationships among the appropriate schema concepts can be modeled very similarly to entities and relationships in a user database and the required schema tables can be derived using standard techniques. Of course, not only logical database design but also physical database design applies to catalog tables, including data placement in in-memory caches for catalog information.

Figure 5.1 shows the entity types table and index together with set-valued attributes for table column and index field. In most database management systems, the relationship between table and index is a one-to-many relationship, i.e., a table can have multiple indexes but an index belong to one table only. A many-to-many relationship would indicate support for join indexes [123]. The relationship between index and B-tree can be a trivial one-to-one relationship that warrants no further elaboration in the ER diagram, a one-to-many relationship if each index may be partitioned into multiple B-trees, or a many-to-one relationship for master-detail clustering, e.g., using merged B-trees [49].

Depending on the facilities provided by a software system, physical database design may be restricted to index tuning or it may also exploit horizontal and vertical partitioning, row- or column-oriented storage formats, compression and bitmap encodings, free space in pages and indexes, sort order and master-detail clustering, and many other options [38]. Given the trends toward automation, it seems that choices about automation should also be included in physical database design, e.g., enabling automatic creation and maintenance of statistics.
(histograms) and soft constraints [60], automatic creation and optimization of indexes, and materialized views with their own indexes, statistics, and soft constraints.

Since database queries specify tables but query execution plans access indexes, a mapping is required. Physical data independence enables and requires choices in this mapping. Query optimization can be resource-intensive and the traditional design for database management software separates compile-time query optimization and run-time query execution. In addition to access path selection, query optimization also chooses execution algorithms (e.g., hash join or merge join) as well as the processing sequence (e.g., the join order).

These choices are captured in a query execution plan. The query execution plan is usually a tree but it may be a dag (directed acyclic graph) in case of common sub-expressions. Note that common sub-expressions might be introduced during query optimization, e.g., for queries with multiple aggregations including some with the “distinct” keyword in SQL, for tables with vertical or horizontal partitioning, and for large updates.

Figure 5.2 shows a simple query execution plan, the data structure constructed by compile-time query optimization and interpreted by run-time query execution. The nodes specify algorithms and the required customizations, e.g., predicates, sort clauses, and projection lists. Ideally, these are compiled into machine code, although special-purpose byte codes and run-time interpretation seem more common today.

In most query execution architectures, control flows down from consumer to producer and data flows up from producer to consumer within

![Fig. 5.2 A query execution plan.](image-url)
each query execution plan. The unit of data flow can be a record, a
“value packet” \cite{78}, a page, or any other unit that is deemed a good
tradeoff between complexity and performance. The control flow in a tree
is often implemented using iterators \cite{39}, resulting in a top-down con-
trol flow. Exceptions to top-down control flow are desirable for shared
(common) sub-plans with multiple consumers and are required query
execution plans with nested iteration, i.e., nested SQL expressions that
were not “flattened” during query optimization due to their complexity
or due to efficient execution strategies based on index search. If query
execution employs multiple threads in a pipeline, bottom-up control
often seems more desirable. Flow control between producer and con-
sumer in a pipeline renders this distinction practically mute. Bottom-
up thread initiation might seem to contradict top-down iterators but
actually does not \cite{40}.

Some operations, most obviously sorting, have nonoverlapping
input and output phases, plus an intermediate phase in many cases.
These operator phases delineate plan phases, e.g., a pipeline from
one sort operation (in its output phase) through a merge join to
another sort operation (in its input phase). These operations are called
stop-and-go algorithms, pipeline-breaking operations, and similar
names; their occurrence in query execution plans obviously affects
resource management such as memory allocation.

Figure 5.3 adds plan phases to the query execution plan of Figure 5.2
as implied by the stop-and-go operation sorting the result of the table
scan. The plan phase in the middle is optional: if the memory allocation

![Fig. 5.3 Plan phases.](image-url)
and input size for the sort operation are such that only a single merge step is required, the middle phase is omitted.

In most traditional systems, a query execution plan is quite rigid: choices are made during query optimization, and the role of query execution is to follow those choices. A variety of techniques exist for choices during query execution, from resource management (e.g., the size of the workspace in sorting and hash join) to limited mutations of a plan [84] and free routing of records [3]. Choices during query execution may benefit from run-time verification of assumptions and estimates employed during query optimization, e.g., record counts and data distributions in intermediate results.

One of the characteristics that distinguish B-tree indexes from other indexes is their support for ordered retrieval. Ordering of index entries supports range predicates and many predicates in multi-column indexes. Ordering of intermediate query results aids index-to-index navigation, retrieval from disk, set operations such as index intersection, merge join, order-based aggregation, and nested iteration.

- Due to a nonprocedural query language and physical data independence, compile-time query optimization chooses a query execution plan, i.e., a dataflow graph composed of query execution operations, based on cardinality estimation and cost calculations for alternative query expressions and execution plans.
- B-tree indexes are exploited in query execution plans both for retrieval (e.g., look-up of literals in the query text) and for ordered scans.
- Query optimization can also improve update execution (index maintenance).

## 5.1 Disk-order Scans

We now turn to specific B-tree techniques for efficient query processing in large data stores.

Most B-tree scans will be guided by the B-tree structure for output in the same sort order as the index ("index-order scan"). Deep, multi-page read-ahead can be guided by the information in parent and
grandparent nodes. If a query must scan all leaves in a B-tree, the scan may be guided by allocation information for the B-tree. Such information is kept in many systems in the context of free space management, often in the form of bitmaps. A scan based on such allocation bitmaps can incur less seek operations on the storage device (“disk-order scan”). In both kinds of scans, the branch nodes of the B-tree must be read in addition to the leaves, so these scans hardly differ in transfer volume. If sorted output is not required, a disk-order scan is usually faster.

Depending on the fragmentation of the B-tree and thus on the number of seek operations required in an index-order scan, a disk-order scan might be faster even if fewer than all B-tree leaves are required. In cases of extreme fragmentation, a disk-order scan followed by an explicit sort operation might be faster than an index-order scan.

Figure 5.4 shows a small, badly fragmented B-tree. A root-to-leaf search is not affected by the fragmentation, but a large range query or complete index-order scan must seek frequently rather than read contiguous disk segments. A disk-order scan guided by allocation information can read 15 contiguous pages, even reading (and then ignoring) the two unused pages (1st row center and 2nd row far right).

Another technique for speeding up scans that is often, although not always, associated with disk-order scans are coordinated scans [33, 132]. This optimization of concurrent scans are now exploited in several database systems. When a new scan is started, the system first checks whether the new scan can consume items in any order and whether
another scan is already active for the same object. If so, the scans are linked and the new scan starts at the current position of the prior scan. After the prior scan completes, the new scan must restart the scan to obtain items previously skipped. This technique works for any number of scans and is sometimes called “merry-go-round” scan because each data consumer takes one tour of a continuously active scan. Potential issues with this type of scan are concurrency control (this works best if all scanners lock the entire table, index, or equivalent) and bandwidth. For example, if two queries process items from the scan at very different speeds, e.g., due to predicates with user-defined functions, the two scans should be unlinked.

Coordinated scans are more sophisticated; they initialize by exploiting pages that remain in the buffer pool due to some prior activity that might not have been a scan, they may link and unlink multiple times, and optimize overall system throughput by sorting required future I/O by relevance, which is based on the amount of sharing, the amount of remaining work in each query, and the danger of starvation for a query. In order to reduce the administration effort, these considerations are applied to “chunks” or groups of pages rather than individual pages.

On the other hand, smart read-ahead and prefetch techniques can improve the performance of index-order scans in fragmented indexes. These techniques optimize the read sequence among the many children referenced in a branch node in order to minimize the number of seek operations in the storage device.

With ever-increasing memory sizes and with more and more data on semiconductor storage such as flash, shared and coordinated scans as well as smart prefetch might lose importance for B-tree indexes and database query processing. It is also possible, however, that these techniques will be needed in the future in order to fully exploit multiple cores sharing large CPU caches.

- If an index is chosen due to its column set, not for its sort order or in support of a predicate, and if the index is fragmented, a disk-order scan guided by allocation information can be faster than an index-order scan.
5.2 Fetching Rows

If a logical table and its rows map directly to a heap file or primary index and its records, many query execution plans obtain references (record identifier or key in the primary index) from secondary indexes and then fetch additional columns for each row. Fetching rows can easily be implemented using an index nested loops join even if that algorithm is more general than fetching as it permits any number of inner result records for each outer input record.

With the most naïve execution, which is the traditional strategy and still common, this can result in a large number of random I/O operations. Thus, secondary indexes may seem valuable only for extremely selective queries. Much recent research in database query execution has focused on scanning large tables without the aid of secondary indexes, for example using coordinated scans [33, 132], data compression [69, 111], columnar storage [122], and hardware support for predicate evaluation, e.g., GPUs or FPGAs [37]. The following techniques may move the break-even threshold in favor of secondary indexes.

Figure 5.5 illustrates the execution costs of three competing plans for a simple operation, selection of a set of rows from a table, for a variety of result sizes. At left, all rows in the table are rejected by the query predicate; at right, all rows satisfy the predicate. Scanning the table (or the data structure containing all rows and columns) cost practically the same, independent of the result size. It requires a sequential I/O per page or per extent (sequence of contiguous pages). In traditional database management system, this is the most robust plan if cardinality estimation for the output is unreliable, which is often the case. The
other traditional plan employs a secondary index, usually a B-tree, to obtain record identifiers of rows satisfying the query predicate. This plan is vastly superior for small result sizes, often 1,000 times faster than a table scan. For large result sizes, however, the traditional execution technique is extremely expensive, because each qualifying row requires a random I/O. Columnar storage and column scans are currently regarded as superior to either traditional plan. They are very robust yet faster than a table scan by a factor equal to the size of a complete row divided by the combined sizes of columns truly required by the given query. This factor is often an order-of-magnitude or more, in particular if the columnar storage format benefits from compression whereas the traditional table format does not.

The ideal situation, of course, would be a technique that gives the performance of a secondary index search for predicates with few qualifying records, the performance of a column scan for predicates with many qualifying records, and graceful degradation in the entire range in between.

- A secondary index can answer a selective query faster than a table scan or even a column scan.
- Unless cardinality estimation during compile-time query optimization is very accurate, a plan with robust performance may be preferable over a plan with better anticipated performance.

### 5.3 Covering Indexes

If a secondary index can supply all the columns required in a query, or all columns from a particular table, then there is no need to fetch records from the heap or the primary index. The common terms for this are “index-only retrieval” or “covering indexes.” The latter term can be confusing because this effect is not due to a property of the index alone but of the combination of index and query.

Figure 5.6 shows a query execution plans for table-to-table navigation without the benefit of covering columns. The query navigates a many-to-many relationship, in this case between courses and students with enrollment as intermediate table. Specifically, the query is
“select st.name from student as st, enrollment as enr, course as co where co.title = “biology 101” and enr.course-number = co.course-number and enr.student-identifier = st.student-identifier.”

This three-table query requires two two-table joins. Each join might require two index retrievals: first, a search in a secondary index produces a row identifier; second, a search in the primary index produces additional required column values. In a query execution plan, each of these index searches appears as nested iteration. Its left input supplies the outer input; within the nested iteration, the outer loop iterates over the items from the left input. Field values from the left input are bound as parameters for the right input. Binding correlation parameters is an exception to dataflow-to-the-root in query execution plans; this is one of the principal difficulties in the implementation of nested iteration, in particular in parallel query execution [42]. The right sub-plan executes once for each distinct parameter binding. The inner loop in the nested iteration operation iterates over the results of its right input.

In each instance in Figure 5.6, the right sub-plan is a single node, an index search. The first (lower-most) nested iteration implements the search for a specific course (“biology 101”) and all required course attribute, specifically course-number; the remaining four instances of nested iteration and index search implement the three-table join using
non-covering secondary indexes. Figure 5.6 shows the worst case: the student table may have a primary index on its primary key student-identifier, saving one of the join operations, and the enrollment table may have a primary index on course-number with a 50% chance since two foreign keys form the primary key of the table.

In order to permit index-only retrieval in more queries, some systems permit adding columns to an index definition that are not part of the search key. The performance gain due to index-only retrieval must be balanced with the additional storage space, bandwidth need during scans, and overhead during updates. Primary keys and foreign keys seem to be the most promising columns to add as they tend to be used heavily in queries with aggregation and multi-table joins, to use small data types (such as integers rather than strings), and to be stable (with rare updates of stored values). In databases for business intelligence, it may also be useful to add date columns to indexes, because time is usually essential in business intelligence. On the other hand, for the same reason, many queries are selective on dates, thus favoring indexes with date columns as search keys rather than as added columns.

Figure 5.7 shows a query execution plan for the same query as Figure 5.6 and database but with the beneficial effect of covering indexes, i.e., key columns added to each secondary index. This plan represents the best case, with student identifier included in the secondary index on enrollment and with student name added to the index on student identifier.

If no single index covers a given query, multiple secondary indexes together might. By joining two or more such indexes on the common reference column, rows can be assembled with all columns required for the query. If the sum of record sizes in the secondary indexes is
smaller than the record size in the primary index, the I/O volume is reduced by joining multiple secondary indexes. If the join algorithm can consume records as fast as the scans can produce them, e.g., an in-memory hash join, this technique requires less I/O than a scan of the primary index, at the expense of substantial memory requirements. This is particularly likely if query predicates reduce the scan in one or multiple of the secondary indexes [52].

Figure 5.8 shows a fragment of a query execution plan in which two indexes of the same table together cover a query. The query here is “select Title, Instructor from Course where Title like ‘biology*’.” The plan exploits two secondary indexes, neither one contains all columns required for the query. The join predicate links the two reference columns. Since both indexes are sorted on key values, not on the reference column, a hash join is the fastest join method, in particular if the two index scans produce very different data volumes due to predicates or due to the sizes of columns preserved from the index scan to the join output [52]. If more than two indexes contribute to cover the query, all of which are joined on the reference column, “interesting orderings” apply to sort-based join algorithms [115] and “teams” in hash-based join algorithms [52].

If multiple indexes are in the same sort order, a simplified merge join is sufficient. This is the standard technique of columnar databases, where each column is stored in the order of references; the references are compressed or even omitted on data pages. In both traditional secondary indexes and in columnar storage, repeated column values can be compressed, e.g., by run-length encoding.

Early relational database management systems did not exploit covering indexes; instead, they always fetched the records from the heap or primary index. Some current systems still do the same. For example,
Postgres relies on multi-version concurrency control. Due to the space overhead of multi-version concurrency control, e.g., timestamps for all records, version control is implemented only in the heap but not in the secondary indexes. Thus, for the purpose of concurrency control, any search in secondary indexes must be followed by fetching records.

- If a secondary index includes all columns required for a query, there is no need to access the table’s primary storage structure (index-only retrieval).
- In some systems, secondary indexes may include columns that are neither index key nor reference to the primary index. Prime candidates for inclusion in secondary indexes are primary key and foreign key columns. Further candidates include columns that are frequently used in predicates but rarely updated, e.g., dates.

5.4 Index-to-index Navigation

If index-only retrieval is not sufficient, various techniques can speed up fetching records in a B-tree index based on values extracted from another index, i.e., for “navigating” from one B-tree index to another and from one record in one index to another record in another index. These techniques are based primarily on two approaches, asynchronous prefetch and sorting. Both approaches have been used alone or together, and in many variants.

Asynchronous prefetch can apply to all individual B-tree pages, including leaf pages, or only to internal pages. For example, prior to an index nested loops join, the upper levels of a B-tree might be read into the buffer pool. Recall that the upper levels typically make up less than 1% of a B-tree, which is less than the typical ratio of memory size and disk size in database servers of 2-3%. The pages may be pinned in the buffer pool in order to protect them against replacement or they may be left for retention or replacement based on the standard buffer pool replacement strategy such as LRU. Small tables and their indexes, e.g., dimension tables in a relational data warehouse with a star schema [77], may even be mapped into virtual memory as in the Red Brick product [33]. In that case, they can be accessed using
memory addresses rather than page identifiers that require search in the buffer pool.

