

Does the Aharonov–Bohm Effect Exist?

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We draw a distinction between the Aharonov–Bohm phase shift and the Aharonov–Bohm effect. Although the Aharonov–Bohm phase shift occurring when an electron beam passes around a magnetic solenoid is well-verified experimentally, it is not clear whether this phase shift occurs because of classical forces or because of a topological effect occurring in the absence of classical forces as claimed by Aharonov and Bohm. The mathematics of the Schroedinger equation itself does not reveal the physical basis for the effect. However, the experimentally observed Aharonov–Bohm phase shift is of the same form as the shift observed due to electrostatic forces for which the consensus view accepts the role of the classical forces. The Aharonov–Bohm phase shift may well arise from classical electromagnetic forces which are simply more subtle in the magnetic case since they involve relativistic effects of the order v^2/c^2 . Here we first review the experimentally observable differences between phenomena arising from classical forces and phenomena arising from the quantum topological effect suggested by Aharonov and Bohm. Second we point out that most discussions of the classical electromagnetic forces involved when a charged particle passes a solenoid are inaccurate because they omit the Faraday induction terms. The subtleties of the relativistic v^2/c^2 classical electromagnetic forces between a point charge and a solenoid have been explored by Coleman and Van Vleck in their analysis of the Shockley–James paradox; indeed, we point out that an analysis exactly parallel to that of Coleman and Van Vleck suggests that the Aharonov–Bohm phase shift is actually due to classical electromagnetic forces. Finally we note that electromagnetic velocity fields penetrate even excellent conductors in a form which is unfamiliar to many physicists. An ohmic conductor surrounding a solenoid does not screen out the magnetic field of the passing charge, but rather the time-integral of the magnetic field is an invariant; this time integral is precisely what is involved in the classical explanation of the Aharonov–Bohm phase shift. Thus the persistence of the Aharonov–Bohm phase shift when the solenoid is surrounded by a conductor does not exclude a classical force-based explanation for the phase shift. At present there is no experimental evidence for the Aharonov–Bohm effect.

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1. INTRODUCTION

The interference intensity pattern which arises on a distant screen when microscopic particles pass through two slits is one of the surprising aspects of quantum behavior. Indeed, Feynman⁽¹⁾ is often quoted as saying that in essence this is the “only mystery” of quantum mechanics. Shifts in this interference pattern have been observed experimentally due to electrostatic forces which are different for particles passing through the two slits⁽²⁾ and also due to gravitational forces which are different for particles passing through the two slits.⁽³⁾ These effects seem to be expected by physicists and are not mentioned in basic textbooks on quantum theory. However, the situation is very different for an interference pattern shift which may well be due to electromagnetic forces and is associated with magnetic energy in exact analogy with the electrostatic energy and gravitational energy of the previously mentioned interference pattern shifts. This is the Aharonov–Bohm phase shift arising when a magnetic solenoid is placed between the slits providing the double-slit particle interference pattern for charged particles. The phase shift is usually obtained by solving the nonrelativistic Schrodinger equation. The mathematics of the equation does not reveal whether the effect is due to classical electromagnetic forces or is due to a new quantum topological effect as claimed by Aharonov and Bohm. However, this Aharonov–Bohm phase shift is now mentioned in the textbooks as a new departure from classical physics. The present article suggests that the great attention given the Aharonov–Bohm phase shift is at least premature and may well be misdirected.

The Aharonov–Bohm phase shift is prominent in the physics literature and is mentioned in the textbooks precisely because it is purported to be an example of the Aharonov–Bohm effect,⁽⁴⁾ namely a quantum-topological interference pattern shift which occurs in the absence of classical forces. Vast numbers of articles have been written on the Aharonov–Bohm effect,⁽⁵⁾ it has been hailed⁽⁶⁾ as another of the great mysteries of nature (comparable to particle interference patterns), and it has been interpreted⁽⁷⁾ as showing us that our classical intuition is just plain wrong. Since the Aharonov–Bohm effect has had such a strong impact on physics, it seems worthwhile to examine the evidence for its existence in nature. Evidence for a quantum-topological Aharonov–Bohm effect depends on experimental observations which can not be accounted for by classical forces. Although there are indeed experimental predictions of the Aharonov–Bohm effect which are different from those based on classical forces, at present none have been observed.

