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The Simplest Dynamic General-Equilibrium Model of an Open Economy

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This paper presents the simplest possible general-equilibrium model of an open economy in which producer and consumer decisions are both intra- and intertemporally consistent. Consumers maximize the present value of the utility of consumption; producers maximize the present value of profits. The model solves for the set of intertemporally consistent prices. The parsimonious structure of the model is achieved by dividing the economy into two producing sectors—exports and domestic goods—and two consumed goods—imports and domestic goods. As a result, there is only one endogenous price per period to be solved for (the price of the domestic good), although “structural” questions, such as the evolution of the real exchange rate, can be posed with the model. Furthermore, with this structural breakdown, the model can be calibrated with national accounts data only. In the paper, we show how to calibrate such a model (including specification of an adjustment-cost function, to avoid “bang-bang” behavior) and use the model to examine various questions where intertemporal issues are important, including terms-of-trade shocks and tariff reform. © 1998 Society for Policy Modeling Published by Elsevier Science Inc.

1. INTRODUCTION

While their popularity has surged over the last two decades, computable general equilibrium (CGE) models have also become the object of some criticism. Consumers of model results complain that the models’ specification is often too complicated and detailed to be comprehensible. People charged with building these models find that CGE models require enormous amounts of data, some

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of which are unavailable. Finally, some academic economists have noted that CGE models, although based in the Walrasian tradition, make several compromises with the “pure” model, leaving the resulting specification difficult to interpret.

A particular form of the last criticism refers to CGE models’ treatment of dynamics. For example, consumers are assumed to save a fixed share of income, and investment decisions are based on historical shares or current rates of return to capital. Savings and investment decisions are not “forward-looking.” Yet, for their within-period decisions, these same consumers and producers solve fairly complicated optimizing problems and take into account the information contained in all the prices in the economy. This apparent contradiction has not escaped several observers (see, for example, Srinivasan, 1982; and Bell and Srinivasan, 1984).

In this paper, we present a model that addresses all three criticisms of CGE models mentioned above. First, it is arguably the simplest possible CGE model of an open economy. There are only three goods: imports, exports, and a domestic good. As has been shown elsewhere in the context of a static model (Devarajan et al., 1990), this is the minimal number of goods required to capture the salient aspects of an open economy. Yet, this three-good model anticipates qualitatively the results obtained from much larger models (see Devarajan et al., 1993). Second, the model can be calibrated using little more than national-accounts data. Finally, and most importantly for this paper, the model solves for the set of intra- and intertemporally consistent prices. Both savings and investment are the result of dynamic optimization based on future prices which are, in turn, consistent with the realized levels of savings and investment.

In addition to addressing some criticisms of CGE models, the model presented in this paper enables us to analyze, in a simple and transparent way, questions that have an intertemporal dimension. We examine the effects of a terms-of-trade shock on savings and investment. The outcome will be different from that obtained from a typical static model, where the effect of this shock on permanent income cannot be captured. Some policies, moreover, are intrinsically intertemporal. For example, we look at the differential impact of tariffs on consumer goods and capital goods, a difference which cannot be captured in a static framework.

To be sure, ours is by no means the first intertemporally consistent CGE model. Several people have examined questions of an

intertemporal nature (such as the effects of tax policy on investment) using perfect-foresight dynamic models (see, for example, Jorgenson and Yun, 1990; and Jorgenson and Wilcoxon, 1990; Goulder and Summers, 1989). Another tradition focuses on open-economy, dynamic models (see Bruno and Sachs, 1985; Bovenberg, 1989; Goulder and Eichengreen, 1989; and Mercenier, 1993). While they have yielded valuable insights, all of these models have been relatively complex and intensive in their use of data and computing power. The purpose of our paper is to boil open-economy dynamic models down to their bare essentials, so that the models can be made more accessible.

In Section 2 of this paper, we show how the dynamic model is specified, demonstrate how the model is calibrated, and conclude with the treatment of terminal conditions. In Section 3, we perform two simulations with the model and interpret them. Section 4 presents some concluding remarks.

2. THE 1-2-3-T MODEL

The framework is a dynamic and expanded version of the “1-2-3 model” described in Devarajan, Lewis, and Robinson (1993). Consumption and investment behavior are intertemporal as in Go (1994). Unlike the latter however, the model is not formulated as a central plan. Explicit expressions or dynamic equations for consumption and investment are derived. Distinction is made among various taxes, particularly import tariffs which generate significant public revenue and confer substantial protection in developing countries. The implementation of the model is kept simple so that only generally available data (e.g., national income and budgetary accounts) are required. In what follows, the model is briefly described. A list of equations is provided in Appendix A.

For the purpose of numerical implementation, the intertemporal problem is formulated in discrete time. Discounting in discrete time requires a dating convention. In order to keep the derivation and calibration simple (see Section 2E), all transactions are assumed to take place at the end of the period (while decisions are made or planned at the beginning of the period). At the beginning of period $t = 0$ for example, income earned in that period has to

be discounted by r_0 , i.e., $\frac{Y_0}{1 + r_0}$; likewise, the present value of

income in the next period is $\frac{Y_1}{(1 + r_0)(1 + r_1)}$; and, the stock of

wealth in period t earns an interest income $r_t W_t$ at the beginning of the next period $t + 1$.

2A. Consumption

The representative consumer maximizes his discounted utility of the temporal sequence of (aggregated) consumption (Equation (1)):

$$\max U_0 = \sum_{t=0}^{\infty} \left(\frac{1}{1 + \rho} \right)^{t+1} \frac{1}{1 - \nu} (C_t)^{1-\nu} \quad (1)$$

This is the familiar homogeneous utility function which is additively separable with constant elasticity of marginal utility, ν . Utility is discounted by the consumer's positive and constant rate of time preference, ρ .¹

To determine the consumer's budget constraint, we first define his wealth, W_0 , as the discounted flow of current income Y_t (Equation (2)):

$$W_0 = \frac{Y_0}{1 + r_0^c} + \frac{Y_1}{(1 + r_0^c)(1 + r_1^c)} + \dots + \frac{Y_t}{\prod_{s=0}^t (1 + r_s^c)} \quad (2)$$

$$+ \dots = \sum_{t=0}^{\infty} \mu(t) Y_t$$

where $\mu(t) = \prod_{s=0}^t (1 + r_s^c)^{-1}$ and r_t^c is the interest rate facing consumers (defined below). It is often convenient to separate wealth into its components: (1) financial wealth, which is the present value of future capital income and which is also equivalent to the amount of capital created, K_t valued at its shadow price q_t ; and (2) nonfinancial wealth, which is the discounted flow of net-of-tax labor income plus net transfers from government and net

¹ Another option is to consider N identical consumers and define the consumer problem as:

$$\begin{aligned} \max U_0 &= \sum_{t=0}^{\infty} N_t \left(\frac{1}{1 + \rho} \right)^{t+1} \frac{1}{1 - \nu} \left(\frac{C_t}{N_t} \right)^{1-\nu} \\ &= \sum_{t=0}^{\infty} N_t^{\nu} \left(\frac{1}{1 + \rho} \right)^{t+1} \frac{1}{1 - \nu} C_t^{1-\nu} \end{aligned}$$

Because N or its base-year level and growth rate [i.e., $N_t = N_0(1 + g)$] are exogenous, the two approaches should lead to the same results. In the aggregated case, one can obviously think of N as set to an index of 1.00 (no population growth) or, alternatively, all relevant variables are scaled and defined in per-capita terms (no population or productivity growth). See footnote 4 also.

remittances from abroad less foreign debt service. The present value of the stream of debt-service payments is the level of foreign debt. Hence, the consumer is assumed responsible for both foreign debt and its interest payments.² The wealth constraint (Equation (3)) of this household requires that the present value of consumption expenditures not exceed its wealth:

$$\sum_{t=0}^{\infty} \mu(t) PC_t C_t \leq W_0 \tag{3}$$

The Lagrangian of the intertemporal problem (Equation (4)) is

$$L = \sum_{t=0}^{\infty} \left(\frac{1}{1 + \rho} \right)^{t+1} u(C_t) + \gamma \left[\sum_{t=0}^{\infty} \mu(t) PC_t C_t - W_0 \right] \tag{4}$$

where $\geq u(C_t) = \frac{1}{1 - \nu} (C_t)^{1-\nu}$. The solution is a consumption sequence in the form of the familiar choice-theoretic consumption model (Equation (5)):³

$$C_t = \frac{W_t(1 + r_t)}{[PC_t(1 + \sum_{j=t+1}^{\infty} \zeta(t,j))]} \tag{5}$$

where

$$\zeta(t,j) = \left[\frac{PC_t}{PC_j} \right]^{1-\nu} \frac{\Pi_{u=t+1}^j (1 + r_u^c)^{1-\nu}}{(1 + \rho)^{j-t}} \Big]^{1/\nu}$$

The intertemporal condition (Equation (6)) requires that the marginal utility of consumption in period t and s ($>t$) satisfy:

$$\frac{u'(C_s)}{u'(C_t)} = \frac{PC_s}{PC_t} \frac{(1 + \rho)^{s-t}}{\Pi_{u=t+1}^s (1 + r_u^c)} \tag{6}$$

Given that $u'(C_t) = C_t^{-\nu}$, the forward change of consumption between two adjacent periods can be derived as a function of the

² Thus we are abstracting from the possibility that foreign debt service imposes an additional cost to society, namely, the deadweight loss associated with transferring resources from the private to the public sector.