Waiting for leaf pages may be reduced by asynchronous prefetch. Some systems fill a small array of incomplete searches and pass them to the storage layer one batch at-a-time [35]; other systems keep the storage layer informed about a fixed number of incomplete searches at all times [34]. If most or even all prefetch hints result in a hit in the buffer pool, it may be appropriate to suppress further hints in order to avoid the overhead of processing useless hints.

In addition to asynchronous prefetch, it often pays to sort the intermediate search information, i.e., sort the set of search keys in the same ordering as the B-tree. In the extreme case, it can collapse prefetch requests for the same leaf page. In many cases, sorting unresolved references enables the storage layer to turn many small read operations into fewer, larger, and thus more efficient read operations.

Figure 5.9 illustrates how references obtained in one index (typically, a secondary index) can be preprocessed before they are resolved in another index (typically, a primary index). In a query execution plan, the set of references are an intermediate result — Figure 5.9 shows some numbers that might be key values in a primary index. In the secondary index, the reference values are obtained in the most convenient or efficient way, with no particular sort order that aids the next processing step. Sorting those references may use any of the usual algorithms and performance techniques. Compression may loose precision; for example, when the first three specific references are collapsed into a range, the specific information is lost and predicate evaluation must be repeated for all records in this range after they are fetched. In the implementation, sorting and compression can be interleaved in various ways, much like duplicate elimination can be integrated into run generation and merging in an external merge sort [16, 39].

<table>
<thead>
<tr>
<th>Intermediate result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted</td>
</tr>
<tr>
<td>Sorted</td>
</tr>
<tr>
<td>Compressed</td>
</tr>
</tbody>
</table>

Fig. 5.9 Preprocessing prefetch hints.
A sorted set of search keys permits efficient navigation within the B-tree. Most importantly, each page is needed only once, leaf pages for a short duration and branch nodes for a longer duration. Once the search has moved to a node’s neighbor with higher keys, the node can be discarded in the buffer pool; there is no need to retain it using a heuristic replacement strategy such as LRU. Thus, the buffer pool needs to provide only one page per B-tree level plus appropriate buffer frames for prefetch and read-ahead. Given that most B-trees have a large fan-out and thus a very small height, it is reasonable to keep all pages on the current root-to-leaf path pinned in the buffer pool. In addition, if the highest key read so far in each level of the B-tree is cached (typically together with the data structures created when the B-tree is “opened” for access by a query execution plan), it is not required that each individual search starts with the root page. Instead, each search can try B-tree pages in leaf-to-root order. Information about the most recent update of each page, e.g., ARIES log sequence numbers [95], can ensure that concurrent updates cannot interfere with the efficient B-tree retrieval. Fence keys in B-tree nodes [44] can ensure that range comparisons are accurate. Cheng et al. [25] describe a slightly more general technique for unsorted sequences of search keys exploiting parent pointers within disk pages rather than pinning in the buffer pool.

Figure 5.10 illustrates parts of the B-tree of Figure 2.4 and the cache that enables efficient navigation within the B-tree during the next search. The right half of Figure 5.10 shows data pages in the buffer pool; the left half of Figure 5.10 shows part of the state associated with the scan or the query operation. If the last search key was 17, the cache contains pointers into the buffer pool for the three nodes shown plus the

![Diagram](https://via.placeholder.com/150)

Fig. 5.10 Cache state in pre-sorted index fetches.
high key and the Page LSN for each node. If the next search pertains
to key value 19, the cache is immediately useful and is best exploited
by starting the search at the leaf level. If the next search pertains to
key value 47, the current leaf must be abandoned but its immediate
parent continues to be useful. The high key of the parent node is 89 as
had been learned from the separator key in the root node. If the next
search key pertains to key value 99, both the leaf and its parent must
be abandoned and only the cached root node of this B-tree is useful.
The optimal starting level for each new search can be found with a
single comparison per B-tree level.

There are several difficulties if modifications of the B-tree structure
can lead to de-allocation of B-tree pages. For example, if the shown
leaf node is merged into a neighboring leaf due to a deletion and an
underflow, something must ensure that the search does not rely on an
invalid page in the buffer pool. This problem can be solved in various
ways, for example by verification of all Page LSNs along the root-to-leaf
path.

In order to enable these optimizations, the search keys may require
an explicit sort operation prior to the B-tree look-ups. In a sort of
search keys, expensive temporary files can be avoided in three ways. The
first way is a large memory allocation. The second one is to perform
merely run generation and invoking the searches with only partially
sorted search keys. This might be called an opportunistic sort, as it
opportunistically exploits the readily available memory and creates as
much ordering in its output as possible.

The third way employs a bitmap for compression and for sorting.
This method only works if there is a sufficiently dense bidirectional
mapping between search arguments and positions in a bitmap. More-
over, this mapping must be order-preserving such that scanning the
bitmap produces sorted search keys. Finally, this works for navigation
from a secondary index to the primary index of the same table, but it
does not work if any information obtained in earlier query operations
must be saved and propagated in the query execution plan.

Figure 5.11 continues Figure 5.9 using bitmaps. The illustration
employs hexadecimal coding for brevity, indicated by the prefix 0x for
each block of 4 digits or 16 bits. Given that bitmaps often contain many
5.4 Index-to-index Navigation

<table>
<thead>
<tr>
<th>Method</th>
<th>Intermediate result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressed</td>
<td>26-34, 42-43, 57, 98</td>
</tr>
<tr>
<td>Bitmap (hexadecimal)</td>
<td>0x0000, 0x0020, 0x2030, ...</td>
</tr>
<tr>
<td>Compressed bitmap</td>
<td>3×0x00, 2×0x20, 1×0x30, ...</td>
</tr>
<tr>
<td>Mixed representation</td>
<td>5×0x00, 0x2020, 0x30, 57, 98</td>
</tr>
</tbody>
</table>

Fig. 5.11 Bitmap compression for prefetch hints.

zeroes, run-length encoding of similar bytes is a standard technique, at least for bytes containing all zeroes. Finally, the most compressed and concise lossless representation interleaves run-length encoding, explicit bitmaps, and lists of explicit values. Moreover, in order to avoid external sorting, the representation may interleave compression techniques without and with information loss.

The various compression methods can be combined. The goal is to reduce the size such that the entire set of references can be retained and sorted in memory, without expensive accesses to lower levels in the storage hierarchy. If compression introduces any information loss for some key range, all B-tree entries in that range must be scanned during B-tree access, including re-evaluation of the appropriate query predicates. In other words, repeated B-tree searches within the affected key range have been turned into a range scan. Figure 5.9 shows an example. If distribution skew is a concern, histograms (commonly used for query optimization) can aid the choice of search keys most promising for compression. If information in addition to the references must be carried along in the query execution plan, i.e., if the intention of the index is to cover the given query, then only lossless compression techniques are applicable.

Sorting the set of search keys improves query execution performance [30, 96] but also the robustness of performance [52]. If cardinality estimation during query optimization is very inaccurate, perhaps even by several orders of magnitude, sorting the search keys into a single sorted stream and optimizing the B-tree navigation ensure that index-to-index navigation performs no worse than a table or index scan.

Figure 5.12 shows the performance of the same two traditional plans shown in Figure 5.5 as well as the cost of a robust plan. For small query results, sorting the references obtained from the secondary index hardly affects the execution cost. For large query results, sorting the
references ensures that the cost never exceeds that of a table scan by too much. The difference is for obtaining and sorting the references from a secondary index. If the record size in the secondary index is a fraction of that in the primary index, the scan cost in the secondary index is the same fraction of the scan cost of the primary index. Appropriate compression ensures that the sort operation can rely on internal sort algorithms and does not contribute I/O operations to the overall query execution cost.

Thus, Figure 5.12 illustrates a prototypical example of robust index-to-index navigation. Similar techniques apply also to complex queries that join multiple tables, not just to navigation among multiple indexes of the same table.

If a secondary index is used to provide an “interesting ordering” [115], e.g., for a subsequent merge join, and if the secondary index is not a covering index for the table and the query, either the merge join must happen prior to fetching full records, or the fetch operation must not sort the references and thus forgo efficient retrieval, or the fetched records must be sorted back into the order in which they were obtained from the secondary index. The latter method essentially requires another sort operation.

One idea for speeding up this second sort operation is to tag the records with a sequence number prior to the first sort such that the second sort can be efficient and independent of the type, collation sequence, etc. in the original secondary index. On the other hand, this sort operation must manage larger records than the sort operation prior to the join and it cannot benefit from compression, particularly from
compression with information loss. Thus, this sort operation may be an external sort even when the initial sort operation is not.

Figure 5.13 shows a fragment of a query execution plan in which this technique could be beneficial. A secondary index provides records in a sort order “interesting” for a subsequent operation such as a merge join (not shown in Figure 5.13) yet, for robust retrieval from a primary index, an alternative sort order is required. With appropriate techniques, the performance of the final sort may be improved by exploiting the sort order in the secondary index.

In order to support index-to-index navigation among multiple tables, a promising heuristic is to include all key columns in all secondary indexes, both primary key columns and foreign key columns. The difference between the query execution plans in Figures 5.6 and 5.7 illustrate the beneficial effect. The technique can be extended by including columns of other tables based of functional dependencies and foreign key constraints. For example, an index on a date column of a table of order details might include a customer identifier; the order details table typically does not include this column but there is an appropriate functional dependency in the orders table.

- If an index needs to be probed with many search keys, sorting the keys avoids duplicate searches and may improve buffer pool effectiveness and thus I/O costs.
- An index search may resume where the preceding search left off, resulting in both upward and downward navigation in a B-tree index.
• These optimizations enable robust query execution plans with a graceful transition from fetching only a few items based on a secondary index to scanning a primary index completely, thus reducing surprising performance due to erroneous cardinality estimation during compile-time query optimization.

• The break-even point between optimized scans (e.g., in compressed column stores) and optimized index retrieval may shift with the transition from traditional high-latency disk storage to semiconductor storage such as flash.

5.5 Exploiting Key Prefixes

Most of the discussion so far has assumed that a search in a B-tree starts with a complete key, i.e., specific values for all the fields in the B-tree key. The output of the search was the information associated with a key, e.g., a pointer to a record in the primary index or the heap file. Since all B-trees are unique, if necessary by adding something to the user-defined key, such B-tree searches produce at most one matching record. While some indexes support only searching with precise values of the entire index key, notably hash indexes, B-trees are far more general.

Most importantly, B-trees support range queries for records with key values between given lower and upper bounds. This applies not only entire fields in a key but also prefixes within a column value, in particular a string value. For example, if a B-tree key field contains names such as “Smith, John,” a query for all persons with the last name “Smith” is effectively a range query, as would be queries for names starting with “Sm” or for “Smith, J.”

It is commonly believed that a B-tree is useful only if the search key in the query specifies a strict prefix of the B-tree key, but this is not the case. For example, in a B-tree organized by zip code (postal code) and (family) name, finding all people with a specific name requires enumeration of all distinct zip code values in the B-tree and searching for the specified name once for each zip code. The enumeration of zip codes can precede the search by name or the two can be interleaved [82]. Moreover, after the search within one zip code is complete, the next step
can search for the next zip code using the B-tree index or it can simply add one to the prior zip code and attempt to find the combination of the incremented zip code and the desired name. If the attempt fails because the incremented zip code does not exist, the root-to-leaf search in the B-tree leads to a record with the next actual value in the zip code column.

Figure 5.14 shows a B-tree index with a two-column key. For a query that specifies only the letter column but not the number column, e.g., “where letter = ‘q’,” an enumeration of the number values is required. The first root-to-leaf traversal of the B-tree finds the number 1 or actually the combined key “1,a”; the first search probes for a combined key of “1,q.” The next root-to-leaf traversal might be employed to find the number 2 in the combined key “2,a” by looking for a B-tree entry with a number larger than 1. In the improved scheme, the number 2 is not found in a search but calculated from the prior value 1, and the next root-to-leaf search focuses immediately on the combined key “2,q.” This works well for number values 2 and 3. The root-to-leaf traversal in search of the combined key “4,q” fails to find an appropriate record but it finds a record with number value 5. Thus, the next search might search directly for the combined key “5,q.”

This technique seems most promising if the number of distinct values is fairly small (zip codes in the example above or number values in Figure 5.14). This is because each distinct leading B-tree key value requires a single random I/O to fetch a B-tree leaf. The number of distinct values is not the true criterion, however. The alternative query execution plan typically is to scan the entire index with large sequential I/O operations. The probing plan is faster than the scanning plan if the data volume for each distinct leading B-tree key value is so large that a scan takes longer than a single probe. Note that this efficiency comparison must include not only I/O but also the effort for predicate evaluation.
In addition to unspecified leading B-tree keys as in the example of a specific name in any zip code area, a leading column may also be restricted by a range predicate rather than a specific value. For example, the zip code might be restricted to those starting “537” without regard for the last two digits. Moreover, the technique also applies to more than two columns: the first column may be restricted to a specific value, the second column unspecified, the third column restricted to a range, the fourth column unspecified, and the fifth column restricted to a specific value again. The analysis of such predicates may be complex but B-trees seem to support such predicates better than other index structures. Perhaps the most complex aspect is the cost analysis required for a cost-based compile-time decision between a full index scan, range scan, and selective probes. A dynamic run-time strategy might be most efficient and robust against cardinality estimation errors, cost estimation errors, data skew, etc.

In a B-tree index with a key consisting of multiple fields, the individual fields may be thought of as forming a hierarchy or as representing dimensions in a multi-dimensional space. Thus, a multi-column B-tree might be considered as a multi-dimensional access method (MDAM) [82]. If a leading dimension is under-specified in the query predicate, its possible values must be enumerated. This strategy works well if the dimensions are measured in integer coordinates and the number of distinct values in the leading dimensions. By transforming the query predicate into appropriate range queries in the sort order of the B-tree, even complex disjunctions and conjunctions can be processed quite efficiently [82].

The techniques illustrated in Figure 5.10 enable dynamic and automatic transitions between probing a B-tree with root-to-leaf traversals and scanning a B-tree in index-order. After a few recent requests could be satisfied by the current leaf or its immediate neighbor, asynchronous read-ahead of subsequent leaf nodes based on parent and grandparent nodes seems promising. If reuse of leaf nodes is rare but parent nodes and their immediate neighbors are useful in successive search requests, leaf nodes might be evicted quickly from the buffer pool but asynchronous read-ahead might be applied to parent nodes.
• Compound (i.e., multi-column) B-tree indexes support equality predicates on any prefix of their columns.
• With clever analysis of predicates, B-trees also support mixtures of range and equality predicates on subsets of their columns, even including cases with no restriction on the leading column.

5.6 Ordered Retrieval

Just like B-trees can provide more kinds of searches than is commonly realized, they can also support a larger variety of sort orders in the output of a scan. Coming back to the earlier idea of a B-tree as a cached state of a sort operation, the most obvious sort order has the B-tree keys as ordering keys. Ascending and descending orders are supported equally well in most implementations. Any sort order on a prefix of the B-tree key is also supported.

If the desired sort key starts with a prefix of the B-tree but also includes columns present in the B-tree but not in a useful way in the B-tree key, then it can be beneficial to execute a sort operation for each distinct value in the useful prefix columns of the B-tree. For example, if a table with columns $A, B, C, \ldots, X, Y, Z$ is stored in a primary index on columns $A, B, C$, and if a query requires output sorted on $A, B, P, Q$, then it is possible to exploit the sort order on $A, B$ and sort records on $P, Q$ for each distinct pair of values of $A, B$. Ideally, this sequence of smaller sort operations can be faster than a single large sort operation, for example because all or most of these small sort operations can be executed in memory whereas the single large sort would require run files on disk. In that case, this execution strategy requires (most of) the run generation effort of the large sort but not its merge effort.

Figure 5.15 illustrates these individual, small sort operations using the dashed brackets. The sizes of individual segments may vary widely such that some but not all segment may fit in the available memory allocation.