This article has three basic parts. In the first part we will review the experimental predictions which, if observed, would distinguish a phase shift

arising in the absence of classical forces from those based upon classical forces. In the second part we will discuss the classical electromagnetic forces which give rise to the observed Aharonov–Bohm phase shift. In the third part we point out that magnetic velocity fields penetrate ohmic conductors in a form which is unfamiliar to most physicists so that experimental attempts to screen solenoids from the fields of passing charges are actually ineffectual. We conclude that there is no experimental evidence for the Aharonov–Bohm effect.

2. DISTINGUISHING EXPERIMENTAL PREDICTIONS

The basis for a double-slit particle interference pattern shift due to classical electromagnetic forces producing a classical lag is seen easily in an electrostatic analogue of the Aharonov–Bohm situation.⁽⁸⁾ We consider a point charge q which passes a line of electric dipoles. The line of dipoles is formed by two line charges $\pm\lambda$, parallel to the z -axis with the plus line charge at $x = \varepsilon$, $y = 0$, and the minus charge at $x = -\varepsilon$, $y = 0$. A point charge q passes the line of dipoles with a displacement from the origin given by $\mathbf{r}_q = \mathbf{i}x_q + \mathbf{j}y_q$, where, in the absence of interaction with the line of dipoles, $x_q = d$, $y_q = v_q t$. The electrostatic energy of interaction is given by Ref. (8)

$$U_E = \frac{1}{8\pi} \int d^3r \, 2\mathbf{E}_{\varepsilon\lambda} \cdot \mathbf{E}_q = [2q(\varepsilon\lambda)] \frac{x_q}{x_q^2 + y_q^2} \quad (1)$$

On the other hand a point charge q passing along the same path with the same velocity past an infinite circular solenoid with axis along the z -axis and with internal magnetic field \mathbf{B}_m and cross-sectional area \mathcal{A} would have an overlap magnetic energy with the magnetic field of the passing charge q given by Ref. (9)

$$U_M = \frac{1}{8\pi} \int d^3r \, 2\mathbf{B}_m \cdot \mathbf{B}_q = \left[\frac{qv_q B_m \mathcal{A}}{2\pi c} \right] \frac{x_q}{x_q^2 + y_q^2} \quad (2)$$

We notice that the functional dependence on the coordinates x_q , y_q of the passing charge q is the same in both equations (1) and (2).

In the electrostatic case, every physicist is able to predict the motion of the charged particle q in the small perturbation approximation. The force on the particle q is given by

$$\begin{aligned}\mathbf{F}_q &= -\nabla_q U_E = [2q(\varepsilon\lambda)] \frac{\mathbf{i}(x_q^2 - y_q^2) + \mathbf{j}(2x_q y_q)}{(x_q^2 + y_q^2)^2} \\ &= [2q(\varepsilon\lambda)] \frac{\mathbf{i}(d^2 - v_q^2 t^2) + \mathbf{j}(2dv_q t)}{(d^2 + v_q^2 t^2)^2}\end{aligned}\quad (3)$$

It follows that the departures from the unperturbed motion are given (to first order in q) by $\delta\mathbf{a}_q$, $\delta\mathbf{v}_q$, $\delta\mathbf{r}_q$, where

$$\delta\mathbf{a}_q(t) = \frac{\mathbf{F}_q}{m_q} = [2q(\varepsilon\lambda)] \frac{\mathbf{i}(d^2 - v_q^2 t^2) + \mathbf{j}(2dv_q t)}{m_q(d^2 + v_q^2 t^2)^2}\quad (4)$$

$$\delta\mathbf{v}_q(t) = \int_{-\infty}^t dt' \delta\mathbf{a}_q(t') = [2q(\varepsilon\lambda)] \frac{\mathbf{i}v_q t - \mathbf{j}d}{m_q v_q (d^2 + v_q^2 t^2)}\quad (5)$$

$$\begin{aligned}\delta\mathbf{r}_q(t_2, t_1) &= \int_{t_1}^{t_2} dt \delta\mathbf{v}_q(t) \\ &= [2q(\varepsilon\lambda)] \frac{1}{2m_q v_q} \left[\mathbf{i} \ln(d^2 + v_q^2 t^2) - \mathbf{j} 2 \arctan\left(\frac{v_q t}{d}\right) \right]_{t=t_1}^{t=t_2}\end{aligned}\quad (6)$$

