Private versus Public Schools in Post-Apartheid South African Cities: Theory and Policy Implications*

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Abstract

Black and white families are heterogenous both in income and ability and simultaneously decide where to locate in the city and which school to send their children to. We show that, in equilibrium, white families reside close to the private school attended by their children whereas black families locate further away and, despite the tuition fees imposed by whites, some black pupils attend the private school. This market solution is shown not to be optimal, one of the reasons being that whites overprice education in order to limit black attendance at the private school, protecting their children from negative human capital externalities. Three types of education policies publicly financed by an income tax are then considered: transportation subsidies, private-school vouchers and public-school spending. The efficiency of such policies depends on the fee-setting behavior of whites which strongly varies from one policy to another.

Key words: fees, education externalities, urban segregation, busing, vouchers.
JEL Classification: H2, I2, R14

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1 Introduction

> 'Most [grammar schools] have benefited enormously from Apartheid and (...) today charge exorbitant school fees to keep Blacks out'
> *South Africa's Minister of education Kader Asmal*[1]

Vryburg High School, a former 'white only' Afrikaner public school under Apartheid, has been plagued with racial tensions since 1996, approximately one year after black students had started to gain admittance into that school. Today, with a black enrollment of only 15%, the school located in a prosperous little town in the North West province of South Africa, has become the much-publicized symbol of whites' hostility towards the post-Apartheid integration of black children into former white schools. Strikingly, one of the main causes of the 1996 racial clash in Vryburg initially revolved around the governing body's decision to increase school fees—clearly the most efficient and legal way for white parents to limit the contact of black learners with their own children. A survey carried out in Vryburg showed that 74% of white children attending the school at that time felt that their parents could afford the increased fees whereas 88% of black children perceived that their parents were unable to afford them (Vally, 1999). The case of Vryburg High School thus epitomizes a major stylized fact in today's South Africa: the high level of fees charged by former white public schools and newly created private schools which, in some cases, have increased fivefold over the past decade (Financial Mail, 2000a). Clearly, in the light of the Vryburg case, the issue of soaring school fees seems crucial to understand the limits of school desegregation in post-Apartheid South Africa.

Another strong barrier to school integration after Apartheid revolves around the sprawling and highly segregated structure of South African cities along ethnic and income lines. Because of the land-use restrictions under Apartheid (blacks and whites were not allowed to reside in the same neighborhoods), schools that were formerly reserved for whites and that people continue to call 'white' schools are generally located in central neighborhoods at a huge distance from the peripheral townships inhabited by blacks. Even though land-use restrictions have been removed, the structure of cities has remained almost entirely segregated, which further limits the possibility of school integration in today's South Africa (see Brueckner, 1996, Selod and Zenou, 2001). However, in spite of long and costly commuting distances, the gap in school quality between former 'black' and former 'white' schools is such that there exist very strong incentives for black parents to send their children to better-resourced white schools if they can afford it.

The first objective of our paper is to model the sharp increase in private-school fees that followed the abolition of Apartheid in order to explain the currently limited integration between black and white pupils. To do that, we take into account two major legacies of Apartheid: South Africa's highly unequal educational system and its highly segregated urban structure.

[1] Notes for remarks by the Minister of Education, Professor Kader Asmal, in debate on President's State of the Nation Address to Parliament, February 9, 2000.

More precisely, in our framework, there are two representative schools: a private one located in the central part of the city and a public one located in the peripheral township. Black and white families are heterogenous both in income and ability and simultaneously decide where to locate in the city and which school to send their child to. The private school's fee is endogenously determined by white families. In South Africa, it is indeed the case that private schools are located in white areas, and white parents very often form a strong majority in the governing bodies of these schools. The white community thus faces the following trade-off: on the one hand, increasing the fee deters black pupils from attending the white school, which reduces negative human capital spillovers while increasing the school's productivity (more money means more school material, lower pupil/teacher ratios, smaller class sizes, better equipment). On the other hand, schooling becomes less affordable for whites too and can weigh down much on their budget. In this paper, we focus on an equilibrium in which some blacks attend the private school. This is possible only if the fee is set at an intermediate value. It is then shown that the market solution does not maximize the total surplus in the economy, in particular because whites overprice education in order to avoid the negative human capital externalities of blacks in the private school.

The second objective of our paper is to analyze the efficiency of education policies publicly financed by an income tax. We use numerical simulations to investigate three types of education policies, namely transportation subsidies, private-school vouchers, and public-school spending. The transportation policy (or 'busing') consists in subsidizing a fixed portion of the transport costs incurred by any black pupil who attends the private school. Therefore, families who live further away from that school receive more than those who reside closer. There are two voucher policies: a uniform-voucher scheme that consists in giving the same fixed amount of money to any black pupil who attends the private school in order to help black families pay tuition fees, and a restricted-voucher scheme that discriminates among black pupils depending on the family's income. Finally, the public-school policy (or 'township improvement') consists in injecting a fixed per-pupil amount of money into the public school located in the township. To compare these policies, we proceed in two steps. First, we compare the optimal level of government intervention for each policy. Then, we compare the different policies for a given cost so as to determine the most cost-effective intervention. The main message our results convey is that whites have the ability to counter policies by modulating the fee charged by the private school. We obtain that both a uniform-voucher policy and a progressive voucher scheme decreasing with income will be inefficient because countered by whites since such policies would have stronger effects not only on private school composition but also on the tax rate and the land rents paid by white families. To the contrary, we show that a restricted-voucher policy that increases with parental income is the policy that is the most efficient and the less costly since it triggers a weaker reaction from whites.

The paper is organized as follows. Section 2 describes some important facts about private and public schools in South Africa. Section 3 presents our main assumptions while section 4 solves the model. Section 5 describes the different policies while section 6 compares them. Finally, section 7 concludes.

2 Private versus public schools in South Africa

In order to fully understand the South African context, a few facts need to be recalled about the specificity and the recent changes concerning education in South Africa. We will first present the educational system under the Apartheid regime and then focus on the major changes that have taken place since Apartheid was abolished.

Under the Apartheid regime, the education of whites and non-whites was overwhelmingly public and any racial mixing in school was forbidden by law. Apartheid's former racial classification distinguished between 'White', 'Indian', 'Coloured' and 'Black' children, and forced them to attend separate schools located in their exclusive portions of the urban space.[2] In the logic of Apartheid, there were stark disparities in the treatment of population groups in *per capita spending*, *class size* and *teacher quality*. Roughly, for every 4 Rands spent on a White child, only 3 Rands were spent on an Indian child, 2 Rands on a Coloured child and 1 Rand on a Black child (Thomas, 1996). During most Apartheid years, the average pupil/teacher ratio in Black schools was commonly in the range of 50:1 and 70:1 and remained at least twice as large as that of White schools (Fedderke et al., 1998, Case and Deaton, 1999, Krige et al., 1994). In 1994, the year Apartheid ended, 46 percent of Black teachers but only 1 percent of White teachers were under- or unqualified (South African Institute of Race Relations, 1997). In fact, it is not excessive to say that under Apartheid 'most African schools had little beyond the shell of their buildings' (Lemon, 1999).

As initially intended, the deprivation of school resources caused extreme human capital discrepancies across population groups. In 1996, nearly one in every four Blacks aged 20 years or more had received no formal schooling whereas only 10 percent of Coloureds, 6 percent of Indians and 1 percent of Whites were in a similar situation. At the higher end of the qualification ladder, 65 percent of Whites and 40 percent of Indians aged 20 years or more had obtained matric (the South African high school certificate) in comparison with only 16 percent of Coloureds and 15 percent of Blacks. It is clear from these figures that the Apartheid educational system served the interest of the white dominant class and exacerbated human capital stratification along the lines of the racial classification it promoted. Empirical studies nevertheless suggest that this was socially very inefficient. In this respect, Mwabu and Schultz (1996) have shown that the private wage returns to schooling in South Africa were twice as high for non-whites than for whites, making a strong case for expanding the education of Blacks. Case and Deaton (1999) and Case and Yogo (1999) suggest that this could have been done by lowering the pupil/teacher ratios in Black schools.

In the last decade however, the public educational system underwent major institutional changes. In 1990, in the last days of Apartheid, most white public schools

[2] We use capital letters when explicitely referring to any of Apartheid's four predefined population groups. Observe that in the South African terminology, the term black can also designate any non-white person. In this sense, it can include Indians/Asians, Coloureds and Blacks/Africans.

were granted the right to appoint teachers, to decide on admission policies and to impose fees (becoming the so-called Model C schools). The main effect of this first reform was to introduce a semi-privatization of the white public educational system and to shift the financing and control of white schools to white parents. In other terms, it enabled the preservation of a privileged white public schooling system in spite of the rising pressure towards racial integration that would eventually lead to the collapse of Apartheid. When Apartheid ended, all restrictions on racial mixing in schools were officially abolished and the 1996 South African Schools Act extended most of the financing and governance provisions of Model C schools to all public schools. Even though this reform aimed to level out all public schools, it has been argued that it only reinforced a system that permits differential fees and maintains a high degree of inequality (Lemon, 1999). Indeed, under this new system, all public schools are now allowed—and expected—to raise funds by themselves, be it through the imposition of school fees. In fact, allowing all schools to raise funds was probably the most direct way of addressing the scarcity of governmental resources available for public education while trying to limit at the same time the growing flight of white children to private schools. The result has been to partly hand out the control of public schools to local communities.

