Journal of the Korean Physical Society, Vol. 50, No. 1, January 2007, pp. 196∼200

Stable Aggregates in the Dynamics of a Competing Population

C. H. Yeung,* Y. P. Ma† and K. Y. Michael Wong‡

Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

(Received 11 August 2006)

We propose a dynamical model of a competing population whose agents have a tendency to balance their decisions in time. The model is applicable to financial markets in which the agents trade with finite capital or to other multiagent systems, such as routers in communication networks attempting to transmit multiclass traffic in a fair way. We find an oscillatory behavior of the model which is explainable by the segregation of agents into two groups. Each group remains winning over *epochs*. The aggregation of *smart* agents is able to explain the lifetime distribution of epochs to 8 decades of probability.

PACS numbers: 02.50.-r, 05.40.-a, 89.20.-a
Keywords: Competing population, Minority game, Dynamical transitions, Epochs, Agent aggregation

I. INTRODUCTION

Many natural and artificial systems involve interacting agents, each making independent decisions to pursue their own short-term objectives, but globally exhibiting long-time correlations beyond the memory sizes or the action cycles of the individuals [1–3]. Examples include the price fluctuations in financial markets [4], traffic patterns on the Internet [5], and the lifespan and metabolic time scales of living organisms. It is interesting to consider the extent to which the inherent properties of the systems can affect the temporal behavior. Correlations beyond individual memory sizes can be consequences of interactions among agents. For example, in a competing population, some agents may find it advantageous to correlate their decisions over an extended period of time. However, due to the dynamics of competition, a group may not be advantageous forever; it only dominates in a certain *epoch*. A very interesting question is how the lifetimes of the winning epochs of the group are related to the dynamics of individual agents.

In this paper, we describe a model of a competing population in which agents aggregate into groups. Agents in the same group make synchronized decisions. In order to maximize their own profit, individual agents employ adaptive strategies occasionally to switch to the winning group. Epochs of winning groups are formed, which are sustained for certain lifetimes [6]. The model is most applicable to financial markets in which agents trade with finite capital, but can be extended to other multiagent systems, such as routers in communication networks attempting to transmit multiclass traffic in a fair way.

As we shall see, the lifetimes of the epochs in our model are closely related to the aggregation of smart agents. There are previous works considering the effects of the attributes of agents. The model of Reents *et al.* [7] corresponds to ours with smart agents only, and there is no differentiation of agent attributes. Marsili considered games with both minority- and majority-seeking agents, but mainly focused on the steady-state behaviour of the game [8]. The model of Xie *et al.* [9] is roughly similar to ours in the sense that the intelligent agents can gain profits from the so-called producers, noisy traders, and conventional agents, but the analysis therein is not applicable to our model due to the absence of producers, and the issue of epoch lifetimes was not considered.

II. SHORT MEMORY LIMIT OF THE RESTORING MINORITY GAME

Specifically, we consider the short memory limit of the Restoring Minority Game (RMG) introduced in Ref. 6. The model consists of a population of N agents competing to maximize their individual utility while striving to maintain a balance of their capital, N being odd. At each time step t, agent i makes a bid $b_i(t) = 1$ or 0, and the minority group among the N agents wins. "1" and "0" may represent "buy" and "sell" decisions in a financial market. Agents make decisions with concerns for both finite capital and adaptive strategies. At each time step, each agent makes a decision according to her capital or strategies with probabilities of k or $1-k$, respectively.

When k is close to 1 such that agents have much stronger concern for capital than adaptive strategies, the

*E-mail: phbill@ust.hk;
†E-mail: yiping.m@gmail.com;
‡E-mail: phkywong@ust.hk

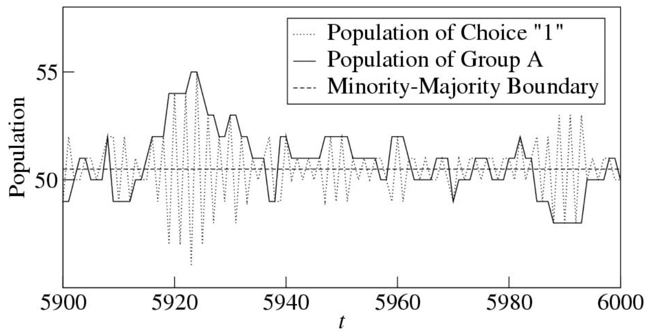

Fig. 1. Population sizes of the buyers and group A for a particular sample ($N = 101$, $m = 1$, $s = 2$, $R = 51$, $k = 0.99$, $\alpha = 0.4$, 10,000 steps).