If, on the other hand, the desired sort order omits a prefix but otherwise matches the B-tree key, the distinct values of the omitted
prefix columns can be thought of identifiers of runs in an external merge sort, and the desired sort order can be produced by merging those runs. Using the same example table and primary index, if a query requires output sorted on \(B, C\), then the B-tree can be thought as many runs sorted on \(B, C\) and the distinct values of \(A\) identify those runs. This execution strategy requires the merge effort of the large sort but not its run generation effort.

Figure 5.16 illustrates the merge step that scans the primary index on \(A, B, C\) once for each individual value of \(A\) and then merges runs identified by distinct values of \(A\). If the count of distinct values of \(A\) is smaller than the maximal merge fan-in (dictated by the memory allocation and the unit of I/O), a single merge step suffices. Thus, all records are read once and, after passing through the merge logic, ready for the next step with run generation, run files on disk, etc.

These two techniques can be combined. For example, if a query requests output sorted on \(A, C\), then the distinct values of \(A\) enable multiple small sort operations, each of which merges runs sorted on \(C\) and identified by distinct values of \(B\). Note that the number of distinct values of \(B\) might vary for different values of \(A\). Thus, the small sort operations might require different merge strategies. Thus, merge planning and optimization require more effort than in a standard sort operation. More complex combinations of these two techniques are also possible.
Sorted output is required not only upon explicit request in the query text but also for many query execution algorithms for joins, set operations such as intersection, grouped aggregation, and more. “Top” queries for the most important, urgent, or relevant data items benefit from the sort and merge strategies described above because the first output is produced faster than by a large sort operation that consumes its entire input before producing its first output. Similar considerations apply to queries with “exists” clauses if the inner query employs an order-based algorithm such as a merge join.

- In addition to any prefix, a B-tree can produce sorted output on many other variations of its column list using merging or segmented execution.
- These techniques assist many order-based query evaluation algorithms and apply to any source of pre-sorted data, not only B-trees.

5.7 Multiple Indexes for a Single Table

Most query execution strategies discussed above exploit at most one index per table. In addition, there are many strategies that exploit multiple indexes, for example for multiple predicates.

For a conjunction of two or more predicate clauses, each matching a different index, the standard strategy obtains a list of references from each index and then intersects these lists. For a disjunction, the union of lists is required. Negated clauses call for difference operations.

Figure 5.17 shows a typical query execution plan for a query with a conjunction predicate executed by intersecting lists of references

Fig. 5.17 Index intersection for a conjunctive predicate.
obtained from multiple indexes for the same table. The result lists from each secondary index are transformed into a bitmap if the indexes are not bitmap indexes, their intersection is computed, and complete rows are fetched for the table using an index nested loops join.

Set operations such as intersection can be processed with standard join algorithms such as hash join or with bitmaps. If bitmaps are not compressed, hardware instructions such as “binary and” can be exploited. If bitmaps are compressed with run-length encoding or one of its variants, appropriate merge logic enables set operations without decompression. Alternatively, disjunctions and negations can be processed using a single bitmap, with multiple index scans setting or clearing bits.

Another type of multi-index strategy exploits multiple indexes on the same table to achieve the performance benefits of covering indexes even when a single covering does not exist. Scanning and joining two or more secondary indexes with few columns and thus short records can be faster than scanning a primary index or heap with long records, depending on the efficiency of the join operation and the availability of sufficient memory [52]. A query execution plan with two indexes covering a query is shown in Figure 5.8 in Section 5.3. It differs from the query execution plan in Figure 5.7 by avoiding the nested iteration to look up rows and by join rather than an intersection operation.

In some join queries, the best plan employs multiple indexes for each of the tables. For example, secondary indexes can be employed for semi-join reduction [14]. The goal is to reduce the list of rows to be fetched not only by single-table selection predicates but also by multi-table join predicates. Thus, secondary indexes from different tables are joined first based on the join predicate in the query. This assumes, of course, that the columns in these secondary indexes cover the join predicate. The cost of then fetching rows from the two tables’ primary indexes is incurred only for rows that truly satisfy the join predicate.

Figure 5.18 shows a query execution plan joining two tables using semi-join reduction. The essential aspect of the query execution plan in Figure 5.18 is that the join predicate is evaluated before rows are fetched from the tables. The order in which tables (or their indexes) are
joined is independent of the order in which tables are accessed to fetch missing column values. In this specific example, referential integrity constraints (courses only exist with enrolled students, enrollments must refer to existing courses) may be defined in the database. If so, semi-join reduction may be ineffective unless there are additional predicates not shown in the diagram, e.g., on titles of courses or on grades in the enrollment table, that invalidate the referential integrity constraints.

The technique applies to complex joins with multiple tables and has proven particularly valuable for complex joins with one particularly large table, also known as the central “fact table” in a “star schema” in relational data warehousing. In the typical “star queries” over such databases, the predicates often apply to the “dimension tables” surrounding the fact table, but the query execution cost is dominated by the accesses to the large fact table. In one of the standard query optimization techniques for such “star joins,” the dimension tables are joined first with secondary indexes of the fact table, the resulting lists of row identifiers in the fact table are combined into a single list of rows that satisfy all predicates in the query, and only the minimal set of rows from the fact table is fetched as a last step in the query execution plan.

Figure 5.19 shows a query execution plan for a star join using both index intersection and semi-join reduction. The original query includes predicates on the two dimensions “customer” (e.g., “city = Big Sky, Montana”) and “part” (e.g., “part name = “boat anchor”). Note that star queries often involve correlations far beyond the semantic capability of database statistics and metadata. For example, few customers
will purchase boat anchors high in the mountains. In the query execution plan of Figure 5.19, each of the two initial join operations performs a semi-join between a dimension table and the appropriate secondary index of the fact table. The result is a set of references in the fact table. The intersection operation produces the references in the fact table for rows satisfying the predicates on both dimensions. The final right semi-join produces these fact rows. If it is desirable that columns from a dimension table are preserved, the appropriate initial semi-join and the final semi-join must be inner joins instead.

A related issue that is often neither understood nor considered is concurrency control among multiple indexes of the same table, in particular if a weak transaction isolation level is chosen. If predicate evaluation is performed in multiple indexes yet concurrency control fails to ensure that matching index entries in multiple indexes are consistent, a query result may include rows that fail to satisfy the overall query predicate and it may fail to include rows that do satisfy the predicate. This issue also applies to predicates across multiple tables, but perhaps users relying on weak transaction isolation levels are more likely to expect weak consistency between tables than within an individual row of a single table. This issue also exists between materialized views and their underlying tables, even if incremental view maintenance is part of each update transaction.

- Index intersection can speed up conjunctive predicates and is commonly used. Index union can support disjunctions and
index difference can support “and not” predicates. Bitmaps can enable efficient implementations of these operations.

- Multiple indexes together may cover a query in a special case of index-only retrieval.
- In data warehouses with star or snowflake schemas, sophisticated index usage may be required for optimal performance.

### 5.8 Multiple Tables in a Single Index

Since join operations are traditionally among the most expensive database query operations, and since most joins are specified with the same equality predicates on foreign keys and primary keys, two techniques suggest themselves that combine multiple tables in a single index.

First, a join index maps values of join columns to references in two (or more) tables [66, 123]. These references may be represented by lists or by bitmaps [105]. Scanning a join index produces a result similar to that of joining two secondary indexes on the respective join columns.

Figure 5.20 illustrates the combined image for 3 tables as proposed by Härder, including a key value, occurrence counters for each of the tables, and the appropriate number of record identifiers in the tables. It is not required that every key value occurs in each table; counters might be zero.

A “star index” [33] is very similar to combined images and to join indexes. The main difference is that the star index contains record references in the dimension tables rather than key values. In other words, a star index maps record references in the appropriate dimension tables to record references in a fact table. If the star index is a B-tree, the order of the dimension tables in records of the star index matters, with the best clustering achieved for the dimension table mentioned.

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**Fig. 5.20** Combined image proposed by Härder.
first. If the record references in all dimension tables are column values, e.g., the primary key or the clustering key of the dimension table, then the star index is practically the same as a multi-column B-tree index on the corresponding foreign key values in the fact table.

Second, merged indexes are discussed in Section 7.3.

- Join indexes and star indexes combine multiple secondary indexes into a single B-tree in order to speed up large joins.

### 5.9 Nested Queries and Nested Iteration

Database and disk sizes continue to grow fast, whereas disk access performance and disk bandwidth improve much more slowly. If for no other reason than that, research into database query processing must refocus on algorithms that grow linearly not with the database size but with the query result size. These algorithms and query execution plans rely very heavily on index navigation, i.e., they start with a constant given in the query predicate, find some records in an index, extract further column values from those records, find more records in another index, and so on. The cost of this type of query plan grows linearly with the number of records involved, which might very well mean that the cost is effectively independent of the database size. Actually, the cost of index look-ups in traditional B-tree indexes grows logarithmically with the database size, meaning the cost doubles as a table grows from 1,000 to 1,000,000 records, and doubles again from 1,000,000 to 1,000,000,000,000 records. The cost barely changes from 1,000,000 to 2,000,000 records, whereas the cost of sort or hash operations doubles.

Moreover, it is well known that scanning “cleans” the I/O buffer of all useful pages, unless the replacement policy is programmed to recognize scans [26, 91, 120]. It is not unlikely, however, that CPU caches and their replacement policies recognize scans; thus, large scans will repeatedly clear out all CPU caches, even level-2 and level-3 caches of multiple megabytes. Based on these behaviors, on the growth rates of disk sizes and disk bandwidths, and on the recent addition of materialized and indexed views to mainstream relational database systems,

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1 Some of this section is derived from [42].
we should expect a strong resurgence of index-based query execution and thus research interest in execution plans relying heavily on nested iteration. The advent of flash storage with access latencies 100 times faster than traditional disk drives will speed this trend.

The key to interactive response times, both in online transaction processing (OLTP) and in online analytical processing (OLAP), is to ensure that query results are fetched, not searched and computed. For example, OLAP products perform well if they can cache the results of prior and likely future queries. In relational database systems, fetching query results directly means index searches. If a result requires records from multiple indexes, index nested loops join or, more generally, nested iteration are the algorithms of choice [63].

Figure 5.21 shows a query evaluation plan for a simple join of three tables, with the “outer” input of nested iterations shown as the left input. The nested iterations are simple Cartesian products here, as the join predicates have been pushed down into the inner query plans. If the filter operations are actually index searches and if table T0 is small relative to T1 and T2, this plan can be much more efficient than any plan using merge join or hash join. An alternative plan uses multiple branches of nested iteration rather than multiple levels as the plan in Figure 5.21. Of course, complex plans might combine multiple levels and multiple branches at any level.

Nested loops join is the simplest form of nested iteration. There is no limit to the complexity of the inner query. It might require searching a secondary index and subsequently fetching from the main

![Diagram of query plan with nested iteration.](image-url)
table; intersecting two or more secondary indexes using bitmap, sort, or hash operations; joining multiple tables each with its own access paths; multiple levels of nested iteration with multiple sub-branches at each level; etc. For all but the most trivial cases, there are complex issues that need to be addressed in a commercial product and in a complete research prototype. These issues exist at the levels of both policies and mechanisms, with avoiding I/O or exploiting asynchronous I/O as well as with managing resources such as memory and threads.

In addition to the techniques described earlier for asynchronous prefetch, sorting references before fetching rows, etc., B-tree indexes can also play an important role for caching the results of inner queries. Conceptually, one might want to employ two indexes, one for previously encountered outer binding values (parameters of the inner query), and one for results of inner query. The latter may contain many records for each of many outer bindings, and thus may grow quite large. The former might include also the result size as well as frequency or recency of usage. Information about empty results may prevent repeated futile searches and information about actual result sizes and usage may guide replacement decisions, e.g., in order to retain the most valuable inner results in a CPU cache or a buffer pool. This index may even be used to guide which outer binding to process next, e.g., producing results immediately for outer bindings with empty inner results. Note that those two indexes might be retained in a single merged B-tree such that any overhead due to separating the cache into two indexes is minimized.

- In query processing based on scans, query execution times grow with the table or database size. In query processing based on index-to-index navigation, query execution times grow with the size of intermediate results and are hardly affected by database or table size.
- Nested loops join is the simplest form of nested iteration. In general, the inner query execution plan can be complex, including multiple levels of nested iteration.
- Sorting the outer bindings is as helpful for general nested iteration as for index nested loops join.
Nested iteration can be very efficient if tables accessed by the inner query are very small or indexed.

5.10 Update Plans

In order to enable these various index-based query execution strategies, indexes must exist and must be up-to-date. Thus, the issue of efficient index creation and index maintenance is a necessary complement to query processing. In the following discussion, updates include modifications of existing records, insertions, and deletions.

Various strategies for efficient index maintenance have been designed. These include sorting changes prior to modifying records and pages in B-trees, splitting each modification of an existing record into a deletion and an insertion, and a choice between row-by-row and index-by-index updates. The row-by-row technique is the traditional algorithm. When multiple rows change in a table with multiple indexes, this strategy computes the delta rows (that include old and new values) and then applies them one by one. For each delta row, all indexes are updated before the next delta row is processed.

The index-by-index maintenance strategy applies all changes to the primary index first. The delta rows may be sorted on the same columns as the primary index. The desired effect is similar to sorting a set of search keys during index-to-index navigation. Sorting the delta rows separately for each index is possible only in index-by-index maintenance, of course.

This strategy is particularly beneficial if there are more changes than pages in each index. Sorting the changes for each index ensures that each index page needs to be read and written at most once. Prefetch of individual pages or large read-ahead can be used just as in read-only queries; in addition, updates benefit from write-behind. If read-ahead and write-behind transfer large sequences of individual pages as a single operation, this strategy is beneficial if the number of changes exceeds the number of such transfers during an index scan.

Figure 5.22 shows parts of a query execution plan, specifically those parts relevant to secondary index maintenance in an update statement. Not shown below the spool operation is the query plan that computes
the delta to be applied to a table and its indexes. In the left branch, no columns in the index’s search key are modified. Thus, it is sufficient to optimize the order in which changes are applied to existing index entries. In the center branch, one or more columns in the search key are modified. Thus, index entries may move within the index or, alternatively, updates are split into deletion and insertion actions. In the right branch, search key columns in a unique index are updated. Thus, there can be at most one deletion and one insertion per search key in the index, and matching deletion and insertion items can be collapsed into a single update item, which might save root-to-leaf B-tree traversals as well as log volume. In spite of the differences among the indexes and how they are affected by the update statement, their maintenance benefits from sorting, ideally data-driven sort operations.

The implementation of index-by-index maintenance may use a plan with multiple operations, which is very similar to a query execution plan. The maintenance operations for the secondary indexes may form a long chain with change rows passing from one to the next, or the change rows may be a shared intermediate result typically implemented with intermediate storage. Both update plans modify one secondary index after another. The long chain might seem easier to schedule but it requires that the earlier sort operations carry along all fields needed in subsequent index updates. In the update plan shown in Figure 5.22, this is avoided, at the expense of writing and reading the change set with all columns in the spool operation.
For additional efficiency, the spool operation can be avoided or, more accurately, integrated into the sort operations. Specifically, a single sort operation at the bottom of this plan fragment could receive input data, save those records in its workspace, and then generate runs for all three branches. For each branch, the sort operation must create an array with one or two surrogates per record in the workspace. Two surrogates are required for branches that show a split operation in Figure 5.22. This multi-function sort operation writes precisely the same records to runs on disk as the multiple sort operations in Figure 5.22. The savings are writing and reading the change set with all its columns in the spool operation and copying data into multiple sort operations’ workspaces; the disadvantage is that all columns are present in the workspace during run generation, resulting in more, smaller runs and possibly additional merge steps.

In this update plan, the sort operation competes for resources, most notably memory, with the operations computing the change set and preparing it, probably sorting it, for updating the primary index. Thus, intermediate runs in the sort operations as shown in Figure 5.22 may be more efficient than an integrated sort operation. Another variant of the algorithm seems desirable if the in-memory workspace is sufficient to sort the entire change set without the need for temporary runs. Thus, it is no surprise that database systems differ in the variety of optimizations implemented for large updates of indexed tables and materialized views.