Despite the implementation of the 1996 South African Schools Act, private schools have mushroomed in response to the flight of many white children from the public educational system and to the rising demand among some middle-class non-white families for private education. Consequently, the number of private schools in South Africa has increased threefold since 1994, reaching 1557 schools in 1999, and enrollment in private schools now stands at 5% of the 12,5 million South African pupils (Financial Mail, 2000b, South African Institute of Race Relations, 1997). Strikingly, there has been a *continuous rise in fees in both private and former Model C schools* over the past decade. These fees have increased on average by 10 to 20 percent a year (Financial Mail, 1998, 2000a, Sunday Times, 2000). The present situation is that schools in affluent white areas charge high fees to maintain high standards and low pupil/teacher ratios, whereas the poorest schools in townships have a very small capacity (if any at all) to raise funds (Lemon, 1999).

In view of these facts, we will now build a model that enables us to better understand the issues at stake.

3 The model

The city is closed, linear with absentee landlords. There are two different centers. The first one, the Central Business District (CBD hereafter), is at the city center and corresponds to the origin of the line. The other one, referred to as the Black Township (BT hereafter), is exactly at the city fringe. The CBD has all the jobs as well as a representative *'white' private school*.[3] The Black Township has no

[3] As we will see below, in equilibrium, both white and black pupils can attend the school located in the city center. However, since this school is managed by white parents who optimally choose its fee, we call it the 'white' school.

jobs but hosts a representative *'black' public school*.[4] This framework is admittedly highly stylized but it captures the essence of many post-Apartheid South African cities in which jobs and 'white' schools are in the vicinity of the city center and its close suburbs (as for example in Cape Town or Durban) whereas 'black' schools are located in peripheral and distant townships.

There is a continuum of families uniformly distributed along the linear city and they all consume the same amount of land (normalized to 1 for simplicity). The density of residential land parcels is taken to be unity so that there are exactly x units of housing within a distance x from the CBD. A family consists of one parent and one child. Parents are working while children are studying. Families belong to two different types, non-whites or blacks (type B) and whites (type W) whose respective masses are given by $\overline{N}_B$ and $\overline{N}_W$ (with $\overline{N}_B + \overline{N}_W = \overline{N}$). The only difference between these two types lies in the income and human capital of adults. Since there was not much variance among whites under Apartheid, we assume that all white adults have the same income y_W and the same human capital h_W. To the contrary, Apartheid discriminated among non-white communities so that the income and human capital of non-whites vary across individuals and groups. We thus assume that the income of black adults is uniformly distributed on $[\underline{y}_B, \overline{y}_B]$ and that it is lower than that of whites, so that $\overline{y}_B < y_W$. It should be clear that income disparities are a strong legacy of the former Apartheid regime that remains in today's South Africa. Similarly, we assume that the human capital of black adults is distributed on $[\underline{h}_B, \overline{h}_B]$ and that it is also lower than that of white adults, with $\overline{h}_B < h_W$. We further assume that there is a one to one relation between income and human capital among blacks. This means that, in accordance with stylized facts and the standard human capital theory, the richer among blacks are also the more educated.

Since land consumption is normalized to 1, each parent of type $i = B, W$ whose child attends school $j = C, T$ (where C stands for the *private school* in the CBD and T for the *public school* in the Black Township), located at a distance x from the CBD, and receiving an income y_i (y_W for whites and $y_B \in [\underline{y}_B, \overline{y}_B]$ for blacks) has the following utility function and budget constraint:

$$U_i = z_i + \alpha h_j \tag{1}$$

$$y_i = z_i + (m + ty_i)\, x + \mu\, |x - \widehat{x}_j| + f_j + R(x) \tag{2}$$

where $\alpha \in [0, 1]$ is the weight ascribed to education in the utility function, z_i denotes the family's consumption of the non-spatial composite good (which price is taken

[4]The use of representative schools is standard in the literature (see for instance the public educational sector in Epple and Romano, 1998). This assumption is justified when all schools in a given sector are identical (i.e. when they charge the same fee, have the same mean ability of the student body and the same per-pupil spending). This is what we have in mind here, both for private and public schools. Moreover, observe that we do not allow for a private school in the Black Township even though it could generate benefits from competition and better tailoring of expenditures to community preferences. This is because, in our framework, black private schools would not be adequate substitutes for private schools in white areas, due to the presence of peer effects from school composition.

as the numeraire), h_j is the human capital acquired by the family's child when attending school $j = C, T$, m and t are respectively the monetary and the time cost of transportation per unit of distance for parents, μ is the monetary cost of transportation per unit of distance for a child, $\widehat{x}_j$ is the location of school j (with $\widehat{x}_C = 0$ and $\widehat{x}_T = \overline{N}$), f_j denotes the fee in school j, and finally $R(x)$ represents the equilibrium land rent at a distance x from the CBD.

The following comments are in order. First, equation (1) assumes that parents care about the human capital of their children which positively affects their utility (α represents the parents' degree of altruism). Parents are thus altruist and children's levels of human capital depend on parental choices as for example in Glomm (1997).

Second, parents bear both a monetary and a time commuting cost, whereas children only incur a monetary cost when traveling to school. This is quite natural since children do not earn any money and in a labor-leisure choice model a unit of commuting time is valued at the wage rate. Here, we do not develop an explicit labor-leisure choice model (as for example in Fujita, 1989, Chapter 2) because such a model is cumbersome to analyze and would not yield any additional insight beyond what can be obtained in our simpler approach. We simply assume that the time cost of commuting increases with income so that ty_i is the opportunity time cost of transportation per unit of distance for a type-i parent with gross income y_i. This way of modeling transport costs is consistent with the empirical literature (see e.g. Small, 1992, and Glaeser, Kahn and Rappaport, 2000).

Third, we assume that the unit monetary cost of commuting is higher for a worker than for a child, i.e. $m > \mu$. This assumption is easy to understand since many children walk or simply use a bicycle to go to school.

Finally, a child's level of human capital is only determined by the quality of the school he or she attends. In this context, the general educational output h_j of a school $j = C, T$ is given by the following CES production function:

$$h_j = \left[\frac{N_{Wj}}{N_{Wj} + N_{Bj}} h_{Wj}^{\rho} + \frac{N_{Bj}}{N_{Wj} + N_{Bj}} h_{Bj}^{\rho} \right]^{\frac{1}{\rho}} p_j^{\eta} \tag{3}$$

where $\eta \in [0, 1]$, N_{Wj} and N_{Bj} are respectively the numbers of white and black children attending school j (with $N_{BC} + N_{BT} \equiv \overline{N}_B$), h_{Wj} and h_{Bj} are the average contributions of white and black children to the educational process in school j as measured by their parents' average level of human capital inherited from Apartheid, and p_j is the per-pupil spending in school j. By simplicity, we do not model the funding of schools by the government but rather focus on school fees which constitute a growing portion if not the major part of the financial resources of both private and public schools in South Africa.[5] In both public and private schools, we assume that all the money perceived in the form of fees is directly invested in the production of education. It follows that, in our model, per-pupil spending in a given school is exactly equal to the fee charged by that school. Consequently, equation (3) can be rewritten with $p_j = f_j$.

[5] Facing the scarcity of government funds available, many public schools have no choice but to charge fees (see our discussion above).

The interesting aspect of this education production function is that it is general enough to capture the key features of South African schools. In this respect, we have the following additional comments:

First, our specification for equation (3) does not depart from standard education production functions used in the literature (see e.g. the survey on education production functions by Hanushek, 1986) and school quality is determined by both *peer quality* and *per-pupil spending* (as in de Bartolome, 1990, Benabou, 1994, 1996). In equation (3), these two inputs are substitutes in the production of education. Observe that, even though we do not explicitly model the pupil/teacher ratio as an input of the education production function, the quantity and quality of teachers are indirectly accounted for through the effect of per-pupil spending on school productivity.

Second, equation (3) takes into account *human capital externalities* or *spillovers* so that each student, regardless of his or her ethnic origin, acquires the same educational output. It is determined by the average human capital contribution of children which is measured by the average human capital of their parents.[6] Observe that all white children have the same contribution to the education production function since all white parents have inherited the same amount of human capital from Apartheid. This implies $h_{Wj} = h_W$. To the contrary, each black child has a different contribution depending on his or her parent's inherited human capital so that h_{Bj} depends on who are the black children that effectively attend school j. Because of the correlation between income and human capital, we will show that it is always the best students among blacks who can first afford to mix with whites. In other words, the private school 'skims off the cream' from the public school (as for instance in Epple and Romano, 1998). This means that the more blacks attending the private school, the lower the average human capital contribution of blacks to that school. Since the human capital of black parents is uniformly distributed on $[\underline{h}_B, \overline{h}_B]$, by computing these averages, we obtain:

$$h_{BC}(N_{BC}) = \begin{cases} \frac{\overline{h}_B + \underline{h}_B}{2} + \frac{\overline{h}_B - \underline{h}_B}{2} \left(\frac{\overline{N}_B - N_{BC}}{\overline{N}_B} \right) & \text{for } N_{BC} \in]0, \overline{N}_B] \\ 0 & \text{for } N_{BC} = 0 \end{cases} \tag{4}$$

where N_{BC} is the number of black children attending the private school. In the peripheral public school, we have:

$$h_{BT}(N_{BC}) = \begin{cases} \underline{h}_B + \frac{\overline{h}_B - \underline{h}_B}{2} \left(\frac{\overline{N}_B - N_{BC}}{\overline{N}_B} \right) & \text{for } N_{BC} \in [0, \overline{N}_B[\\ 0 & \text{for } N_{BC} = \overline{N}_B \end{cases} \tag{5}$$

[6]The effect of parental education on school achievement is well documented (see Murnane et al., 1981, Datcher, 1982, Corcoran et al., 1990, Altonji and Dunn, 1996). In South Africa, Case and Deaton (1999) find that the education of the household head is a strong predictor of school attainment for both blacks and whites. According to those authors, a household head completing 12 years of education (i.e. with a highschool qualification) as opposed to 8 years (i.e. with a secondary education) is predicted to raise the educational attainment of members of the household by a quarter of a school year. This is indirectly confirmed by Case and Yogo (1999) who show that years of completed schooling of older adults in a magestrial district (a county-sized region) have a significant effect on the schooling of younger adults in the same geographic area.