behavior of the system becomes independent of the specific strategies adopted by the agents [6]. This is the case to be considered in this paper. When the agents have very limited capital (or stock), most often they have to buy immediately after they have sold (or sell immediately after they have bought). This immediate *restoring* decision corresponds to the short memory regime of the RMG. Thus, when k is close to 1, we are considering the *strongly restoring limit* of the RMG.

Occasionally with a probability of $1 - k$, agents have chances to pause their buy-and-sell cycle and to make decisions according to their adaptive strategies, such as those used in the Minority Game (MG) [3] or the Evolutionary Minority Game (EMG) [10]. In the MG, strategies are binary functions mapping an m-bit signal to an output bit, the signal here being the winning bits of the most recent m steps. Before the game starts, each agent randomly picks s strategies. At each time step, the *cumulative payoffs* of strategies that give a correct prediction increase by 1 while the payoffs of those predicting incorrectly decrease by 1. Each agent then follows the strategy with the highest cumulative payoff among those she owns. All agents initially hold equal value of capital and stocks, and the cumulative payoffs of strategies are randomized with variance R [11]. The EMG differs from the MG in details, but shares the same feature that the adopted strategies evolve according to historical performances. In this paper, we focus on the case of the MG.

III. EMERGENCE OF EPOCHS

As Fig. 1 shows, the buyer population is essentially oscillating with period 2. These oscillations arise from the intentions of the agents to balance their budget in consecutive steps, causing them to segregate into two groups, A and B, analogous to the crowd-anticrowd picture of multiagent systems [12]. Group A consists of agents making alternating buy and sell bids at odd and even time steps, respectively, and group B consists of agent making opposite alternating bids.

At each time step, an average of $(1 - k)N \equiv \tilde{k}$ agents make decisions according to adaptive strategies and, thus, have a chance to switch from group A to B or vice versa. Then, groups A and B change from majority to minority, or vice versa, on a time scale longer than the period-2 oscillations. The emergence of these multiple time scales is illustrated in Fig. 1, in which occasional phase slips in the buyer population signify switches from group A being the winners to group B or vice versa. This corresponds to instants in Fig. 1 where the population of group A crosses the minority-majority boundary. The lifetime of an *epoch* is the time during for which group A or B wins continuously.

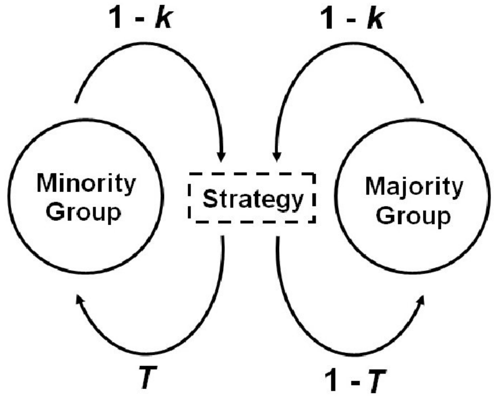

Fig. 2. Mean-rate approximation.

IV. MEAN-RATE APPROXIMATION

We first analyze the dynamics in the *mean-rate* approximation as shown in Fig. 2. At each time step, each agent pauses the restoring decision and uses her adaptive strategy with probability $1 - k$. Let T be the average probability that a strategy makes a minority decision. Then, the probability that an agent switches sides is $(1-k)T$ from majority to minority and $(1-k)(1-T)$ in the opposite direction. The master equation for the distribution of the number N_{A} of agents in group A can then be solved numerically or by using the Monte Carlo method.

To calculate the average probability T, we consider the case $m = 1$. Each agent holds s strategies randomly drawn from the following four functions f: (1) *contrarian*: $f(b) = 1 - b$; (2) *buyer*: $f(b) = 1$; (3) *seller*: $f(b) = 0$; (4) *follower*: $f(b) = b$. Since the history is dominated by a series of alternating "0" and "1", the contrarian strategy acquires the highest cumulative payoffs, and the follower strategy the lowest. The buyer and seller strategies predict correctly half of the time in an epoch, and their cumulative payoffs are intermediate.