Another technique enabled by index-to-index maintenance is per-index protection against the Halloween problem [90]. In general, searching and updating an index in the same plan phase may lead to incorrect updates or even infinite loops. Since a sort operation creates separate plan phases, it is possible to exploit one or more secondary indexes to compute the change rows and to apply those changes immediately to the primary index.

Finally, the basic ideas of index-by-index maintenance and update plans also apply to foreign key constraints, materialized view and their indexes, and even triggers. Specifically, foreign key constraints may be verified using branches in an update plan quite similar to maintenance of an individual secondary index, and cascading of information for
foreign key constraints and materialized views can be achieved in additional branches. Plan creation in complex databases is a challenge in its own right; for the purposes of this survey focused on B-tree indexes, it is sufficient to realize that various techniques for very efficient index maintenance are already implemented in some database management systems.

- Index maintenance can be optimized as much as index-to-index navigation during read-only query execution. The principal choice is index maintenance row-by-row or index-by-index. Similar choices pertain to checking integrity constraints.
- Sorting updates for each index can speed up index maintenance. Sorting can also provide protection from the Halloween problem due to its separation of input phase and output phase.

### 5.11 Partitioned Tables and Indexes

Partitioning divides a single table into multiple data structures. Among the basic forms of partitioning, horizontal partitioning defines subsets of rows or records whereas vertical partitioning defines subsets of columns or fields. In each form of partitioning, these subsets usually are disjoint. In other words, partitioning does not introduce redundancy.

In a partitioned table or index, each partition is kept in its own data structure, often a B-tree. Each partition and its B-tree are registered in the database catalogs. Since creation and removal of a partition must be recorded in the database catalogs, appropriate permissions and concurrency control in the catalogs are required. In contrast, a partitioned B-tree [43] keeps multiple horizontal partitions in a single B-tree. B-tree entries belong to the partition identified by the artificial leading key field in each record. Partitions are created and removed simply by insertion or deletion of appropriate records and their key values.

Horizontal partitioning can be used for manageability and as a coarse form of indexing. Manageability can be improved by partitioning on a time attribute such that load and purge operations, also known
as roll-in and roll-out, always modify entire partitions. As a form of
indexing, partitioning directs retrievals to a subset of a table. If most
queries need to scan data equivalent to a partition, partitioning can be
a substitute for indexing. In some cases, multi-dimensional partitioning
can be more effective than single-dimensional indexing.

Vertical partitioning, also known as columnar storage, enables effi-
cient scans when appropriate indexes are not available to guide the
search. Some claim that a data structure with only one column and
data type permits more effective compression and therefore faster scans;
others believe that equally effective compression is possible in both row
storage and column storage [108]. In addition to compression, scan effi-
ciency benefits from transferring only those columns truly needed in
query execution plan. On the other hand, result rows must be assem-
bled from column scans and exploiting shared or coordinated scans
might be more complex for columnar storage.

Query execution plans often must treat partitioning columns and
partition identifiers quite similarly to search keys in primary indexes.
When sorting search keys in index-to-index navigation, the partition
identifier should be a minor sort key; if the partition identifier is the
major sort key, partitions will be searched one after the other rather
than in parallel. Depending on key distributions, a more sophisticated
mapping from reference to sort key might be required in some cases.

Many such considerations and techniques also apply to updates.
When updating the partitioning key, index entries might need to move
from one partition to another; this requires splitting the update into
deletion and insertion. In other words, what is a performance optimiza-
tion in non-partitioned indexes might be a requirement in partitioned
indexes.

- Tables and indexes can be partitioned horizontally (by rows)
or vertically (by column).
- A secondary index is local if it is partitioned horizontally
in the same way as the primary data structure of the table.
References from the secondary index to the primary data
structure do not require partition identifiers as in a global
index.
• Most strategies for index-to-index navigation during query execution and for index maintenance require some adaptation in case of partitioned tables and indexes.

5.12 Summary

In summary, there is a large variety of techniques to make optimal use of B-tree indexes in query execution plans and update execution plans. Naïve or incomplete implementation of query optimization or query execution reduces the value of B-tree indexes in databases. It seems that all actual implementations fall short in one dimension or another. Therefore, creation, maintenance, scans, search, and joins of indexes continue to be a field of innovation and of competition. With databases stored on flash devices instead of traditional disk drives, hundred-fold faster access times will increase the focus on index data structures and algorithms.
In addition to the wide variety of efficient transactional techniques and query processing methods exploiting B-tree indexes in databases, perhaps it is really the large set of existing utilities that separates B-trees from alternative indexing techniques. For example, a B-tree index structure in a database or on some backup media can be verified with a single scan over the data pages, in any order, and with limited amounts of temporary information retained in memory or on disk. Even efficient index creation for large data sets often takes years to develop for a new index structure. For example, more and more efficient strategies for construction and bulk loading of R-tree indexes have been forthcoming over a long time [23, 62] compared to simply sorting for efficient construction of a B-tree index, which also applies to B-tree indexes adapted to multiple dimensions [6, 109]. Comparable techniques for alternative index structures are usually not part of initial proposals for new indexing techniques.

Index utilities are often set operations, whether index creation by preparing a set of future index entries or structural verification by processing a set of index facts after extracting them from an index such as a B-tree. Thus, many traditional query processing techniques
can be employed in index utilities including query optimization (e.g.,
the choice among joining two secondary indexes of the same table
or scanning the table’s primary index), partitioning and parallelism,
workload management for admission control and scheduling, and
resource management for memory and temporary disk space or disk
bandwidth. Similarly, many transactional techniques can be employed
in index utilities that have already been discussed above. Thus, the
following discussion does not address implementation issues related
to space management, partitioning, non-logged operation, online
operation with concurrent updates, etc.; instead, the primary focus of
the following discussion is on aspects not yet covered but relevant for
database utilities for indexes in general and for B-trees in particular.

- Utilities are crucial for efficient operation of a database and
  its applications. More techniques and implementations exist
  for B-trees than for any other index format.
- Utilities often affect entire databases, tables, or indexes.
  They often run a long time. Some systems employ techniques
  and code from query optimization and query execution.

6.1 Index Creation

While some products initially relied on repeated insertions for index
creation, the performance of index creation is much improved by first
sorting the future index entries. Therefore, techniques for efficient index
creation can be divided into techniques for fast sorting, techniques for
B-tree construction from a sorted stream, techniques enabling parallel
index construction, and transactional techniques for index creation.

During B-tree construction from a sorted stream, the entire “right
gate” of the future B-tree may be kept and even pinned in the buffer
pool at all times in order to avoid effort on redundant search in the
buffer pool. Similarly, during online index creation, “large” locks [48]
may be retained as much as possible and released only in response
to a conflicting lock request. Alternatively, transactional locks may be
avoided entirely as index creation does not modify the logical database
contents, only their representation.
A new index is usually created with some free space for future insertions and updates. Free space within each leaf node enables insertions without leaf splits, free space within branch nodes enables splits in the next lower B-tree level without splits cascading up the tree, and free pages within allocation units (such as extents or disk cylinders) enable splits without expensive seek operations during the split and, much more importantly, during all subsequent range queries and index-order scans. Not all systems give explicit control over these forms of free space because it is difficult to predict which parameter values will turn out to be optimal during future index usage. Moreover, the control parameters might not be followed precisely. For example, if the desired free space in each leaf is 10%, prefix B-trees [10] might choose the key ranges of neighboring leaves such that the separator key is short and the left node contains anywhere from 0% to 20% free space.

Figure 6.1 shows fixed-size pages with variable-size records immediately after index creation. All pages are fairly but not necessarily entirely full. After some number of pages (here 3), an empty page remains readily available for future page splits.

In an index with non-unique key values, the sort order should include sufficient reference information to render entries unique, as discussed earlier. This will aid in concurrency control, in logging and recovery, and in eventual deletion of entries. For example, when a logical row is deleted in a table, all records pertaining to that row in all indexes must be deleted. In a non-unique secondary index, this sort order enables finding the correct entry to delete efficiently.

If a secondary index is “very non-unique,” i.e., there is a large set of references for each unique key value, various compression methods can reduce the size of the index as well as the time to write the initial index to disk, to scan the index during query processing, or to copy the index during a replication or backup operation. The most traditional representation associates a counter and a list of references to each unique

![Fig. 6.1 Free space immediately after index creation.](image-url)
key value [66]. In many cases, the numeric differences between neighboring references can be represented more compactly than complete reference values. The proper sort enables efficient construction of such lists of differences; in fact, construction of such lists can be modeled as an aggregation function and can thus reduce the data volume written to temporary storage in an external merge sort. Bitmaps can also be employed. Grouping future index entries based on equal key values as early as possible not only enables compression and thus less I/O during sorting but also fewer comparisons during the merge steps, because only a single representative key value needs to participate in the merge logic after entries have been grouped.

Figure 6.2 (from [46]) illustrates this point for a three-way merge. Underlined keys are representatives of a group in the merge inputs and in the merge output. Values 1, 2, and 3 are struck out in the merge inputs because they have already gone through the merge logic. In the inputs, both copies of value 2 are marked as representatives of a group within their runs. In the output, only the first copy is marked whereas the second one is not, to be exploited in the next merge level. For value 3, one copy in the input is already not marked and thus did not participate in merge logic of the present merge step. In the next merge level, two copies of the value 3 will not participate in the merge logic. For value 4, the savings promises to be even greater: only two of six copies will participate in the merge logic of the present step, and only one in six in the next merge level.

Another possible issue during creation of large indexes is their need for temporary space to hold run files. Note that run files or even individual pages within run files may be “recycled” as soon as the merge process has consumed them. Some commercial database systems
therefore store the runs in the disk space designated for the final index, either by default or as an option. During the final merge, pages are recycled for the index being created. If the destination space is the only disk space available, there is no alternative to using it for the runs, although an obvious issue with this choice is that the destination space is often on mirrored disks or redundant RAID disks. Moreover, sorting in the destination space might lead to a final index that is rather fragmented because pages are recycled from merge input to merge output effectively in random order. Thus, an index-order scan of the resulting index, for example a large range query, would incur many disk seeks.

Figure 6.3 illustrates a situation in which the data source for the sort operation is larger than the temporary space. Thus, the temporary run files cannot be placed in the standard place for temporary data. Instead, the run files are placed in the destination space. As a merge step consumes the run files, disk space must be released immediately in order to create free space for the output of the merge step.

There are two possible solutions. First, the final merge can release pages to the global pool of available pages, and the final index creation attempts to allocate large contiguous disk space from there. However, unless the allocation algorithm’s search for contiguous free space is very effective, most of the allocations will be in the same small size in which space is recycled in the merge. Second, space can be recycled from initial runs to intermediate runs, among intermediate runs, and to the final index in larger units, typically a multiple of the I/O unit. For example, if this multiple is 8, disk space not exceeding 8 times the size of memory might be held for such deferred group recycling, which is typically an acceptable overhead when creating large indexes.
The benefit is that a full index-order scan or a large range scan of the completed index requires 8 times fewer seeks in large ordered scans.

If the sort operation for an index creation uses the final B-tree space for temporary runs, recovery from a system or media failure must repeat the original sort operation very precisely. Otherwise, recovery might place B-tree entries differently than the original execution and subsequent log records describing B-tree updates cannot be applied to the recovered B-tree index. Specifically, sizes of initial runs, the sequence of merge steps, merge fan-in, choice of merge inputs, etc. all must be either logged or implied by some information that is logged for the index creation, e.g., the memory allocation granted to the sort operation. Thus, during recovery, the same memory must be made available to the sort as during the original execution. The scan providing data for the sort must also be repeated precisely, without permuting input pages or records, e.g., due to asynchronous read-ahead. This could be an issue if recovery is invoked on different hardware as the original execution, e.g., after a catastrophic hardware failure such as a flood or fire. The need for precisely repeated execution may also inhibit adaptive memory allocation during index creation, i.e., the memory allocation, initial run size, merge fan-in all responding to fluctuations in memory contention during index creation.

Not only grouping and aggregation but a large variety of techniques from query processing can be employed for index creation, both query optimization and query execution techniques. For example, the standard technique to create a new secondary index is to scan the base structure for the table; if, however, two or more secondary indexes exist that contain all required columns and together can be scanned faster than the base structure, query optimization might select a plan that scans these existing indexes and joins the result to construct the entries for the new index. Query optimization plays an even larger role during view materialization, which in some systems is modeled as index creation for a view rather than a table.

For parallel index creation, standard parallel query execution techniques can be employed to produce the future index entries in the desired sort order. The remaining problem is parallel insertion into the new B-tree data structure. One method is to create multiple separate
B-trees with disjoint key ranges and to “stitch” them together with a single leaf-to-root pass and load balancing among neighboring nodes.

- Efficient B-tree creation relies on efficient sorting and on providing transactional guarantees without logging the new index contents.
- Commands for index creation usually have many options, e.g., about compression, about leaving free space for future updates, and about temporary space for sorting the future index entries.

### 6.2 Index Removal

Index removal might appear to be fairly trivial, but it might not be, for a variety of reasons. For example, if index removal can be part of a larger transaction, does this transaction prevent all other transactions from accessing the table, even if the index removal transaction might yet abort? Can index removal be online, i.e., may concurrent queries and updates be enabled for the table? For another example, if a table has both a primary (nonredundant) index plus some secondary indexes (that point to records in the primary index by means of a search key), how much effort is required when the primary index is dropped? Perhaps the leaves of the primary index may become a heap and merely the branch nodes are freed, but how long does it take to rebuild the secondary indexes? Again, can index removal be online?

Finally, an index may be very large and updates to the allocation information (e.g., free space map) may take considerable time. In that case, an “instant” removal of the index might merely declare the index obsolete in the appropriate catalog records. This is somewhat similar to a ghost record, except that a ghost indicator pertains only to the record in which it occurs whereas this obsolescence indicator pertains to the entire index represented by the catalog record. Moreover, whereas a ghost record might be removed long after its creation, space of a dropped index ought to be freed as soon as possible, since a substantial amount of storage space may be involved. Even if a system crash occurs before or during the process of freeing this space, the process ought to continue quickly after a successful restart. Suitable log records in the
recovery log are required, with a careful design that minimizes logging volume but also ensures success even in the event of crashes during repeated attempts of recovery.

An alternative to the obsolescence indicator in the catalog record, an in-memory data structure may represent the deferred work. Note that this data structure is part of the server state (in memory), not of the database state (on disk). Thus, this data structure works well unless the server crashes prior to completion of the deferred work. For this eventuality, both creation and final removal of the data structure should be logged in the recovery log. Thus, this alternative design does not save logging effort. Moreover, both designs require that intermediate state be support both during normal processing and during recovery after a possible crash and the subsequent recovery.

- Index removal can be complex, in particular if some structures must be created in response.
- Index removal can in instantaneous by delaying updates of the data structure used for management of free space.

Many other utilities could use this execution model but index removal seems to be the most obvious candidate.

### 6.3 Index Rebuild

There are a variety of reasons for rebuilding an existing index, and some systems require rebuilding an index when an efficient defragmentation would be more appropriate, in particular if index rebuilding can be online or incremental.

Rebuilding a primary index might be required if the search keys in the primary index are not unique and new values are desired in the artificial field that ensure unique references. Moving a table with physical record identifiers similarly modifies all references. Note that both operations require that all secondary indexes be rebuilt in order to reflect modified reference values.

Rebuilding all secondary indexes is also required when the primary index changes, i.e., when the set of key columns in the primary index changes. If merely their sequence changes, it is not strictly required that new references be assigned and the secondary indexes rebuild.
Updating all existing secondary indexes might be slower than rebuilding the indexes, partially because updates require full information in the recovery log whereas rebuilding indexes can employ non-logged index creation (allocation-only logging).