Third, in equation (3), it is easy to see that *the level of fees affects the quality of the school* since a higher fee means a higher per-pupil spending (i.e. more teachers, better quality material) and a higher productivity. In particular, when the private school's fee is high enough, white children can obtain a higher human capital level than their parents even if they mix with pupils of weaker socioeconomic backgrounds.

Fourth, we assume that white students never attend the township's public school so that $N_{WC} = \overline{N}_W$ and $N_{WT} = 0$.[7] By using (4) and (5), this implies that the quality of the private school is given by:

$$h_C = \left[\frac{\overline{N}_W}{\overline{N}_W + N_{BC}} h_W^{\rho} + \frac{N_{BC}}{\overline{N}_W + N_{BC}} \left[\frac{\overline{h}_B + \underline{h}_B}{2} + \frac{\overline{h}_B - \underline{h}_B}{2} \left(\frac{\overline{N}_B - N_{BC}}{\overline{N}_B} \right) \right]^{\rho} \right]^{\frac{1}{\rho}} f_C^{\eta} \tag{6}$$

and the general level of human capital in the public school is equal to:

$$h_T = \left[\underline{h}_B + \frac{\overline{h}_B - \underline{h}_B}{2} \left(\frac{\overline{N}_B - N_{BC}}{\overline{N}_B} \right) \right] f_T^{\eta} \tag{7}$$

Fifth, our CES formulation of the education production function allows for possible aggregate gains or losses associated with heterogeneity. In particular, the lower ρ, the more negatively whites are affected by the presence of black pupils in 'their' school. For high values of ρ, the negative externalities are weaker and 'good' students tend to pull other students upwards.

Finally, we assume that none of our representative schools are capacity constrained. To illustrate the implications of this assumption, let us briefly discuss what it means in the case of the private school. All things else being equal, if the number of black pupils attending that school increases a lot, then one can imagine that new schools of identical quality (i.e. with the same ratio of whites and blacks, the same human capital contributions, and the same financial input per pupil) are created. Therefore, assuming no replication costs, we do not face a capacity problem and interpreting our single school in the CBD as a representative school is not problematic.

Let us now solve our model. The timing has two stages. In the first stage, white families chose the fee to be charged by the private school they control. They cannot discriminate between pupils and the same fee applies to any child wishing to attend that school. The fee in the public school is exogenously determined. In the second stage, families simultaneously decide which school to send their child to and where to locate in the city. As usual in a framework with successive stages, we solve the model backwards, starting with the second stage.

[7] As acknowledged by the former Education Minister Sibusiso Bengu, 'one never finds whites trying to assimilate by attending black schools'. Indeed, it is truly the case that there are very few white children attending former black schools in South Africa (if any at all). This is not only due to prejudice. In fact, whites just do not have incentives to attend very low quality schools located far away from their homes in unsafe poverty-stricken neighborhoods ridden with criminality.

4 The equilibrium

4.1 Choosing location and school

To solve this stage, we treat f_C as a parameter (since it will be determined in the first stage of the model) and successively focus on residential location and school attendance. Therefore, we first solve the *residential equilibrium* (i.e. we determine the exact location of families in the city, their land rents and utilities) treating N_{BC}, the number of black children attending the central school, as a parameter. This determines a unique city structure which, as we will see, happens to be independent of N_{BC}. We then focus on *school decisions* by assuming from the outset that the city structure corresponds to that unique urban configuration. This enables us to derive the value of N_{BC} and the corresponding land rents and utilities as functions of f_C.

4.1.1 Choosing location: the land-use urban equilibrium

As explained above, we treat both f_C and N_{BC} as parameters. For a given choice of school attendance, each family chooses the location in the city which maximizes its utility subject to the corresponding budget constraint. By using (1) and (2), we can rewrite the indirect utility function of a white family as:

$$U_W = y_W - (m + t\, y_W)\, x - \mu\, x - f_C - R(x) + \alpha h_C \tag{8}$$

where h_C is given by (6). Similarly, for a black family which sends its child to the private school and with income $y_B \in [\underline{y}_B, \overline{y}_B]$, we have:

$$U_{BC} = y_B - (m + t\, y_B)\, x - \mu\, x - f_C - R(x) + \alpha h_C \tag{9}$$

whereas a black family which sends its child to the public school and has an income $y_B \in [\underline{y}_B, \overline{y}_B]$ obtains:

$$U_{BT} = y_B - (m + t y_B)\, x - \mu\, (\overline{N} - x) - f_T - R(x) + \alpha h_T \tag{10}$$

where h_T is given by (7).

Inverting indirect utilities (8), (9) and (10), we obtain the following bid rents:[8]

$$\Psi_W(x, v_W) = y_W - (m + t\, y_W)\, x - \mu\, x - f_C - v_W + \alpha h_C \tag{11}$$

$$\Psi_{BC}(x, v_{BC}(y_B)) = y_B - (m + t\, y_B)\, x - \mu\, x - f_C - v_{BC}(y_B) + \alpha h_C \tag{12}$$

$$\Psi_{BT}(x, v_{BT}(y_B)) = y_B - (m + t y_B)\, x - \mu\, (\overline{N} - x) - f_T - v_{BT}(y_B) + \alpha h_T \tag{13}$$

where $v_W, v_{BC}(y_B)$ and $v_{BT}(y_B)$ are the respective equilibrium utilities of white families, of black families with income y_B which send their children to the private

[8] Bid rents are functions $\Psi_i(x, y, v)$ defined as the maximum rent that a family of type i and earning a given income y would be willing to pay at a given location x so as to reach a given level of utility v. Observe that all white families have the same income so that, for whites, it is not necessary to express income as an argument of their bid rent.

school located in the city center, and of black families with income y_B which send their children to the public school located in the black township.

Inspection of (11), (12) and (13) shows that the bid rent functions are decreasing in x (for (13), even though the school is located in the suburbs, the negativity of the slope is guaranteed by $m > \mu$). In the present model, this reflects the combined influence of the time and the monetary costs of transportation to work and to school. Further inspection of (12) and (13) shows that, at a given location x, an increase in y_B steepens the slopes of these two bid rents ($\partial^2 \Psi_{Bj}/\partial y_B \partial x < 0$, $j = C, T$). This means that richer black families have steeper bid-rent curves than poorer ones. The intuitive reason is that *an extra mile of commuting reduces the utility of a rich family more than it reduces the utility of a poor family* since the former have a higher gross income and thus a greater opportunity cost of commuting. Therefore, richer families are willing to pay more than poorer families to get closer to the city center.

When comparing the residential locations of families, we use the well-known result that the group with the steeper bid-rent curve locates closer to the CBD (see Fujita, 1989, Chapter 2). In the present model, this implies that *richer families locate closer to the city center than poorer families.* However, solving this location problem is not standard (as for example in Hartwick, Schweizer and Varaiya, 1976, in which the number of income classes is finite). Indeed, in our framework, blacks form a continuum of heterogenous families so that we face a continuum of bid rents. Brueckner, Thisse and Zenou (2002) have solved a similar type of location problem and we use their method to solve ours.