³ Alternatively, if transactions are made at the beginning of each period $C_t = \frac{W_t}{[PC_t(1 + \sum_{j=t+1}^{\infty} \zeta(t,j))]}$, which is the more familiar form. The difference is the term $(1 + r_t)$ on the right-hand side. In our dating convention, consumption expenditure is realized at the end of the period and has to be discounted to the beginning of the period when decisions are made.

relative prices of the two periods, the rate of time preference, and the rate r_i^c by which current consumption is transformed into future consumption (Equation (7)):⁴

$$\frac{C_{t+1}}{C_t} = \left(\frac{PC_{t+1}(1 + \rho)}{PC_t(1 + r_{t+1}^c)} \right)^{\frac{1}{v}} \tag{7}$$

A large r^c makes future consumption cheaper, so that future consumption will increase. The intertemporal rate r^c is determined by the opportunity cost of savings, which in this case is the cost of foreign borrowing. The latter, Equation (8), is defined as the world interest rate i^* plus the forward percent change in the real exchange rate, e_i^c . The real exchange rate, e_i^c , in turn, is the relative price between imports and domestic goods (the two goods purchased by the consumer) Equation (9):

$$r_i^c = i^* + \frac{e_{i+1}^c - e_i^c}{e_i^c} \tag{8}$$

where

$$e_i^c = PM_i/PD_i \tag{9}$$

2B. Investment

As in the work of Abel (1980), Hayashi (1982), and Summers (1981), the dynamic decision problem of the firm is to choose a time path of investment that maximizes the value of the firm V_t , defined as the present value of net income, Equation (10):

⁴This is the discrete-time version of the more familiar, continuous-time case $\frac{C_t}{C_t} = \frac{1}{v} \left(r_i^c - \rho_t - \frac{PC_t}{PC_t} \right)$. Note also that if the economy is growing at an exogenous balanced-growth rate g , a $(1 + g)$ term has to be added to these derivations:

$$\begin{aligned} \zeta(t,j) &= \left[\left(\frac{PC_t}{PC_j} \right)^{1-v} \frac{\prod_{u=t+1}^j (1 + r_u^c)^{1-v}}{(1 + \rho)^{j-t}} \right]^{1/v} (1 + g)^{j-t} \\ \frac{u'(C_s)}{u'(C_t)} &= \frac{PC_s}{PC_t} \frac{(1 + \rho)^{s-t}}{\prod_{u=t+1}^s (1 + r_u^c)} (1 + g)^{s-t} \\ \frac{C_{t+1}}{C_t} &= \left(\frac{PC_{t+1}(1 + \rho)}{PC_t(1 + r_{t+1}^c)} \right)^{\frac{1}{v}} (1 + g) \end{aligned}$$

Equations (5), (6), and (7) imply either one of these cases: (1) $g = 0$, so that the economy is defined in per-capita terms and there is neither population nor productivity growth; or (2) all growth rates in the economy are detrended by g , the balanced growth rate.

$$\max V_0 = \sum_{t=0}^{\infty} \mu(t)R(t) \quad (10)$$

subject to Equation (11):

$$K_{t+1} - K_t = I_t - \delta K_t \quad (11)$$

Equation (11) is the familiar capital accumulation equation, with δ being the depreciation rate. $R(t)$ is gross profit less investment expenditures. $J(\cdot)$, i.e., $R(t) = J(\cdot)$. Investment expenditures, Equation (12), are affected by the replacement cost of capital PK_t as well as investment tax credits tc_t and adjustment costs $\theta(x_t)$, Equation (13):

$$J[I_t, \theta(t) \dots] = I_t PK_t [1 - tc_t + \theta(x_t)] \quad (12)$$

$$\theta(x_t) = \left(\frac{\beta}{2}\right) \frac{(x_t - \alpha)^2}{x_t} \quad (13)$$

The variable $\theta(\cdot)$ signifies the presence of adjustment costs in investment and increases as a function of the ratio $\left(\frac{I_t}{K_t}\right)$, defined as x_t above. A quadratic function with parameters α and β , $\theta(\cdot)$, is treated as external to the firm. It implies that production does not adjust instantaneously to price changes and that desired capital stocks are only attained gradually over time.

The interest rate used in the discount factor $\mu(\cdot)$ is the interest rate affecting the producer, r^p . Like the discount rate for consumption, r^p (Equation (14)) is defined by the world interest rate and the forward percent change in the real exchange rate. However, the real exchange rate affecting the producer, e^p , is the relative price between exports and domestic goods (the two goods sold by the producer) Equation (15):

$$r_t^p = i^* + \frac{e_{t+1}^p - e_t^p}{e_t^p} \quad (14)$$

where

$$e_t^p = PE_t/PD_t \quad (15)$$

The current-value Hamiltonian of the firm's problem, Equation (16), is

$$R(t) + q_t(I_{t-1} - \delta K_{t-1}) \quad (16)$$

The intertemporal optimal conditions (a) and (b) below have a natural economic interpretation: (a) firms invest until the marginal

cost of investment $J'(I_t)$ is equal to the shadow price of capital q_t ; and (b) the required return to capital $r_t^p q_t$ is equal to the marginal revenue product of the added capital R_k plus capital gains Δq net of depreciation loss δq_{t+1} .

$$\begin{aligned} \text{a) } \frac{\partial H}{\partial I_t} = 0 & \Rightarrow J'(I_t) = q_t \\ & \text{or } PK_t(1 - tc_t + [\theta(\cdot) + x_t \theta'(\cdot)]) = q_t \\ \text{b) } \mu(t+1)q_{t+1} - \mu(t)q_t = -\frac{\partial H}{\partial K_t} & \Rightarrow (1 + r_t^p)q_t = (1 - \delta)q_{t+1} + R_k(t) \\ & \text{or } r_t^p q_t = R_k(t) + \Delta q - \delta q_{t+1} \end{aligned}$$

The difference equation above (subject to the transversality condition $\lim_{T \rightarrow \infty} \mu(t, T)q_{T+1}K_T = 0$) can be solved to yield the shadow price of capital expressed as the present value of the future marginal revenue products of capital (Equation (17)):

$$q_t = \sum_{s=t}^{\infty} \mu(s) [R_k(s) (1 - \delta)^{s-t}] \quad (17)$$

The solution of the dynamic problem is an investment sequence dependent on the tax-adjusted Tobin's q and the parameters of the adjustment cost function (Equation (18)):

$$\begin{aligned} \frac{I_t}{K_t} &= h(Q_t^T) \\ &= \alpha + \frac{1}{\beta} Q_t \end{aligned} \quad (18)$$

where

$$Q_t^T = \frac{q_t}{PK_t} - (1 - tc_t)$$

where Q_t^T is the ratio of the shadow price of capital q_t and the replacement cost of capital PK_t , adjusted for various taxes. Moreover, by Hayashi's identity (1982), the shadow price of capital q_t also is equal to the average q , the ratio of the value of the firm to its capital stock, that is, $q_t \equiv \frac{V_t}{K_t}$. In a simple case without tax credits, Q_t in the investment function becomes Equation (19):

$$Q_t = \frac{q_t^a}{PK_t} - 1$$

$$= \frac{V_t}{PK_t K_t} - 1 \quad (19)$$

The first term on the right is simply's Tobin's q , the ratio of the value of the firm to the replacement cost of capital. Thus, if Tobin's q , or the shadow price of capital deflated by the replacement cost of capital, is greater than 1, investment will be positive, and vice-versa if it is less than 1.

2C. The Static 1-2-3 Framework

The static (within-period) model is a slightly extended version of the 1-2-3 model in Devarajan, Lewis and Robinson (1993). It is identical to the model in Devarajan et al. (1997). As mentioned earlier, there are two produced goods, exports, E_t , and domestic goods, D_t . Output is a fixed coefficient combination of valued added and intermediate imports. Value added is a CES composite of labor and installed capital.

There are three types of imports, each assessed with a unique import duty. In addition to intermediate imports, there are capital imports, which is a fixed coefficient requirement in investment, and final imports, which compete with the domestic good.

Imperfect substitution characterizes the competition between foreign and domestic goods. This is reflected in the Armington CES function between domestic goods and final imports and a constant elasticity of transformation (CET) between sales to the domestic market and sales to the export markets. Moreover, reflecting the country's endowments, trade specialization, and past policies, the baskets of export goods and competitive import goods are different. This dichotomy implies that the real exchange rate affecting the demand side depends on the prices of domestic goods and imports while the real exchange rate affecting the supply side relies on the prices of domestic and export goods.

Government revenue comes from tariffs, domestic indirect taxes, income taxes, plus external borrowing. Government current expenditures include public consumption, transfers, and subsidies, all of which are assumed to be exogenous. The difference between government revenue and expenditure equals government savings.⁵

⁵ For simplicity, we do not distinguish between private and public investment in this paper.

2D. Equilibrium Conditions

To arrive at a solution, both the intertemporal and general-equilibrium conditions have to be satisfied simultaneously. At every point in time, the usual general-equilibrium conditions require that: (1) material balance in the demand and supply of all goods in the economy holds; (2) the demand for total labor equals its supply; (3) the balance in the external current account must be offset by flows in the capital account;⁶ and (4) government revenue is allocated between public expenditures and savings. By Walras' law, the savings-investment identity is implied by the above equations. Because investment and domestic savings decisions are independent and separate, foreign savings balances the equality. Note, however, that the stream of debt-service payments arising from an increase in foreign savings will be incorporated into the consumer's decision. Alternatively, if foreign savings are given as a borrowing constraint, then fiscal policy, in the form of tax or public expenditure adjustment, will be endogenous.

The intertemporal conditions ensure that future prices and quantities are fully anticipated and factored into the behavior of consumption and investment. They also guarantee the path towards a new steady state is unique. However, the intertemporal rate of exchange for consumption and the intertemporal rate of transformation of production may diverge. This is because the consumer faces the cost of the domestic good and that of the competitive import in his decisions, while the producer looks at the sale prices of the domestic and export good. The existence of two discount rates implies that the shadow prices of capital used by the producer and by the consumer may also diverge. On the supply side, investment responds to q_t as defined; once made, the capital stock is fixed and can be altered only by depreciation and further investment. On the demand side, the consumer receives net-of-investment capital income in addition to labor and other income and discounts his income flows using the consumer discount rate. The distinction between the export and import good, the presence of adjustment costs in investment, and the immobility of the capital stock at any point in time prevent immediate arbitrage. Of course, in the steady

⁶ The model treats capital flows as equal to the balance of trade, adjusted for net foreign remittances/transfers and debt service payments. It does not distinguish among different types of foreign flows that may complicate the story without enriching the analysis.

state, all prices and exchange rates cease to change and all asset prices converge to the world interest rate.