Figure 6.4 illustrates a query execution plan for rebuilding a table’s primary index and subsequently its three secondary indexes. The scan captures the data from the current primary index or heap file. The sort prepare filling the new primary index. The spool operation retains in temporary storage only those columns required for the secondary indexes. The spool operation can be omitted if it is less expensive to scan the new primary index repeatedly than to write and re-read the spool data. Alternatively, the individual sort operations following the spool operation can serve the purpose of the spool operation, as discussed earlier in connection with Figure 6.4.

In addition to non-logged index creation, index rebuild operations can also employ other techniques from index creation such as partitioning, parallel execution, stitching B-trees with disjoint key ranges, etc. For online index rebuild operations, the same techniques for locking, anti-matter, etc. apply as for online index creation.

- An index may be rebuilt after a corruption (due to faulty software or hardware) or when defragmentation is desired but removing and rebuilding the index is faster.
• When a primary index is rebuilt, the secondary indexes usually must be rebuilt, too. Various optimizations apply to this operation, including some that are usually not exploited for standard query processing.

6.4 Bulk Insertions

Bulk insertion, also known as incremental load, roll-in, or information capture, is a very frequent operation in many databases, in particular data warehouses, data marts, and other databases holding information about events or activities (such as sales transactions) rather than states (such as account balances). Performance and scalability of bulk insertions sometimes decides among competing vendors when the time windows for nightly database maintenance or for the initial proof-of-concept implementation are short.

Any analysis of performance or bandwidth of bulk insertions must distinguish between instant bandwidth and sustained bandwidth, and between online and offline load operations. The first difference is due to deferred maintenance of materialized views, indexes, statistics such as histograms, etc. For example, partitioned B-trees [43] enable high instant load bandwidth (basically, appending to a B-tree at disk write speed). Eventually, however, query performance deteriorates with additional partitions in each B-tree and requires reorganization by merging partitions; such reorganization must be considered when determining the load bandwidth that can be sustained indefinitely.

The second difference focuses on the ability to serve applications with queries and updates during the load operation. For example, some database vendors for some of their releases recommended dropping all indexes prior to a large bulk insertion, say insertions larger than 1% of the existing table size. This was due to the poor performance of index insertions; rebuilding all indexes for 101% of the prior table size can be faster than insertions equal to 1% of the table size.

Techniques optimized for efficient bulk insertions into B-trees can be divided into two groups. Both groups rely on some form of buffering to delay B-tree maintenance and to gain some economy of scale. The first group focuses on the structure of B-trees and buffers insertions in
branch nodes [74]. Thus, B-tree nodes are very large, are limited to a small fan-out, or require additional storage “on the side.” The second group exploits B-trees without modifications to their structure, either by employing multiple B-trees [100] or by creating partitions within a single B-tree by means of an artificial leading key field [43]. In all cases, pages or partitions with active insertions are retained in the buffer pool. The relative performance of the various methods, in particular in sustained bandwidth, has not yet been investigated experimentally. Some simple calculations below highlight the main effects on load bandwidth.

Figure 6.5 illustrates a B-tree node that buffers insertions, e.g., a root node or a branch node. There are two separator keys (11 and 47), three pointers to child nodes within the same B-tree, and a set of buffered insertions with each child pointer. In a secondary index, index entries contain a key value and reference to a record in the primary index of the table, indicated by “ref” here. In other words, each buffered insertion is a future leaf entry. The set of buffered insertions for the middle child is much smaller than the one for the left child, perhaps due to skew in the workload or a recent propagation of insertions to the middle child. The set of changes buffered for the right child includes not only insertions but also a deletion (key value 72). Buffering deletions is viable only in secondary indexes, after a prior update of a primary index has ensured that the value to be deleted indeed must exist in the secondary index.

The essence of partitioned B-trees is to maintain partitions within a single B-tree, by means of an artificial leading key field, and to reorganize and optimize such a B-tree online using, effectively, the merge step well known from external merge sort. This key field probably should
be an integer of 2 or 4 bytes. By default, the same single value appears in all records in a B-tree, and most of the techniques for partitioned B-trees rely on exploiting multiple alternative values, temporarily in most cases and permanently for a few techniques. If a table or view in a relational database has multiple indexes, each index has its own artificial leading key field. The values in these fields are not coordinated or propagated among the indexes. In other words, each artificial leading key field is internal to a single B-tree, such that each B-tree can be reorganized and optimized independently of all others. If a table or index is horizontally partitioned and represented in multiple B-trees, the artificial leading key field should be defined separately for each partition.

Figure 6.6 illustrates how the artificial leading key field divides the records in a B-tree into partitions. Within each partition, the records are sorted, indexed, and searchable by the user-defined key just as in a standard B-tree. In this example partition 0 might be the main partition whereas partitions 3 and 4 contain recent insertions, appended to the B-tree as new partitions after in-memory sorting. The last partition might remain in the buffer pool, where it can absorb random insertions very efficiently. When its size exceeds the available buffer pool, a new partition is started and the prior one is written from the buffer pool to disk, either by an explicit request or on demand during standard page replacement in the buffer pool. Alternatively, an explicit sort operation may sort a large set of insertions and then append one or multiple partitions. The explicit sort really only performs run generation, appending runs as partitions to the partitioned B-tree.

The initial load operation should copy the newly inserted records or the newly filled pages into the recovery log such that the new database
contents is guaranteed even in the event of a media or system failure. Reorganization of the B-tree can avoid logging the contents, and thus log only the structural changes in the B-tree index, by careful write ordering. Specifically, pages on which records or pointers have been removed may over-write their earlier versions in their old location only after new copies of the records have been written in their new location. Minimal logging enabled by careful write ordering has been described for other forms of B-tree reorganization [44, 89, 130] but it also applies to merging in partitioned B-trees after bulk insertion by appending new partitions.

For some example bandwidth calculations, consider bulk insertion into a table with a primary index and three secondary indexes, all stored on a single disk supporting 200 read–write operations per second (100 read–write pairs) and 100 MB/s read–write bandwidth (assuming large units of I/O and thus negligible access latency). In this example calculation, record sizes are 1 KB in the primary index and 0.02 KB in secondary indexes, including overheads for page and record headers, free space, etc. For simplicity, let us assume a warm buffer pool such that only leaf pages require I/O. The baseline plan relies on random insertions into 4 indexes, each requiring one read and one write operation. 8 I/Os per inserted row enable 25 row insertions per second into this table. The sustained insertion bandwidth is $25 \times 1 \text{ KB} = 25 \text{ KB/s} = 0.025 \text{ MB/s}$ per disk drive. This is both the instant and the sustained insertion bandwidth.

For the plan relying on index removal and re-creation after insertion, assume index removal is practically instant. With no indexes present, the instant insertion bandwidth is equal to the disk write bandwidth. After growing the table size by 1%, index creation must scan 101 times the insertion volume, then write 106% of that data volume to run files for the 4 indexes (sharing run generation when sorting for the secondary indexes), and finally merge those runs into indexes: for a given amount of data added, the I/O volume is $1 + 101 \times (1 + 3 \times 1.06) = 423$ times that amount; at 100 MB/s, this permits $100/423 \text{ MB/s} = 0.236 \text{ MB/s}$ sustained insertion bandwidth. While this might seem poor compared to 100 MB/s, it is about ten times faster than random insertions, so it is not surprising that vendors have recommended this scheme.
For a B-tree implementation that buffers insertions at each branch node, assume buffer space in each node for 10 times more insertions than child pointers in the node. Propagation upon overflow focuses on the child with the most pending insertions; let us assume that 20 records can be propagated on average. Thus, only every 20th record insertion forces reading and writing a B-tree leaf, or 1/20 of a read–write pair per record insertion. On the other hand, B-tree nodes with buffer are much larger and thus each record insertion might require reading and writing both a leaf node and its parent, in which case each record insertion forces $2/20 = 1/10$ of a read–write pair. In the example table with a primary index and three secondary indexes, i.e., 4 B-trees in total, these assumptions lead to $4/10$ of a read–write pair per inserted record. The assumed disk hardware with 100 read–write pairs per second thus support 250 record insertion per second or 0.250 MB/s sustained insertion bandwidth. In addition to the slight bandwidth improvement when compared to the prior method, this technique retains and maintains the original B-tree indexes and permits query processing throughout the load process.

With the assumed disk hardware, partitioned B-trees permit 100 MB/s instant insertion bandwidth, i.e., purely appending new partitions sorted in the in-memory workspace using quicksort or replacement selection. B-tree optimization, i.e., a single merge level in a partitioned B-tree that reads and writes partitions, can process 50 MB/s. If reorganization is invoked when the added partitions reach 33% of the size of the master partition, adding a given amount of data requires an equal amount of initial writing plus 4 times this amount for reorganization (reading and writing). 9 amounts of I/O for each amount of data produce a sustained insertion bandwidth of 11 MB/s for a single B-tree. For the example table with a primary index and three secondary indexes, this bandwidth must be divided among all indexes, giving $11 \text{ MB/s} \div (1 + 3 \times 0.02) \text{ KB} = 10 \text{ MB/s}$ sustained insertion bandwidth. This is more than an order of magnitude faster than the other techniques for bulk loading. In addition, query processing remains possible throughout initial capture of new information and during B-tree reorganization. A multi-level merge scheme might increase this bandwidth further because the master partition
may be reorganized less often yet the number of existing partitions can remain small.

In addition to traditional databases, B-tree indexes can also be exploited for data streams. If only recent data items are to be retained in the index, both bulk insertion techniques and bulk deletion techniques are required. Therefore, indexing streams is discussed in the next section.

- Efficiency of bulk insertions (also known as load, roll-in, or information capture) is crucial for database operations.
- Efficiency of index maintenance is so poor in some implementations that indexes are removed prior to large load operations. Various techniques have been published and implemented to speed up bulk insertions into B-tree indexes. Their sustained insertion bandwidths differ by orders of magnitude.

6.5 Bulk Deletions

Bulk deletion, also known as purging, roll-out, or information de-staging, can employ some of the techniques invented for bulk insertion. For example, one technique for deletion simply inserts anti-matter records with the fastest bulk insertion technique, leaving it to queries or a subsequent reorganization to remove records and reclaim storage space.

Partitioned B-trees permit reorganization prior to the actual deletion. In the first and preparatory step, records to be deleted are moved from the main “source” partition to a dedicated “victim” partition. In the second and final step, this dedicated partition is deleted very efficiently, mostly by simply de-allocation of leaf nodes and appropriate repair of internal B-tree nodes. Note that the first step can be incremental, rely entirely only system transactions, and can run prior to the time when the information truly should vanish from the database.

Figure 6.7 shows the intermediate state of a partitioned B-tree after preparation for bulk deletion. The B-tree entries to be deleted have all been moved from the main partition into a separate partition such that their actual deletion and removal can de-allocate entire
leaf pages rather than remove individual records distributed in all leaf pages. If multiple future deletions can be anticipated, e.g., daily purge of out-of-date information, multiple victim partitions can be populated at the same time.

The log volume during bulk deletion in partitioned B-trees can be optimized in multiple ways. First, the initial reorganization (“un-merging”) into one or more victim partitions can employ the same techniques as merging based on careful write ordering. Second, after the victim partitions have been written back to the database, turning multiple valid records into ghost records can be described in a single short log record. Third, erasing ghost records does not require contents logging if the log records for removal and commit are merged, as described earlier. Fourth, de-allocation of entire B-tree nodes (pages) can employ similar techniques turning separator keys into ghost records. If the victim partitions are so small that they can remain in the buffer pool until their final deletion, committed deletion of pages in a victim partition permits writing back dirty pages in the source partitions.

Techniques for bulk insertion and bulk deletion together enable indexing of data streams. Data streaming and near-real-time data processing can benefit from many database techniques, perhaps adapted, e.g., from demand-driven execution to data-driven execution [22]. Most stream management systems do not provide persistent indexes for stream contents, however, because the naïve or traditional index maintenance techniques would slow down processing rates too much.

With high-bandwidth insertions (appending partitions sorted in memory), index optimization (merging runs), partition splitting (by prospective deletion date), and deletions (by cutting entire partitions), B-tree indexes on streams can be maintained even on permanent
storage. For example, if a disk drive can move data at 100 MB/s, new data can be appended, recent partitions merged, imminently obsolete partitions split from the main partition, and truly obsolete partitions cut at about 20 MB/s sustained. If initial or intermediate partitions are placed on particularly efficient storage, e.g., flash devices or nonvolatile RAM, or if devices are arranged in arrays, the system bandwidth can be much higher.

A stream with multiple independent indexes enables efficient insertion of new data and concurrent removal of obsolete data even if multiple indexes require constant maintenance. In that case, synchronizing all required activities imposes some overhead. Nonetheless, the example disk drive can absorb and purge index entries (for all indexes together) at 20 MB/s.

Similar techniques enable staging data in multiple levels of a storage hierarchy, e.g., in-memory storage, flash devices, performance-optimized “enterprise” disks and capacity-optimized “consumer” disks. Disk storage may differ not only in the drive technology but also in the approach to redundancy and failure resilience. For example, performance is optimized with a RAID-1 “mirroring” configuration whereas cost-per-capacity is optimized with a RAID-5 “striped redundancy” configuration or a RAID-6 “dual redundancy” configuration. Note that RAID-5 and -6 can equal each other in cost-per-capacity because the latter can tolerate dual failures and thus can be employed in larger disk arrays.

- Bulk deletion is less important than bulk insertion; nonetheless, various optimizations can affect bandwidth by orders of magnitude.
- Indexing data streams requires techniques from both bulk insertion and bulk deletion.

6.6 Defragmentation

1Defragmentation in file systems usually means placing blocks physically together that belong to the same file; in database B-trees,

1Much material in this section is copied from [55].
defragmentation encompasses a few more considerations. These considerations apply to individual B-tree nodes or pages, to the B-tree structure, and to separator keys. In many cases, defragmentation logic can be invoked when some or all affected pages are in the buffer pool due to normal workload processing, resulting in incremental and online defragmentation or reorganization [130].

For each node, defragmentation includes free space consolidation within each page for efficient future insertions, removal of ghost records (unless currently locked by user transactions), and optimization of in-page data compression (e.g., de-duplication of field values). The B-tree structure might be optimized by defragmentation for balanced space utilization, free space as discussed above in the context of B-tree creation, shorter separator keys (suffix truncation), and better prefix truncation on each page.

B-tree defragmentation can proceed in key order or in independent key ranges, which also creates an opportunity for parallelism. The key ranges for each task can be determined a priori or dynamically. For example, when system load increases, a defragmentation task can commit its changes instantaneously, pause, and resume later. Note that defragmentation does not change the contents of a B-tree, only its representation. Therefore, the defragmentation task does not need to acquire locks. It must, of course, acquire latches to protect in-memory data structures such as page images in the buffer pool.

Moving a node in a traditional B-tree structure is quite expensive, for several reasons. First, the page contents might be copied from one page frame within the buffer pool to another. While the cost of doing so is moderate, it is probably faster to “rename” a buffer page, i.e., to allocate and latch buffer descriptors for both the old and new locations and then to transfer the page frame from one descriptor to the other. Thus, the page should migrate within the buffer pool “by reference” rather than “by value.” If each page contains its intended disk location to aid database consistency checks, this field must be updated at this point. If it is possible that a de-allocated page lingers in the buffer pool, e.g., after a temporary table has been created, written, read, and dropped, this optimized buffer operation must first remove from the buffer’s hash table any prior page with the new page identifier.
6.6 Defragmentation

Alternatively, the two buffer descriptors can simply swap their two page frames.

Second, moving a page can be expensive because each B-tree node participates in a web of pointers. When moving a leaf page, the parent as well as both the preceding leaf and the succeeding leaf must be updated. Thus, all three surrounding pages must be present in the buffer pool, their changes recorded in the recovery log, and the modified pages written to disk before or during the next checkpoint. It is often advantageous to move multiple leaf pages at the same time, such that each leaf is read and written only once. Nonetheless, each single-page move operation can be a single system transaction, such that locks can be released frequently both for the allocation information (e.g., an allocation bitmap) and for the index being reorganized.

