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NETWORK EFFECTS ON CASH-CARD SUBSTITUTION IN TRANSACTIONS AND LOW INTEREST RATE REGIMES*

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We develop a Nash game on the adoption of a new EFTPOS (Electronic Fund Transfer at Point of Sale) card payment media given cash is dispensed by a competitive ATM (Automated Teller Machine) network. Equilibrium conditions when cash and card use coexist entail a specific relationship between the card network coverage parameter (π^k) and the proportion (ω) of cash financed expenditures. We derive a payments innovation technology constrained transactions demand for money which is highly interest rate sensitive in low interest rate regimes. Data on cash-card use in the UK (1990–97) is used to calibrate the model.

In small payments technology which operates in a highly decentralised way, cash has dominated as the means of payment in retail expenditures. This is primarily because till the advent of the EFTPOS (Electronic Fund Transfers at Point of Sale) all paper based non-cash payment instruments posed an additional cost of verifying cash balances of the payor[1] and methods of guaranteeing payment have been too costly relative to the value of the trade. In contrast, the electronic networks that link points of sale in the retail sector directly to bank balances of customers has drastically reduced costs in payment guarantees. Preceding the growth of networks in EFTPOS, the banking system had revolutionised cash dispensation via Automated Teller Machine (ATM) networks that enhanced the convenience yield of cash by increasing its accessibility to point of use. This also set in motion the irreversible trend in lowering labour costs of many banking services and other production costs to monetary services from economies of scale (Walker, 1978; Revell, 1983). Further, the opportunity cost of holding cash has increased with the introduction of interest bearing current accounts and highly liquid deposit accounts. While the latter drives the need to economise on transaction cash balances, ATM networks facilitate the means to do so by reducing shoe leather costs.

Humphrey *et al.* (1996) in a recent empirical study on trends in cashlessness have noted that the above network benefits of ATMs to the consumer is being eroded by an overall decline in cash use because of the emergence of debit and credit card payments via the EFTPOS. Nevertheless, in recent international comparisons on value of ATM cash and card use over 1990–98, Markose and Loke (2000) find that the dominance of card use has clearly emerged only in 4 out of the 14 countries, viz. Canada, the US, Finland and France. Further, there is the somewhat surprising finding in Markose and Loke (2002*a*) that in the latter half of the 1990s, of the

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[1] Till recently the Clower (1967) cash-in-advance constraint was of crucial practical significance for this reason. The cheque guarantee card was the first milestone in the direction of making the cash-in-advance constraint a non-binding one.

countries in the vanguard of EFTPOS developments, Canada, France and Finland are experiencing a resurgence in ATM cash use. Finally, as indicated in the 1999 BIS Report on retail payment systems, despite the overall trend away from cash payments, cash continues to dominate in all countries in terms of up to 75–90% of total volume of transactions. Typically these are of low average value compared to non-cash purchases. One of the major stumbling blocks here is that unlike card payments there are no records of cash purchases due to anonymity of cash use.

Against the backdrop of the above facts and problems regarding data on the volume and value of cash use in retail expenditures, this paper aims to give a framework of analysis for the consumer's adoption of the new EFTPOS payments media given the status quo of universally accepted currency. Crucial to the cost comparisons for using the cash or the card networks is the consumer's expectation of the proportion, π^k, of merchants in the economy who are EFTPOS linked. The installation of the latter in turn depends on the merchant's expectation of the proportion of per capita retail expenditures that will be card financed, $(1 - \omega)$. Similar strategic issues have been raised in Kiyotaki and Wright (1993) and Berentsen (1997).[2] In Section 1, we develop a Nash equilibrium of such a game in which both cash and card use coexist under conditions of optimal money management when costs of cash and card use are equated at the margin.

In Section 1.1, the standard Baumol–Tobin model (Baumol, 1952; Tobin, 1956) of optimal cash management that determines transactions demand for money is extended in the spirit of Prescott (1987) and Santomero and Seater (1996). Unlike the traditional assumption that 100% of the value of expenditures is cash financed, on including the possibility of substitution by card payments, the transactions demand for cash balances is reduced by a factor of $\sqrt{\omega}$, the square root of the proportion of cash financed expenditures. The smaller is ω, the larger the substitution away from cash in payments.

The focus of this paper is on the transactions demand for ATM networked cash and how substitution with the EFTPOS card takes place on the basis of their relative network costs arising from incompatibility of cards with networks and or as a result of inadequate proliferation of networks. Thus in terms of their network effects, ATM cash and EFTPOS card are taken to be perfect substitutes to the consumer with no subjective preferences governing the demand for either. Undoubtedly, cash enjoys the distinctive feature of anonymity in its use for which a consumer may have positive preference when engaged either in the black economy or 'bad behaviour' such as a visit to the brothel (Goodhart, 2000; Drehmann *et al.*, 2002). In so far as ATMs do not dispense large denomination notes that are typically associated with the more nefarious sections of the black economy, ATM cash use arguably does not constitute what many, Rogoff (1998), regard to be the bulk of the demand for currency that arises from subjective preference for anonymity in transactions in the black economy. Recently, Dutta and Weale (2001) specified a consumption payments model with a constant elasticity of substitution

[2] Berentsen (1997) was unable to incorporate the network costs of cash and card use to the consumer in the specification of the Nash equilibrium of the game on the adoption of electronic POS payments media.

(CES) utility function for cash and non-cash financed consumption and a payments technology that incorporated the extent of acceptance of non-cash payments. Our paper and that of Dutta and Weale (2001), share the novel feature of developing a payments innovation technology constraint on transactions demand for money. We arrive at the conclusion that the Nash equilibrium conditions that determine the competitive provision of ATM cash along with its EFTPOS substitute has increased the interest rate elasticity of the transactions demand for cash.

To define an equilibrium in which ATM cash and EFTPOS payments media are perfect substitutes with their simultaneous use in vogue, requires a specification in which their cost schedules have to be equal at the margin. To specify this correctly, it was found necessary to address explicitly what bearing the observed feature that cash purchases are low in value and high in frequency/volume has on the money management problem of a consumer. In Section 1.2, we present this problem as a capital budgeting one of how to expend two given funds in terms of the respective present values of the stream of expenditures that are of different frequency and value over the year. This approach yields a simple rule for estimating the volume of cash purchases given that the volume of card purchases is known. This overcomes the problem of the lack of data on per capita volume of cash purchases. In the case of cash purchases, these present value calculations introduce a non-zero 2% deposit interest rate floor at which the incentive to economise on cash ceases.

The main result of the paper shows that in an equilibrium in which cash and card use coexists there is a unique relationship between the observed per capita proportion of ATM cash financed expenditures, ω, and the unobserved parameter of card network coverage, π^k. In Section 2, this result is used to estimate the density of card network coverage of an economy and derive the iso π^k-curves for the combinations of interest rates and cash-card substitution possibilities given by ω. The features of a payments innovation technology constrained transactions demand for money are also explored here.

The main policy related implication of a high EFTPOS linked economy is that the payments technology constrained transactions demand for money is found to be highly interest rate sensitive in low interest rate regimes. In Section 3, UK payments data for 1990–97 on EFTPOS and ATM transactions is used to test and simulate the results of the theoretical model. This is followed by a brief concluding section.