2E. Calibration of the Model

2E-1. DATA. The data requirements for model calibration are modest. There is no tedious balancing of social-accounts matrices; only national income data are employed. For the present model, we use 1990 data for the Philippines. In the base year, the model utilizes three sets of information: (1) GNP/GDP at market prices and their components, i.e., private consumption expenditure, gross investment, government consumption, exports, imports, and aggregate factor income of labor and capital; (2) tax revenues by major components—indirect taxes (consisting of domestic indirect taxes, import tariffs and export duties) and aggregate direct income taxes;⁷ and (3) balance of payments data pertaining to the foreign exchange rate, debt, and interest payments. In order to distinguish three types of imports we also need the distribution of imports according to the broad categories of consumer, intermediate, and capital imports and some estimates of their corresponding tariff levels.⁸ Using the tax information, we derive the components of GNP at factor prices and the various tax ratios and purchase price indices in the model. The substitution elasticities used are 0.50 in the Armington function, 0.60 in the CET transformation, and 0.90 in net output.⁹ The derivation of the scale and share parameters of the CES and CET functions follows the usual procedure (see Condon et al., 1987). We now discuss the calibration of the dynamic run.

2E-2. STEADY-STATE REFERENCE RUN. Calibration of all CGE models begins with the assumption that the data are obtained from an economy in some type of “equilibrium.” In static models, this is usually an equilibrium point. In our dynamic model, we assume the entire *path* of our reference run represents an equilibrium or steady state of the economy. Parameters are then calibrated for this reference run, which ensures that the model will

⁷ Direct taxes may be broken down further as needed.

⁸ The latter are usually available. Otherwise, the original 1-2-3 formulation of using one aggregate import good may be employed.

⁹ These values were tested and used in Go (1994).

generate an equilibrium solution with values that match the benchmark data of the economy in question, the Philippines in this case. In this manner, a change in policy or the advent of an external shock traces an alternative path reflecting the deviation from the steady-state reference run.

For simplicity, we assume the balanced-growth rate, g , to be zero. This is in fact not an unreasonable assumption for the Philippines during the 1980s. Specifying a positive g requires eliminating or detrending the exogenous, balanced-growth trends in the reference run for easy interpretation. We in fact scaled all relevant data to per-capita terms (i.e., divided by N_0) and setting $g = 0$ then implies no productivity change (see footnote 4).

2E-3. UTILITY. The parameters in the utility function can then be calibrated from the intertemporal condition for consumption, Equation (7). In the steady state, $PC_{t+1} = PC_t = PC_{ss}$ and $C_{t+1} = C_t = C_{ss}$ so that $\rho = r_{ss}^c$. The discount rate r_t^c at any time t is defined by Equation (8) as the world interest rate i^* and the rate of change in the appropriate exchange rate e_t^c . However, all prices including e^c cease to change in the steady-state so that $r_{ss}^c = i^*$. Hence, $\rho = i^*$. The world interest rate i^* is derived as the ratio of the external debt service to total external debt (or it can be set exogenously). We set $\nu = 0.90$.

2E-4. ASSET PRICES AND THE VALUE OF THE FIRM. The asset market equilibrium at any point in time requires that the return to the firm be the market discount rate, i.e., Equation (20):

$$\frac{R(t)}{V_t} = r_t^p \quad (20)$$

where r^p , as defined in Equation (14), is the world interest rate i^* and the rate of change in the appropriate exchange rate affecting production e_t^p . In the steady-state, the value of the firm V_{ss} , Equation (21), is simply the present value of an infinite stream of the base-year net revenue R_{ss} with i^* as the discount rate (recall that, in the steady state, $\frac{e_t^p}{e_t^p} \equiv 0$):¹⁰

¹⁰ In the alternative dating convention, the asset price in the steady state is not exactly i^* but $\frac{i^*}{(1+i^*)}$, i.e., $\frac{R}{V} = \frac{i^*}{(1+i^*)}$.

$$\begin{aligned}
 V_{ss} &= \frac{R_{ss}}{(1+i^*)} + \frac{R_{ss}}{(1+i^*)^2} + \dots \\
 &= \frac{R_{ss}}{i^*}
 \end{aligned}
 \tag{21}$$

Hence, the value of the firm in the reference run is easily calibrated with data on R_{ss} , which is defined as capital income Π_{ss} less investment expenditures $J(ss)$.

2E-5. WEALTH. From the budget constraint, the consumer's wealth is likewise derived as Equation (22):

$$\begin{aligned}
 W_{ss} &= \frac{PC_{ss}C_{ss}}{(1+i^*)} + \frac{PC_{ss}C_{ss}}{(1+i^*)^2} + \dots \\
 &= \frac{PC_{ss}C_{ss}}{i^*}
 \end{aligned}
 \tag{22}$$

Since we observe $PC_{ss}C_{ss}$ from the national accounts, we can immediately infer W_{ss} .

The baseline wealth is also defined from its components of financial wealth (value of the firm or VF) and nonfinancial wealth (human wealth, HW , and other wealth, OW). This is equivalent to the present value of household income from capital, labor and transfers, less income tax and investment expenditures, plus an adjustment factor Y_{adj} to make the two definitions of wealth consistent, Equation (23),

$$\begin{aligned}
 W_{ss} &= VF_{ss} + HW_{ss} + OW_{ss} \\
 &= \frac{1}{i^*} [\Pi_{ss} - J(ss) + w_{ss}L_{ss} + Y_{tr} - Y_{tax} + Y_{adj}] \\
 &= \frac{1}{i^*} [Yh_{ss} + Y_{adj} - J(ss)]
 \end{aligned}
 \tag{23}$$

Setting the two definitions of wealth equal to each other and rearranging, we have Equation (24):

$$\begin{aligned}
 J_{ss} &= Yh_{ss} - PC_{ss}C_{ss} + Y_{adj} \\
 &= S_p + Y_{adj} \\
 &= S_p + S_g + S_f
 \end{aligned}
 \tag{24}$$

Note that $(Yh_{ss} - PC_{ss}C_{ss})$ is equal to private savings S_p . Moreover, the two definitions are consistent only if the savings-investment

identity holds, i.e., Y_{adj} must equal government savings S_g plus foreign savings S_f . This makes good intuitive sense because the representative consumer is responsible for financing the total investment in the economy which he owns. His current income Y_t is therefore 'national' in scope, including net transfers of resources from the public and the external sectors.¹¹ In this formulation, both the Ricardian implications of fiscal deficits and the intertemporal impact of external borrowing are part of the consumer's optimal decisions.

Another way of looking at it is to substitute various definitions (note equations for S_g , S_f etc. from the appendix) back into the budget constraint. This shows that wealth is simply the present value of GNP less investment, government consumption, and the resource balance ($E-M$), which reduces to the first definition above, the present value of consumption:

$$\begin{aligned} W_{ss} &= \frac{1}{i^*} [\Pi_{ss} + w_{ss}L_{ss} - J(ss) - PC_{ss}G_{ss} - (M_{ss} - E_{ss})er] \\ &= \frac{1}{i^*} [GNP - J(ss) - PC_{ss}G_{ss} - (E_{ss} - M_{ss})er] \end{aligned}$$

In calibrating wealth, it does not matter which definition is used because income and expenditure flows are already reconciled in any consistent systems of national accounts.

2E-6. CALIBRATING q_t AND K_t . Parameters on the supply side require iterations to maintain consistency in the alternative definitions of f and K_t . The variable q in Tobin's definition is Equation (25):

$$q_{ss}^1 = \frac{V_{ss}}{K_{ss}} \quad (25)$$

Given real investment, the depreciation rate of δ , and exogenous growth rate g ($= 0$ by assumption), the steady-state capital stock must satisfy Equation (26):

$$\frac{I_{ss}}{K_{ss}} = g + \delta \quad (26)$$

From Equation (12), real investment is determined by current

¹¹ Hence, of the total investment expenditure J , only $(J - S_g - S_f)$ comes from private resources.

investment expenditures, the adjustment cost function, and tc (which is set to zero initially).

By Hayashi's identity, a second definition of q is the present value of the marginal revenue product of capital, Equation (17), which reduces to the following in the steady state:

$$q_{ss}^2 = \frac{R_{K_{ss}}}{(i^* + \delta)}$$

$R_{K_{ss}}$, is, of course, dependent on the steady-state level of capital.

There are two definitions of Q_{ss} : (1) the ratio of the shadow price and replacement cost of capital, adjusted for tax incentives, Equation (18) and (2) a function of α and β (parameters in the adjustment cost function) and the investment-capital ratio, Equation (12).

To calibrate all these variables, we first set α to zero and β to 2.0, which reduces the adjustment cost function to a linear form as in Bruno and Sachs (1985). These values were found to be reasonable in the context of the Philippines and correspond to fairly responsive investment behavior (see Go, 1994). We then solve for the consistent δ , I , K , R_K , q , and Q from the constraints: Equations (12), (13), (17), (18), (25), and (26). If, in some situations, a predetermined value of the shadow price of capital is preferred, then one of the parameters in the adjustment cost function has to be freed. Alternatively, a new parameter bb can be introduced in Equations (18) and (12) to capture existing distortions or incentives to investment not captured by tc or θ .

2E-7. TERMINAL CONDITIONS FOR I_t AND C_t . To solve a growth model that has an infinite horizon such as our framework, we follow the usual procedure of imposing steady-state conditions at some future terminal period, tf .

As long as the transversality conditions are satisfied, the sums of various infinite series pertaining to the consumption function (e.g., utility and wealth) and the investment equation (e.g., the shadow price of capital or value of the firm) will be finite and well defined. A sufficient condition is that the discount rate and the rate of time preference be positive and greater than the balanced-growth rate by the terminal period.

On the supply side, the required condition is simply that Equation (26) be operating at the terminal period for all simulations, both the reference and alternative paths. Defining I_{tf} imposes terminal values for q_{tf} and Q_{tf} .

The shadow price of capital, q_{tf} , in turn sets the terminal value of consumer's wealth, W_{tf} . The variable W_{tf} consists of non-human wealth ($q_{tf}K_{tf}$) and human wealth (the present value of after-tax non-capital income at period tf), which works out to be $\frac{Y_{ss}}{i^*}$.)