Let us now introduce the definition of a residential equilibrium. Strictly speaking, the definition of such an equilibrium encompasses three different cases or regimes depending on the number of black children N_{BC} that attend the central school. Indeed, we have either *partial school integration* when $0 < N_{BC} < \overline{N}_B$ or *complete school segregation* when $N_{BC} = 0$ or *complete school integration* when $N_{BC} = \overline{N}_B$. To simplify matters, and also because it is the most plausible and interesting situation, in this paper, we will only analyze the equilibrium of the *partial school integration* regime.[9]

Lemmas 1, 2 and 3 in Appendix 1 characterize the urban properties of the equilibrium and justify the mapping that assigns a particular location to a particular family income. Let us now present the definitions and characterizations of that unique *urban equilibrium.*

Definition 1 *A residential equilibrium with partial school integration (Figure 1) consists of: a mapping $y_B(\cdot)$ that assigns a black family with income y_B to a location*

[9] Formally, complete school segregation and complete school integration can be viewed as particular cases of the more general partial school integration equilibrium with $N_{BC} = 0$ or $N_{BC} = \overline{N}_B$. In fact, all the subsequent properties that we will derive for the partial school integration regime will remain valid in the other regimes (with adequate changes in notation and accounting for the fact that, under complete school segregation, no black family sends its child to the private school in the city center, or that under complete school integration, no black child attends the public school in the township). For further details, a complete characterization of all three regimes is presented in Selod and Zenou (2002).

x; a set of utility levels v_W, $v_{BC}(y_B)$ and $v_{BT}(y_B)$; and a land-rent curve $R(x)$ such that:

(i) at each $x \in \left[0, \overline{N}\right]$:

$$R(x) = \max \left\{ \Psi_W(x, v_W), \max_{y_B} \Psi_{BC}(x, v_{BC}(y_B)), \max_{y_B} \Psi_{BT}(x, v_{BT}(y_B)) \right\} \tag{14}$$

(ii)

$$\Psi_W(\overline{N}_W, v_W) = \Psi_{BC}(\overline{N}_W, v_{BC}(y_B(\overline{N}_W))) \tag{15}$$

(iii)

$$\Psi_{BC}(\overline{N}_W + N_{BC}, v_{BC}(y_B(\overline{N}_W + N_{BC}))) = \Psi_{BT}(\overline{N}_W + N_{BC}, v_{BT}(y_B(\overline{N}_W + N_{BC}))) \tag{16}$$

(iv)

$$\Psi_{BT}(\overline{N}, v_{BT}(y_B(\overline{N}))) = 0 \tag{17}$$

(v) at each $x \in \left[\overline{N}_W, \overline{N}\right]$, the mapping is given by:

$$y_B(x) = \overline{y}_B - \frac{(x - \overline{N}_W)}{\overline{N}_B}(\overline{y}_B - \underline{y}_B) \tag{18}$$

This general definition corresponds to the case of *partial school integration* in which there are three different types of families competing for land: whites, blacks who send their child to the private school in the city center (type BC) and blacks who send their child to the public school in the township (type BT). Equation (14) states that the land rent $R(x)$ at a distance x from the CBD is the maximum of all bid rents across family types. In other words $R(.)$ is the upper envelope of all bid-rent functions. In the equilibrium configuration, white families live between $x = 0$ and $x = \overline{N}_W$. Type-BC blacks live next to whites between $x = \overline{N}_W$ and $x = \overline{N}_W + N_{BC}$, whereas type-$BT$ blacks locate at the outskirts of the city, between $x = \overline{N}_W + N_{BC}$ and $x = \overline{N}$. The mapping (18) says that income type and distance are perfectly correlated over the segment $\left[\overline{N}_W, \overline{N}\right]$, so that richer blacks live closer to jobs and the private school (see Appendix 1). Equations (15), (16) and (17) ensure that the land rent is continuous. In particular, equation (17) says that the land rent at the city's periphery is equal to the opportunity cost of land, which is assumed to be zero without loss of generality.

[*Insert Figure* 1 *here*]

We have the following property:

Proposition 1 (Urban Land Equilibrium) *There exists a unique residential equilibrium with partial school integration. The utilities $v_{BT}(y_B)$, $v_{BC}(y_B)$ and v_W are respectively given by (41), (43) and (45).*

Proof. See Appendix 1.

Let us now turn to school choices and the (simultaneous) determination of N_{BC}, keeping in mind that N_{BC} can be equal to 0 (*comple school segregation*) or equal to $\overline{N}_B$ (*complete school integration*) or strictly comprised between these two values (*partial school integration*).

4.1.2 Choosing the school

In our model, the choice of a school (which only concerns black families since white children always attend the private school) is simultaneously decided along with the location choice described in the previous subsection.

Observe that, *a priori*, people can choose a school either because it leads to a higher utility or because it is the only school they can afford to pay for. In the South African context, the gap in education and income is such that it is most reasonable to consider that *all black families would rather send their child to the (good) private school and that it is their tight budget constraint which might prevent them from doing so.*[10] Thus, in our model, the number of black children attending the private school is determined by the number of families who can afford to send their child to that school. In this section, we will present the conditions under which each regime ($N_{BC} = 0$ or $N_{BC} = \overline{N}_B$ or $N_{BC} \in \left]0, \overline{N}_B\right[$) will prevail. These (mutually exclusive) conditions are parameterized by the endogenous school fee f_C and guarantee that the disposable income of black families corresponds to each one of the three possible situations. Appendix 2 details how these conditions are derived. To keep our presentation simple, only the two following results need to be recalled:

First, in Lemma 4 in Appendix 2, we show that the disposable income of a black family (whatever its type) is a decreasing function of the distance to the city center. Second, using Lemma 4, we are able to characterize the conditions under which black families have or do not have the means to send their child to the private school. By using the following notations,

$$\underline{f} \equiv \underline{y}_B(1 - t\overline{N}) - (m + \mu)\overline{N}$$

and

$$\overline{f} \equiv \overline{y}_B(1 - t\overline{N}) + \frac{(\overline{y}_B - \underline{y}_B)}{2} t\overline{N}_B - m\overline{N} + \mu(\overline{N}_B - \overline{N}_W)$$

we have:

Proposition 2 (School Fees) *If $\underline{f} < f_C < \overline{f}$, then there exists a unique $N_{BC} \in \left]0, \overline{N}_B\right[$ given by (50). Moreover, N_{BC} is a decreasing function of f_C.*

Proof. See Appendix 2.

N_{BC} is a decreasing function of f_C because, when the fee is increased, fewer black parents can afford to send their child to the private school.

[10] Mwabu and Schultz (1996) suggest that significant credit constraints among South African Blacks limit their investments in education. Case and Deaton (1999) confirm this finding.

At this point of our presentation, we have solved the second stage of the model. We have shown that three regimes can occur depending on the number of black children that attend the private school. Whatever the regime, whites occupy the core of the city and bid away black families to the suburbs. Among blacks, since time costs increase with income, the richer always outbid the poorer so that we have a mapping in which physical distance to the CBD and the income of workers is negatively correlated. In this context, only the richest among black families may have the means to send their child to the private school located in the CBD.

4.2 The optimal fee choice in the private white school

We now have to solve the first stage of our model in which whites choose the level of the fee in the private school, anticipating the residential equilibrium and the corresponding school choices. This enables us to define a *market equilibrium* which requires solving the two stages of the model. This involves, first, a fee-setting problem and, then, simultaneously, a location problem—which solution is described by a residential equilibrium—and a school choice problem (see section 4.1). We have the following definition:

Definition 2 (Market Equilibrium) *A **market equilibrium** is a quintuple $(f_C^*, N_{BC}^*, v_W^*, v_{BC}^*, v_{BT}^*)$ such that white families choose the private school's fee f_C so as to maximize their utility v_W (which is defined by (52)) anticipating the ensuing residential equilibrium and school choices. The partial school integration regime is characterized by: $\underline{f} < f_C^* < \overline{f}$, Proposition 1, the two following participation constraints:*

$$y_B(x) - [m + y_B(x)t]\, x - \mu x - f_C^* - R(x) \geq 0 \quad \textit{for } x = \overline{N}_W + N_{BC}^* \tag{19}$$

and

$$y_B(x) - [m + y_B(x)t]\, x - \mu(\overline{N} - x) - f_T \geq 0 \quad \textit{for } x = \overline{N} \tag{20}$$

as well as the following incentive compatibility constraint:

$$v_{BC}^*(\overline{N}_W + N_{BC}^*) > v_{BT}^*(\overline{N}_W + N_{BC}^*) \tag{21}$$

This definition of equilibrium is quite long since it must consider all the possible deviations from equilibrium. In our framework, whites choose the private school's fee anticipating the residential equilibrium and the school choices made by blacks. Technically, whites choose the fee f_C that maximizes (52). Their optimal choice determines the regime, depending on the value of the f_C chosen relative to $\underline{f}$ and $\overline{f}$. We have written the equilibrium definition for partial school integration, which prevails if and only if the f_C chosen belongs to $\left]\underline{f}, \overline{f}\right[$. In this regime, black students are partially integrated ($0 < N_{BC}^* < \overline{N}_B$) and the residential equilibrium is such that whites occupy central locations, black families of type BT reside in the suburbs, and black families of type BC live in between them (see Proposition 1 and Figure 1).

In order to check that this urban configuration is indeed an equilibrium, we have to verify a set of conditions.

First, we have to check that all black families can afford to pay the fee at the school they send their children to. Condition (19) (respectively (20)) states that the poorest black family which sends its child to the private school (respectively the public school) and which resides in $\overline{N}_W + N_{BC}^*$ (respectively $\overline{N}$) has a positive disposable income.[11] Observe that, as soon as (19) is satisfied, the disposable income of whites is strictly positive since it can easily be shown that white families have a higher disposable income than any black family in the city.

Second, we have to check that the black families which send their child to the private school are better off than those which send their child to the public school. Condition (21) states that the black family of type BC with the lowest utility (i.e. the black family residing in $\overline{N}_W + N_{BC}^*$) obtains a higher utility by sending its child to the private school than by sending him or her to the public school. Observe that, using (41) and (43) in Appendix 1, the incentive compatibility constraint (21) can be written as:

$$\alpha(h_C^* - h_T^*) > (f_C^* - f_T) + \mu(\overline{N}_W - \overline{N}_B + 2N_{BC}^*) \tag{22}$$

This is more intuitive since it states that, for the black family residing in $\overline{N}_W + N_{BC}^*$, the gain in human capital $(h_C^* - h_T^*)$ obtained from sending its child to the private school rather than to the public school must outweigh the higher cost of education $(f_C^* - f_T)$ and the possibly higher transportation costs $\mu(\overline{N}_W - \overline{N}_B + N_{BC}^*)$ that may stem from an increase in the distance traveled by the child.