1. Microstructure of Cash-Card Networks

We develop a simple framework for the Nash equilibrium of a game for the mutual adoption by merchants and consumers of the new EFTPOS payments media. As is in the nature of network goods, the extent to which consumers adopt a new EFTPOS card payment and reduce their holdings of cash, ω, being the proportion of cash financed expenditures, depends on the expectation, $E\pi^k$, of the proportion of merchants who will accept EFTPOS card payments. The reaction function of the representative consumer is denoted by $R_C[(1 - \omega); E\pi^k]$. Likewise, the merchant's decision to invest in the EFTPOS network is influenced by the relative costs of handling cash and card and the expectation of the proportion of per capita total expenditures that is card financed, $E(1 - \omega)$. The reaction function of the representative merchant is denoted by $R_M[\pi^k; E(1 - \omega)]$. The year on year estimates for EFTPOS network density, π^k, for an economy is obtained from the Nash equilibrium condition for a series of formally identical one shot games between a representative consumer and merchant who are faced with the historical per capita data pertinent to their respective decision problem at each date.

1.1. *The Consumer's Decision Problem*

The representative consumer is assumed to receive a fixed income α at the beginning of a year. This is intermediated by a bank and unspent bank deposits receive a return at the per annum deposit interest rate of r. The consumer makes a fixed volume of purchases, V, in the course of a year and each good takes one shopping trip. The value of purchases made by cash (card) is referred to as the cash (card) fund and is denoted, respectively, by

$$\omega\alpha = \alpha_c, \quad (1 - \omega)\alpha = \alpha_k. \tag{1}$$

The volume of purchases using card is denoted by V_k and the volume of cash purchases is denoted by V_c, with $V = V_k + V_c$.

1.1.1. *Network costs for cash and card use*

The expected total cost of using the cash network for implementing $\omega\alpha = \alpha_c$ value and V_c volume of annual expenditures is given by,

$$\begin{aligned} T_c &= \mu V_c[t + (1 - \mathrm{E}\pi^c)\varepsilon] + \frac{1}{2}\frac{r\alpha_c}{\mu V_c} \\ &= \mu V_c T_c^{\#} + \frac{1}{2}\frac{r\alpha_c}{\mu V_c}. \end{aligned} \tag{2}$$

Here, $T_c^{\#} = [t + (1 - \mathrm{E}\pi^c)\varepsilon]$ includes the standard shoe leather cost, t, to each cash withdrawal and the ATM incompatibility cost, ε. Typically a consumer will suffer extra service charges, ε, if he has to use an ATM which is not compatible with the ATM network of his bank. This occurs with probability $(1 - \pi^c)$ where π^c is the proportion of ATM terminals that belongs to the consortia of his bank relative to the total number of ATM terminals. We denote by μ in (2), the rate of cash withdrawals, $\mu = W_c/V_c$, where W_c is the total number of cash withdrawals per annum. The last term in (2) is the annual interest rate costs on the per person average annual cash balances, $B = \frac{1}{2}(W_c/V_c)$.

Note in (2), with $\mu = 1$, we have full frequency of cash withdrawals to correspond to all the V_c shopping trips. In this case, ATM cash use in transactions is functionally closest to the EFTPOS with the shoe leather cost of cash use at a maximum of tV_c and the annual interest rate costs at a minimum of $r(\omega\alpha/2V_c)$.

RESULT 1. *The consumer deals optimally with the above trade off between the shoe leather and interest rate costs by minimising* (2) *with respect to the rate of cash withdrawals,* μ. *This gives the Baumol–Tobin type optimal square root rule,*

$$\mu^* = \sqrt{\frac{r\omega\alpha}{2V_c^2 T_c^{\#}}}.$$

Thus, the optimal number of cash withdrawals for a given V_c volume of cash purchases is

$$W_c^* = \mu^* V_c = \sqrt{\frac{r\omega\alpha}{2T_c^{\#}}}. \quad (3)$$

On substituting μ^* into (2), the expected optimal total cost of cash use simplifies to

$$T_c = 2\mu^* V_c T_c^{\#}. \quad (4)$$

Further, on setting the optimal W_c^* to equal the historical per capita data on the number of ATM cash withdrawals, with data on r, ω and α, from (3) the unit ATM transaction costs to the consumer is estimated as

$$T_c^{\#} = \frac{r\omega\alpha}{2W_c^2}. \quad (5)$$

The per capita optimal transaction balances is given by

$$B^*(\omega) = \frac{\omega\alpha}{2W_c^*} = \sqrt{\frac{\omega\alpha T_c^{\#}}{2r}} = \sqrt{\omega}B^*. \quad (6)$$

RESULT 2. *The second equality in* (6) *is obtained by substituting for* W_c^* *from* (3). *The optimal cash balance curve in the (B, r) plane shifts downward by* $\sqrt{\omega}$ *relative to the curve* B^* *with* $\omega = 1$ *in a pure cash economy. In other words, for any given level of interest rate, the transactions balances are less when cash-card substitution is allowed.*

The total expected cost of implementing the $(1 - \omega)\alpha$ value and V_k purchases using the card network given the consumer's expected value of card network coverage $\mathrm{E}\pi^k$ is

$$T_k = V_k(1 - \mathrm{E}\pi^k)T_c^{\#}. \quad (7)$$

For card use in (7), with probability $(1 - \mathrm{E}\pi^k)$ a merchant does not have EFTPOS facilities and hence the customer has to have cash at hand and thereby incurs the cost of cash withdrawal, $T_c^{\#}$. The important point to be noted here is that if π^k, the probability of card network coverage, goes to zero, the expected unit network cost of a card purchase equals that of a cash purchase. Further, as $\pi^k \to 1$, the payments system tends to full EFTPOS coverage of retail nodes. As will be seen, a corner solution with zero cash use when $\pi^k \to 1$, is a feature of our model if the interest rate is above a critical value.

1.1.2. *Volume of cash and card purchases*

Here we model the commonplace feature of modern payments systems that cash purchases are higher in volume but lower in value than card purchases, viz. on average $\alpha_c/V_c < \alpha_k/V_k$, and in terms of their volumes, $V_c > V_k$. This results in differential flow effects of disbursements from the cash and card funds and the respective unit costs of using cash and card derived from (4) and (7) also apply differentially over time. The respective present discounted value for cash and card purchases per annum is given by

$$\alpha_j \mathrm{PVE}_j = \alpha_j \frac{1}{V_j} \sum_{i=1}^{V_j} \frac{1}{(1 + r/V_j)^i} \leq \alpha_j, \quad j = c, k. \tag{8}$$

The present values above can be seen to be obtained from fixed but multiple coupon payments (equal to α_j/V_j, $j = c, k$) with appropriate discount rates.[3] Note, on a unit fund of \$1, the present values of the respective disbursements worth $1/V_c$ and $1/V_k$ must be the same to specify a no arbitrage relationship between the use of cash and card in payments. Thus,

$$\mathrm{PVE}_c = \mathrm{PVE}_k = \frac{1}{V_j} \mathrm{PV}_j^u = \frac{1}{V_j} \sum_{i=1}^{V_j} \frac{1}{(1 + r/V_j)^i}, \quad j = c, k. \tag{9}$$

Here, PV_c^u and PV_k^u are the present discounted values of a dollar worth of purchases made by cash and card, respectively. Trivially, if $V_c = V_k$, PV^u is the same for both streams, and $\mathrm{PVE}_c = \mathrm{PVE}_k$. Using simple principles of a capital budgeting problem we determine $V_c > V_k$ such that $\mathrm{PVE}_c = \mathrm{PVE}_k$. Thus, given that the volume of card purchases are recorded, this approach enables us to determine the volume of cash purchases for which records do not exist.