Consumption expenditures at tf should be equal to the interest income of total wealth at the world interest rate or equal to the consumer income (note this is net of investment expenditures by definition.) This can be shown from the consumption function [Equation (5)]. Given W_{tf} and recalling that $r_{ss}^c = \rho = i^*$ and $PC_{t+1} = PC_{tf}$ in the steady state:

$$C_{tf} = \frac{W_{tf}(1 + i^*)}{[PC_{tf}(1 + \sum_{j=tf+1}^{\infty} \zeta(tf,j))]}$$

where

$$\zeta(tf,j) = \left[\frac{\prod_{u=tf+1}^j (1 + i^*)^{1-\nu}}{(1 + i^*)^{j-tf}} \right]^{1/\nu} = (1 + i^*)^{-(j-tf)}$$

$$\begin{aligned} PC_{tf}C_{tf} &= \frac{W_{tf}(1 + i^*)}{\left[1 + \frac{1}{(1 + i^*)} + \frac{1}{(1 + i^*)^2} + \dots \right]} \\ &= \frac{W_{tf}(1 + i^*)}{\frac{(1 + i^*)}{i^*}} = i^*W_{tf} = i^* \left(\frac{Y_{tf}}{i^*} \right) = Y_{tf} \end{aligned}$$

Thus, $C_{tf} = \frac{Y_{tf}}{PC_{tf}}$.

2E-8. ALTERNATIVE EQUATIONS FOR C_t AND I_t . Finally, a few words on the alternative ways of accounting for C_t and I_t in the model. With a terminal condition already defined for C_{tf} above, a simple way to account for consumption is to employ the intertemporal condition Equation (7). A second approach is to use the consumption function directly Equation (5). The latter, however, involves defining wealth components as present values of future incomes in each period, which often expands the numerical dimension of the problem and slows down computation. One solution is to define the evolution of the discount factor and the wealth variable in the function. For example, defining $\Omega_t = [1 + \sum_{j=t+1}^{\infty} \zeta(t,j)]$ and recalling that $C_t = W_t(1 + r)/\Omega_t$, it can then be shown that

$$\Omega_t = 1 + \Omega_{t+1} \left[\left(\frac{PC_t}{PC_{t+1}} \right)^{1-\nu} \frac{(1 + r_{t+1}^c)^{1-\nu}}{(1 + \rho)} \right]^{1/\nu}$$

Wealth accumulation in turn can be derived as follows:

$$W_t(1 + r_t^c) = W_{t+1} + Y_t$$

or

$$\begin{aligned} W_t &= FW_t + NFW_t \\ FW_t * (1 + r_t^c) &= FW_{t+1} + \Pi(t) - J(t) \\ NFW_t * (1 + r_t^c) &= NFW_{t+1} + YH_t(1 - ty) \end{aligned}$$

where FW is financial wealth, NFW is nonfinancial wealth, $\Pi(t) - J(t)$ is capital income less investment expenditures, and YH is income from labor plus net transfers from the other sectors. Both approaches are implemented but we prefer the first one which is simpler and quicker.¹²

Similarly, the determinant of investment, q_t , can be solved using Equation (17). Having defined the terminal q_{tf} however, we find it easier to use the difference equation involving Δq_t . Both alternatives are available in our implementation.¹³

Choice of Terminal Period

Our implementation allows for choice in the number of periods. While there are disagreements regarding how long it takes for a steady state to be reached, the choice in practice is defined by convenience and the amount of computing resources available.¹⁴ In this framework, it is also dependent on the parameters selected for the adjustment cost function, which, if set at a high level implies that it will require more periods for a given exogenous shock to diffuse through the system. How accurate the results should be also depends on whether one is interested in using numerical calculations for qualitative interpretation or for precise estimates of magnitudes. Because our interest is mainly the former—the

¹² A third approach is the central-plan formulation discussed in Abel and Blanchard (1983) and Go (1994).

¹³ Our model is solved using the General Algebraic Modeling System (GAMS). The GAMS files are available by request to the authors from the e-mail address "sdevarajan@worldbank.org" and/or "dgo@worldbank.org". See also Dixon et al. (1992), chapter 5, for a discussion of alternative numerical methods to intertemporal modeling.

¹⁴ Mercenier and Michel (1994) discussed the implications of time-aggregation assumptions and the accuracy of the computed optimal solution as an approximation to the underlying continuous- or more disaggregated discrete-time formulation.

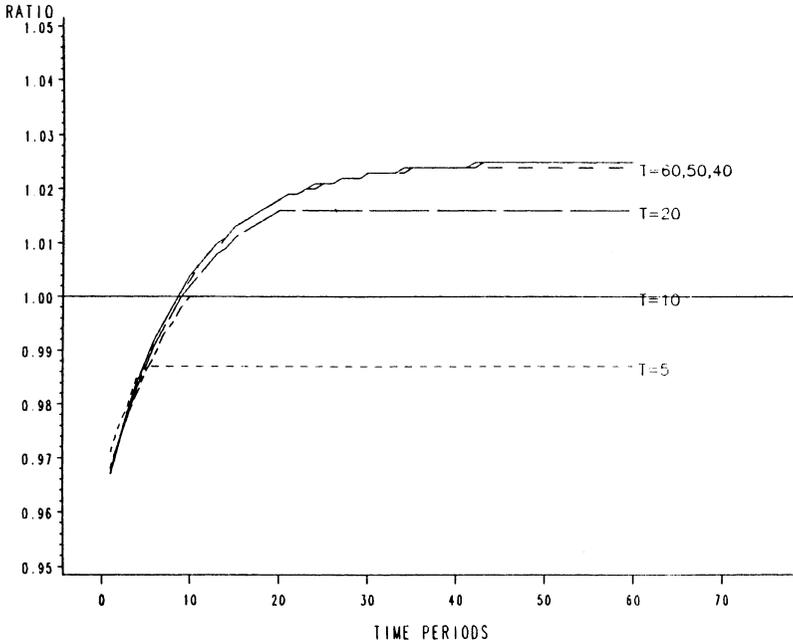


Figure 1. Effects of alternate terminal periods on consumption (reference run = 1.00).

latter rests on good estimates of the parameters as well—we experiment with different time periods to determine the sensitivity of the model results to the choice of terminal time period.

In Figures 1 to 4 we examine the effects of an external terms-of-trade shock¹⁵ on consumption, investment, the real shadow price of capital $\frac{q_t}{PK_t}$ or Tobin's q , and the amount of capital stock in the economy using different numbers of time periods (T): 10, 20, 40, 50, 60. In the figures, the steady-state levels of plots with $T < 60$ are simply extended to period 60 for easy comparison.

Given the parameters chosen in the model, the steady state is reached in about 40 periods. The results show no significant gains are made by extending the number of time periods from 40 to 50 or 60. With capital fixed once installed, the accumulation of capital

¹⁵ The shock is a permanent 10% increase in the price of exports described in simulation 1. See next section for analysis and discussion.

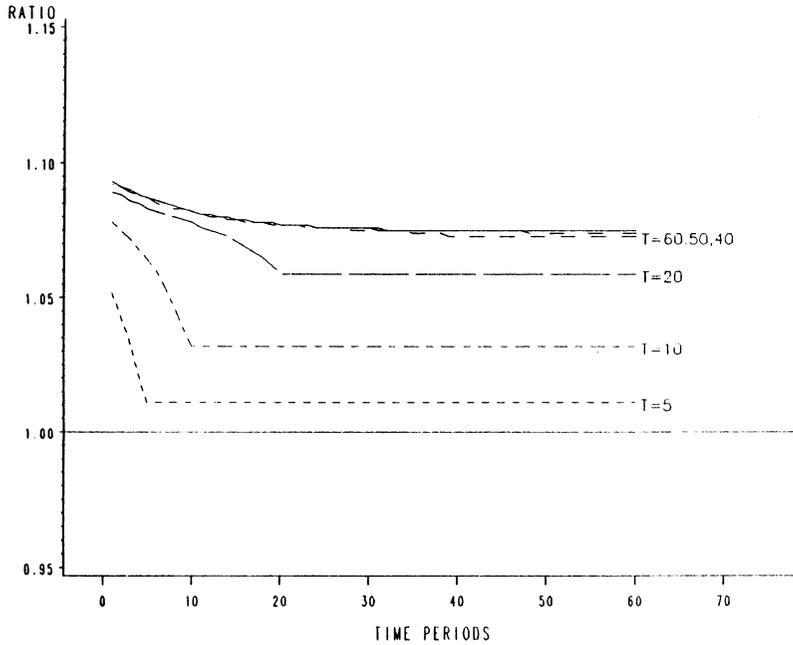


Figure 2. Effects of alternate terminal periods on investment (reference run = 1.00).

(and hence wealth) for $T = 5, 10, 20$ follows closely the pattern in the early periods for $T \geq 40$ but will understate the eventual steady-state level of capital stock (Figure 4). The same is true of consumption, which is dependent on wealth (Figure 1). There are more differences in the path $\frac{q_t}{PK_t}$ (Figure 3) and its effects on investment (Figure 2).¹⁶ Note, however, that the qualitative changes in $\frac{q_t}{PK_t}$ and I_t are not affected by choosing $T \geq 40$. If the steady-state levels are important, the minimum number of periods should be at least 40.

It is of course entirely plausible that a more severe shock may take more time to work itself out. In that case, the key variable

¹⁶ The differences will be more pronounced if the terminal condition for investment is imposed as an endpoint rather a terminal curve as in Equation (26).