If B-tree nodes within each level do not form a chain by physical page identifiers, i.e., if each B-tree node is pointed to only by its parent node but not by neighbor nodes, page migration and therefore defragmentation are considerably less expensive. Specifically, only the parent of a B-tree node requires updating when a page moves. Neither its siblings nor its children are affected; they are not required in memory during a page migration, they do not require I/O or changes or log records, etc.

The third reason why page migration can be quite expensive is logging, i.e., the amount of information written to the recovery log. The standard, “fully logged” method to log a page migration during defragmentation is to log the page contents as part of allocating and formatting a new page. Recovery from a system crash or from media failure unconditionally copies the page contents from the log record to the page on disk, as it does for all other page allocations.

Logging the entire page contents is only one of several means to make the migration durable, however. A second, “forced write” approach is to log the migration itself with a small log record that contains the old and new page locations but not the page contents, and to force the data page to disk at the new location prior committing the page migration. Forcing updated data pages to disk prior to transaction commit is well established in the theory and practice of logging and recovery [67]. A recovery from a system crash can safely assume that a
committed migration is reflected on disk. Media recovery, on the other hand, must repeat the page migration, and is able to do so because the old page location still contains the correct contents at this point during log-driven redo. The same applies to log shipping and database mirroring, i.e., techniques to keep a second (often remote) database ready for instant failover by continuously shipping the recovery log from the primary site and running continuous redo recovery on the secondary site.

The most ambitious and efficient defragmentation method neither logs the page contents nor forces it to disk at the new location. Instead, this “non-logged” page migration relies on the old page location to preserve a page image upon which recovery can be based. During system recovery, the old page location is inspected. If it contains a log sequence number lower than the migration log record, the migration must be repeated, i.e., after the old page has been recovered to the time of the migration, the page must again be renamed in the buffer pool, and then additional log records can be applied to the new page. To guarantee the ability to recover from a failure, it is necessary to preserve the old page image at the old location until a new image is written to the new location. Even if, after the migration transaction commits, a separate transaction allocates the old location for a new purpose, the old location must not be overwritten on disk until the migrated page has been written successfully to the new location. Thus, if system recovery finds a newer log sequence number in the old page location, it may safely assume that the migrated page contents are available at the new location, and no further recovery action is required.

Some methods for recoverable B-tree maintenance already employ this kind of write dependency between data pages in the buffer pool, in addition to the well-known write dependency of write-ahead logging. To implement this dependency using the standard technique, both the old and new page must be represented in the buffer manager. Differently than in the usual cases of write dependencies, the old location may be marked clean by the migration transaction, i.e., it is not required to write anything back to the old location on disk. Note that redo recovery of a migration transaction must re-create this write dependency, e.g., in media recovery and in log shipping.
The potential weakness of this third method are backup and restore operations, specifically if the backup is “online,” i.e., taken while the system is actively processing user transactions, and the backup contains not the entire database but only pages currently allocated to some table or index. Moreover, the detailed actions of the backup process and page migration must interleave in a particularly unfortunate way. In this case, a backup might not include the page image at the old location, because it is already de-allocated. Thus, when backing up the log to complement the online database backup, migration transactions must be complemented by the new page image. In effect, in an online database backup and its corresponding restore operation, the logging and recovery behavior is changed in effect from a non-logged page migration to a fully logged page migration. Applying this log during a restore operation must retrieve the page contents added to the migration log record and write it to its new location. If the page also reflects subsequent changes that happened after the page migration, recovery will process those changes correctly due to the log sequence number on the page. Again, this is quite similar to existing mechanisms, in this case the backup and recovery of “non-logged” index creation supported by some commercial database management systems.

While a migration transaction moves a page from its old to its new locations, it is acceptable for a user transaction to hold a lock on a key within the B-tree node. It is necessary, however, that any such user transaction must search again for the B-tree node, with a new search pass from B-tree root to leaf, in order to obtain the new page identifier and to log further contents changes, if any, correctly. This is very similar to split and merge operations of B-tree nodes, which also invalidate knowledge of page identifiers that user transactions may temporarily retain. Finally, if a user transaction must roll back, it must compensate its actions at the new location, again very similarly to compensating a user transaction after a different transaction has split or merged B-tree nodes.

- Most implementations of B-trees (as of other storage structures) require occasional defragmentation (reorganization) to ensure contiguity (for fewer seeks during scans), free space, etc.
• The cost of page movement can be reduced by using fence keys instead of neighbor pointers (see also Sections 3.5 and 4.4) and by careful write ordering (see also Section 4.10).
• Defragmentation (reorganization, compaction) can proceed in many small system transactions and it can ‘pause and resume’ without wasting work.

6.7 Index Verification

There obviously is a large variety of techniques for efficient data structures and algorithms for B-tree indexes. As more techniques are invented or implemented in a specific software system, omissions or mistakes occur and must be found. Many of these mistakes manifest themselves in data structures that do not satisfy the intended invariants. Thus, as part of rigorous regression testing during software development and improvement, verification of B-trees is a crucial necessity.

Many of these omissions and mistakes require large B-trees, high update and query loads, and frequent verification in order to be found in a timely manner during software development. Thus, efficiency is important in B-tree verification.

To be sure, verification of B-trees is also needed after deployment. Hardware defects occur in DRAM, flash devices, and disk devices as is well known [114]. Software defects can be found not only in database management systems but also in device drivers, file system code, etc. [5, 61]. While some self-checking exists in many hardware and software layers, vendors of database management system recommend regular verification of databases. Verification of backup media is also valuable as it enhances the trust and confidence is those media and their contents, should they ever be needed.

For example, Mohan described the danger of partial writes due to performance optimizations in implementations of the SCSI standard [94]. His focus was on problem prevention using appropriate page modification, page verification after each read operation, logging,
log analysis, recovery logic, etc. The complexity of these techniques, together with the need for ongoing improvements in these performance-critical modules, reinforces our belief that complete, reliable, and efficient verification of B-tree structures is a required defensive measure.

For example, Figure 6.8 shows the result of an incorrect splitting of a leaf node. When leaf node b was split and leaf node c was created, the backward pointer in successor node d incorrectly remained unchanged. A subsequent (descending) scan of the leaf level will produce a wrong query result, and subsequent split and merge operations will create further havoc. The problem might arise after an incomplete execution, an incomplete recovery, or an incomplete replication of the split operation. The cause might be a defect in the database software, e.g., in the buffer pool management, or in the storage management software, e.g., in snapshot or version management. In other words, there are many thousands of lines of code that may contain a defect that leads to a situation like the one illustrated in Figure 6.8.

As B-trees are complex data structures, efficient verification of all invariants has long been elusive, including in-page invariants, parent-child pointers and neighbor pointers, and key relationships, i.e., the correct ordering of separator keys and keys in the leaf nodes. The latter problem pertains not only to parent-child relationships but to all ancestor-descendent relationships. For example, a separator key in a B-tree’s root node must sort not only the keys in the root’s immediate children but keys at all B-tree levels down to the leaves. Invariants that relate multiple B-trees, e.g., the primary index of a table and its secondary indexes or a materialized view and its underlying tables and views, can usually be processed with appropriate joins. If all aspects of database verification are modeled as query processing problems, many query processing techniques can be exploited, from resource management to parallel execution.
In-page invariants are easy to validate once a page is in the buffer pool but exhaustive verification requires checking all instances of all invariants, e.g., key range relationships between all neighboring leaf nodes. The cross-page invariants are easy to verify with an index-order sweep over the entire key range. If, however, an index-order sweep is not desirable due to parallel execution or due to the limitations of backup media such as tapes, the structural invariants can be verified using algorithms based on aggregation. While scanning B-tree pages in any order, required information is extracted from each page and matched with information from other pages. For example, neighbor pointers match if page $x$ names page $y$ as its successor and page $y$ names page $x$ as its predecessor. Key ranges must be included in the extracted information in order to ensure that key ranges are disjoint and correctly distinguished by the separator key in the appropriate ancestor node. If two leaf nodes share a parent node, this test is quite straightforward; if the lowest common ancestor is further up in the B-tree, and if transitive operations are to be avoided, some additional information must be retained in B-tree nodes.

Figure 6.9 shows a B-tree with neighboring leaf nodes with no shared parent but instead a shared grandparent, i.e., cousin nodes $d$ and $e$. Shaded areas represent records and their keys; two keys are different (equal) if their shading is different (equal). Efficient verification of keys and pointers among cousin nodes $d$ and $e$ does not have an immediate or obvious efficient solution in a traditional B-tree implementation. The potential problem is that there is no easy way to verify that all keys

![Diagram](image_url)
in leaf page $d$ are indeed smaller than the separator key in root page $a$ and that all keys in leaf page $e$ are indeed larger than the separator key in root page $a$. Correct key relationships between neighbors ($b - c$ and $d - e$) and between parents and children ($a - b, a - c, b - d, c - e$) do not guarantee correct key relationships across skipped levels ($a - d, a - e$).

Figure 6.10 shows how fence keys enable a simple solution for the cousin problem in B-tree verification, even if fence keys were originally motivated by write-optimized B-trees [44]. The essential difference to traditional B-tree designs is that page splits not only post a separator key to the parent page but also retain copies of this separator key as high and low “fence keys” in the two post-split sibling pages. Note that separators and thus fence keys can be very short due to prefix and suffix truncation [10]. These fence keys take the role of sibling pointers, replacing the traditional page identifiers with search keys. Fence keys speed up defragmentation by eliminating all but one page identifier that must be updated when a page moves, namely the child pointer in the parent node. Fence keys also assist key range locking since they are key values that can be locked. In that sense, they are similar to traditional ghost records, except that fence keys are not subject to ghost cleanup.

The important benefit here is that verification is simplified and the cousin problem can readily be solved, including “second cousins,” “third cousins,” etc. in B-trees with additional levels. In Figure 6.10, the following four pairs of facts can be derived about the key marked...
by horizontal shading, each pair derived independently from two
pages.

1. From page \(a\), the fact that \(b\) is a level-1 page, and its high
   fence key
2. From page \(a\), the fact that \(c\) is a level-1 page, and its low
   fence key
3. From page \(b\), the fact that \(b\) is a level-1 page, and its high
   fence key; this matches fact 1 above
4. From page \(b\), the fact that \(d\) is a leaf page, and its high fence
   key
5. From page \(c\), the fact that \(c\) is a level-1 page, and its low
   fence key; this matches fact 2 above
6. From page \(c\), the fact that \(e\) is a leaf page, and its low fence
   key
7. From page \(d\), the fact that \(d\) is a leaf page, and its high fence
   key; this matches fact 4 above
8. From page \(e\), the fact that \(e\) is a leaf page, and its low fence
   key; this matches fact 6 above

No match is required between cousin pages \(d\) and \(e\). Their fence
keys are equal due to transitivity among the other comparisons. In
fact, matching facts derived from pages \(d\) and \(e\) could not include page
identifiers, because these pages do not carry the other’s page identifiers.
At best, the following facts could be derived, although they are implied
by the ones above and thus do not contribute to the quality of B-tree
verification:

9. From page \(b\), the fact that a level-1 page has a specific high
   fence key
10. From page \(c\), the fact that a level-1 page has a specific low
    fence key; to match fact 9 above
11. From page \(d\), the fact that a leaf page has a specific high
    fence key
12. From page \(e\), the fact that a leaf page has a specific low
    fence key; to match fact 11 above
The separator key from the root is replicated along the entire seam of neighboring nodes all the way to the leaf level. Equality and consistency are checked along the entire seam and, by transitivity, across the seam. Thus, fence keys also solve the problem of second and third cousins etc. in B-trees with additional levels.

These facts can be derived in any order; thus, B-tree verification can consume database pages from a disk-order scan or even from backup media. These facts can be matched using an in-memory hash table (and possibly hash table overflow to partition files on disk) or they can be used to toggle bits in a bitmap. The former method requires more memory and more CPU effort but can identify any error immediately; the latter method is faster and requires less memory, but requires a second pass of the database in case some facts fail to match equal facts derived from other pages. Moreover, the bitmap method has a miniscule probability of failing to detect two errors that mask each other.

Fence keys also extend local online verification techniques [80]. In traditional systems, neighbor pointers can be verified during a root-to-leaf navigation only for siblings but not for cousins, because the identity of siblings is known from information in the shared parent node but verification of a cousin pointer would require an I/O operation to fetch the cousin’s parent node (also its grandparent node for a second cousin, etc.). Thus, earlier techniques [80] cannot verify all correctness constraints in a B-tree, no matter how many search operations perform verification. Fence keys, on the other hand, are equal along entire B-tree seams, from leaf level to the ancestor node where the key value serves as separator key. A fence key value can be exploited for online verification at each level in a B-tree, and an ordinary root-to-leaf B-tree descent during query and update processing can verify not only siblings with a shared parent but also cousins, second cousins, etc. Two search operations for keys in neighboring leaves verify all B-tree constraints, even if the leaves are cousin nodes, and search operations touching all leaf nodes verify all correctness constraints in the entire B-tree.

For example, two root-to-leaf searches in the index shown in Figure 6.10 may end in leaf nodes $d$ and $e$. Assume that these two root-to-leaf passes occur in separate transactions. Those two searches can verify correct fence keys along the entire seam. In the B-tree
that employs neighbor pointers rather than fence keys as shown in Figure 6.9, the same two root-to-leaf searches could verify that entries in leaf nodes $d$ and $e$ are indeed smaller and larger than the separator key in the root node but they cannot verify that the pointers between cousin nodes $d$ and $e$ are mutual and consistent.

B-tree verification by extracting and matching facts applies not only to traditional B-trees but also to B$^{\text{link}}$-trees and their transitional states. Immediately after a node split, the parent node is not yet updated in B$^{\text{link}}$-tree and thus generates the facts as above. The newly allocated page is a normal page and also generates the facts as above. The page recently split is the only one with special information, namely a neighbor pointer. Thus, a page with a neighbor pointer indicating a recent split not yet reflected in the parent must trigger derivation of some special facts. Since this old node provides the appropriate “parent facts” for the new node, the old node could be called a “foster parent” if one wants to continue the metaphor of parent, child, ancestor, etc.

Figure 6.11 illustrates such a case, with node $b$ recently split. The fact about the low fence key of node $d$ cannot be derived from the (future) parent node $a$. The fact derived from node $d$ must be matched by a fact derived from node $b$. Thus, node $b$ acts like a temporary parent for the new node $d$ not only in terms of the search logic but also during verification of the B-tree structure. Note that the intermediate state in Figure 6.11 could also be used during node removal from a B-tree, again with the ability to perform complete and correct B-tree verification at any time, in any sequence of nodes, and thus on any media.

In addition to verification of a B-tree structure, each individual page must be verified prior to extraction of facts, and multiple B-tree indexes may need to be matched against one another, e.g., a secondary index against the appropriate identifier in the primary index. In-page

Fig. 6.11 Fence keys and verification in a B$^{\text{link}}$-tree.
verification is fairly straightforward, although it might be surprising how many details are worth validating. Matching multiple indexes against one another is very similar to a join operation and all standard join algorithms can be employed. Alternatively, a bitmap can be used, with a miniscule probability of two errors masking each other and with a second pass required if the bitmap indicates that an error exists.

Automatic repair of B-tree indexes is not well studied. Techniques may rely on dropping and rebuilding an entire index, replaying the recovery log for just one page, or adjusting a page to match its related pages. A systematic study of repair algorithms, their capabilities, and their performance would be useful for the entire industry.

- Verification of B-tree indexes protects against software and hardware faults. All commercial database systems provide such utilities.
- B-trees can be verified by a single index-order scan, which may be expensive due to fragmentation.
- Verification based on a disk-order scan requires aggregation of facts extracted from pages. A bit vector filter can speed the process but, in case an inconsistency is found, cannot identify the inconsistency precisely (due to possible hash collisions).
- Query execution may (as a side effect) verify all B-tree invariants if nodes carry fence keys rather than neighbor pointers.