RESULT 3. *The consumer equates the present values of average per dollar payments over the year for the cash and card purchases defined in* (9), *such that* $V_c > V_k$. *This yields*

$$V_c = V_k(r|D_k| + 1). \tag{10}$$

Proof. see Appendix 1.

Here, $|D_k|$ denotes the absolute value of the duration of the card fund in (9) and $r|D_k|$ is the interest elasticity of the present value flow. The dollar duration is the measure of interest rate sensitivity of a present value flow and is defined as the percentage change in the present value for a 1% change in the interest rate. This formula is given[4] in the Appendix (equation (A3)). From (10), we see that the volume of cash purchases for a fixed value of the cash fund will rise considerably as

[3] While making disbursements from his cash and card funds, the consumer adopts an average time interval between each of the cash purchases to be $365/V_c$ while that for card purchases is $365/V_k$. This results in respective discount rates in (8) of r/V_c and r/V_k where r is the per annum interest rate. The flow of purchases over time are then discounted at a compounded rate equal to the frequency of purchases. This corresponds to a capital budgeting problem with multiple but fixed value coupon payments in a year.

[4] Note that a 1% change in the interest rate means a change from say 8% to 9% rather than 8% to 8.08%. That is, when r is 5% and $|D| = 0.50$, $r|D| = 5 \times 0.50 = 2.5$. The relevant duration values for the card fund will be tabulated in the empirical Section 4, Table 2. Duration here includes the frequency weighted average of the present values of a fixed value of purchases, $\$1/V_j$. Further, as is well known, duration decreases as the frequency of coupon payments per year increases. Thus, as V_k ranged from 1 to about 50, $|D_k|$ fell from 1 to a minimum value of 0.5. In Appendix 1, it is analytically shown why $|D_j|$, $j = (c, k)$, for our problem converges to 0.5 for not so large frequency/volume of purchases (numerically found to be about 50 per annum irrespective of interest rates). See also Table 2.

the interest rate rises. This has the effect of reducing the average per dollar value of a cash purchase and hence interest rate costs by increasing the frequency of cash purchases relative to those made by card. Note that in Result 3, the formula in (10) for the volume of cash purchases V_c for a given V_k is obtained in a way that it is *independent* of the size of the cash fund, $\omega\alpha$.

1.1.3. *Consumer's reaction function* $R_C[(1-\omega); \mathrm{E}\pi^k]$

Since flow effects in disbursements discussed above are important to the consumer in the use of cash and card, his profit or net return functions denoted by PVR_j, $j = c,k$, is given by,

$$\max_{V_k} \ \mathrm{PVR} = \mathrm{PVR}_k + \mathrm{PVR}_c.$$

$$\mathrm{PVR}_k = \left[\frac{(1-\omega)}{V_k}(1+r) - \left(1 - \mathrm{E}\pi^k\right) T_c^{\#}\right] \mathrm{PV}_k^u. \tag{11}$$

$$\mathrm{PVR}_c = \left[\frac{\omega}{V_c}(1+r) - 2\frac{W^*}{V_c} T_c^{\#}\right] \mathrm{PV}_c^u. \tag{12}$$

In (11) and (12), the first terms are the portfolio weighted average per dollar gross return on deposit balances from the card fund and cash funds respectively and the second terms are the per unit (volume) cost of using the card and cash network obtained from (7) and (4) respectively. As, $\mathrm{PVE}_k = \mathrm{PVE}_c$ in (9) to equalise present value of flow effects of disbursements from a unit fund, the net revenue function in (11) and (12) simplifies to

$$\mathrm{PVR} = \left[\frac{1}{V_k}(1+r) - \left(1 - \mathrm{E}\pi^k\right) T_c^{\#}\right] \mathrm{PV}_k^u - 2\frac{W_c^*}{V_c} T_c^{\#} \mathrm{PV}_c^u. \tag{13}$$

The objective function above can be shown to be a special case of a more general CES utility function for cash and card disbursements when the two payments media are taken to be perfect substitutes. This is given in Appendix 2. From (13) we see that the optimal ω is indeterminate which is again a generic result with a CES utility function in the case of perfect substitutes. Then, only relative costs matter and an unique reaction function $R_C[(1-\omega); \mathrm{E}\pi^k]$ can be shown to exist when the volume of cash and card use are such that their network unit costs are equalised at the margin.

RESULT 4. *In an equilibrium when cash and card use coexists, the marginal present value of their cost functions in* (13) *are equal. Thus,*

$$\frac{\mathrm{d}[(1-\mathrm{E}\pi^k) T_c^{\#} \mathrm{PV}_k^u]}{\mathrm{d}V_k} = \frac{\mathrm{d}\left(2\mu^* T_c^{\#} \mathrm{PV}_c^u\right)}{\mathrm{d}V_c}. \tag{14}$$

The above condition yields,

$$(1-\mathrm{E}\pi^k) T_c^{\#} r|D_k| = 2\mu^* T_c^{\#}(r|D_c| - 1). \tag{15}$$

The consumer's implicit reaction function $R_C[(1-\omega); E\pi^k]$ (*plotted in Figure* 1) *is obtained from the above relationship between the expected card network density parameter,* $E\pi^k$*, and the possibility of substitution between cash and card determined by* ω*. Thus,*

$$E\pi^k = 1 - 2\mu^* \frac{(r|D_c| - 1)}{r|D_k|} = 1 - 2\sqrt{\frac{r\omega\alpha}{2V_c^2 T_c^{\#}}}\frac{(r|D_c| - 1)}{r|D_k|}. \qquad (16)$$

The second equality in (16) is obtained by substituting for the optimal rate of cash withdrawals, $\mu^* = W_c/V_c$, from (3). Note that in arriving at (16), the appropriate unit cost comparisons between cash and card use must entail their respective volumes, V_c and V_k. Hence, for determining π^k, $0 \le \pi^k \le 1$, what is relevant is the optimal rate $\mu^* = W_c/V_c$ of cash withdrawals rather than the number of cash withdrawals W_c. Further, the last term in (16), $(r|D_c| - 1)/(r|D_k|)$, which yields the reciprocal of the ratio of the interest elasticity of PVE_k, $r|D_k|$, and the volume elasticity of PVE_c, $(r|D_c|-1)$ respectively, appears to be essential to get the correct trend in the values for π^k over the years. It also incorporates the relative flow effects of cash and card use.