In an epoch, the agents making minority strategic de-

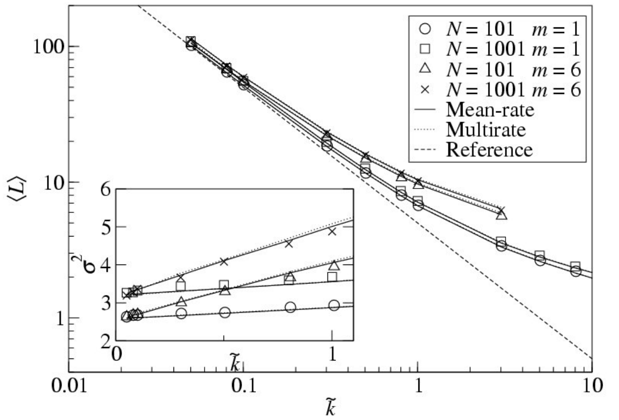

Fig. 3. Lifetime $\langle L\rangle$ as a function of $\tilde{k}$ ($s = 2$, $R = 51$, $\alpha = 0.4$, 30,000 steps with 2,000 samples) compared with the mean-rate and multirate approximations. Reference: $\langle L\rangle = 5\tilde{k}^{-1}$. Inset: Variance σ^2 as a function of $\tilde{k}$.

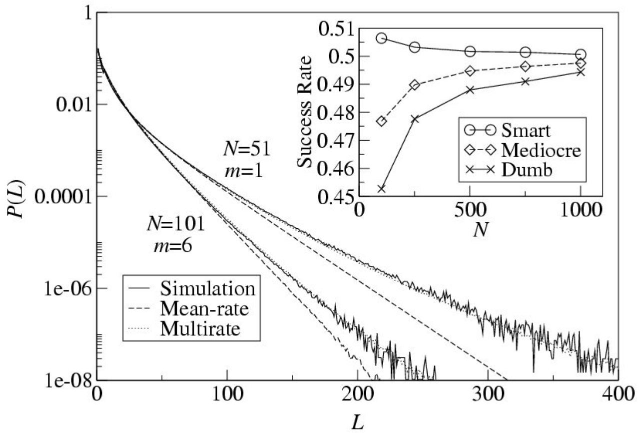

Fig. 4. Lifetime distribution ($s = 2$, $R = 51$, $k = 0.99$, $\alpha = 0.4$, 30,000 steps with 30,000 samples). Inset: Success rate versus N for $\tilde{k} = 0.101$ ($s = 2$, $R = 51$, $\alpha = 0.4$, 30,000 steps with 200 samples).

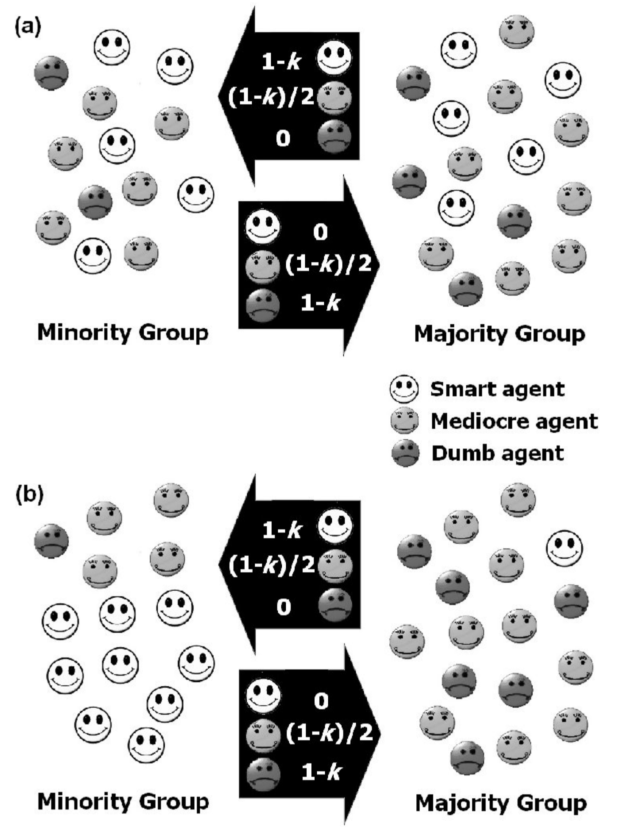

Fig. 5. (a) Agent dynamics in the multirate approximation. (b) Aggregation of smart agents in the minority group.