We notice that (in contrast to the x -component) the y -component of force and hence the y -component of $\delta\mathbf{r}_q$ changes sign when the displacement d changes sign corresponding to the particle passing on opposite sides of the line of electric dipoles. If particles pass on opposite sides of the line of dipoles, then they experience forces in a different time-order along the direction of motion. On one side, a particle q is first speeded up as it approaches and then slowed down as it recedes, while on the other side, a particle q is first slowed down and then later speeded up. Thus if charged particles start out side by side but pass on opposite sides of the line of dipoles, then after passing the dipoles there is a relative displacement between the particles given by

$$\Delta\mathbf{r}_{qE} = \delta\mathbf{r}_q(+\infty, -\infty)_{x=-d} - \delta\mathbf{r}_q(+\infty, -\infty)_{x=+d} = \mathbf{j} \frac{2\pi[2q(\varepsilon\lambda)]}{m_q v_q^2}\quad (7)$$

This relative displacement is the basis for a particle interference pattern phase shift given by the phase change

$$\phi_{qE} = \frac{m_q \mathbf{v}_q \cdot \Delta\mathbf{r}_{qE}}{\hbar} = \frac{m_q v_q}{\hbar} \frac{2\pi[2q(\varepsilon\lambda)]}{m_q v_q^2} = \frac{2\pi[2q(\varepsilon\lambda)]}{v_q \hbar}\quad (8)$$

Precisely this same phase shift could also be obtained by substituting the potential energy of Eq. (1) into the Schrodinger equation and then using

the WKB approximation for particles which pass on opposite sides of the line of dipoles. The phase shift corresponds to that seen in the experimental work of Matteucci and Pozzi.⁽²⁾

We notice immediately that the analogy in functional form for the interaction energy between the case of a line of dipoles and the case of a solenoid invites us to follow a path of calculation analogous to that given above. Thus the magnetic interaction energy in Eq. (2) suggests a force analogous to (3), a particle acceleration analogous to (4), a change of velocity analogous to (5), and a relative displacement analogous to (7),

$$\mathbf{F}_q = \left[\frac{qv_q B_m \mathcal{A}}{2\pi c} \right] \frac{\mathbf{i}(d^2 - v_q^2 t^2) + \mathbf{j}(2dv_q t)}{(d^2 + v_q^2 t^2)^2} \quad (9)$$

$$\delta \mathbf{v}_q(t) = \left[\frac{qv_q B_m \mathcal{A}}{2\pi c} \right] \frac{\mathbf{i}v_q t - \mathbf{j}d}{m_q v_q (d^2 + v_q^2 t^2)} \quad (10)$$

$$\Delta \mathbf{r}_{qM} = \frac{\mathbf{j}2\pi}{m_q v_q^2} \left[\frac{qv_q B_m \mathcal{A}}{2\pi c} \right] \quad (11)$$

The relative displacement (11) suggests a phase shift analogous to Eq. (8)

$$\phi_{qM} = \frac{2\pi}{v_q h} \left[\frac{qv_q B_m \mathcal{A}}{2\pi c} \right] = \frac{qB_m \mathcal{A}}{hc} = \frac{q}{hc} \oint \mathbf{A}_m \cdot d\mathbf{r} \quad (12)$$

But Eq. (12) is precisely the magnitude observed for the Aharonov–Bohm phase shift when electrons pass a magnetic solenoid. It can be obtained from the nonrelativistic Schroedinger equation by using the vector potential \mathbf{A}_m corresponding to a long solenoid. If one uses the WKB approximation to calculate the phase shift for particles passing on opposite sides of the solenoid, one finds the phase shift of Eq. (12). Thus although the Aharonov–Bohm phase shift is usually cited as evidence for the topological Aharonov–Bohm effect, it is not at all clear that the phase shift does not arise from the forces, velocity changes, and displacements comparable to those given above. Perhaps the Aharonov–Bohm phase shift is analogous to the observed electrostatic interference pattern shift.