RESULT 5. *Taking the relevant cost function for cash purchases to be the present discounted value of the per unit* (*volume*) *costs of ATM cash use within a year with discounting being done as in the case of multiple coupon payments* (*see footnotes 3 and 4*), *the marginal cost of cash use in transactions on the RHS of* (14) *and* (15) *becomes zero and then negative at precisely* 2% *deposit interest rate and below.*

Result 5 follows immediately from the marginal cost condition on RHS of (14) and (15) as the duration term $|D_c|$ is approximately 1/2 and the volume elasticity, $(r|D_c|-1)$, of the present value of cash use PVE_c become zero or negative at $r = 2\%$ or below. Hence, a straight forward consideration that consumers are concerned

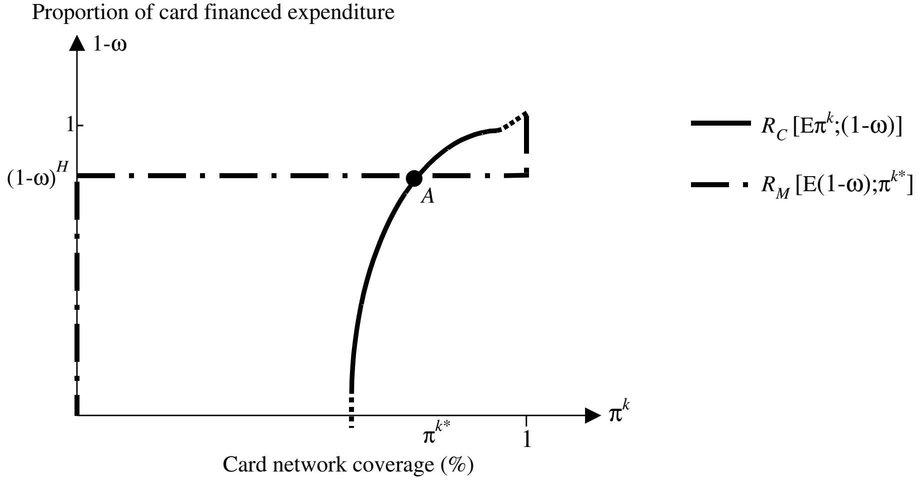

Fig. 1. *Reaction Functions of Consumer and Merchant: Nash Equilibrium*

about the present value flow effects over a fixed period of time for purchases made by different payments media introduces a remarkable non-zero deposit interest rate floor at which incentives to economise on cash cease.

1.2. *The Merchant's Decision Problem*

The merchant's reaction function $R_M[\pi^k; \mathrm{E}(1-\omega)]$ is obtained from the factors that determine the profitability of installing EFTPOS as opposed to not installing it.[5] The profit function of the representative merchant is given as:

$$P = (1-\pi^k)(\mathrm{E}\omega\alpha - T_c - T_c^m) + \pi^k[\mathrm{E}(1-\omega)(\alpha - T_k^m) + \mathrm{E}\omega(\alpha - T_c^m) - T_c - F]. \tag{17}$$

Here, the first term denotes the net revenue from cash transactions in the case when with probability $(1-\pi^k)$ a merchant has not installed the EFTPOS. In this situation, the merchant gets a gross return of only the proportion $\omega\alpha$ of cash expenditures less T_c and T_c^m which are respectively the total cost of cash transactions incurred by the consumer given in (4) and the total cost to the merchant of for cash handling. The second term in (17) denotes the net revenue to the merchant obtained with probability π^k that he has installed the EFTPOS. In this case, the representative merchant effectively gets the full per capita α of annual income less the weighted sum of costs of card and cash handling T_k^m and T_c^m, the costs to the consumer for using cash T_c and a fixed cost F for the installation of an EFTPOS connection.

RESULT 6. *Optimizing* (17) *with respect to* π^k *yields the following decision rules for the merchant.*

$$\mathrm{E}(1-\omega)\begin{cases} > & \Rightarrow \pi^k = 1 \\ = \dfrac{F}{\alpha + (T_c^m - T_k^m)} & \Rightarrow 0 < \pi^k < 1. \\ < & \Rightarrow \pi^k = 0 \end{cases} \tag{18}$$

The RHS of (18) gives the ratio of the fixed cost to the per capita income α and the cost difference of handling cash transactions and card transactions. Equation (18) states that if the merchant's expectation of the proportion of per capita card expenditures is greater than the cost factors on the RHS of (18) then he will invest in EFTPOS. In the absence of heterogeneous expectations, this also implies that there is full EFTPOS coverage with $\pi^k = 1$. When there is equality in (18), the relevant section of the merchant's reaction function is horizontal in the $((1-\omega), \pi^k)$ plane, see Figure 1, and the economy can have any π^k, $0 < \pi^k < 1$ consistent with the per capita $(1-\omega)$ given by the consumer's reaction function in (16). Finally, if $\mathrm{E}(1-\omega)$ is less than the cost factors in (18), no merchant will invest in EFTPOS and the economy has zero EFTPOS.

[5] Unlike papers by Matutes and Padilla (1994) and McAndrews and Rob (1996), we are not concerned about the economies of scale aspects of payment networks from the supplier's side nor do we consider second order strategic problems about what type of electronic POS instrument to provide, viz. on line or off line and so on.

1.3. *Nash Equilibrium*

In Figure 1 we plot the consumer's reaction function $R_C[\mathrm{E}\pi^k; (1-\omega)]$ from (16) with that of the merchant $R_M[\mathrm{E}(1-\omega); \pi^k]$ given in (13). As specified in (18), the merchant's reaction function R_M has three segments.

The Nash equilibrium points in the $((1-\omega), \pi^k)$ plane are at the intersection between the R_C curve and the horizontal segment of the R_M curve. Thus, as shown in Figure 1, as $R_C[\mathrm{E}\pi^k, (1-\omega)]$ exists strictly for, $0 < (1-\omega) < 1$, viz. only for cases when both cash and card are in use by the consumer, the R_C curve cannot intersect at the points where $\pi^k = 1$ and the horizontal axis where $(1-\omega) = 0$.

RESULT 7. *In the Nash equilibrium, the merchant's expectation* $\mathrm{E}(1-\omega)$ *has to equal the historically observed per capita* $(1-\omega)^H$ *for the economy. The* ω *consistent with this then determines the Nash equilibrium* $\mathrm{E}\pi^k = \pi^{k*}$ *in terms of the consumer's implicit reaction function in* (16). *For given historical payments data for* ω, α, r, V_k *and* W_c, *we have an unique* $R_C[(1-\omega)^H; \mathrm{E}\pi^k]$ *curve from* (16) *and hence an unique point A, in Figure* 1, *on the horizontal segment of the* R_M *curve which denotes the Nash equilibrium pair* $[(1-\omega)^H, \pi^{k*}]$ *for the economy.*

2. Payments Innovation Technology Constrained Transactions Demand for Money and Monetary Policy Implications

Using (16) and Result 7, the card network coverage parameter π^k for any economy can be calibrated by using the formula in the first equality which only requires historical W_c, r and estimated values for V_c and $|D_j|$, $j = c,k$. The main premise of this framework is that the level of π^k of an economy and the equilibrium relationship between π^k and ω in the second equality in (16) constrains the extent of cash-card substitution that is technologically feasible and economically optimal. To understand how the payments technology constrains cash-card substitution, in particular its interest rate sensitivity, we develop the so called iso π^k-curve for the equilibrium level of π^k given by (16).

2.1. *The Iso* π^k*-Curves*

RESULT 8. *Using condition* (16), *the combinations of* (ω, r) *that keep* π^k *unchanged for different parametrically given levels of* π^k *with all other variables fixed, yields a family of iso* π^k*-curves that satisfies the equation,*

$$\mathrm{d}\pi^k = \frac{\partial \pi^k}{\partial r}\mathrm{d}r + \frac{\partial \pi^k}{\partial \omega}\mathrm{d}\omega = 0.$$

Then, the interest rate sensitivity of cash-card substitution is given by,

$$\left.\frac{\mathrm{d}\omega}{\mathrm{d}r}\right|_{\mathrm{d}\pi^k=0} = -\frac{\omega}{r}\left(\frac{r|D_c|+1}{r|D_c|-1}\right) = -\frac{\omega}{r}\left[\frac{2\mu^*(r|D_c|+1)}{r|D_k|(1-\pi^k)}\right]. \tag{19}$$

The interest rate sensitivity of cash-card substitution, $d\omega/dr$, can be directly expressed as a function of π^k as shown in the second equality in (19).[6] Figure 2 plots a family of π^k-curves in the (ω,r) plane using the 1997 UK historical payments data for α, V_k, and model derived values for $T_c^{\#}$ and V_c.