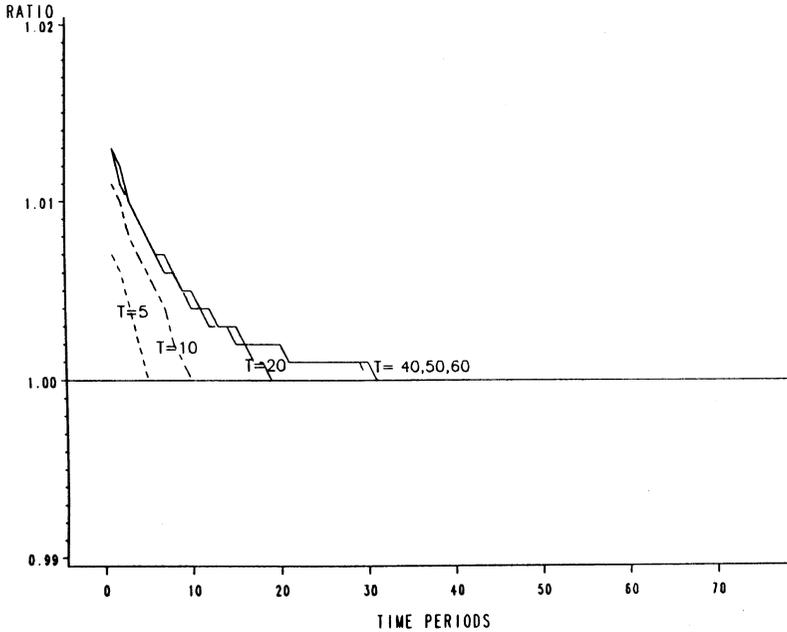


Figure 3. Effects of alternate terminal periods on real shadow price of capital (reference run = 1.00).

to look at is $\frac{q_t}{PK_t}$, which must return to the value 1.00 before the terminal conditions are met. Otherwise, investment and capital stock will not approach their steady-state levels naturally (and will be forced by the terminal conditions at the final period).¹⁷ In the experiments below, we use $T = 40$ which is more than adequate for the types of parameters and shocks chosen.

3. SIMULATIONS

We now report on three simulations with the model which illustrate the types of results that can be obtained with this dynamic framework. In each case, we show how the results are different from those that would have been obtained from a static model.

¹⁷ In Figure 3, $\frac{q_t}{PK_t}$ returns to 1.00 before the terminal conditions in about period 32. Hence, $T = 30$ will provide close approximations.

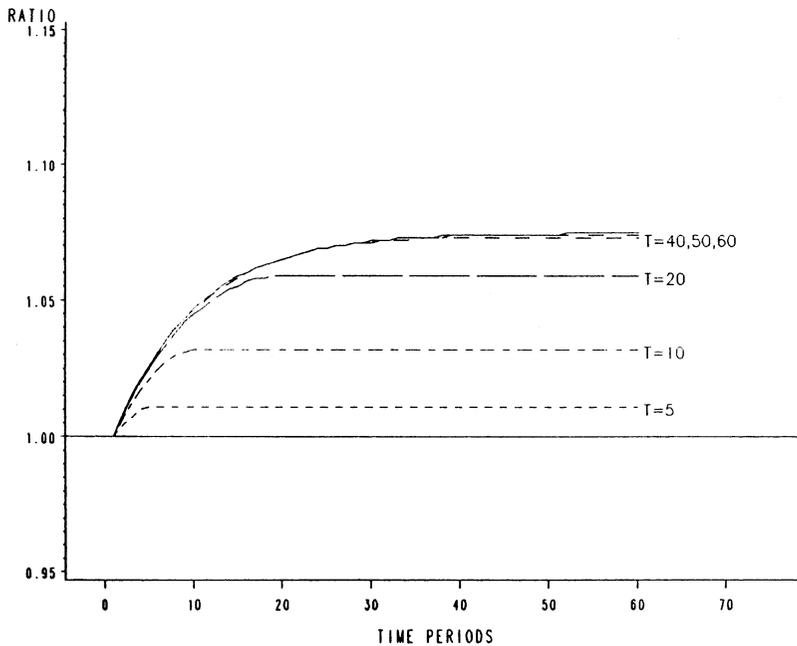


Figure 4. Effects of alternate terminal periods on capital stock (reference run = 1.00).

3A. Terms-of-Trade Shock

We begin with a permanent terms of trade shock. Suppose the Philippines faced a trajectory of world export prices which was 10 percent higher than in the reference run every year for the next 20 years. This is a favorable terms of trade shock which would in general lead to an expansion of the economy. Furthermore, how the structure of the economy would respond to this shock can also be predicted from our knowledge of the “Dutch disease,” based primarily on static models. The typical pattern would be for the export boom to bid up the price of domestic goods making imports more competitive and hence increasing the volume of imports. The real exchange rate would appreciate.

Qualitatively, this pattern is reflected in our dynamic simulation (Figures 5–8), with some notable differences. First, the real exchange rate (defined by PM/PD) does appreciate, initially by 3 percent. Note, though, that as the economy adjusts to the new terms of trade, the real exchange rate gradually depreciates and returns to the level in the reference run.

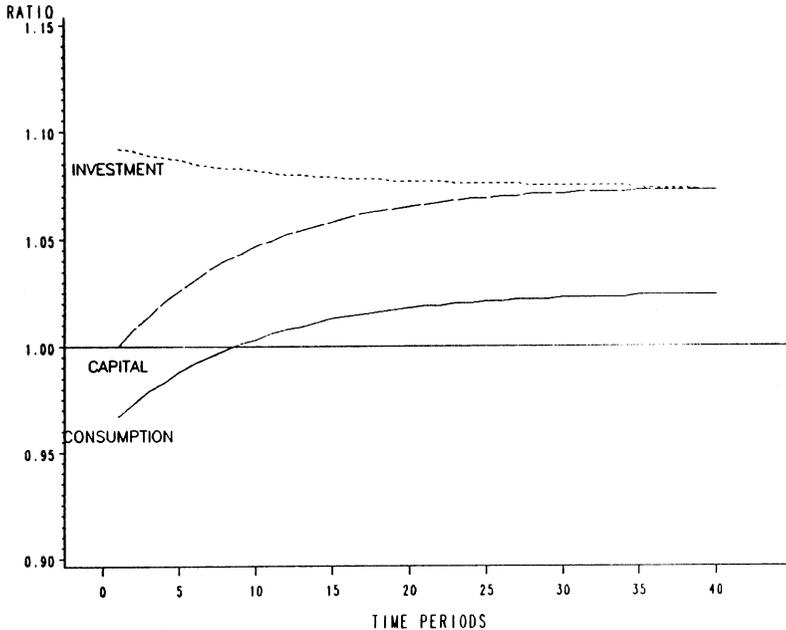


Figure 5. Terms-of-trade shocks effects on consumption and investment (reference run = 1.00).

Second, imports, exports and domestic production follow the predicted pattern. Exports and imports jump and continue to increase over time as the economy grows, thanks to the investment boom accompanying this shock (see below). Production of domestic goods also grows, although in the first few years it is actually below the reference levels. The reason for this is that the favorable terms-of-trade shock draws resources towards exports immediately.

Third, the economy sharply reduces its borrowing to the point that in the later years it is actually running a current account surplus. This is not surprising because exports rise by more than imports in the later years, once the investments undertaken in this sector come on stream.

The main difference from static models arises in the behavior of investment and consumption. In a static model, one would expect the levels of consumption and investment to rise. If investment responded to current rates of return to capital, more investment would get allocated to the exporting sector. However, the overall level of investment would be determined by available savings which in this case would be (roughly) a fixed share of total

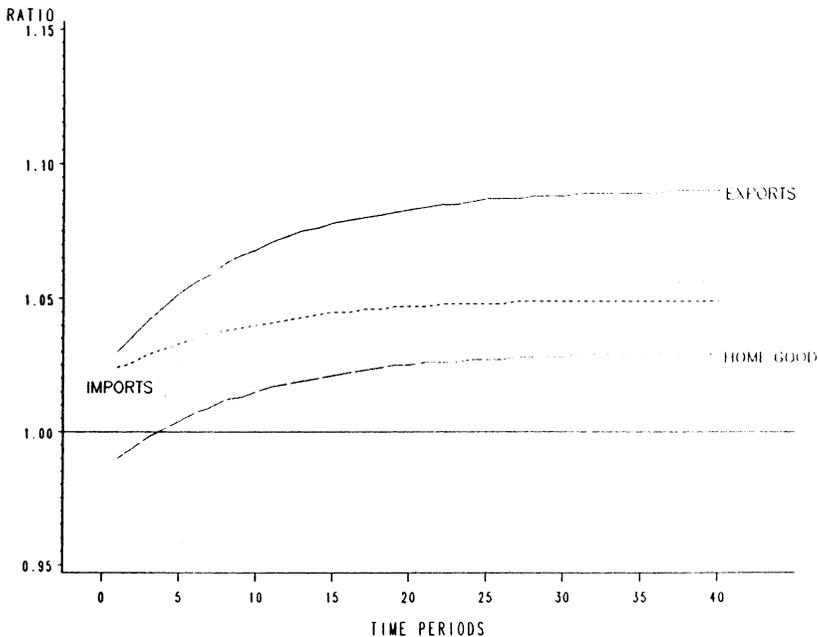


Figure 6. Terms-of-trade shocks effects on goods markets (reference run = 1.00).

income. By contrast, in this dynamic model, the level of investment surges in the first year before settling in at a level which is 7 percent higher than in the base run. The reason is that the increase in the world price of exports affects the discount rate used by firms (see Equation (20)). In order to maintain asset equilibrium, firms invest more as the real exchange rate depreciates initially (recall that the firms' real exchange rate is PE/PD). Similarly, the rate of growth of consumption is affected by the discount rate for consumption Equation (7). The effect of the terms of trade shock is to increase r_{ct} , thereby increasing the growth rate of consumption. The consumption trajectory has to tilt by so much that in the initial years consumption is actually lower than in the reference run. This is surely in contrast to the static model where consumption unambiguously rises. The requirement that consumption be intertemporally consistent, therefore, leads to a sign-reversal in the impact on consumption of a favorable terms of trade shock.