Let us now study the existence of the market equilibrium. We have the following result:

Proposition 3 (Existence) *There always exists a market equilibrium.*

Proof. See Appendix 3.

The following comments are in order. First, we are not able to prove that the market equilibrium is unique because the properties of the utility function of whites (52) are very difficult if not impossible to characterize analytically (see Appendix 3). However, for all the numerical simulations we have computed (some of which are presented below), uniqueness is always verified. Second, the choice of the private school's fee by the white community calls for a more detailed analysis. For that, let us examine how the utility of whites varies with f_C. Using (52), we obtain:

$$\frac{\partial v_W}{\partial f_C} = -1 + \alpha \frac{\partial h_C(N_{BC}(f_C), f_C)}{\partial f_C} - 2\mu \frac{\partial N_{BC}}{\partial f_C} \qquad \text{for } \underline{f} < f_C < \overline{f} \tag{23}$$

where $\frac{\partial h_C(\cdot)}{\partial f_C} > 0$ and $\frac{\partial N_{BC}}{\partial f_C} < 0$.

The variations in the utility of whites associated with a change in the private school's fee f_C is influenced by forces of opposite directions. In the above decomposition, the first term is the only negative force. It corresponds to the marginal cost -1

[11] Note that the land rent does not appear in (20) because $R(\overline{N}) = 0$.

associated with the higher school fee incurred by white children when f_C is increased (the *cost effect*). The second term measures the marginal effect of an increase in f_C on h_C, the human capital level of the central school, through two channels. Firstly, a higher f_C directly increases h_C since it means more financial input per pupil and better school productivity. Secondly, there is also *indirect* positive effect of f_C on h_C through N_{BC}: when whites increase the private school's fee, fewer black pupils are able to attend the private school because some parents cannot cope with the increase (their disposable income would become negative if they did not switch to the public school). These two effects are valued in the utility of white parents with a weight α. The last term corresponds to a positive land-market effect (it is indeed easy to verify that for $x \in \left[0, \overline{N}_W\right]$, $\partial R(x)/\partial f_C = 2\mu(\partial N_{BC}/\partial f_C) < 0$): a rise in f_C induces a fall in N_{BC}, which means less competition for central locations among black families and thus lower land rents and a higher utility for whites.

If, for some parameter values, the cost effect is very high and rapidly outweighs the positive human capital effect, then f_C may be set at a very low value so that complete school integration can occur in theory ($N_{BC} = \overline{N}_B$). However, if parameters are such that the cost effect is very low in comparison with the positive effects on human capital and the decrease in land rents, then whites may choose a fee that is high enough to discourage all black families from sending their child to the private school (complete segregation, with $N_{BC} = 0$). If, for other parameter values, the cost effect is not too high in comparison with the positive effects, then whites may choose an intermediate value of f_C for which positive and negative marginal effects cancel out for an interior value of N_{BC}. In this case, $\underline{f} < f_C < \overline{f}$, so that we have partial integration in the private school.

The question now is to identify the key parameters that affect the choice of white families through the different above-mentioned effects. In fact, there are (at least) four parameters that are crucial here: η, which measures the human capital return on financial inputs, α, which measures how much parents value education, ρ, the degree of substitutability between black and white pupils in the education production function, and $\underline{h}_B$ and $\overline{h}_B$, which measure the level and dispersion of human capital contributions among black pupils. It is clear that if η and α are high, then white parents have much interest in setting high fees in order to make an efficient use of the financial input in the education production function and because it is important for them to secure a high level of human capital for their children by isolating them from poorly educated individuals. In this case, complete school segregation or partial school integration are more likely to occur. Similarly, the lower ρ, the more sensitive the education production function with respect to the negative human capital externalities of blacks, and thus the higher the fee chosen by whites and the more likely school segregation will occur.[12] Finally, the effects of $\underline{h}_B$ and $\overline{h}_B$ are also easy to trace. To understand that, recall that human capital and income

[12]Indeed, if $\rho \to +\infty$, then $h_C = \max\{h_W, h_{BC}\} f_C^\eta = h_W f_C^\eta$ so that whites are not affected by the presence of blacks within the school and whites may choose a low fee. If, on the contrary, $\rho \to -\infty$, then $h_C = \min\{h_W, h_{BC}\} f_C^\eta$ so that blacks strongly affect the human capital of the central school and whites are willing to choose a high fee. For moderate values of ρ, some integration will take place.

are perfectly correlated, so that when $\overline{h}_B$ and $\underline{h}_B$ are both very low, the incomes of black workers are also very low. Because of the strong negative human capital externalities of black children, whites will be willing to set the private school's fee at a high level, which will all the more restrain integration since blacks families are poor and cannot afford to pay much for school fees.

In order to illustrate these intuitions, we will now proceed to some numerical simulations and initiate a surplus analysis. By denoting by S_L, S_W, S_{BC} and S_{BT}, the respective surpluses of landlords, of whites, of blacks attending the private school, and of blacks attending the public school, we have:

$$S_L = \int_0^{\overline{N}} R(x)dx \quad , \quad S_W = \overline{N}_W v_W \tag{24}$$

$$S_{BC} = \int_{\overline{N}_W}^{\overline{N}_W + N_{BC}} v_{BC}(x)dx \quad , \quad S_{BT} = \int_{\overline{N}_W + N_{BC}}^{\overline{N}} v_{BT}(x)dx \tag{25}$$

Therefore, by denoting by $S_B = S_{BC} + S_{BT}$ the surplus of all black families, we define the total surplus in the economy S_{Total} as follows:[13]

$$S_{Total} = S_W + S_B + S_L \tag{26}$$

4.3 Numerical simulations of the model

In all the following simulations, we assume strictly identical distributions for human capital and income so that $y = h$. This assumption is the simplest possible way to account for the correlation between a family's income and the inherited human capital of parents.

We will first present a base case in which we normalize the city's total population to one hundred. In this base case, blacks (i.e. non-whites) represent 60% of the total population ($\overline{N}_B = 60$).[14] We assume large income disparities between population groups so that the income of the richest black family amounts to two thirds of any white family's income ($\overline{y}_B = 8$ while $y_W = 12$). The income of the poorest black family stands at one third of the same amount ($\underline{y}_B = 4$).[15] This means that, on

[13]Observe that adding these terms is perfectly legitimate since all utilities are expressed in terms of monetary equivalent (or consumption equivalent). The reason we have treated landlords as a separate group is to avoid the arbitrary issue of land allocation among families. However, in South African cities, because of history, much of the land is owned by whites. It would therefore be possible to 'close' our model by considering that whites own most or all the land in the city. This approximation would alter the choice of the private school's fee by whites but would not change the main results of our model.

[14]In 1996, the population of Pretoria for instance was 62% non-white (Source: computed from Census 1996).

[15]In Cape Town in 1995, it is estimated that two thirds of White households earned more than R3500 per month whereas three quarters of African households earned less than R1500. A majority of Indian and Coloured households earned between R1500 and R3500 (Moving Ahead, 1996).

average, the income of a white family is twice that of a black family. Similarly, the inherited human capital of a white parent is twice the inherited human capital of an average black parent ($h_W = 12 > (\overline{h}_B + \underline{h}_B)/2 = 6$).[16] Furthermore, we assume that the monetary transport cost per unit distance is two times higher for workers than it is for children ($m = 20/1000$ and $\mu = 10/1000$) while the time cost of transportation per unit distance t is equal to $2/1000$.[17] Since city size is normalized to 100, this means that the opportunity cost of commuting all the way from the city fringe to the city center amounts to 20% of one's income. Finally, the elasticity of substitution between human capital inputs in the education production function is negative and quite low in absolute terms ($\sigma = -.25$, or equivalently $\rho = 5$). This corresponds to a case in which negative human capital externalities are not too harsh and good students are able to pull other students towards better results.[18] The elasticity η stands at 0.2. This means that, for a given composition of a school, increasing per-pupil spending by 1% (by e.g. buying books, hiring teachers or investing in the school's infrastructure) raises the school's human capital output by 0.2%. All parents value education in their utility function with a weight $\alpha = 1/2$. By simplicity, we have set the exogenous fee at the township school to unity ($f_T = 1$).

We start with a base case simulation and then introduce simple variations which enable us to comment the role played by our main parameters. All our simulations satisfy the conditions in Definition 2. The results are presented in Table 1a.

[*Insert Table* 1a *here*]

Our comments are in order. First, under the base case, we obtain that whites set a fee f_C^* below the level $\overline{f}$ that would exclude all blacks from the central school ($f_C^* = 2.65 < \overline{f} = 4.84$). In our theoretical framework, this situation corresponds to the partial integration regime we have presented in the previous subsections. In the simulation, 45.8% of the city's non-white students have access to the private school. In a city such as Cape Town in which more than half of non-whites are Coloureds or Asians, this means that many of them have access to private schools or former model C schools located in central areas whereas very few Blacks—if any at all—benefit from such an opportunity. Moreover, it is the best students among non-whites that flee the township, which explains the low output of the public school ($h_T^* = 5.1$). To the contrary, the human capital output of the private school is much higher than

[16] In South Africa, in 1991, the average number of years of education was 9.5 for White men, 7.9 for Indian men, 5.5 for Coloured men and 3.9 for Black men (Thomas, 1996). Since the South African educational system has 12 years of schooling, this corresponds to a high school qualification for Whites and Indians and a primary school qualification for Coloureds and Blacks.