Properties of iso π^k-curves:

(i) Note that the downward sloping π^k-curves implies an inverse relationship between r and ω.

(ii) Further, as the π^k-curves shift leftward in Figure 2, for higher levels of card network coverage it implies that card use dominance defined as $\omega < 1/2$ is feasible at lower interest rates. Thus, if card network coverage is only 50%, card dominance is feasible only with high interest rates of over 8%. Whereas if $\pi^k = 0.79$, that is 79% of all merchants are EFTPOS linked, then card dominance can prevail with low interest rates of below 4%.

(iii) Also note that the maximum degree of substitution of cash to card is determined by the shape of the π^k-curve. An example of this is indicated by the arrow in Figure 2 where $d\omega/dr \to 0$.

(iv) The convexity of the π^k-curves in the (ω, r) plane implies an asymmetric rate of cash-card substitution to interest rate changes depending on if there is an interest rate cut or an interest rate rise and whether the initial regime

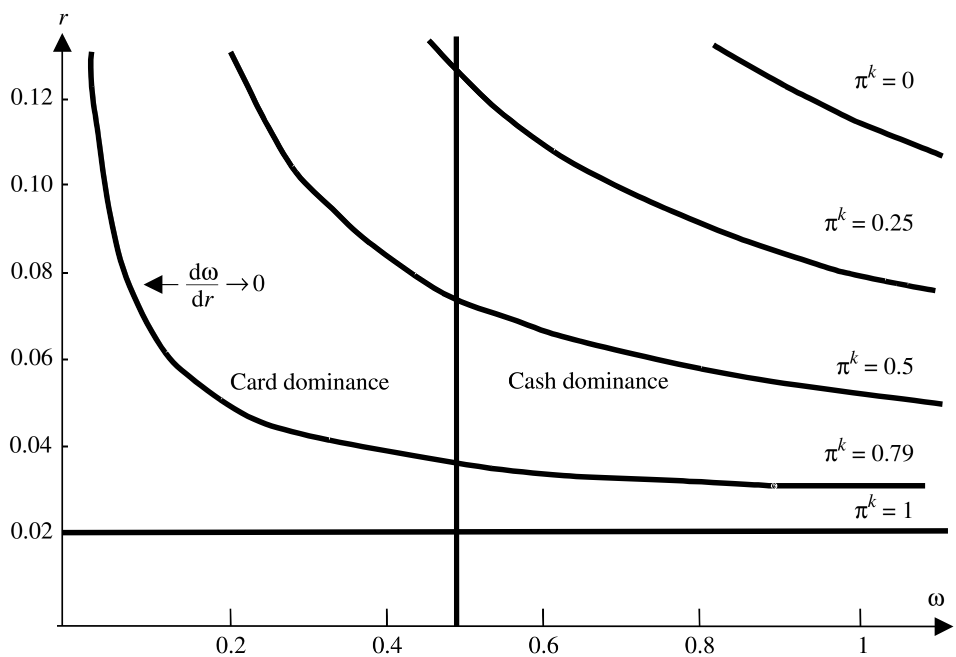

Fig. 2. *Card Network Coverage* (π^k), *Interest Rate* (r) *and the Rate of Cash-Card Substitutions* (ω)

[6] This is done by solving for $(r|D_c|-1)$ by using the first equality in (16).

is a high or low interest rate one. From (19) it can also be verified that $\mathrm{d}\omega/\mathrm{d}r$ is greater at lower initial interest rates than at higher ones.

(v) From Figure 2, we see that at high π^k levels, the π^k-curves are relatively flat at low interest rates making cash-card substitution highly sensitive to an interest rate change. This can be verified analytically from the second equality in (19).

(vi) From (16) and as the absolute value of the dollar duration, $|D_c|$ is close to a $\frac{1}{2}$ (see footnote 4), in the limit when $\pi^k \to 1$, we see the π^k-curve is horizontal at the critical value of 2% nominal interest rate.

RESULT 9. *For any interest rate increase from the critical interest rate of 2%, when* $\pi^k \to 1$, *there exists a corner solution with zero use of ATM cash. In other words, as can be verified from* (19), *there is infinite substitution away from cash to card at this point as* $\mathrm{d}\omega/\mathrm{d}r = -\infty$. *The converse is true when interest rates are cut from the critical 2% nominal interest rate.*

2.2. *Implications for the Shape and Dynamics of Cash Balance Equation*

The shifts in the optimal cash balance equation is given by,

$$\mathrm{d}B(r, \omega, \alpha, T^{\#}) = -\frac{1}{2}\frac{B}{r}\mathrm{d}r + \frac{1}{2}\frac{B}{\alpha}\mathrm{d}\alpha + \frac{1}{2}\frac{B}{\omega}\mathrm{d}\omega + \frac{1}{2}\frac{B}{T_c^{\#}}\mathrm{d}T_c^{\#}. \tag{20}$$

Equation (20) confirms the standard results of Baumol–Tobin type optimal cash balance equations that interest rate elasticity is $-\frac{1}{2}$ and the income elasticity is $+\frac{1}{2}$. However, the crucial difference arises from $\mathrm{d}\omega$ in (19). The latter cannot be determined independently of a given π^k-curve for an economy.

RESULT 10. *The interest rate sensitivity of transaction balances when possibilities for cash card substitution are constrained by a given iso-π^k curve for the economy is defined as,*

$$\left.\frac{\mathrm{d}B}{\mathrm{d}r}\right|_{\mathrm{d}\pi^k=0} = \frac{\partial B}{\partial r} + \left.\frac{\partial B}{\partial \omega}\frac{\mathrm{d}\omega}{\mathrm{d}r}\right|_{\mathrm{d}\pi^k=0} = -\frac{B}{r}\left[\frac{1}{2} + \frac{\mu^*(r|D_c|+1)}{r|D_k|(1-\pi^k)}\right]. \tag{21}$$

Here the first term is the standard Baumol–Tobin interest rate sensitivity while the second term incorporates the π^k constrained interest rate sensitivity in cash-card substitution from (19). Note that the payments innovation technology constrained demand for cash balances has interest rate elasticity (the term inside the bracket in (21)) which is a time varying function of the level of π^k in the economy.

For purposes of graphical illustration, we use the iso-B curve in the (ω, r) plane (with all other variables unchanged). The iso-B curve which plots the combinations of (ω, r) that yield the same value of transactions cash balances defined in (6) is a straight line through the origin with slope

$$\left.\frac{\mathrm{d}r}{\mathrm{d}\omega}\right|_{\mathrm{d}B=0} = \frac{T_c^{\#}\alpha}{B^2} = \frac{r}{\omega} > 0.$$

Examples of this are given in Figure 3*a*.

2.3. *Implications for Monetary Policy*[7]

Using Figure 3 we illustrate the implications of the interest rate sensitivity of the π^k-constrained transaction balances derived in Result 10. Consider an attempt by monetary authorities to curb a credit boom by an increase in the repo rate. In Figure 3, this is assumed to result in an increase in the deposit rate from r_o to r^+. Starting from an initial (ω_o, r_o) marked as point A in Figure 3*a,* as the interest rate rises, the B-curve fans upwards implying smaller transaction cash balances.