The Aharonov–Bohm effect suggested by Aharonov and Bohm is different from all the other effects based on classical forces in its claim that the particles experience no forces while traveling along their paths to the screen; the particles do not change velocity and are not relatively displaced. Thus any experiment which shows that the charged particles passing a solenoid do not have different velocities as they pass the solenoid or are not relatively displaced between the sides or do not have a relative time lag

would confirm the Aharonov–Bohm prediction. However, any experiment which showed that the charged particles did indeed have different velocities on opposite sides of the solenoid or did have a relative displacement between the two sides or did have a relative time lag would show that the force-free Aharonov–Bohm effect is not involved, but rather the Aharonov–Bohm phase shift is a response to classical forces and displacements. If the suggested velocity change were large enough, one could imagine deflecting the particles as they passed on opposite sides of the solenoid and in this way measuring any difference in velocity. If the time delay were sufficiently large, this would become classically measurable. If the relative displacement were large enough so as to exceed the particle coherence length, then one could imagine breaking down the double-slit interference pattern between particles passing on opposite sides of the solenoid while leaving the single-slit envelope unaffected. According to the Aharonov–Bohm-effect interpretation, this breakdown would not occur because there is no relative displacement of the particles no matter how large the solenoid flux is made. We conclude that there are indeed experimental effects which, if observed, would confirm that the Aharonov–Bohm phase shift is a new and different type, not dependent upon velocity changes leading to relative displacements. However, at present the only experimental evidence we have is the Aharonov–Bohm phase shift itself, and this phase shift is indistinguishable from those due to classical electromagnetic forces.

3. CALCULATIONS OF ELECTROMAGNETIC FORCES

Although there is no experimental evidence that the Aharonov–Bohm phase shift is not based upon forces and velocity changes, the Aharonov–Bohm-effect interpretation has achieved wide credence because the classical electromagnetic forces involved are hard to calculate; proponents of the effect claim that the forces are lacking entirely. The forces are hard to calculate because they involve multiparticle, relativistic effects of order v^2/c^2 .

The usual argument given by proponents of the Aharonov–Bohm effect goes something like this.^(4, 6) A very long neutral solenoid has negligible electric and magnetic fields outside its current-winding. The charged particles which pass by the solenoid are therefore traveling in a region where there are neither electric nor magnetic fields. Therefore the passing charges experience no classical electromagnetic force, and so can not give a force-dependent phase shift. Thus the experimentally observed Aharonov–Bohm phase shift is evidence for the quantum-topological Aharonov–Bohm effect.

In this form, the argument is obviously flawed. The same analysis could be given to suggest that there was no force experienced by a charged particle passing a neutral conductor with no currents. Thus a neutral conductor with no currents produces no electromagnetic fields, and therefore a passing charged particle experiences no electromagnetic forces. Actually, of course, there are electromagnetic forces. The electric fields carried by the passing charge act so as to polarize the conductor. The separation of charges of the conductor then gives rise to electromagnetic fields back at the position of the passing charge, and the passing charge indeed experiences electromagnetic forces.

In exactly the analogous fashion there is an interaction of a solenoid with the electromagnetic fields of a passing particle. The particles of the solenoid accelerate and produce electromagnetic fields back at the position of the passing charge. Thus the passing charge experiences forces associated with the presence of the solenoid. And perhaps it is these forces on the passing charge which lead to velocity changes and produce the Aharonov–Bohm phase shift.

Classical electromagnetic forces between a point charge and a solenoid are notoriously hard to calculate and so give rise to apparent “paradoxes.” For example, the paradox of Shockley and James⁽¹⁰⁾ was answered in a now well-known article by Coleman and Van Vleck.⁽¹¹⁾ In Ref. 12 we consider some of the complications associated with the evaluation of electromagnetic forces involving a charged particle and a solenoid. Here we will merely outline the theoretical calculations which strongly suggest that classical electromagnetic forces provide the basis for the observed Aharonov–Bohm phase shift.