[17] We assume a higher unit transportation cost for parents than for children because, in South Africa, children often resort to cheaper transport modes (such as "minibus taxis" or bicycles) or even walk to school. For more detailed information on transport costs in South African cities, see Selod and Zenou (2001).

[18] This is in accordance with the economic literature on education production functions and peer group effects (See for instance Summers and Wolfe, 1977, or Henderson, Mieszkowski and Sauvageau, 1978, who suggest that mixing children with heterogenous abilities raises the overall production of education). Even though similar studies have not been carried out for South Africa, we have no reason to believe that these results would not apply there.

that of the public school and even higher than the human capital of white parents ($h^*_C = 13.3 > h_W = 12$). This is because, in the private school, per-pupil spending is high while negative human capital externalities are weak (because of the negative elasticity of substitution that pulls education upwards).

Second, Table 1a shows that the weight parents put on education has a strong impact on our results as anticipated in the previous section. Indeed, when education is not so important for parents ($\alpha = 1/3$ instead of $1/2$), whites set a lower fee ($f^*_C = 1.14$ instead of 2.65) and accept more school integration ($N^*_{BC}/\overline{N}_B = 78.9\%$ instead of 45.8%).[19] The intensified flight from the township school (and the lower per-pupil spending in the private school) causes the human capital output to decrease in both schools and the average land rents to rise.[20] To the contrary, when parents put a lot of weight on education ($\alpha = 2/3$ instead of $1/2$), we have exactly the opposite effects: whites set sufficiently high fees to prevent black families from sending their children to the central school ($f^*_C = \overline{f} = 4.84$ so that $N^*_{BC} = 0$) and complete school segregation prevails. It follows that human capital outputs are now higher in both schools (because of better school composition and higher per-pupil spending) and landlords have a lower surplus due to the depressing effect of school segregation on land rents.

Third, it can be seen from Table 1a that variations in ρ also play a major role. When ρ tends towards infinity (or equivalently when σ is negative and tends towards zero, implying $h_C = \max\{h_W, h_{BC}\} f^\eta_C = h_W f^\eta_C$) then there are no negative education externalities in the education production function: the educational output at the private school is only determined by the human capital contribution of its white students and the level of per-pupil spending. In this context, whites have no human capital incentive to isolate themselves from blacks. In our particular simulation, the fee chosen ($f^*_C = 1.84 < \overline{f} = 4.84$) leads to more school integration ($N^*_{BC}/\overline{N}_B = 63.5\%$) but not complete integration. To the contrary, when ρ tends towards $-\infty$ (or equivalently σ is positive and tends towards zero, implying $h_C = \min\{h_W, h_{BC}\} f^\eta_C = h_{BC} f^\eta_C$) then the negative education externalities are very intense: the educational output of the private school is now determined by the average human capital of its black students and the level of per-pupil spending. Whites now have a very strong incentive to isolate themselves from blacks and decide to set the private school's fee so as to exclude all blacks ($f^*_C = \overline{f} = 4.84$).

Lastly, when η takes higher values ($\eta = .3$ instead of $\eta = .2$), the marginal rate of return on per-pupil spending is increased. It implies that whites will prefer to invest more money in the private school and raise its fee. In our simulation, the marginal gain of raising fees outweighs the marginal cost as long as there are black pupils in the school. In equilibrium, whites set the fee at the exact value that excludes all blacks ($f^*_C = \overline{f} = 4.84$) and obtain a higher human capital output than under the

[19] For lower values of α, the private school's governing body may even accept complete school integration.

[20] This is an important feature of our model: when more integration occurs, more black parents would like to locate close to the school attended by their child in order to save on commuting costs, and thus bid for land accordingly. More integration implies fiercer competition for central locations, higher land rents and a higher surplus for landowners.

base case.

To briefly summarize our results, the simulations confirm that, in a post-Apartheid context, *school segregation is all the more likely to occur when parents put a high value on education, when the education production function exhibits a strong complementarity between inputs, and when the elasticity of capital in the education production function is high.*

It remains to be seen however how city composition (in terms of population group and human capital distribution) affects the predictions of our model. This is presented in Table 1b.

[*Insert Table* 1*b here*]

There are two important effects. In the first four lines of Table 1b, we present the impact of a change in the relative number of whites on integration and human capital outputs. These simulations suggest that *school integration is more likely to occur* and that *non-white children obtain a higher average human capital in cities which have a numerous white population* rather than in cities where whites are relatively less numerous. Indeed, when whites represent half the population in the city, they set the fee at a low level ($f_C^* = 2.24 < \overline{f} = 4.6$) which causes more integration ($N_{BC}^*/\overline{N}_B = 52.4\%$) than under the base case. To the contrary, when whites represent only 25% of the population, they choose the level of fee that excludes all blacks from the central school ($f_C^* = \overline{f} = 5.2$). This is because when white children are few in the private school, each additional black pupil attending that school yields a strong negative human capital externality. In other words, the fewer whites, the stronger their incentive to exclude blacks from the private school.

In the last four lines of Table 1b, we present the impact of a change in the distribution of human capital. Keeping $h_W = 12$ as in the base case, our simulations progressively increase $\overline{h}_B$ and $\underline{h}_B$ and show that *when the human capital distribution of non-whites is shifted upwards, whites set lower fees and integration is boosted.* Since Indians/Asians were less discriminated against under Apartheid and have inherited a higher human capital than Coloureds or Africans, this means that cities which have a higher proportion of Indians/Asians should exhibit more integration than other cities. In our simulations, when the ratio $h_W/\overline{h}_B$ decreases from 1.5 to 1.25 and $h_W/\underline{h}_B$ decreases from 3 to 2.5, the private-school fee decreases from 2.65 to 2.04 and the proportion of black children attending the private school increases from 45.8% to 76.6%. This is both because whites are less induced to set high fees (since their children now incur weaker negative human capital externalities) and because non-white families are richer and can afford to spend more money on school fees.

Let us now verify if these predicted effects of city composition on human capital acquisition are consistent with what can be observed in South African cities. This is done in Table 2a which presents the population, human capital, and income distributions of each population group in the main metropolitan areas of South Africa in 1996. In order to get an indication of how h_W, $\overline{h}_B$ and $\underline{h}_B$ compare, we have computed the proportion of individuals aged between 45 and 64 holding a high school diploma (matriculation)[21] or a higher qualification in each population group. Similarly, the same indicator for individuals aged 19 to 24 measures the average human capital outputs of non-white and white children in the model. The percentages of each population group that stand above the city's median income also give a rough measure of y_W, $\overline{y}_B$ and $\underline{y}_B$. These indicators show that all cities present strong income and human capital imbalances that favor Whites and Indians to Coloureds and Africans. Furthermore, even though it is quite difficult to disentangle the effect associated with the size of the white population from the effect associated with the human capital distribution of non-whites, we have the following comments: Having ranked cities by order of white concentration, the first predicted effect of our model is blatant since cities such as Pretoria, Cape Town, and Johannesburg have a white population of respectively 38%, 22% and 18% while their proportions of educated non-whites aged 19 to 24 stand at 46%, 41%, and 29%. This is consistent with our model's prediction that *non-whites are more integrated and obtain a higher human capital output in cities in which whites are more numerous.* The second prediction of the model is also easily verified by comparing Port Elizabeth, Johannesburg and Durban which have relative Indian populations of respectively 1%, 3% and 24%, while their proportions of educated non-whites aged 19 to 24 amount to 29%, 36%, and 41%. This is consistent with our second prediction that *when non-white parents in the city have a higher human capital* (i.e. when the proportion of Indians/Asians is higher), *non-white children are more integrated and acquire a higher human capital.*

[*Insert Table 2a here*]

In accordance with our model, it is most likely that school fees play an important role in explaining these two striking effects associated with city composition. Unfortunately, school-fee data are scarce in South Africa, and we have not been able to compare the level of private-school fees for different metropolitan areas (this would have been a test of how school fees change with city composition). There exists however an interesting study in the metropolitan area of Cape Town which suggests that school fees and school location explain segregation (see Lemon, 1999). Table 2b presents the fees and the racial composition of the schools investigated in that study. While fees range from $R30$ to $R4400$ per annum, school composition varies from 100% African to 62% White. It is striking that white children only attend the two most expensive schools and that there are more whites in more expensive schools. This is in accordance with our model which predicts that the higher the school fee, the lower the number of non-whites in the school. In Table 2b, schools like Usasazo and Elda Mahlente could correspond to our township public school (100% black and very low tuition fees) whereas former model C schools like Buren and Rustenberg could correspond to the central private school in our analysis (a majority of whites and high tuition fees).

[*Insert Table 2b here*]

[21] Matriculation or 'matric' in South Africa is equivalent to the A level in the UK, the Baccalauréat in France or the SAT in the US.

5 Market inefficiencies and policy implications

In this section, we show that the market equilibrium is socially suboptimal and simulate appropriate policy measures.