However, note that point F at (ω_o, r^+) is out of equilibrium as it is off the given π^k-curve of the economy. The partial equilibrium point marked as E yields a substantially smaller ω^+ which can be read off at the point of intersection between the new B_2^+-curve and the π^k-curve.

Figure 3*b* compares the response of the B-curves in a pure cash economy and that of a cash-card one. At the initial r_o, following Result 2, the B_2-curve is $\sqrt{\omega_o}$ less along the length of the B_1-curve of the pure cash economy. However, the major difference between the pure cash economy and a cash-card one is as follows. In the absence of substitution possibilities, with an interest rate rise, B_1^+ is at point E_1 in Figure 3*b,* viz. a movement along the same B_1 curve. However, for the cash-card economy, transactions demand shifts from the $B_2(\omega_o)$ curve to the $B_2^+(\omega^+)$ curve with the partial equilibrium point marked as E_2 on Figure 3*b*. The new ω^+ must satisfy (19). We see that the contraction in the optimal demand for transaction

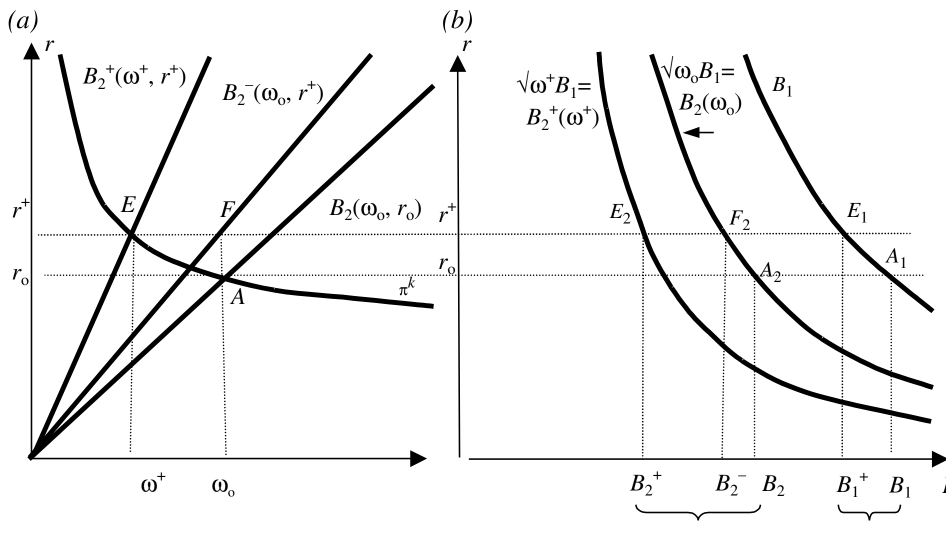

Fig. 3. *Interest Rate Sensitivity of Transaction Balances, B*

[7] To make comparisons between an economy with cash-card substitution ($0 < \omega < 1$) and a pure cash economy ($\omega = 1$), we use subscript 1 for the latter and subscript 2 for the former.

balances in the cash card economy, $(B_2 - B_2^+)$, is substantially larger than $(B_1 - B_1^+)$ for the pure cash economy.

From (21) and Property (v) of iso π^k-curves, we find that there will be large interest rate sensitive swings in transaction balances in high π^k economies. Thus, a small interest rate rise may lead to a large contraction in transaction balances. With lower cash in circulation and higher per capita amounts in depository institutions viz. a lower cash to deposit ratio, raising interest rates from a low initial interest rate may fail to curtail a credit boom if banks can fully lend the increase in deposits. In other words, raising interest rates in an economy with high π^k and a low initial interest rate may have a perverse impact on the loanable funds market. Only if the initial interest rate is high enough on the π^k-curve where $d\omega/dr \to 0$ (see, Property (iii) of iso π^k-curves) will these adverse effects from cash-card substitution on the effectiveness of interest rises to curb bank liquidity be mitigated. A quantification of these implications of our model is undertaken in the empirical Section 3.

3. Empirical Results: Cash-Card Payments Data for UK and Monetary Analysis

3.1. *Historical Data and Estimates of Cash-Card Use in the UK*

Table 1 gives the historical per capita cash and card payments data for the UK for 1990–97.[8] Here, $\alpha = \alpha_c + \alpha_k$ is the annual per person total value of networked expenditures with α_c being the value of ATM cash transactions and α_k is the value of card use. The ratio $\omega = \alpha_c/\alpha$ gives the historical per capita value of the proportion of ATM cash financed expenditures in each year.

As seen from Table 1, while the total value of networked expenditures per person, α, has more than doubled in the years 1990–97, and proportion of the value of cash financed expenditures, ω, has declined about 17% from 0.57 in 1990 to 0.47 in 1997. The recorded annual per person number of ATM transactions that proxies for cash withdrawals, W_c, has risen from 17 per person in 1990 to about 30 in 1997. V_k, the annual number of recorded card transactions per person has more than trebled from 15 to 45 over this period. V_k proxies for the number of (composite) card financed transactions. On the other hand, V_c, the number of cash financed purchases is unobserved and we estimate this from the formula in (10), $V_c = V_k(r|D_k| + 1)$. In this formula for V_c, we use the historical values of V_k, the volume of card purchases, in Table 1.

[8] As the focus of this paper is on the network aspects of the cash payments system, the value and volume of cash use is taken to be those related only to ATM transactions. For this we use Table 6 of the European Central Bank/EMI publication *Payment Systems in the EU Countries* (1994, 1996 and 1999). The data on the value and volume of card payments is from Table 14 and Table 15 of the EMI publication. EFTPOS related card payments data include both debit and credit cards. Note, the UK data on the value and volume of card use from the above sources also coincide with those given in the APACS *Yearbook of Payment Statistics* Table 6.5 (for card value) and Table 6.7 for value of ATM transactions. Per capita figures are obtained by dividing the aggregate data by total population. The latter are given in Table 1 in the EMI source. The data on the value of card and cash use has further been converted into USD to facilitate international comparisons undertaken in Markose and Loke (2000, 2002*a*). The per annum deposit interest rate is obtained from *International Financial Statistics*, Yearbook, 1998.

Table 1

Historical U.K. Per Capita Networked Cash-Card Payments Data: 1990–1997 (USD)

Year	1990	1991	1992	1993	1994	1995	1996	1997
α_c: Value of ATM cash purchases	1338.21	1504.32	1639.62	1495.44	1704.46	1945.21	2122.87	2498.49
α_k: Value of card purchases	1020.77	1194.25	1369.39	1325.27	1570.73	1912.79	2337.81	2864.94
$\alpha = \alpha_c + \alpha_k$: Total value of purchases	2358.98	2698.57	3009.01	2820.71	3275.19	3858	4460.68	5363.43
$\omega = \alpha_c/\alpha$	0.57	0.56	0.54	0.53	0.52	0.50	0.48	0.47
W_c: Volume of cash withdrawals	17	19	20	21	23	25	27	30
α_c/W_c: Value of ATM cash withdrawal	77.43	81.28	81.07	70.08	74.56	77.23	78.06	84.47
B: Average cash balances $\alpha_c/(2W_c)$	38.74	40.67	40.42	34.98	37.37	38.66	39.74	41.67
V_k: Volume of card purchases	15	18	22	24	28	33	39	45
r: Interest rate	0.1254	0.1028	0.0746	0.0397	0.0366	0.0411	0.0305	0.0363

From Table 2 we see that the estimated $T_c^{\#}$ unit costs of using the cash network which is inclusive of shoe leather costs of cash withdrawal and costs of ATM incompatibility (derived in (5)) have fallen substantially from 29 cents in 1990 to about 5 cents in 1997. On using these costs in the Baumol–Tobin formula for optimal rate of cash withdrawals, we verify that given the V_c estimates discussed above, the optimal μ^* is identical to the rate of cash withdrawals estimated as W_c/V_c.