In the situation discussed by Coleman and Van Vleck, the changing magnetization of the solenoid puts an obvious Lorentz force on the external charged particle while there is no apparent force back on the solenoid. In the situation proposed by Aharonov and Bohm the reverse situation occurs; there is an obvious magnetic Lorentz force on the solenoid due to the magnetic field of the passing charged particle while there is no apparent force back on the particle. In both cases the forces are of order v^2/c^2 . In both cases the analysis of Coleman and Van Vleck indicates that there must be forces on both the particle and the solenoid in order to conserve energy, linear momentum, and constant motion of the system center of energy.

The Lorentz force due to the magnetic field of the passing charge q acting on the currents of the solenoid can be rewritten by partial integration and the use of Maxwell’s equations in the form⁽¹²⁾

$$\mathbf{F}_m = -\frac{q}{c}(\mathbf{v}_q \cdot \nabla_q) \mathbf{A}_m(\mathbf{r}_q) = -\nabla_q U_M \quad (13)$$

where \mathbf{v}_q is the velocity of the passing charge q , $\mathbf{A}_m(\mathbf{r}_q)$ is the vector potential of the solenoid in the Coulomb gauge evaluated at the position of the passing charge q , and U_M is exactly the magnetic energy given above in Eq. (2). If we use this force in Newton's law $\mathbf{F} = M\mathbf{a}$ to obtain the acceleration of the solenoid and then integrate twice, we obtain a displacement of the center of energy of the solenoid exactly analogous to Eq. (6) above.⁽¹³⁾ However, there is no obvious displacement of the passing charge. We have already noted that the *unperturbed* solenoid has no electric or magnetic fields outside its winding, and therefore the *unperturbed* solenoid produces no force on the passing charge. However, if the passing charge is not displaced from its unperturbed motion, then the principle of the constant velocity of the center of energy of the entire solenoid-point charge system has been violated, although this principle is required by general considerations of relativistic electromagnetic theory.

In this case we suggest that the force on the magnetic moment has been correctly calculated in Eq. (13) while the force on the passing charge q has not been obtained correctly. The reasoning is as follows. The Lorentz force acting on a charge distribution depends upon the instantaneous density of charge and current of the distribution and also upon the electromagnetic fields at the distribution due to the other objects. The electromagnetic fields depend upon the charges, currents, and also charge accelerations of the sources. The electromagnetic fields calculated naively from the unperturbed behavior of the solenoid and the passing charge omit the Faraday induction terms when charges accelerate due to the perturbation. Now the external passing charge q does not accelerate much since the solenoid is electrically neutral and so provides no electrostatic forces on q to lowest order in q . Since q has only a very small acceleration, the electromagnetic fields q produces at the magnetic moment are given correctly by the electric and magnetic fields produced by q in its unperturbed motion. On the other hand, the charges of the solenoid experience both the electric and magnetic fields of the passing charge. These fields would give each of these individual solenoid charges a significant acceleration were it not for the collective nature of their response to the force; the accelerations of the many interacting solenoid particles can produce significant acceleration fields back at the passing particle q . These Faraday acceleration fields are omitted entirely in calculations which use fields arising from the unperturbed motions of the solenoid charges. Thus we can not trust the electric and magnetic fields associated with the unperturbed motion of the solenoid particles when calculating the fields back at the passing charge. Exactly this situation is familiar for the situation of a Faraday-induced emf in a circuit where the resistance is small and the mutual inductance is high.