cisions are those holding at least one contrarian strategy. They are referred to as *smart* agents and are found with probability $1-(3/4)^s$. Otherwise, if agents have at least one buyer or seller strategy, their strategic bid follows the minority decision in an epoch with probability 1/2, and those agents are referred to as *mediocre* agents. The probability of this case is $(3/4)^s - (1/4)^s$. With probability $(1/4)^s$, agents have only follower strategies and are referred to as the *dumb* agents. By considering the transition probabilities for each of these agent types and taking the average, we find $T = 1-(3/4)^s/2-(1/4)^s/2$.

As Fig. 3 shows, the average epoch lifetime $\langle L\rangle$ decreases with the average number $\tilde{k}$ of agents using adaptive strategies. Furthermore, the prediction of the mean-rate approximation has an excellent agreement with the simulation results for both $\langle L\rangle$ and the variance σ^2 of the buyer population. Fig. 4 shows that the epoch lifetimes follow an exponential distribution near the maximum at small L. The mean-rate approximation has an excellent agreement with simulation results in this regime. When L increases further, the distribution crosses over to an exponential one with a lower decay rate. However, long epochs are *more frequent* than the prediction of the mean-rate approximation!!!

V. MULTIRATE APPROXIMATION

The discrepancy that long epochs are more frequent than predicted by the mean-rate approximation, arises from the inability of the mean-rate approximation to differentiate the aggregation effects of various agents in the majority and the minority groups. Due to the strategies held by different agent types, the probabilities that smart, mediocre, and dumb agents switch from the majority to the minority group are $1-k$, $(1-k)/2$, and 0, respectively. For opposite switches, the probabilities are 0, $(1-k)/2$, and $1-k$, respectively. As the inset of Fig. 4 shows, these agents have different winning rates. The physical picture of this agent dynamics is illustrated in Fig. 5(a), in which the switching probabilities of different agent types are represented by the arrows between the minority and the majority groups. This leads to the

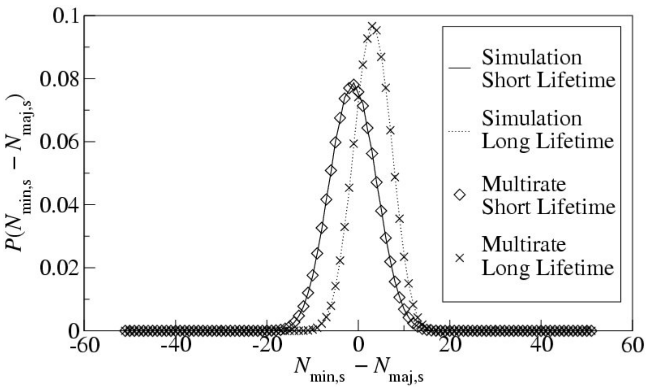

Fig. 6. Distribution of the difference ($N_{\rm min,s} - N_{\rm maj,s}$) between the numbers of smart agents in the minority and the majority groups ($N = 51$, $m = 6$, $s = 2$, $R = 51$, $\tilde{k} = 0.51$, $\alpha = 0.4$, 30,000 steps with 30,000 samples). The average lifetime $L \approx 11$. Epochs with short and long lifetimes are defined by $L \leq 20$ and $L \geq 100$, respectively.

second approximation of our analysis, the *multirate* approximation. We denote by $N_{\rm As}$, $N_{\rm Am}$, and $N_{\rm Ad}$ the numbers of smart, mediocre, and dumb agents in group A, respectively. The master equation for the probability $P(N_{\rm As}, N_{\rm Am}, N_{\rm Ad})$ can be solved by using the Monte Carlo method. As Fig. 4 shows, the multirate approximation yields a significantly higher probability for long epochs than the mean-rate approximation does, resulting in an excellent agreement with simulation results over 8 decades of probability.