6.8 Summary

In summary, utilities play an important role in the usability and total cost of ownership of database systems. B-trees are unique among index structures because a large number of techniques for efficient utilities are well known and widely implemented. Newly proposed index structures must compete with B-trees for performance and scalability not only during query processing and updates but also during utility operations from index creation to defragmentation and index verification.
Advanced Key Structures

As the prior sections have demonstrated, a large amount of planning and coding is required in order to fully support database index structures such as B-trees. No other index structure has received as much attention from database researchers and software developers. By careful and creative construction of B-tree keys, however, additional indexing capabilities can be enabled with few, if any, modifications within the core B-tree code. The present section surveys several of them.

The present section does not consider indexes on computed columns, i.e., values derived from other columns in the same table and given a name in the database schema. These columns are computed as needed and need not be stored in the main data structure for the table. If there is an index on such a column, however, appropriate key values are stored in this secondary index.

Similarly, the present section does not consider partial indexes, i.e., indexes with fewer entries than rows in the underlying table based on a selection predicate. A typical predicate ensures that only values other than Null are indexed. Both topics, computed columns and partial indexes, are orthogonal to advanced key structures in B-trees.
An earlier discussion (Section 2.5) gave an example of unusual B-tree keys, namely hash indexes implemented as B-trees on hash values. A few small adaptations in the B-tree code emulate the main performance benefits traditionally associated with hash indexes, namely a single I/O in the worst case, direct address calculation within the hash directory, and efficient key value comparisons. The first can be emulated with a very large root page pinned in the buffer pool (very similar to a large hash directory); the other two benefits can be mirrored by appropriate key values including poor man’s normalized keys.

- B-trees with advanced key structures retain all the advantages of B-trees, e.g., theory and implementation of key range locking, optimizations of logging and recovery, efficient index creation, and other utilities for efficient database operations.
- A B-tree on hash values has many advantages over a traditional hash index with comparable performance.

### 7.1 Multi-dimensional UB-trees

By their nature, B-trees support only a single sort order. If multiple key columns are used, they may form a hierarchy of major sort key, minor sort key, etc. In this case, queries that restrict the leading key columns perform better than those that do not. If, however, the key columns represent dimensions in a space, e.g., a geometric space, queries may restrict the leading column only by a range predicate or not at all. Leslie et al. [82] describe sophisticated algorithms for accessing B-trees in those cases.

An alternative approach projects multi-dimensional data onto a single dimension based on space-filling curves. The principal tradeoff in the design of space-filling curves is conceptual and computational simplicity on one hand and preservation of locality and thus search efficiency on the other hand. The simplest construction of a space-filling curve first maps each dimension into an unsigned integer and then interleaves individual bits from these integers. When drawn as a line in a 2-dimensional space, this space-filling curve resembles nested Z shapes, which is why it is also called the z-order. This is the design underlying
multi-dimensional indexing and query processing in Probe [106] and Transbase [109]. Alternatives to this Morton curve include the Hilbert curve and others.

Figure 7.1 (copied from [106]) illustrates z-order curves. The original x- and y-coordinates have 3 bits and therefore 8 distinct values; the interleaved z-values contain 6 bits and therefore 64 points. The “Z” shape is repeated at 3 scales. The technique applies to any number of dimensions, not only 2-dimensional spaces as shown in Figure 7.1.

UB-trees are B-tree indexes on such z-values. Each point and range query against the original dimensions is mapped to the appropriate intervals along the z-curve. The crucial component of this mapping is determining the z-values at which the z-curve enters and exits the multi-dimensional range defined by the query predicate. Known algorithms are linear in the number of original dimensions, their resolution (i.e., the number of bits in the z-values), and the number of entry and exit points [109].

In addition to points in multi-dimensional space, space-filling curves, z-order mappings, and UB-trees can index multi-dimensional rectangles (boxes) by treating start and end points as separate dimensions. In other words, UB-trees can index not only information about points but also about intervals and rectangles. Both space and time (including intervals) can be indexed in this way. For moving objects, location and speed (in each spatial dimension) can be treated as
separate dimensions. Finally, even precision in location or speed can be indexed, if desired. Unfortunately, indexing based on space-filling curves loses effectiveness with the number of dimensions, just as the performance of traditional B-tree applications suffers when a B-tree contains many columns but only a few of them are specified in a query predicate.

- Z-values (or other space-filling curves) provide some multi-dimensional indexing with all the advantages of B-trees.
- The query performance of specialized multi-dimensional indexes is probably better, but load and update performance of B-trees are not easy to match.

### 7.2 Partitioned B-trees

As discussed earlier in the section on bulk insertions and illustrated in Figure 6.6, the essence of partitioned B-trees [43]¹ is to maintain partitions within a single B-tree, by means of an artificial leading key field. Partitions and the artificial leading key field are hidden from the database user. They exist in order to speed up large operations on B-trees, not to carry any information. Partitions are optimized using the merge step well known from external merge sort. By default, the same single value appears in all records in a B-tree, and most of the specific techniques rely on exploiting multiple alternative values, but only temporarily. If a table or index is represented in multiple B-trees, the artificial leading key field should be defined separately for each such B-tree.

The leading artificial key column effectively defines partitions within a single B-tree. Each existing distinct value implicitly defines a partition, and partitions appear and vanish automatically as records are inserted and deleted. The design differs from traditional horizontal partitioning using a separate B-tree for each partition in an important way: Most advantages of the design depend on partitions (or distinct values in the leading artificial key column) being created and removed very dynamically. In a traditional implementation of partitioning (using

¹The text in this subsection is adapted from this reference.
Advanced Key Structures

multiple B-trees), creation or removal of a partition is a change of the table’s schema and catalog entries, which requires locks on the table’s schema or catalog entries and thus excludes concurrent or long-running user accesses to the table, as well as forcing recompilation of cached query and update plans. If partitions within a single B-tree are created and removed as easily as inserting and deleting rows, smooth continuous operation is relatively easy to achieve. It is surprising how many problems this simple technique can help address in data management software and its real-world usage.

First, it permits putting all runs in an external merge sort into a single B-tree (with the run number as the artificial leading key field), which in turn permits improvements to asynchronous read-ahead and to adaptive memory usage. In SAN and NAS environments, hiding latency by exploiting asynchronous read-ahead is important. With striped disks, forecasting multiple I/O operations is important. Finally, in very large online databases, the ability to dynamically grow and shrink resources dedicated to a single operation is very important, and the proposed changes permit doing so even to the extremes of pausing an operation altogether and of letting a single operation use a machine’s entire memory and entire set of processors during an otherwise idle batch window. While sorting is used to build B-tree indexes efficiently and B-trees are used to avoid the expense of sorting and to reduce the expense of searching during query processing, the mutually beneficial relationship between sorting and B-trees can go substantially further.

Second, partitioned B-trees can substantially reduce, by at least a factor of two, the wait time before a newly created index is available for query answering. While the initial form of an index does not perform as well as the final, fully optimized index or a traditional index, at least it is usable by queries and permits replacing table scans with index searches, resulting in better query response time as well as a smaller “locking footprint” and thus a reduced likelihood of deadlocks. Moreover, the index can be improved incrementally from its initial form to its final and fully optimized form, which is very similar to the final form after traditional index creation. Thus, the final index is extremely similar in performance to indexes created offline or with traditional online methods; the main difference is cutting in half (or better) the
delay between a decision to create a new index and its first beneficial impact on query processing.

Third, adding a large amount of data to a large, fully indexed data warehouse so far has created a dilemma between dropping and rebuilding all indexes or updating all indexes one record at a time, implying random insertions, poor performance, a large log volume, and a large incremental backup. Partitioned B-trees resolve this dilemma in most cases without special new data structures. A load operation simply appends a number of new partitions to each affected index; the size of these partitions is governed by the memory allocation for the in-memory run generation during the load operation. Updates (both insertion and deletions) can be appended to an existing B-tree in one or multiple new partitions, to be integrated into the main partition at the earliest convenient time, at which time deletions can be applied to the appropriate old records. Appending partitions is, of course, yet another variation on the theme of differential files [117]. Batched maintenance in a partitioned B-tree reduces the overall update time; in addition, it can improve the overall space requirements if pages of the main partition are filled completely with compressed records; and it may reduce query execution times if the main partition remains unfragmented and its pages optimized for efficient search, e.g., interpolation search.

While a partitioned B-tree actually contains multiple partitions, any query must search all of them. It is unlikely (and probably not even permitted by the query syntax) that a user query limits itself to a subset of partitions or even a single one. On the other hand, a historic or “as of” query might map to a single partition even when newer partitions are already available. In general, however, all existing partitions must be searched. As partitioning is implemented with an artificial leading key field in an otherwise standard B-tree implementation, this is equivalent to a query failing to restrict the leading column in a traditional multi-column B-tree index. Efficient techniques for this situation are known and not discussed further here [82].

Partitions may remain as initially saved or they may be merged. Merging may be eager (e.g., merging as soon as the number of partitions reaches a threshold), opportunistic (e.g., merging whenever there is idle time), or lazy (e.g., merging key ranges required to answer actual
queries). The latter is called adaptive merging [53]. Rather than merging partitions in preparation of query processing, merging can be integrated into query execution, i.e., be a side effect of query execution. Thus, even if key ranges are left as parameters in query predicates, this technique merges only key ranges actually queried. All other key ranges remain in the initial partitions. No merge effort is spent on them yet they are ready for query execution and index optimization should the workload and its access pattern change over time.

- In partitioned B-trees, partitions are identified by an artificial leading key field. Partitions appear and disappear simply by insertion and deletion of B-tree entries with appropriate key values, without catalog updates.
- Partitioned B-trees are useful for efficient sorting (e.g., deep read-ahead), index creation (e.g., early query processing), bulk insertion (append-only data capture with in-memory sorting), and bulk deletion (victim preparation).
- Query performance equals that of traditional B-trees once all partitions have been merged, which is the default state.

### 7.3 Merged Indexes

As others have observed, “optimization techniques that reduce the number of physical I/Os are generally more effective than those that improve the efficiency in performing the I/Os” [70]. It is a common belief that clustering related records requires pointers between records. An example relational database management system with record clustering is Starburst [20], which uses hidden pointers between related records and affects their automatic maintenance during insertions, deletions, and updates. The technique serves only tables and their primary storage structures, not secondary indexes, and it requires many-to-one relationships defined with foreign key integrity constraints.

The desirability of clustering secondary indexes is easily seen in a many-to-many relationship such as “enrollment” as many-to-many relationship between “courses” and “students.” In order to support table-to-table, index-to-index, and record-to-record navigation both from students to courses and from courses to students, the enrollment
7.3 Merged Indexes

Table requires at least two indexes, only one of which can be the primary index. For efficient data access in both directions, however, it would be desirable to cluster one enrollment index with student records and one enrollment index with course records.

Merged indexes \cite{49}\textsuperscript{2} are B-trees that contain multiple traditional indexes and interleave their records based on a common sort order. In relational databases, merged indexes implement “master-detail clustering” of related records, e.g., orders and order details. Thus, merged indexes shift de-normalization from the logical level of tables and rows to the physical level of indexes and records, which is a more appropriate place for it. For object-oriented applications, clustering can reduce the I/O cost for joining rows in related tables to a fraction compared to traditional indexes, with additional beneficial effects on buffer pool requirements.

Figure 7.2 shows the sort order of records within such a B-tree. The sort order alone keeps related records co-located; no additional pointers between records are needed. In its most limited form, master-detail clustering combines two secondary indexes, e.g., associating two lists of row identifiers with each key value. Alternatively, master-detail clustering may merge two primary indexes but not admit any secondary indexes. The design for merged indexes accommodates any combination of primary and secondary indexes in a single B-tree, thus enabling clustering of entire complex objects. Moreover, the set of tables, views, and indexes can evolve without restriction. The set of clustering columns can also evolve freely. A relational query processor can search and

\begin{verbatim}
...  
Order 4711, Customer "Smith", ...  
Order 4711, Line 1, Quantity 3, ...  
Order 4711, Line 2, Quantity 1, ...  
Order 4711, Line 3, Quantity 9, ...  
Order 4712, Customer "Jones", ...  
Order 4712, Line 1, Quantity 1, ...  
...
\end{verbatim}

Fig. 7.2 Record sequence in a merged index.

\textsuperscript{2}The text of this subsection is adapted from this reference.
update index records just as in traditional indexes. With these abilities, the proposed design may finally bring general master-detail clustering to traditional databases together with its advantages in performance and cost.

In order to simplify design and implementation of merged indexes, a crucial first step is to separate implementation of the B-tree structure from its contents. One technique is to employ normalized keys, discussed and illustrated earlier in Figure 3.4, such that the B-tree structure manages only binary records and binary keys. In merged indexes, the mapping from multi-column keys to binary search keys in a B-tree is a bit more complex than in traditional indexes, in particular if adding and removing any index at any time is desired and if individual indexes may have different key columns. Thus, it is essential to design a flexible mapping from keys in the index to byte strings in the B-tree. A tag that indicates a key column’s domain and precedes the actual key fields, as shown in Figure 7.3, can easily achieve this. In other words, when constructing a normalized key for a merged index, domain tags and field values alternate up to and including the identifier for the index.

In practice, different than illustrated in Figure 7.3, a domain tag will be a small number, not a string. It is possible to combine the domain tag with the Null indicator (omitted in Figure 7.3) such that the desired sort order is achieved yet actual values are stored on byte boundaries. Similarly, the index identifier will be a number rather than a string.

<table>
<thead>
<tr>
<th>FIELD VALUE</th>
<th>FIELD TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Customer identifier”</td>
<td>Domain tag</td>
</tr>
<tr>
<td>123</td>
<td>Data value</td>
</tr>
<tr>
<td>“Order number”</td>
<td>Domain tag</td>
</tr>
<tr>
<td>4711</td>
<td>Data value</td>
</tr>
<tr>
<td>“Index identifier”</td>
<td>Domain tag</td>
</tr>
<tr>
<td>“Orders.OrderKey”</td>
<td>Identifier value</td>
</tr>
<tr>
<td>“2006/12/20”</td>
<td>Data value</td>
</tr>
<tr>
<td>“Urgent”</td>
<td>Data value</td>
</tr>
<tr>
<td>…</td>
<td>Data values</td>
</tr>
</tbody>
</table>

Fig. 7.3 B-tree record in a merged index.
Domain tags are not required for all fields in a B-tree record. They are needed only for key columns, and more specifically only for those leading key columns needed for clustering within the merged index. Following these leading key columns is a special tag and the identifier of the individual index to which the record belongs. For example, in Figure 7.3, there are only 2 domain tags for key values plus the index identifier. If there never will be any need to cluster on the line numbers in order details, only leading key fields up to order number require the domain tag. Thus, the per-record storage overhead for merged indexes is small and may indeed be hidden in the alignment of fields to word boundaries for fast in-memory processing. An overhead of 2–4 single-byte domain tags per record may prove typical in practice.

- Merging multiple indexes into a single B-tree provides master-detail clustering with all the advantages of B-trees. A single B-tree may contain any number of primary and secondary indexes of any number of tables.
- The B-tree key alternates domain tags and values up to and including the index identifier.
- Merged indexes permit tables in traditional normal forms with the performance of free denormalization.
- Merged indexes are particularly valuable in systems with deep storage hierarchies.

7.4 Column Stores

Columnar storage has been proposed as a performance enhancement for large scans and therefore for relational data warehouses where ad-hoc queries and data mining might not find appropriate indexes. Lack of indexes might be due to complex arithmetic expressions in query predicates or to unacceptable update and load performance. The basic idea for columnar storage is to store a relational table not in the traditional format based on rows but in columns, such that scanning a single column can fully benefit from all the data bytes in a page fetched from disk or in a cache line fetched from memory.

\[\text{Some text in this section is copied from [47].}\]
If each column is sorted by the values it contains, values must be tagged with some kind of logical row identifier. Assembling entire rows requires join operations, which may be too slow and expensive. In order to avoid this expense, the columns in a table must be stored all in the same order. This order might be called the order of the rows in the table, since no one index determines it, and B-trees can realize column storage using tags with practically zero additional space.