5.1 Inefficiency of the market equilibrium

Before getting into the analysis, observe that there are several sources of distortion in the model:

First, there are human capital externalities which are not internalized by families when choosing their child's school: blacks are not penalized for their negative human capital externalities nor are whites rewarded for their positive human capital externalities towards blacks. Second, school choices are limited by the credit constraints of black families. Third, whites have an institutional advantage over blacks since they choose the fee at the private school. Thus, when setting the private-school fee, whites do not take into account the impact their choice has on the well-being of blacks.

We have the following result:

Proposition 4 (Efficiency) *The market equilibrium is not efficient and the level of tuition fees chosen by white families is locally excessive in equilibrium if f_T is sufficiently low, i.e. if*

$$f_T < \left(\frac{4\mu \overline{N}_W}{\alpha(\overline{h}_B - \underline{h}_B)} \right)^{1/\eta} \tag{27}$$

Proof. See Appendix 3.

The market equilibrium is suboptimal in the sense that it does not maximize the social surplus. The proposition shows that it may be so because whites overprice education since they are mainly concerned with reducing human capital externalities in the private school and preventing the rise in land rents associated with school integration. When condition (27) is satisfied, a marginal decrease in the equilibrium fee f_C^* will always increase the total surplus. Observe that this proposition is only valid at the vicinity of f_C^*. However, even though we have not been able to assess the condition under which the equilibrium fee f_C^* is always higher than the socially optimal fee (see Appendix 3), it is always the case in the simulations we have computed. Finally, observe that (27) is only a sufficient condition for whites to locally overprice education and that whites may locally overprice education even if (27) is not satisfied. Moreover, they may also choose a school fee that is higher than the socially optimal fee, even when (27) is not satisfied. In the base case for instance, even though (27) is not satisfied, whites still have strong incentives to globally overprice education and a simulation presented in Table 3a show that the social optimum would be attained for a private-school fee ($f_C = .87$) lower than that of the market solution ($f_C = 2.65$). The total surplus would then be 5.6% higher than under the market solution and more integration would occur ($N_{BC}/\overline{N}_B = 84.9\% > 45.8\%$).

Thus, even in situations in which there would be much aggregate gains from school integration (i.e. for high values of the elasticity of substitution ρ), whites may chose a high school fee that leads to a number of black students at the private school that is too small. In fact, it should be clear that a socially preferable situation would be reached if, in the first stage of the model, the private school's fee was chosen by the government (instead of whites) so as to maximize the total surplus (including the utilities of landlords and those of all black families in the city). Thus, the lower surplus of the market equilibrium may justify the intervention of the government. In South Africa however, it is doubtful that the government has the power to directly interfere with the way private schools choose their fees. We believe that the government is left with little leeway so that it can only *consider several education policies subject to the constraint that private schools may freely choose their fees*. To model this type of intervention, *we need to add an initial step to our model in which the government announces the policy it intends to implement.* The rest of the timing is as previously: whites choose the private school's fee, thus controlling the level of school integration, but they now take into account the costs and effects the announced policy has for them. Families then simultaneously choose their place of residence and their child's school.

In order to understand our approach, it is important to note that *our goal is not to present Pareto-improving policies*. Indeed, in South Africa, the government seems more intent on 'redressing past imbalances' than on improving the utilities of all agents in the economy. In this context, we will evaluate the impact of several education policies on the total surplus and calculate their effects on the level of school integration.

5.2 The different policies

The policies that we will consider are: (i) subsidizing the transport of black children to the private school, (ii) injecting more financial inputs into the township school, (iii) handing out private-school vouchers to the best students among blacks.

Any of these policies is self-financed by a tax rate τ applied to the gross income of all workers in the city. Thus, tax resources are equal to:

$$T = \tau \left[\overline{N}_W y_W + \int_{\overline{N}_W}^{\overline{N}} y_B(x) dx \right] \tag{28}$$

5.2.1 Transportation subsidies (or busing)

There already exists a transport to school policy in South Africa, since children in rural or city-edge areas that live more than five kilometers from the closest school are entitled to subsidized transportation and organized busing. In practice, this policy comes down to busing black children to black schools and has no effect on racial integration in schools. What we have in mind is a different policy that pays for part or all of the transport cost of the black children who attend the private school (and who pay the full private school's fee).[22] In our framework, the government subsidizes the transport costs of black children who attend the private school at a rate $0 < \theta < 1$. This means that traveling a distance x will effectively cost those families $\mu(1-\theta)x$ instead of μx if there were no subsidies. Thus, for the government's budget constraint to be satisfied, it must hold that:

$$\int_{\overline{N}_W}^{\overline{N}_W+N_{BC}} \theta\,\mu\,x\,dx = T \tag{29}$$

where T is defined by (28).

To better understand the way white families will react to this policy, let us derive formally their utility function. Solving the augmented model (that includes the government), we obtain:

$$v_W^B = (1-\tau^B)\left[y_W(1-t\overline{N}_W) - \frac{(\overline{y}_B+\underline{y}_B)}{2}t\overline{N}_B\right] - m\overline{N} - \mu\left(\overline{N}_W - \overline{N}_B\right) - f_C^B + \alpha h_C^B - \mu N_{BC}^B(2-\theta)$$

where the superscript B stands for 'busing policy' and where, using (28) and (29), τ^B is given by:

$$\tau^B = \frac{\int_{\overline{N}_W}^{\overline{N}_W+N_{BC}^B} \theta\,\mu\,x\,dx}{\overline{N}_W y_W + \int_{\overline{N}_W}^{\overline{N}} y_B(x)dx}$$

It is easy to verify that $\partial\tau^B/\partial N_{BC}^B > 0$ and thus

$$\frac{\partial\tau^B}{\partial f_C^B} = \frac{\partial\tau^B}{\partial N_{BC}^B}\frac{\partial N_{BC}^B}{\partial f_C^B} < 0$$

This is quite intuitive since higher tuition fees limit school integration and thus reduces the taxes needed to finance the busing policy. Now, for $\underline{f} < f_C < \overline{f}$, we have:

$$\frac{\partial v_W^B}{\partial f_C^B} = -\frac{\partial\tau^B}{\partial f_C^B}\left[y_W(1-t\overline{N}_W) - \frac{(\overline{y}_B+\underline{y}_B)}{2}t\overline{N}_B\right] - 1 + \alpha\frac{\partial h_C^B}{\partial f_C^B} - (2-\theta)\mu\frac{\partial N_{BC}^B}{\partial f_C} \tag{30}$$

Thus, in comparison with the model without policy (see (23)), there is an additional effect that white families will take into account when choosing the private school's fee: the impact on the taxes they will have to pay in order to finance the busing policy (this corresponds to the first term on the right hand side of (30)). If one compares (23) and (30), it can also be seen that the effect on the land market is now different. Indeed, the last term on the right hand side of (30) is now lower

[22] It is worth noting that few theoretical papers have studied the efficiency of transportation policies on education or labor market outcomes (exceptions include Zenou, 2000, Martin, 2001, and Wasmer and Zenou, 2002).

$(2-\theta<2)$, which means that an increase in f_c reduces less the land rents paid by whites because of the depressing effect transport subsidies have on the competition for central locations. For whites, there is less to gain in terms of land rents by limiting school integration.

To summarize, with a transportation policy, an increase in the school fee only has one negative effect for whites (the marginal cost effect (-1)) and the three following positive effects:

(i) **a school-composition effect:** a higher f_C reduces black integration and ensures that the private school's composition remains predominantly white.

(ii) **a land-rent effect:** a higher f_C reduces integration and dampens the pressure exerted on land rents at the vicinity of the private school where whites live.

(iii) **a fiscal effect:** a higher tuition fee dissuades additional black students from attending the private school, which reduces the fiscal burden borne by whites (who represent a large fraction of the tax base since they are richer).

5.2.2 Township improvement

Injecting more money in the township school is the second policy that our model enables us to assess. It is obviously an important issue in the political context of 'redressing past imbalances' in South Africa. In the initial stage of our framework, the government announces that it will inject an amount Ω_T in the township school for each child attending that school. This means that the human capital output there is now given by:

$$h_T=\left[\underline{h}_B+\frac{\overline{h}_B-\underline{h}_B}{2}\left(\frac{\overline{N}_B-N_{BC}}{\overline{N}_B}\right)\right](\ f_T+\Omega_T)^{\eta}$$

while the effective township fee incurred by families remains unchanged.

Since $\overline{N}_B-N_{BC}$ children attend the township school, balancing the government's budget constraint for this policy requires that:

$$\Omega_T(\overline{N}_B-N_{BC})=T \tag{31}$$

As with the previous policy, we present the utility function of whites under a township improvement policy. We have:

$$v_W^T=(1-\tau^T)\left[y_W(1-t\overline{N}_W)-\frac{(\overline{y}_B+\underline{y}_B)}{2}t\overline{N}_B\right]-m\overline{N}-\mu\left(\overline{N}_W-\overline{N}_B\right)-f_C^T+\alpha h_C^T-2\mu N_{BC}^T$$

where the superscript T stands for 'township improvement' and where, using (28) and (31), τ is given by:

$$\tau^T=\frac{\Omega_T(\overline{N}_B-N_{BC}^T)}{\overline{N}_W y_W+\int_{\overline{N}_W}^{\overline{N}} y_B(x)dx}$$

It is easy to check that $\partial\tau^T/\partial N_{BC}^T < 0$ and thus

$$\frac{\partial\tau^T}{\partial f_C^T} = \frac{\partial\tau^T}{\partial N_{BC}^T}\frac{\partial N_{BC}^T}{\partial^T f_C} > 0$$

Observe that an increase in the private school's fee now raises the tax rate in equilibrium. This is also quite intuitive since higher tuition fees limit black integration in the private school and increases school attendance in the township school, which raises the cost of a township improvement policy. For $\underline{f} < f_C < \overline{f}$, we now have:

$$\frac{\partial v_W^T}{\partial f_C^T} = -\frac{\partial\tau^T}{\partial f_C^T}\left[y_W(1-t\overline{N}_W) - \frac{(\overline{y}_B + \underline{y}_B)}{2}t\overline{N}_B\right] - 1 + \alpha\frac{\partial h_C^T}{\partial f_C^T} - 2\mu\frac{\partial N_{BC}^T}{\partial f_C^T} \tag{32}$$

In comparison with the model without policy, an increase in the private school's fee now has two negative effects for whites (a marginal cost effect and a fiscal effect), and two positive effects (a school-composition effect and a land rent effect).