To explain the higher probability of long epochs predicted by the multirate approximation compared with the mean-rate approximation, we consider what happens to the composition of the majority and the minority groups when an epoch prolongs. Since smart and dumb agents choose the minority and the majority groups, respectively, the majority group in a long epoch has fewer smart agents than the minority group. This situation is illustrated in Fig. 5(b), in which smart agents aggregate in the minority group, leading to a smaller number of smart agents present in the majority group. It follows that when an agent in the majority group chooses to make a strategic decision, it is less likely that she switches to the minority group. This aggregation of smart agents reduces the flow of agents from the majority to the minority group and, hence, reduces the exponential decay rate in the tail of the lifetime distributions. This effect is most significant in the mesoscopic regime when the fluctuations of the smart agent aggregate and the minority group size are important.

The evidence that smart agents aggregate in the minority group at the expense of the majority group is demonstrated in Fig. 6. We measure the difference in the numbers of smart agents between the minority and the majority groups ($N_{\rm min,s} - N_{\rm maj,s}$) averaged over an epoch. Compared with the distribution for short epochs, the peak of the distribution for long epochs is positively biased. This shows that, on average, long epochs have more smart agents distributed in the minority group. Furthermore, the distribution of smart agents derived from the multirate approximation has an excellent agreement with simulation results, as shown in Fig. 6. This shows that the aggregation of smart agents is crucial in explaining the distribution of long epochs.

Further interesting effects of agent aggregation arise when the analysis is generalized to cases of large memory size ($m > 1$), in which the existence of the *super* agents further refines the lifetime distribution of short epochs. The details are presented in Ref. 6.

VI. CONCLUSION

In summary, we have proposed a model of a competing population with multiple time scales. When the tendency to balance the decisions of the agents is strong and short term, there are oscillations with two groups of agents making alternating, but opposite, decisions. On a longer time scale, the history is characterized by epochs dominated by one of the two groups. Epochs end when sufficient numbers of agents follow adaptive strategies and switch to the winning side. Epochs are stabilized by the aggregation of smart agents on the winning side. The study of the epoch lifetime is important to agents who want to stay on the winning side as long as possible or at least, who, want to know how long they will prevail. These generic features are relevant to multiagent systems, such as futures markets in which agents switch between long and short positions, distributed control of multiclass traffic in communication networks, and other systems with competing aggregates of agents.

ACKNOWLEDGMENTS

We thank S. W. Lim, D. Saad, B. Meister, and Z. Struzik for discussions. This work is supported by the Research Grant Council of Hong Kong (HKU ST6062/02P, DAG04/05.SC25 and DAG05/06.SC36).

REFERENCES

[1] P. W. Anderson, K. J. Arrow and D. Pines, *The Economy as an Evolving Complex System* (Addison Wesley, Redwood City, 1988).
[2] F. Schweitzer (ed.), *Modeling Complexity in Economic and Social Systems* (World Scientific, Singapore, 2002).
[3] D. Challet, M. Marsili and Y. C. Zhang, *Minority Games* (Oxford University Press, Oxford, 2004).
[4] R. N. Mantegna and H. E. Stanley, Nature **376**, 46 (1995).
[5] P. Abry, R. Baraniuk, P. Flandrin, R. Riedi and D. Veitch, IEEE Signal Processing Magazine **19**, 28 (2002).

[6] C. H. Yeung and K. Y. M. Wong, Europhysics Lett. **75**, 357 (2006).
[7] G. Reents, R. Metzler and W. Kinzel, Physica A **299**, 253 (2001).
[8] M. Marsili, Physica A **299**, 93 (2001).
[9] Y. B. Xie, B. H. Wong, C. K. Hu and T. Zhou, Eur. Phys. J. B **47**, 587 (2005).
[10] N. F. Johnson, P. M. Hui, R. Jonson and T. S. Lo, Phys. Rev. Lett. **82**, 3360 (1999).
[11] K. Y. M. Wong, S. W. Lim and Z. Gao, Phys. Rev. E **70**, 025103(R) (2004).
[12] N. F. Johnson, M. Hart and P. M. Hui, Physica A **269**, 1 (1999).