Using the formula (16), we estimate the equilibrium values for the card network coverage parameter π^k. From Table 2, we see that by 1990 card network coverage of retail nodes in the UK had already reached over 70%. In 1996–97 this figure for EFTPOS coverage seems to have jumped to about 79–80% of all retail nodes.

3.3. *Partial Equilibrium Estimates of Interest Rate Sensitivity of Transaction Balances*

In Table 3 column (5) we estimate the interest rate elasticity for the payments technology constrained transactions demand for cash derived in (21). As can be observed, this is substantially greater than the Baumol–Tobin elasticity of one half. Further, note that the greater interest rate elasticity of transaction balances in column (5) of Table 3, corresponds with higher π^k in the UK in the latter part of the last decade.

Columns (6) and (8) compare the dollar changes in the transaction balances of a cash-card economy with a pure cash one. Here we take the year on year historical changes in the deposit interest rates reported in column (4) of Table 3. Note that the dollar changes in transactions balances in a cash-card economy is substantially greater than in a pure cash economy. Consider the 349 basis point cut in deposit interest rates around the time of the 1992–93 recession. *Ceteris paribus* in column 7 Table 3, we see that there could have been potentially large expansions in transactions balances. Similarly in 1996–97 when interest rates rose due to overheating and excessive consumer lending, we find a large percentage contraction in transaction balances of over 50% in the cash-card economy, while in the pure cash economy only a modest change can occur.

Table 2

Estimated U.K. Per Capita Cash-Card Payments Data: 1990–1997 (USD)

Year	1990	1991	1992	1993	1994	1995	1996	1997
$\|D_k\|$: Duration	0.519	0.516	0.515	0.517	0.514	0.511	0.510	0.508
$r\|D_k\|^{\dagger}$	6.50	5.30	3.84	1.89	2.04	2.10	1.56	1.84
V_c: Volume of cash purchases	113	114	106	69	85	102	100	128
$V = V_c + V_k$: Total volume of purchases	128	132	128	93	113	135	139	173
V_c/V	0.88	0.86	0.83	0.74	0.75	0.76	0.72	0.74
$T_c^{\#}$: Shoe leather cost	0.29	0.21	0.16	0.07	0.06	0.06	0.05	0.05
$\mu^* = W_c/V_c$	0.151	0.167	0.188	0.303	0.270	0.244	0.271	0.235
π^{k*}	0.74	0.73	0.72	0.71	0.72	0.74	0.80	0.79

†For calculation of $|D_k|$ refer to equation (A3) in Appendix 1 and footnote (4). Note $|D_c| = 0.5$.

Table 3

Partial Equilibrium Estimates of Interest Sensitivity of Transaction Balances

					Cash-card economy: $\omega < 1$			Pure cash economy: $\omega = 1$	
Year	π^k (1)	B_2 ATM cash balance (US$) (2)	Initial interest rate: r_0 (3)	Changes in interest rate: dr (4)	Interest rate elasticity (5)	ΔB_2 (US$) (6)	ΔB_2 (%) (7)	ΔB_1 (US$) (8)	ΔB_1 (%) (9)
1990	0.74	38.74	0.1254	−0.0226	−1.16	8.08	20.86	4.62	9.01
1991	0.73	40.67	0.1028	−0.0228	−1.23	13.77	33.87	7.27	13.72
1992	0.72	40.42	0.0746	−0.0349	−1.35	25.48	63.05	12.99	23.39
1993	0.71	34.98	0.0397	−0.0031	−2.10	5.73	16.39	1.91	3.90
1994	0.72	37.37	0.0366	0.0045	−1.93	−8.88	−23.76	−3.16	−6.15
1995	0.74	38.66	0.0411	−0.0106	−1.89	18.80	48.63	7.07	12.90
1996	0.80	39.74	0.0305	0.0058	−2.70	−20.41	−51.34	−5.42	−9.51
1997	0.79	41.67	0.0363	0.0085	−2.20	−21.47	−51.53	−7.24	−11.71
Average					−1.82				

Note: Columns 1–3 contain data from Tables 1 and 2. Column (5) uses the formula for interest rate elasticity given in the square bracket of (21). Column (6) = Column (5) × Column (4) × (B_2/r). Column (8) = $-(1/2r)B_1\,\mathrm{d}r = -(1/2r)\sqrt{\alpha T_c^{\#}/2r}\,\mathrm{d}r$ and column (9) = $-(1/2r)\mathrm{d}r$.

These excessive per capita swings of cash in circulation reported above can alter the liquidity of depository institutions. The interest rate increase aimed at curbing bank lending, for instance, may consequently have little impact on this as the scope for cash economisation by the consumer in a high EFTPOS economy enhances liquidity of depository institutions. However, this model cannot handle the fuller implications of payments technology innovation on bank lending or consumer spending. Finally, it must be noted that the dominance of interest rate effects given in columns (6) and (7) of Table 3 on total year on year changes in transactions balances (see, (20)) will manifest only in high π^k-economies when the scope for further improvements in π^k and reductions in unit ATM costs $T_c^{\#}$ have been exhausted. Table 2 has indicated that the latter was not the case in the UK in the 1990s where from 1991 and well into the second half of the decade the growth of EFTOS networks along with falling ATM costs worked to keep cash balances on a downward trend obscuring thereby the pure interest rate effects on cash balances.

4. Summary and Conclusion

To conclude, this is a work horse type paper, long overdue in the literature, that sets out the consequences for traditional transactions demand for cash from card type payment media which obviate the need for cash at point of sale. A payments innovation technology constrained optimal demand for transactions balances has been developed. Money demand functions specified solely in terms of income and the rate of interest are known to have broken down in the late 1970s due to innovations in the payments technology that allowed substitution away from cash.

Such models chronically over predicted money demand and failed to explain the observed volatility in the velocity of monetary base; see Mishkin (1997).

The main results of the paper are summarised here.

(i) A simple Nash equilibrium framework is used to develop the determinants for the mutual adoption of the new EFTPOS payments media by the representative consumer and merchant in the face of universally accepted currency that is being competitively dispensed.
(ii) In Result 3 we explicitly include the dominance of cash use in volume terms and incorporate the differential flow effects of purchases with cash and card over the year. This enabled us to obtain estimates on the volume of cash purchases for which there are no records.
(iii) In the Nash equilibrium specified in Result 7, there is a unique relationship between the parameter π^k of EFTPOS coverage and the proportion, ω, $0 < \omega < 1$, of cash financed expenditures.
(iv) Due to the impact of the flow effects of cash and card use in (16) and (19), when $\pi^k \rightarrow 1$, possibilities arise for infinite cash-card substitution at a critical 2% nominal interest rate at which incentives to economise on cash ceases.
(v) Estimates of the π^k parameter using UK data on cash-card transactions in Table 2 indicate that by 1997, there is some 79% coverage by EFTPOS of all retail outlets. Table 2 also reports how unit costs of cash network use have fallen from 29 cents in 1990 to 5 cents in 1997.
(vi) The family of iso π^k-curves in Figures 2 and 3, which constitutes the main didactic contribution of the paper, shows that higher levels of card network coverage when combined with low costs of ATM cash use, magnify the interest rate sensitivity of cash-card substitution. This can have crucial implications for monetary policy in that for high π^k economies in low interest rate regimes, interest rate rises (cuts) targeted at curbing (expanding) bank lending may prove to be difficult.