These tags are in many ways similar to row identifiers, but there is an important difference between these tags and traditional row identifiers: tag values are not physical but logical. In other words, they do not capture or represent a physical address such as a page identifier, and there is no way to calculate a page identifier from a tag value. If a calculation exists that maps tag values to row address and back, this calculation must assume maximal length of variable-length columns. Thus, storage space would be wasted in some or all of the vertical partitions, which would contradict the goal of columnar storage, namely very fast scans.

Since most database management systems rely on B-trees for most or all of their indexes, reuse and adaptation of traditional storage structures mean primarily adaptation of B-trees, including their space management and their reliance on search keys. In order to ensure that rows and their columns appear in the same sequence in all B-trees, the search key in all indexes must be the same. Moreover, in order to achieve the objectives, the storage requirement for search keys must be practically zero, which seems rather counter-intuitive.

The essence of the required technique is quite simple. Rows are assigned tag values sequentially numbered in the order in which they are added to the table. Note that tag values identify rows in a table, not records in an individual partition or in an individual index. Each tag value appears precisely once in each index, i.e., it is paired with one value for each column in the table. All vertical partitions are stored in B-tree format with the tag value as the leading key. The important aspect is how storage of this leading key is reduced to practically zero.

\[^4\]Note that a materialized view may be stored in a different sort order. If so, a row's position in the materialized view is, of course, not useful for retrieving additional information in the base table.
The page header in each B-tree page stores the lowest tag value among all entries on that page. The actual tag value for each individual B-tree entry is calculated by adding this value and the slot number of the entry within the page. There is no need to store the tag value in the individual B-tree entries; only a single tag value is required per page. If a page contains tens, hundreds, or even thousands of B-tree entries, the overhead for storing the minimal tag value is practically zero for each individual record. If the size of the row identifier is 4 or 8 bytes and the size of a B-tree node is 8 KB, the per-page row identifier imposes an overhead of 0.1% or less.

If all the records in a page have consecutive tag values, this method not only solves the storage problem but also reduces “search” for a particular key value in the index to a little bit of arithmetic followed by a direct access to the desired B-tree entry. Thus, the access performance in leaf pages of these B-trees can be even better than that achieved with interpolation search or in hash indexes.

Figure 7.4 illustrates a table with 2 columns and 3 rows and columnar storage for it. The values in parentheses indicate row identifiers or tags. The right part of the diagram shows two disk pages, one for each column. The column headers of each page (dashed lines) show a row count and the lowest tag in the page.

The considerations so far have covered only the B-tree’s leaf pages. Of course, the upper index pages also need to be considered. Fortunately, they introduce only moderate additional storage needs. Storage needs in branch nodes is determined by the key size, the pointer size, and any overhead for variable-length entries. In this case, the key size is equal to that of row identifiers, typically 4 or 8 bytes. The pointer size is equal to a page identifier, also typically 4 or 8 bytes. The overhead for managing variable-length entries, although not strictly needed for the B-tree indexes under

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Fig. 7.4 Table and columnar storage.
consideration, is typically 4 bytes for a byte offset and a length indicator. Thus, the storage needs for each separator entry is 8 to 20 bytes. If the node size is, for example, 8 KB, and average utilization is 70%, the average B-tree fan-out is 280 to 700. Thus, all upper B-tree pages together require disk space less than or equal to 0.3% of the disk space for all the leaf pages, which is a negligible in practice.

Compared to other schemes for storing vertical partitions, the described method permits very efficient storage of variable-length values in the same order across multiple partitions. Thus, assembly of entire rows in a table is very efficient using a multi-way merge join. In addition, assembly of an individual row is also quite efficient, because each partition is indexed on the row identifier. All traditional optimizations of B-tree indexing apply, e.g., very large B-tree nodes and interpolation search. Note that interpolation search among a uniform data distribution is practically instant.

Figure 7.5 illustrates the value of B-trees for columnar storage, in particular if column values can vary in size either naturally or due to compression. The alphabet strings are actual values; the dashed boxes represent page headers with record count and lowest tag value. The upper levels of the B-tree indicate the lowest tag value in their respective subtrees. Leaf pages with varying record counts per page can readily be managed and assembly of individual rows by look-up of tags can be very efficient. Depending on the distributions of key values and their sizes, further compression may be possible and is often employed in relational database management system with columnar storage.

- With appropriate compression adapting run-length encoding to series of row identifiers, columnar storage may be based on B-trees.

Fig. 7.5 Columnar storage using a B-tree.
7.5 Large Values

In addition to B-trees containing many records, each smaller than a single leaf page, B-trees can also represent large binary objects or byte strings with many bytes. In that case, the leaf nodes contain data bytes and the branch nodes contain sizes or offsets. The data bytes in the leaf nodes can be divided into records as in traditional B-tree indexes or they can be without any additional structure, i.e., byte strings. In the latter case, most or all size information is kept in the branch nodes. Sizes or offsets serve as separator keys within branch nodes. In order to minimize effort and scope of update operations, in particular insertion and deletion of individual bytes or of substrings, sizes and offsets are counted locally, i.e., within a node and its children, rather than globally within the entire large binary object.

Figure 7.6, adapted from [18, 19], illustrates these ideas. In this example, the total size of the object is 900 bytes. The tree nodes at the leaf level indicate byte ranges. The values are shown in the leaf nodes only for illustration here; instead, the leaf nodes should contain the actual data bytes and possibly a local count of valid bytes. The branch nodes of the tree indicate sizes and offsets within the large object. Key values in the left half of the figure and in the root node are fairly obvious. The most interesting entries in this tree are the key values in the right parent node. They indicate the count of valid bytes in their child nodes; they do not indicate the position of those bytes within the entire object. In order to determine absolute positions, one needs to add the key values from the root to the leaf. For example, the absolute position of the left-most byte in the right-most leaf node is \( 421 + 365 = 786 \).

Fig. 7.6 A large string as a tree of small sub-strings.
Similarly, search may use binary search (or even interpolation search) within a node but must adjust for key values in upper nodes. For example, a root-to-leaf traversal in search of byte 698 may use a binary search in the right parent node but only after subtracting the key value in the root (421) from the search key (698), i.e., searching for key value $698 - 421 = 277$ within the right parent node and finding the interval between 192 and 365. With that leaf, local byte position $277 - 192 = 85$ corresponds to global byte position 698.

Insertion or deletion of some bytes in some leaf node affect only the branch nodes along one root-to-leaf path. For example, deletion of 10 bytes at position 30 reduces the values 120, 282, and 421 in Figure 7.6. Although such a deletion changes the absolute positions of the data bytes in the right subtree, the right parent node and its children remain unchanged. Similarly, insertion or deletion of an entire leaf node and its data bytes affect only along a single root-to-leaf path. Maintenance of the key values along the path can be part of the initial root-to-leaf traversals in search of the affected leaves or it can follow maintenance of the data bytes in the leaf nodes. All nodes can be kept 50–100% full using algorithms very similar to traditional B-trees. Aggressive load balancing among sibling nodes can delay node splits. A B-tree representing a large object enables such a merge-before-split policy more than a standard B-tree because a parent contains sufficient information to decide whether or not sibling leaves are promising candidates for load balancing.

- With relative byte offsets as key values, a B-tree can be adapted to store large objects spanning many pages, even permitting efficient insertions and deletions of byte ranges.

### 7.6 Record Versions

Many applications require notions of “transaction time” and “real-world time,” i.e., information about when a fact has been inserted into the database and when the fact is valid in the real world. Both notions of time enable what is sometimes called “time travel,” including “what result would this query have had yesterday?” and “what is
known now about yesterday’s status?” Both types of queries and their results can have legal importance.5

The former type of query is also used for concurrency control. In those schemes, the synchronization point of each transaction is its start time. In other words, transactions may run in serializable transaction isolation but the equivalent serial schedule orders the transactions by their start times, not by their end times as in common locking techniques. For long-running transactions, it may be required to provide an out-of-date database state. This is often achieved by retaining old versions of updated records. Thus, the name for this technique is multi-version concurrency control [15]. Closely related is the concept of snapshot isolation [13, 32].

Since most transactions in most applications require the most up-to-date state, one implementation technique updates database records in place and, if required for an old transaction, rolls back the data page using a second copy in the buffer pool. The rollback logic is very similar to that for transaction rollback, except that it is applied to a copy of the data page. Transaction rollback relies on the chain of log records for each transaction; efficient rollback of a data page requires a chain of log records pertaining to each data page, i.e., each log record contains a pointer to the prior log record of the same transaction and another pointer to the prior log record pertaining to the same data page.

An alternative design relies on multiple actual records per logical record, i.e., versions of records. Versioning might be applied to and managed in a table’s main data structure only, e.g., the primary index, or it can be managed in each data structure, i.e., each secondary index, each materialized view, etc. If a design imposes substantial overheads in terms of space or effort, the former choice may be more appropriate. For greatest simplicity and uniformity of data structures and algorithms, it seems desirable to reduce overheads such that versioning can be applied in each data structure, e.g., each B-tree index in a database.

5Michael Carey used to explain the need for editing large objects in a database with the following play on US presidential politics of the 1970s: “Suppose you have an audio object representing a recorded phone conversation and you feel the need to erase 18 minutes in the middle of it…” Playing on US presidential politics of the 1980s, one might say here: “What did the database know, and when did he know it?”
Figure 7.7 illustrates how some designs for record versioning tag each version record with the version’s start time, its end time, and a pointer to the next record in the chain of versions. In the example, changing a single small field to reflect a worker’s increased hourly wage requires an entire new record with all fields and tags. In a secondary index with few fields in each index entry, three additional fields impose a high overhead. By an appropriate modification of B-tree keys, however, two of these three fields can be avoided. Moreover, new versions can require much less space than complete new copies of the versioned record.

Specifically, if the start time provides the least significant part of a B-tree key, all versions of the same logical record (with the same user-defined key value) are neighbors in the sequence of records. Pointers or a version chain are not required as the sequence of versions is simply the sequence of B-tree entries. End times can be omitted if one version’s start time is interpreted as the prior version’s end time. Upon deletion of a logical record, a ghost record is required with the appropriate start time. This ghost record must be protected as long as it carries information about the logical record’s history and final deletion.

Figure 7.8 illustrates the design. The record keys are underlined. Start times are the only additional required field in version records, avoiding 2 of 3 additional fields required by the simplest design for version records with timestamps and naturally ensuring the desired placement of version records.
7.6 Record Versions

Start times can be compressed by storing, in each B-tree leaf, a base time equal to the oldest record version within the page. In that case, start times are represented within each record by the difference from the base time, which hopefully is a small value. In other words, an additional key field appended to the B-tree key can enable record versioning with a small number of bytes, possibly even a single byte.

Moreover, record contents can be compressed by explicitly storing only the difference between a version and its predecessor. For fastest retrieval and assembly of the most recent version, version records should store the difference between a version and its successor. In this case, retrieval of an older version requires multiple records somewhat similar to “undo” of log records. Alternatively, actual log records could be used, leading to a design similar to the one based on rollback of pages but applied to individual records.

Figure 7.9 illustrates these techniques. A field in the page header indicates the time of the oldest version record currently on the page. The individual records store the difference from this base time rather than a complete timestamp. Moreover, unchanged field values are not repeated. The order of version records is such that the current record is most readily available and older versions can be constructed by a local scan in forward direction. In the diagram, the absolute value of the prior value is shown, although for many data types, a relative value could be used, e.g., “−$3.” Further optimizations and compression, e.g., prefix truncation, may be employed as appropriate.

If “time travel” within a database can be limited, for example to one year into the past, all version records older than this interval can be interpreted as ghost records. Therefore, they are subject to removal and space reclamation just like traditional ghost records, with all rules

```
2011-1-1 0:00

Doc, John +3M 123 Easy Street $18

Doc, John +0  wage $15
```

Fig. 7.9 Version records with compression.
and optimizations for locking and logging during ghost removal. When all other versions for a logical record have been thus removed, the ghost record indicating deletion of a logical record can also be removed and its space can be reclaimed.

If versioning in secondary indexes is independent from versioning in the table’s primary index, pointers in a secondary index can refer only to the appropriate logical record (unique user-defined key value) in the primary index. The transaction context must provide a value for the remaining key field in the primary index, i.e., the time value for which the record from the primary index is desired. For example, a secondary index might contain two versions due to an update two days ago, whereas the primary index might contain three versions due to an additional update of a non-indexed field only one day ago. A transaction might query the index as of four days ago and determine that the old index entries satisfies the query predicate; following the pointer from the secondary index into the primary index leads to all three version records, among which the transaction chooses the one valid four days ago. If most transactions require the most recent record version, and if forward scans are more efficient than backward scans, it might be useful to store this record first among all versions, i.e., to sort version records by decreasing start time as shown in Figure 7.9.

-Appending a version number to each key value and compressing neighboring records as much as possible turns B-trees into a version store efficient in space (storage) and time (query and update).

7.7 Summary

In summary, B-trees can solve a wide variety of indexing, data movement, and data placement problems. Not every issue requires changes in the index structure; very often, a carefully chosen key structure enables new functionality in B-tree indexes. A B-tree with a newly designed key structure retains the traditional operational benefits of B-trees, e.g., index creation by sorting, key range locking, physiological logging, and more. Thus, when new functionality is required, enabling
this functionality by a new key structure for B-tree indexes may be
easier than definition and implementation of a new index structure.

Advanced key structures can be derived from user-defined keys in
various ways. The preceding discussion includes adding artificial pre-
fixes (partitioned B-trees) or suffixes (record versions), interleaving
user keys with each other (UB-trees) or with artificial key components
(merged indexes). While this list of alternative key enhancements might
seem exhaustive, new key structures will probably be invented in the
future in order to expand the power of indexing without mirroring the
effort already spent on B-trees in form of research, development, and
testing.

Obviously, many of the techniques discussed above can be combined.
For example, a merged index can hold multiple complex objects by
combining object or attribute identifiers with offsets within individual
objects. Also, an artificial leading key field can be added to UB-trees
or to merged indexes, thus combining efficient loading and incremental
index optimization with multi-dimensional indexing or master-detail
clustering. Similarly, merged indexes may contain (and thus cluster)
not only traditional records (from various indexes) but also bitmaps or
large fields. The opportunities for combinations seem endless.
In summary, the core design of B-trees has remained unchanged in 40 years: balanced trees, pages or other units of I/O as nodes, efficient root-to-leaf search, splitting and merging nodes, etc. On the other hand, an enormous amount of research and development has improved every aspect of B-trees including data contents such as multi-dimensional data, access algorithms such as multi-dimensional queries, data organization within each node such as compression and cache optimization, concurrency control such as separation of latching and locking, recovery such as multi-level recovery, etc.

Among the most important techniques for B-tree indexes seem to be:

- Efficient index creation using sorting and append-only B-tree maintenance
- Space management within nodes with variable-size records
- Normalized keys
- Prefix and suffix truncation
- Data compression including order-preserving compression
- Fence keys
• Separation of logical contents and physical representation — user transactions versus system transactions, locking versus latching, etc.
• Key range locking for true synchronization atomicity (serializability)
• B\textsuperscript{link}-trees with temporary “foster parents”
• Ghost records for deletion and insertion
• Non-logged (yet transactional) index operations, in particular index creation
• Covering indexes and index intersection during query processing.
• Sorting search keys prior to repeated search (e.g., in index nested loops join) for performance and scalability
• Optimized update plans, index-by-index updates
• Bulk insertion (and deletion) for incremental loading
• B-tree verification

Gray and Reuter believed that “B-trees are by far the most important access path structure in database and file systems” [59]. It seems that this statement remains true today. B-tree indexes are likely to gain new importance in relational databases due to the advent of flash storage. Fast access latencies permit many more random I/O operations than traditional disk storage, thus shifting the break-even point between a full-bandwidth scan and a B-tree index search, even if the scan has the benefit of columnar database storage. We hope that this tutorial and reference of B-tree techniques will enable, organize, and stimulate research and development of B-tree indexing techniques for future data management systems.
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