If the displacement of the center of energy of the solenoid is correctly calculated, then we can use the principle of the constant motion of the system center of energy in order to obtain the motion of the passing charged particle.⁽¹²⁾ Since the solenoid is neutral, there are no cross terms when calculating the motion of the center of energy of the solenoid-charge system. Since the changes in solenoid velocity following from the magnetic force given above in (13) are already of order $1/c^2$, the constant motion of the system center of energy requires that⁽¹²⁾

$$M_m \delta \mathbf{v}_m + m_q \delta \mathbf{v}_q = 0 \quad \text{and} \quad M_m \delta \mathbf{r}_m + m_q \delta \mathbf{r}_q = 0 \quad (14)$$

But then the relative displacement of the passing charge follows from Eqs. (13) and (14) as

$$\begin{aligned} \delta \mathbf{r}_q(t_2, t_1) &= -\frac{M_m}{m_q} \delta \mathbf{r}_m(t_2, t_1) \\ &= -\left[\frac{qv_q B_m \mathcal{A}}{2\pi c} \right] \frac{1}{2m_q v_q} \left[\mathbf{i} \ln(d^2 + v_q^2 t^2) - \mathbf{j} 2 \arctan\left(\frac{v_q t}{d}\right) \right]_{t=t_1}^{t=t_2} \end{aligned} \quad (15)$$

Now except for sign, this condition is exactly what was used to calculate the magnitude of the relative lag (11) of particles passing on opposite sides of the solenoid which gave the magnitude of the Aharonov–Bohm phase shift (12). The present calculation, in contrast to the suggestive analogy in Sec. 2 above, actually gives both the magnitude and sign for the Aharonov–Bohm phase shift. We conclude that classical electromagnetic theory indeed suggests that the Aharonov–Bohm phase shift may be a force-based effect.

4. ATTEMPTS AT SHIELDING BY CONDUCTORS

In order to counter suggestions (such as those of the present article) that the Aharonov–Bohm phase shift is actually based on classical electromagnetic forces, there have been experiments proposed⁽¹⁴⁾ which attempt to shield the solenoid from the electromagnetic fields of the passing charges. The argument is made that if the electromagnetic fields of the passing charge q are screened by a conductor so that the fields never interact with the solenoid, then there can be no classical electromagnetic interaction between the passing particle and the solenoid, and hence no classical

basis for the Aharonov–Bohm phase shift. In one experiment,⁽¹⁵⁾ a superconducting solenoid was used and the Aharonov–Bohm phase shift was still seen (at the now-quantized flux of the solenoid). Although it was recognized that at the high frequencies associated with the rapid passage of the particle the superconductor might behave as a normal metal, it was argued that the magnetic field of a passing charge would be screened out in a very thin surface layer and hence could not produce any forces back on the passing charge.⁽¹⁶⁾ In another experiment,⁽¹⁷⁾ a permalloy toroid was covered with both superconducting material and copper in order to prevent the magnetic flux from leaking out and the electrons from penetrating. Many physicists assume that the fields of the passing electrons would not penetrate to the permalloy magnetic material. However, the shielding of magnetic velocity fields, in contrast with electromagnetic wave fields, is an area unfamiliar to most physicists. It turns out that the magnetic fields of the passing charge do indeed penetrate through an ohmic conductor.⁽¹⁸⁾

Electromagnetic wave fields arise from the acceleration of a charged particle and have a spectrum associated with the time-Fourier transform of the particle acceleration. These fields are absorbed in ohmic conductors with a penetration depth which is dependent upon the frequency spectrum. In contrast with wave fields, electromagnetic velocity fields depend only upon the position and velocity of a charged particle at the retarded time, they falloff as $1/r^2$, and they allow superposition to obtain the fields of a steady current. It is these velocity fields which are involved in the interaction between a solenoid and a passing charge. In a recent article,⁽¹⁸⁾ “Understanding the Penetration of Electromagnetic Velocity Fields into Conductors,” a new time-integral invariant was noted for the magnetic field. It was proved that the time-integral of the magnetic field at a given point in space due to a charged particle moving with constant velocity is independent of any ohmic conductors which are present. Thus, while an ohmic conductor will indeed decrease the magnitude of the penetrating magnetic field, in compensation the magnetic field lasts for a longer time so as to make the time-integral of the magnetic field an invariant. It turns out that the time-integral which is involved in the classical calculation of the Aharonov–Bohm phase shift is precisely the same time-integral which is invariant despite the presence of ohmic conducting materials. Thus the phase shift (12) depends upon the relative displacement (11) which takes the form