A similar calibration of cash-card substitution in other OECD countries has been done. This yields interesting results on early and late adopters of EFTPOS (Markose and Loke, 2002*a*). For a more thoroughgoing analysis of the monetary policy implications of the high interest rate sensitivity of the payments technology constrained transactions demand for money, the model developed here has been extended in Markose and Loke (2002*b*) to include an explicit banking sector with a household oriented loanable funds market.

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Appendix 1: Proof for Result 3

We require that $V_c > V_k$, yet $\text{PVE}_c = \text{PVE}_k$ in (9). Starting from an initial volume of purchases equal to that of the card purchases V_k which is known, we have to determine

(i) the precise increase in the volume of purchases for the cash fund relative to the card fund, $\text{d}V_k = V_c - V_k$ and
(ii) the corresponding fall in value, $\text{d}(1/V_k) < 0$, that maintains the same present value, PVE_k, as for the card fund.

For fixed interest rates, the total derivative of the present value function in (9) with $j = k$, is set equal to zero. This has to be identical whether in volume, V_k, or value $\$1/V_k$ terms. Further by definition

$$\frac{1}{V_k}\text{d}V_k = -V_k\text{d}\frac{1}{V_k}. \tag{A1}$$

Hence, the iso-PVE curve in the (PV^u, V_k) plane is given by

$$\frac{\text{dPVE}_k(V_k)}{\text{PVE}_k} = \frac{\partial\text{PVE}_k/\partial V_k}{\text{PVE}_k}\text{d}V_k + \frac{\partial\text{PVE}_k/\partial\text{PV}_k^u}{\text{PVE}_k}\text{dPV}_k^u = \frac{1}{V_k}(r|D_k| - 1)\text{d}V_k + \frac{\text{dPV}^u}{\text{PV}^u} = 0. \tag{A2}$$

Here, $|D_j|$ denotes the absolute value of the duration of the jth fund, $j = c, k$, and $r|D_j|$ is the interest elasticity of its present value flow. Also note that, $(r|D_j|-1)$ is the volume elasticity of its present value flow. The duration of a flow value of a fund defined as the percentage change in the present value for a 1% change in the interest rate is given by the formula

$$\begin{aligned} D_j = \frac{\text{dPVE}_j/\text{d}r}{\text{PVE}_j} &= -\frac{[1/(1+r/V_j)](1/V_j)\sum_{i=1}^{V_j} i/[V_j(1+r/V_j)^i]}{\text{PVE}_j} \\ &\equiv -\frac{1}{V_j^2}\sum_{i=1}^{V_j} i, \quad j = c, k.^9 \end{aligned} \tag{A3}$$

From (A2), we see that the following relationships must hold for the iso-level PVE_k curve,

$$\frac{\text{dPV}_k^u}{\text{PV}^u} = \frac{1}{V_k}(1 - r|D_k|)\text{d}V_k = V_k(r|D_k| - 1)\text{d}\frac{1}{V_k}. \tag{A4}$$

The second equality in (A4) is obtained by using (A1). On rewriting (A4), we have

$$\frac{1}{V_k}\text{d}V_k = -V_k\text{d}\frac{1}{V_k} + \left(\frac{1}{V_k}r|D_k|\text{d}V_k + V_k r|D_k|\text{d}\frac{1}{V_k}\right). \tag{A5}$$

To satisfy (A1), in (A5) the expression in parenthesis is zero. This implies that

$$\frac{\text{d}V_k}{\text{d}(1/V_k)} = \frac{\text{dPV}^u/\text{PV}^u\text{d}(1/v_k)}{\text{dPV}^u/\text{PV}^u\text{d}V_k}, \text{ as } \frac{\text{dPV}_k^u}{\text{PV}^u\text{d}(1/V_k)} = V_k r|D_k| \text{ and } \frac{\text{dPV}_k^u}{\text{PV}^u\text{d}V_k} = -\frac{1}{V_k}r|D_k|.$$

Thus,

$$\text{d}V_k = V_c - V_k = \frac{\text{dPV}_k^u}{\text{PV}^u\text{d}(1/V_k)} = V_k r|D_k| \tag{A6}$$

[9] The derivation in the second line of (A3) follows as PVE_j, $j = c,k$, in (8) and the discount factor $1/(1 + r/V_j)$ are approximately equal to 1 for large V_j. As $\sum_{i=1}^{V_j} i = V_j/2(1 + V_j) = V_j/2 + V_j^2/2$ in (A3) is the formula for an arithmetic sum, $D_j = -1/V_j^2(V_j/2 + V_j^2/2) = -(1/2V_j + 1/2) \approx -1/2$, for large V_j. That is $|D_j|$ is 1 and falls to a minimum of 1/2 as V_j ranges from 1 to about 50.

and

$$\mathrm{d}\frac{1}{V_k} = \frac{\mathrm{dPV}_k^u}{\mathrm{PV}^u \mathrm{d}V_k} = -\frac{1}{V_k} r|D_k|. \tag{A7}$$

From (A6) we derive Result 3, that $V_c > V_k$ which consistent with the iso-PVE$_k$ for a given V_k is

$$V_c = V_k(r|D_k| + 1).$$

Appendix 2: The CES Utility Function and the Case of Perfect Substitutes

Consider the general CES utility function for cash and card disbursements with the coefficient $\eta = 1$ implying cash and card are perfect substitutes:[10]

$$\mathrm{Max}_{\alpha_c,\alpha_k}[(\alpha_c \mathrm{PVE}_c)^\eta + (\alpha_k \mathrm{PVE}_k)^\eta]$$

subject to the budget constraint $Z \geq \alpha_c + 2W_c^* T_c^\# + \alpha_k + V_k(1 - \pi^k)T_c^\#$ with α_jPVE$_j$ being defined in (8) and (9). With the constraint above being exactly met, note that $\alpha_c = q_0 Z - 2W_c^* T_c^\#$ and $\alpha_k = (1 - q_0)Z - V_k(1 - \pi^k)T_c^\#$. Now substitute the latter identities for the expenditure shares (α_c, α_k) in the CES utility function. On using (9) and setting $\eta = 1$, we have the objective function in (13) with the proviso that Z ≡ (1 + r) as α has been normalised to one in our model. The generic indeterminacy of optimal expenditures shares when $\eta = 1$ and cash and card are perfect substitutes can be noted from the following. On using the Lagrangian function L for the utility maximization problem above, we have $(\partial \mathrm{L}/\partial\alpha_c)/(\partial \mathrm{L}/\partial\alpha_k) = (\alpha_c/\alpha_k)^{\eta-1} = 1 + r/2W_c^*$, from which the required result follows when $\eta = 1$.

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[10] The version of the CES utility function used in Dutta and Weale (2001) incorporates the payments technology parameter, π^k, into the utility function as follows: $\mathrm{Max}_{\alpha_c,\alpha_k}[(1-\pi)^{1-\eta}(\alpha_c \mathrm{PVE}_c)^\eta + \pi^{1-\eta}(\alpha_k \mathrm{PVE}_k)^\eta]$.

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