3B. Tariff Liberalization and Fiscal Policy

Next, we revisit the issue of trade liberalization. Because import tariffs are major sources of public revenue in most developing

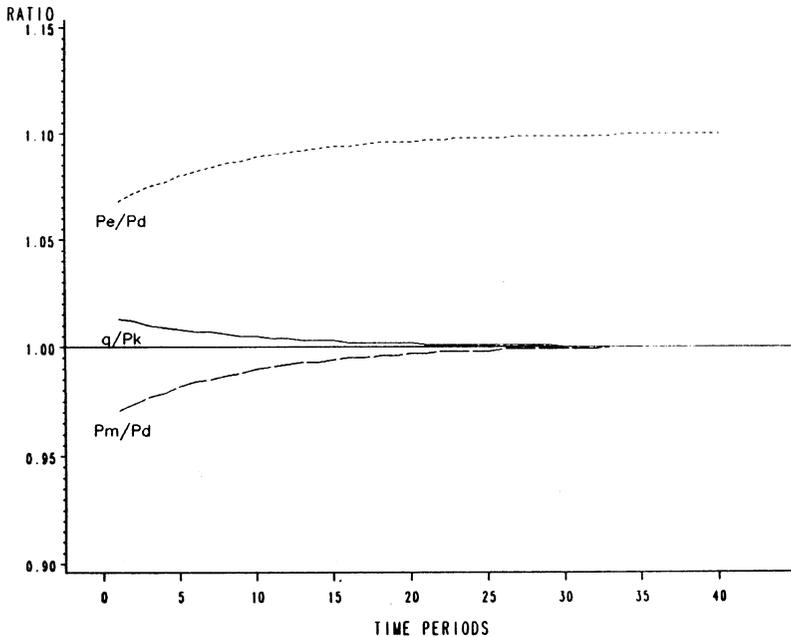


Figure 7. Terms-of-trade shocks effects on prices (reference run = 1.00).

countries, policymakers are often rightly apprehensive about incurring greater macro deficits (in the balance-of-payment or fiscal account) if any tariff reductions are not matched by offsetting revenue measures of fiscal adjustments.¹⁸ In fact, this is a typical result found in most static, tax models employed in estimating the amount of tax adjustment necessary in order to maintain a certain revenue level or some sustainable external or fiscal account balance during a tariff reform.¹⁹ In the simulations below, we examine whether this result is maintained or altered in the dynamic case with intermediate and capital imports.

To obtain comparable, static results, we remove the intertemporal consumption and investment specifications of the model and replace them with the following: consumption is a fixed share of

¹⁸ See, for examples, Greenaway and Milner (1991) and Mitra (1992).

¹⁹ The result is also true in a dynamic case, Go (1994), in which imports are disaggregated by sectoral category (primary goods, manufacturing, and services) but are all treated as final goods.

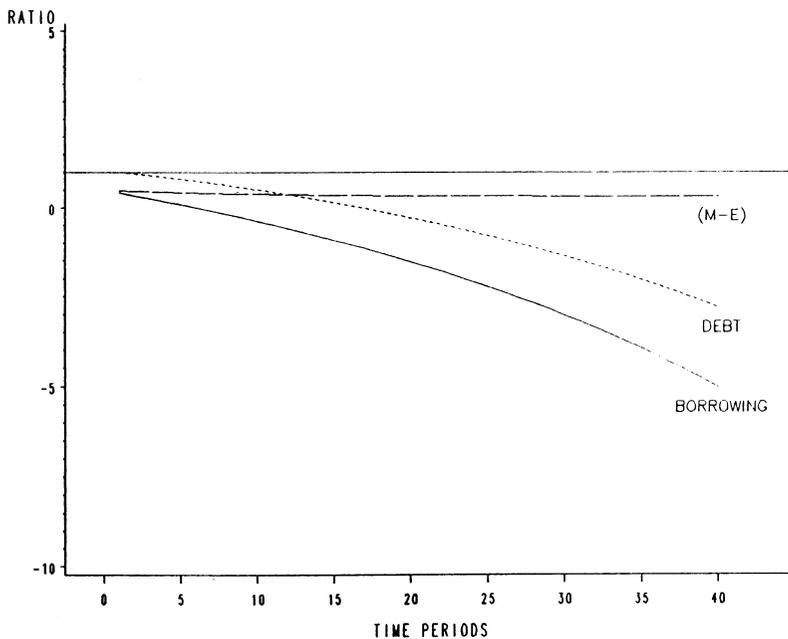


Figure 8. Terms-of-trade shocks effects on debt (reference run = 1.00).

private income while real investment is held constant at the base-year level. The model calculates the amount of tax adjustment, taken to mean the change in domestic indirect taxes (5.4 percent of domestic final demand in the base year), required so that the current-account deficit remains at the reference level in real terms (the deficit is about 5 percent of GDP in the base year). In the dynamic runs, it matters only that the current-account constraint is maintained at the steady-state, terminal period through a one-time, across-the-periods, but fully anticipated tax adjustment.²⁰

The Philippine data used in the model show that import tariffs constitute about 28 percent of tax revenue; the average tariff (collection) rates in the base year are about 10.0 percent for

²⁰ It is possible to fix revenue or foreign savings for every period matched by an endogenous fiscal adjustment for every period. However, this assumes that governments can initiate and fine-tune tax legislations every year or vary easily their tax administrations. Furthermore, changing the tax adjustments every year would cause cycles in consumption and investment.

Table 1: Welfare Impact

	Static case	Dynamic case
Reference run	121.987273	121.987273
Case I	121.989172	121.962597
Case Ib=All taxes replaced by a lump-sum tax	121.991598	122.129600
Case Ic=Case Ib plus $tm^r=0.30$	121.980530	122.118382

To make the welfare measurement in the state case consistent with the dynamic case (which is a present value of consumer utility), we assume that the one-period utility in the static run is unchanging in perpetuity so that the welfare $\Theta = \frac{1}{r}U(C)$ where $r = \rho = i^*$. i^* is calibrated as the rate of external debt service, which is about 10.5 percent in the base year.

imports of final goods, 14.3 percent for imports of intermediate goods, and 17.8 percent for imports of capital goods.²¹

3B-1. CASE I—REDUCING THE TARIFFS OF FINAL IMPORTS. In the experiment, we reduce the tariff rate of final imports to 8 percent. In the static run, the current-account balance is maintained by raising the domestic tax rate of 5.8 percent. Imports increase by 0.4 percent while exports go up by 0.6 percent. The fall in domestic prices and the tax adjustment offset one another in the purchase price of the consumption good so that consumption hardly change. The one-time adjustment is generally moderate. Welfare shows a very slight improvement of 0.0016 percent over the base case. The small change is typical in static results (see Table 1).

The tax adjustment in the dynamic run (reported in Figures 9 and 10) is much more substantial—the domestic indirect tax rate is raised to 7.1 percent. Investment falls initially by 3.5 percent below the reference level and by 2.7 percent at the steady state. The capital stock and output fall. Consumption, which rises initially by 0.9 percent, eventually falls by 1.3 percent from the loss of wealth. A slight gain in exports is registered early as they grow 0.3 percent initially, but fall 1.9 percent when the capital stock declines. Imports decline throughout. The contraction of tax base requires a greater tax adjustment in order to maintain the current-account target. Welfare interestingly declines by 0.0202 percent.

²¹ In mapping import and tariff data to these categories, we make the simple assumptions that imports of consumer goods are purely final goods and that actual imports of intermediate and capital goods do not contain final goods (i.e., non-competitive).

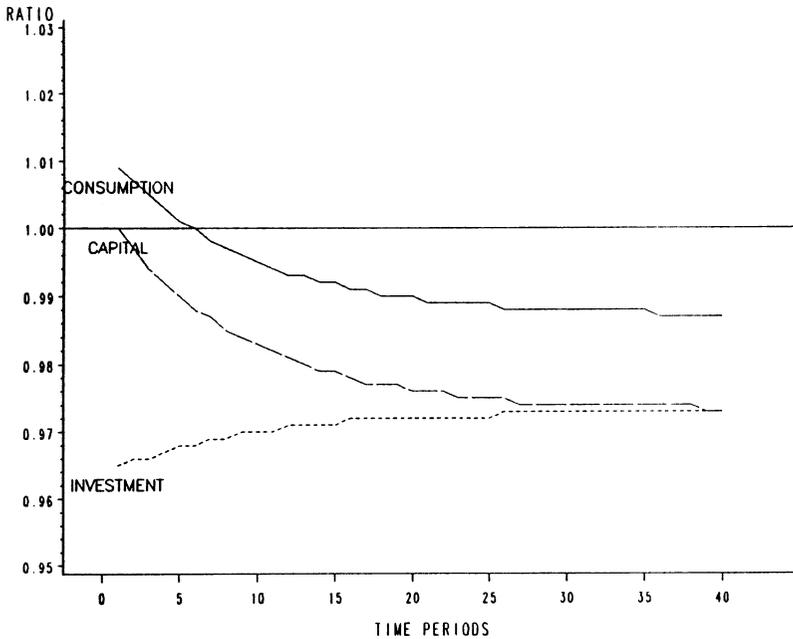


Figure 9. Reducing trade protection effects on consumption and investment—Case I (reference run = 1.00).

The surprising results in the dynamic case requires some explanations. Normally, the substitution of tariffs with less distortionary taxes should increase welfare. However, in the second best world of the dynamic model, the relative rates of taxation of imports matter. When the tariff rate of final imports tm^c are reduced, the protection/subsidy to producers through higher domestic prices will decline and q (the present value of future marginal revenue products of capital) will fall. On the other hand, the tariff rate on capital goods tm^k remains fixed so that PK will be higher than the producer price; hence $\frac{q}{PK}$ and I will fall. Furthermore, the substitute tax (the domestic indirect tax) does not operate like a tax on inelastically-supplied factors as in the static case because capital endowment, no longer fixed, is changing with investment. The decline in wealth in the dynamic case eventually leads to a decline in consumption. We confirm this reasoning with a few sensitivity tests.