$$\Delta \mathbf{r}_{qM} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{t'=t} dt' \frac{1}{m_q} \left\{ \nabla_q U_M(x_q = -d, y_q = v_q t) \right. \\ \left. - \nabla_q U_M(x_q = +d, y_q = v_q t) \right\}$$

$$\begin{aligned}
&\approx \int_{-\infty}^{\infty} dt \frac{1}{m_q v_q} \int_{-\infty}^{y=v_q t} dy \mathbf{j} \left\{ \frac{\partial}{\partial y} U_M(x_q = -d, y) \right. \\
&\quad \left. - \frac{\partial}{\partial y} U_M(x_q = +d, y) \right\} \\
&= \frac{\mathbf{j}}{m_q v_q} \left(\int_{-\infty}^{\infty} dt U_M(x_q = -d, y_q = v_q t) \right. \\
&\quad \left. - \int_{-\infty}^{\infty} dt U_M(x_q = +d, y_q = v_q t) \right) \tag{16}
\end{aligned}$$

Now the time-integral here can be written as

$$\begin{aligned}
&\int_{-\infty}^{\infty} dt U_M(x_q, y_q = v_q t) \\
&= \int_{-\infty}^{\infty} dt \frac{1}{8\pi} \int d^3r 2\mathbf{B}_m(x, y, z) \cdot \mathbf{B}_q(x, y - v_q t, z) \\
&= \frac{1}{8\pi} \int d^3r 2\mathbf{B}_m(x, y, z) \cdot \int_{-\infty}^{\infty} dt \mathbf{B}_q(x, y - v_q t, z) \tag{17}
\end{aligned}$$

But this last time-integral is an invariant⁽¹⁸⁾ independent of any ohmic conducting materials which may be present.

It seems astonishing that the time-integral of the magnetic field energy which was used to calculate the relative particle displacements for charges passing on opposite sides of a solenoid is actually independent of the presence of any intervening ohmic conductors. However, such is indeed the case. We conclude that it is not at all clear that the presence of conducting or superconducting materials eliminates the possibility of a force-based Aharonov–Bohm phase shift.

5. CLOSING SUMMARY

The phase shifts observed experimentally when an electron beam passes around a line of electric dipoles or a line of magnetic dipoles are both accurately described by the Schroedinger equation. The classical limits of the effects can be investigated by using the Hamilton–Jacobi limit of the Schroedinger equation. In the case of the line of electric dipoles, the Hamilton–Jacobi limit gives exactly the electrostatic lag in Eq. (7) of this article, based on electrostatic forces. In the case of the line of magnetic

dipoles (a solenoid), a careful Hamilton–Jacobi analysis has never been adequately explored and is not obvious. It is not at all clear just what approximation the nonrelativistic Schroedinger equation represents in relation to the classical, relativistic, multiparticle interaction of a charged particle and a solenoid.

Aharonov and Bohm claim that the phase shift in the case of the solenoid corresponds to a new quantum topological effect. This point of view has entered the textbook literature as a major departure from classical physics. However, is yet, there seems to be no experimental evidence for the effect since the experimentally observed Aharonov–Bohm phase shift takes the same form as an interference pattern shift which is due to forces which produce velocity changes and particle displacements. Moreover, classical physics contains strong suggestions that relativistic electromagnetic forces of order v^2/c^2 produce the observed Aharonov–Bohm phase shift. Finally, experimental attempts to eliminate classical electromagnetic interactions between the solenoid and the passing particle are not unambiguous because electromagnetic velocity fields penetrate through ohmic conductors in an unexpected fashion which is completely different from the familiar exponential damping of wave fields.

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14. H. Erlichson, *Am. J. Phys.* **38**, 162 (1976).
15. B. Lischke, *Z. Physik* **239**, 360 (1970). Although a superconductor expells the magnetic field lines of a time-independent magnetic field in the Meissner effect, a superconductor acts similarly to a normal metal for high frequency fields.
16. The erroneous point of view regarding the role of conducting materials appears on p. 426 of the review by Olariu and Popescu listed in Ref. 5, and on p. 123 of the review by Tonomura.
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