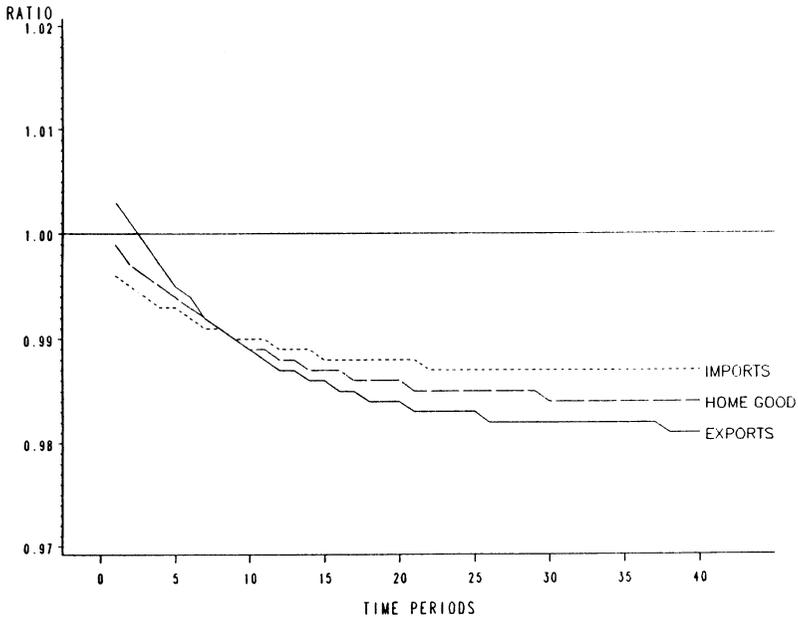


Figure 10. Reducing trade protection effects on goods markets—Case I (reference run = 1.00).

First, we replace all taxes from the reference levels with a lump-sum tax T on household income while keeping the current-account constraint. The results indeed show that the removal of tax distortions will lead to a welfare improvement of 0.1167 percent (Case Ib versus I in Table 1). Second, relative to this first-best world of lump-sum taxation, we increase the distortion by imposing a 30 percent tm^c and allow T to change in order to meet the current-account target. This time, the after-tariff welfare does decline by 0.0092 percent (Case Ic versus Ib in Table 1). For comparison, the welfare numbers for the static case, properly adjusted to make it comparable with the dynamic case (see footnote) are also shown in Table 1. Even with appropriate adjustment to the static case, the welfare levels of the dynamic runs are still higher for Case Ib and Ic. The excess burden of the import tariff (in percent or absolute change between Case Ib and Ic) are however comparable in both cases. Furthermore, in the absence of export externalities or factor-productivity growth, the deadweight loss of the import tariff, even in the dynamic case, is still numerically small. It should

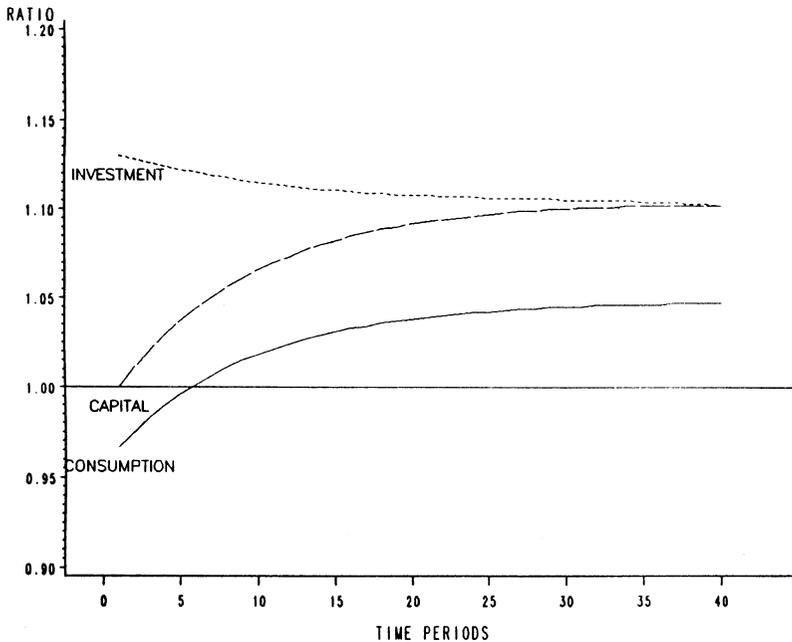


Figure 11. Reducing trade protection effects on consumption and investment—Case III (reference run = 1.00).

be remembered that in our dynamic model, investment decisions are decentralized with a q -type formulation and with lump-sum taxation changing in this sensitivity case to meet the current-account constraint. Hence, increasing tm^c raises the implicit subsidy to the firm and raises investment. The latter in turn dampen the welfare loss by increasing wealth and hence consumption eventually. In addition, there is the added issue of how to impose a revenue-neutral (the current-account constraint in this case) in a dynamic framework and any method chosen will certainly affect the results as well.

3B-2. CASE II—REDUCING THE TARIFFS OF INTERMEDIATE IMPORTS. Next, we reduce the tariff rate of intermediate imports to 8 percent. In the static run, the domestic tax rate is raised to about 5.8 percent. As may be recalled, intermediate imports are also fixed-coefficient inputs of production, so that we are replacing one indirect tax with another. Hence, there is very little change in the real economy.

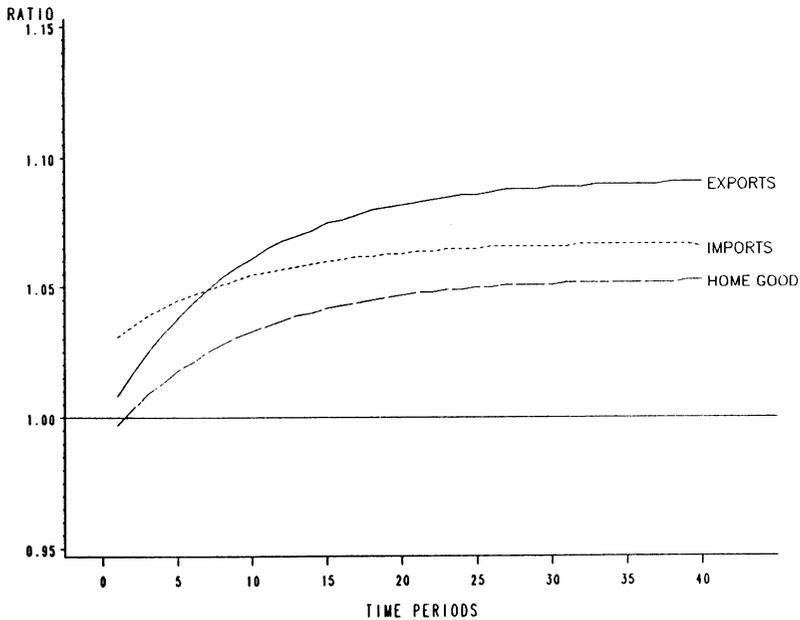


Figure 12. Reducing trade protection effects on goods markets—Case III (reference run = 1.00).

The Leontief technology relating to the use of intermediate imports affect the results of the dynamic run in the same way—no changes were registered in consumption, investment, exports and imports. The indirect tax is raised to the same rate at 5.8 percent in order to maintain the current-account balance at the steady-state.

3B-3. CASE III—REDUCING THE TARIFFS OF CAPITAL-GOOD IMPORTS. Imports of capital goods also form a fixed-coefficient share in the investment good. In the static case, the reduction of their tariffs reduces the cost of investment and thereby offsets the additional savings (tax adjustment) required in the public sector. The indirect tax rate falls slightly at 5.3 percent. Consumption, exports, and imports remain the same.

In the dynamic case (Figures 11 and 12), the relative rates of import taxation matter. The benefit of reducing the cost of investment through a cut in the import tariff of capital goods while trade protection is maintained is very substantial. Investment jumps by 12.7 percent and remain a full 10 percent higher than

the reference level at the steady state. The expansion of output benefits exports, which rise eventually by 10 percent. Imports also expand. The shift of resources toward investment initially hurts consumption but it eventually rises by 4.6 percent above the old steady-state level. The rapid expansion of the tax base implies that the domestic indirect tax rate can be reduced almost to 1 percent.²²

The analysis above suggests two points: first, by and large, static results generally emphasize the revenue requirement of trade policy reform, although in a conservative way; and second, the gains and pains of trade liberalization, captured in the dynamic runs, depend on which tariffs are being reformed. The reduction of protection while the real cost of investment is kept high with an import tariff on capital goods has severe adverse impact and requires a higher fiscal adjustment than the static case. But reducing the tariffs of capital-good imports while maintaining trade protection has the significant and beneficial impact of trade liberalization that economists talked about (even without recourse to assuming an increase in productivity).

4. CONCLUSION

The purpose of this paper has been to describe how to specify, calibrate and run simulations with the simplest possible dynamic model of an open economy. By cutting the size of the within-period model down to the minimum, we are able to focus on the intertemporal aspects in greater depth. The model can be (and was) used to analyze questions where the response of intertemporal variables—especially savings and investment—is important, but also the structure of the economy—such as the breakdown between tradable and nontradable production—is relevant.

Simulations with the model revealed that the response of the economy to a terms of trade shock could be quite different from what static models would predict. For instance, with an increase in world export prices, investment rises by so much (to capitalize on the greater profitability of the economy) that consumption initially falls. This result stands in sharp contrast with the static models of the “Dutch disease” where consumption rises with the export boom. Our result also indicates that the actual response

²² However, like the results in Table 1, welfare will deteriorate if a tariff on the imports of capital goods is imposed over Case Ib.

of many countries experiencing a favorable terms of trade shock may have been suboptimal. The simulations with tariff reform show that which tariff rate is lowered can have a bearing on the outcome. In particular, there is a marked difference between consumption- and capital-good tariffs, since the effect of lowering the latter is to stimulate investment and thereby improve the growth performance of the economy. Again, static models treat these two kinds of tariffs more or less symmetrically. Finally these results have implications for the widely-prescribed policy of unifying tariff rates.

The usual cliché that the model can be extended applies almost by definition to the model in this paper. Our goal, however, has not been to present the most general model possible, but to expose a tool that is simple enough that the benefits of using dynamic models are transparent, and the costs of building them are reasonably low. Our hope is that others will extend and develop the model to suit their individual purposes.

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APPENDIX A. THE 1-2-3-T MODEL

A.1. List of Equations

A.1.1. CONSUMPTION

$$\frac{C_{t+1}}{C_t} = \left(\frac{PC_{t+1}(1 + \rho)}{PC_t(1 + r_{t+1}^c)} \right)^{\frac{1}{\nu}} \quad (\text{A.1})$$

A.1.2. INVESTMENT AND TOBIN'S q

$$\frac{I_t}{K_t} = \alpha + \frac{1}{\beta} Q_t^T \quad (\text{A.2})$$

$$Q_t^T = \left[\frac{q_t}{PK_t} - (1 - bb - tc_t) \right] \quad (\text{A.3})$$

$$r_t q_t = R_k(t) + \Delta q - \delta q_{t+1} \quad (\text{A.4})$$

$$J_t = [1 - tc + \theta_t(I/K)]I_t \quad (\text{A.5})$$

$$Rk_t = rk_t - PK_t(I_t/K_t)^2\theta'_t(I/K) \quad (\text{A.6})$$

$$\theta(x_t) = \left(\frac{\beta}{2}\right) \frac{(x_t - \alpha)^2}{x_t} \quad (\text{A.7})$$

$$K_t = (1 - \delta)K_{t-1} + I_{t-1} \quad (\text{A.8})$$

A.1.3. TERMINAL CONDITIONS

$$\frac{I_{tf}}{K_{tf}} = \delta^R \quad (\text{A.9})$$

$$PC_{tf}C_{tf} = Y_{tf} \quad (\text{A.10})$$

$$r_{tf}^c \equiv r_{tf}^p \equiv i^* \quad (\text{A.11})$$

A.1.4. TRANSFORMATION RATES

$$r_t^p = i_t^* + \frac{e_t^p}{e_t^p} \quad (\text{A.12})$$

$$r_t^c = i_t^* + \frac{e_t^c}{e_t^c} \quad (\text{A.13})$$

$$e_t^p = \frac{PE_t}{PD_t} \quad (\text{A.14})$$

$$e_t^c = \frac{PM_t}{PD_t} \quad (\text{A.15})$$

A.1.5. PRICES

$$PE_t = \frac{pe_t^*er}{1 + te_t} \quad (\text{A.16})$$

$$PM_t = pm_t^*(1 + tm_t^c)er \quad (\text{A.17})$$

$$PMK_t = pmk_t^*(1 + tm_t^k)er \quad (\text{A.18})$$

$$PMN_t = pmn_t^*(1 + tm_t^n)er \quad (\text{A.19})$$

$$PK_t = [a_k PMK_t + (1 - a_k)P_t] (1 + \varphi_t tx_t) \quad (\text{A.20})$$

$$PC_t = P_t(1 + tx_t) \quad (\text{A.21})$$

A.1.6. ARMINGTON CES FUNCTION

$$P_t X_t = PD_t D_t + PM_t M_t \quad (\text{A.22})$$

$$X_t = \alpha_c [\delta_c M_t^{-\rho_c} + (1 - \delta_c) D_t^{-\rho_c}]^{-1/\rho_c} \quad (\text{A.23})$$

$$\frac{M_t}{D_t} = \left[\frac{\delta_c}{(1 - \delta_c)} \frac{PD_t}{PM_t} \right]^{1/(1+\rho_c)} \quad (\text{A.24})$$

A.1.7. CET TRANSFORMATION

$$PQ_t Q_t = PD_t D_t + PE_t E_t \quad (\text{A.25})$$

$$Q_t = \alpha_e [\delta_e E_t^{\rho_e} + (1 - \delta_e) D_t^{\rho_e}]^{1/\rho_e} \quad (\text{A.26})$$

$$\frac{E_t}{D_t} = \left[\frac{(1 - \delta_e) PE_t}{\delta_e PD_t} \right]^{1/(\rho_e - 1)} \quad (\text{A.27})$$

A.1.8. VALUE ADDED

$$PV_t = PQ_t / (1 + ts_t) - a_n PMN_t \quad (\text{A.28})$$

$$PV_t Q_t = w_t L_t + rk_t \bar{K}_t \quad (\text{A.29})$$

$$Q_t = \alpha_v [\delta_v L_t^{-\rho_v} + (1 - \delta_v) \bar{K}_t^{-\rho_v}]^{-1/\rho_v} \quad (\text{A.30})$$

$$\frac{L_t}{K_t} = \left[\frac{\delta_v}{(1 - \delta_v)} \frac{rk_t}{w_t} \right] \quad (\text{A.31})$$

A.1.9. HOUSEHOLD BUDGET

$$\begin{aligned} YH_t &= w_t L_t + rk_t K_t \\ &+ GTRS_t P_t \\ &+ FTRS_t er \\ &- i^* DEBT_t er \end{aligned} \quad (\text{A.32})$$

$$Y_t = (1 - ty)[YH_t - (PK_t J_t - B_t er - SG_t)] \quad (\text{A.33})$$

A.1.10. GOVERNMENT BUDGET

$$\begin{aligned} TAX_t &= tm_t^c (M_t pm_t^* er) \\ &+ tm_t^k (MK_t pmk_t^* er) \\ &+ tm_t^n (MN_t pmn_t^* er) \\ &+ te_t (E_t pe_t^* er) \\ &+ tx_t [P_t (C_t + G_t + J_t)] \\ &+ ty_t [YH_t - (PK_t J_t - B_t er - SAVG_t)] \end{aligned} \quad (\text{A.34})$$

$$\begin{aligned} SG_t &= TAX_t + ts_t / (1 + ts_t) PQ_t Q_t \\ &- G_t PC_t + GTRS_t P_t \end{aligned} \quad (\text{A.35})$$

A.1.11. BALANCE OF PAYMENTS

$$pm_i^* M_t + pmk_i^* MK_t + pmn_i^* MN_t + i^* DEBT_t \equiv pe_i^* E_t + FRTS_t + B_t \quad (\text{A.36})$$

$$DEBT_t = DEBT_{t-1}(1 - d_{adj}) + B_t \quad (\text{A.37})$$

$$MN_t = a_n Q_t \quad (\text{A.38})$$

$$MK_t = a_k J_t \quad (\text{A.39})$$

A.1.12. LABOR MARKET

$$L_t \equiv \overline{L S}_t \quad (\text{A.40})$$

A.1.13. GOODS MARKET

$$X_t \equiv C_t + G_t + J_t(1 - ak) \quad (\text{A.41})$$

A.1.14. ALTERNATIVE SPECIFICATIONS FOR C_t AND I_t . Instead of the intertemporal first-order condition (equation A.1) and the terminal conditions (equations A.9 and A.10), we can specify the full consumption function as follows:

$$C_t = \frac{W_t(1 + r_t^c)}{PC_t \Omega_t} \quad (\text{A.42})$$

$$\Omega_t = 1 + \Omega_{t+1} \left[\left(\frac{PC_t}{PC_{t+1}} \right)^{1-\nu} \frac{(1 + r_{t+1}^c)^{1-\nu}}{(1 + \rho)} \right]^{1/\nu} \quad (\text{A.43})$$

$$W_t(1 + r_t^c) = W_{t+1} + Y_t \quad (\text{A.44})$$

or

$$W_t = FW_t + NFW_t \quad (\text{A.45})$$

$$FW_t * (1 + r_t^c) = FW_{t+1} + \Pi(t) - J(t)$$

$$NFW_t * (1 + r_t^c) = NFW_{t+1} + YH_t(1 - ty)$$

Similarly, the differential equation for the shadow price of capital (equations A.4) may be rewritten as the present value of marginal revenue product of capital:

$$q_t = \sum_{s=t}^{\infty} \frac{Rk_s(1 - \delta)^{s-t}}{\prod_{k=t}^s (1 + r_k^e)} \quad (\text{A.46})$$

APPENDIX B. GLOSSARY

B.1. Parameters

α_q shift parameter in the CES function for Q

α_t shift parameter in the CET function for Q

α_v shift parameter in the CES function for V
 α_n coefficient of intermediate imports
 α_k coefficient of capital imports
 δ depreciation rate of capital
 δ_q share parameter in the CES function for Q
 δ_i share parameter in the CET function for Q
 δ_v share parameter in the CES function for V
 α a parameter in the adjustment cost function
 er nominal exchange rate, price numeraire
 g growth rate
 i_t^* world interest rate
 φ a parameter in the purchase price of investment goods
 ρ rate of consumer time preference
 ρ_q exponent parameter in the CES function for Q
 ρ_i exponent parameter in the CET function for Q
 ρ_v exponent parameter in the CES function for V
 pe_t^* world export price
 pm_t^* world price of final imports
 pmk_t^* world price of capital imports
 pmn_t^* world price of intermediate imports
 tc_t rate of new tax credits to investment
 te_t export tax or subsidies rate
 ty_t direct income tax
 tm_t^c import duty for final goods
 tm_t^k import duty for capital goods
 tm_t^n import duty for intermediate goods
 tx_t domestic indirect tax rate

B.2. Prices

P_t price of supply
 PD_t price of domestic goods
 PE_t domestic price of exports
 PK_t price of capital
 PMC_t domestic price of final imports
 PMK_t domestic price of capital imports
 PMN_t domestic price of intermediate imports
 PQ_t price of gross output
 PV_t price of value added
 e_t^p real exchange rate for supply
 e_t^c real exchange rate for demand
 q_t shadow price of capital

Q_t^T tax adjusted Tobin's q
 r_t^p discount rate for supply
 r_t^c discount rate for demand
 rk_t gross rate of return to capital
 μ_t discount factor
 w_t wage rate

B.3. Quantities

C_t aggregate consumption at time t
 D_t domestic goods
 E_t exports
 G_t government consumption
 I_t investment
 K_t capital stock
 L_t labor demand
 L_0 base year labor supply
 LS_t labor supply at time t
 M_t final imports
 MK_t capital imports
 MN_t intermediate imports
 Q_t gross output
 V_t value added
 Rk_t marginal net revenue product of capital
 X_t aggregate supply

B.4. Values

B_t foreign borrowings or capital inflows
 $DEBT_t$ outstanding foreign debt at time t
 SG_t government savings
 $GTRS_t$ government transfers to households
 J_t total investment expenditures, including adjustment cost
 $FRTS_t$ foreign remittances
 $\theta(x_t)$ adjustment cost function