5.2.3 Private-school vouchers

The third policy consists in subsidizing the fee paid by the black children who attend the private school by handing them publicly funded private-school vouchers. This is now a widespread practise in many countries (West, 1996) even though it is quite controversial, especially in the United States. Opponents argue that resorting to vouchers is detrimental to the public school system because it 'skims off the cream' from public schools and exacerbates a two-tier system without yielding tangible results (Epple and Romano, 1998). To the contrary, proponents of vouchers assert that vouchers are welfare-enhancing because they increase school choice and force public schools to improve if they want to retain good students (Friedman, 1955, Manski, 1992, Hoxby, 1996). Our model incorporates both types of arguments: vouchers skim the cream from township schools but increase school choice among black children (since vouchers loosen the budget constraint of black families). We want to assess whether a voucher policy is adapted to the South African context. Two types of vouchers can be considered depending on how funds may be differentially allocated to recipients.

A *uniform-voucher* scheme is a program in which all recipients get the same amount of money. Under such a scheme, the government's policy consists in announcing the level Ω_C of a fixed voucher which will be allocated to any black child attending the private school.[23] This means that the effective fee paid by a black child who attends the private school will be $f_C - \Omega_C$ whereas all white children will still have to pay f_C. In this framework, the government's budget constraint becomes:

$$\Omega_C N_{BC} = T \tag{33}$$

[23] It is not clear whether a race-based voucher is politically feasible in South Africa even though the government is intent on 'redressing past imbalances'. In practice, an equivalent program could be implemented by announcing an income threshold I (with $y_W > I > \overline{y}_B$) under which families would be eligible for a voucher.

A *restricted-voucher* scheme is a program in which the amount given to a black child is gradual and depends on a chosen criterion such as the family's income or the child's ability. We will consider an income-based voucher program so that the government's policy now consists in announcing how much will be given to a black child attending the private school as a function of his parent's income. A simple way to do that is to announce that a family with income y_B is eligible to a voucher $\omega(y_B)$ to send its child to the private school, with:

$$\omega(y_B) = \frac{\overline{\omega}(\overline{y}_B - y_B) + \underline{\omega}(y_B - \underline{y}_B)}{\overline{y}_B - \underline{y}_B}$$

This is a simple linear function of y_B that assigns a voucher $\underline{\omega}$ to the richest black child (with parental income $\overline{y}_B$) and $\overline{\omega}$ to the poorest black child (with parental income $\underline{y}_B$). All families with income between $\underline{y}_B$ and $\overline{y}_B$ are given a voucher comprised between $\underline{\omega}$ and $\overline{\omega}$ if they send their child to the private school. This means that the effective fee paid by a black child of parental income y_B will be $f_C - \omega(y_B)$ if he attends the private school.[24] There are two possible restricted-voucher policies: either *a restricted-voucher scheme decreasing with income* so that $\underline{\omega} < \overline{\omega}$ or *a restricted-voucher scheme increasing with income* so that $\underline{\omega} > \overline{\omega}$.

Using the fact that in the augmented model with economic policies, families are still spatially ranked in order of relative income, it can easily be shown that the government's budget constraint is balanced when we have:

$$\frac{N_{BC}}{2\overline{N}_B}\left[N_{BC}(\overline{\omega} - \underline{\omega}) + 2\overline{N}_B\underline{\omega}\right] = T \tag{34}$$

Let us now present the utility function of whites under a voucher policy (be it uniform or restrictive). We obtain:

$$v_W^V = (1-\tau^V)\left[y_W(1 - t\overline{N}_W) - \frac{(\overline{y}_B + \underline{y}_B)}{2}t\overline{N}_B\right] - m\overline{N} - \mu\left(\overline{N}_W - \overline{N}_B\right) - f_C^V + \alpha h_C^V - 2\mu N_{BC}^V$$

where the superscript V stands for 'voucher policy'. Under a uniform-voucher policy, using (28) and (33), τ^V is given by:

$$\tau^V = \frac{\Omega_C N_{BC}}{\overline{N}_W y_W + \int_{\overline{N}_W}^{\overline{N}} y_B(x)dx}$$

whereas, under a restricted-voucher policy, using (28) and (34), we have:

$$\tau^V = \frac{N_{BC}}{2\overline{N}_B}\frac{\left[N_{BC}(\overline{\omega} - \underline{\omega}) + 2\overline{N}_B\underline{\omega}\right]}{\overline{N}_W y_W + \int_{\overline{N}_W}^{\overline{N}} y_B(x)dx}$$

[24] In our framework, a uniform voucher can thus be viewed as a particular restricted voucher with $\overline{\omega} = \underline{\omega} = \Omega_C$.

It is easy to check that in all cases (i.e. with a uniform-voucher policy or a restricted-voucher policy with $\overline{\omega} > \underline{\omega}$ or a restricted-voucher policy with $\overline{\omega} < \underline{\omega}$), we have $\partial \tau^V / \partial N_{BC} > 0$ and thus

$$\frac{\partial \tau^V}{\partial f_C^V} = \frac{\partial \tau^V}{\partial N_{BC}^V} \frac{\partial N_{BC}^V}{\partial f_C^V} < 0$$

Just as for a busing policy, an increase in the private school's fee reduces integration and the tax needed to finance the voucher program. Now, for $\underline{f} < f_C < \overline{f}$, we have:

$$\frac{\partial v_W^V}{\partial f_C^V} = -\frac{\partial \tau^V}{\partial f_C^V} \left[y_W (1 - t\overline{N}_W) - \frac{(\overline{y}_B + \underline{y}_B)}{2} t\overline{N}_B \right] - 1 + \alpha \frac{\partial h_C^V}{\partial f_C^V} - 2\mu \frac{\partial N_{BC}^V}{\partial f_C^V} \quad (35)$$

So, whatever the voucher policy implemented, an increase in tuition fees only has one negative effect for whites (the marginal cost of effect) and three positive effects (a school-composition effect, a land-rent effect, and a fiscal effect).

6 Comparing the different policies

Solving analytically the augmented model is very cumbersome and difficult to analyze. We thus resort to numerical simulations using the same parameters as in the base case and try to compare the different policies. To do that, we will proceed in two steps. First, we will present the optimal level of government intervention for each policy. Then, we will compare the effects of each policy for an equal outlay so as to determine the most cost-effective intervention. All our results are summarized in Table 3a and Table 3b at the end of the present section.

6.1 Comparing optimal policies

6.1.1 Transportation subsidies (busing)

The government chooses the level of θ that maximizes the surplus in the economy by anticipating the reaction of the white school's governing body. A unique solution exists for $\theta = 58.25\%$ as reported in Table 3a. In equilibrium, in order to finance this optimal busing policy, the government will need to levy 1.25% of every family's income in the city. Whites react to the announcement by choosing a fee ($f_C = 2.66$) which is almost exactly the same as under the market equilibrium but which enables more integration in the private school. Indeed, in this new framework, it can be calculated that $\overline{f}$ is now significantly higher than under the market solution.[25] This

[25] In the augmented model with transportation subsidies, we have

$$\begin{aligned} \overline{f} \equiv \; & \overline{y}_B(1 - t\overline{N})(1 - \tau) + \frac{(\overline{y}_B - \underline{y}_B)}{2} t\overline{N}_B(1 - \tau) \\ & - m\overline{N} + \mu(\overline{N}_B - (1 - \theta)\overline{N}_W) \end{aligned}$$

means that, due to the busing policy, blacks are less constrained by their budget and an additional 7.5% of black families are now able to send their children to the private school. Consequently, a majority of non-whites (53.4%) now have access to the 'good' school. Observe that due to the limited amount of additional school integration, human capital outputs are only slightly lower than under the market equilibrium. However, since more blacks have access to a better education, their average human capital h^{child}_{black} raises by more than 5% and reaches 9.3. Also observe that the surplus of landlords decreases by approximately 3% in spite of greater school integration (and the associated land rent effect). This is because *subsidized transport costs have a depressing effect on land rents which more than offsets the upward pressure on land rents exerted by greater integration.* The surplus of whites decreases by slightly more than 1% while blacks gain 3%. The overall effect is a 0.2% increase in the total surplus.

6.1.2 Township improvement

The government now maximizes the total surplus in the economy by choosing the level of per-pupil subsidy to the township school. Our simulations show that a unique optimum is attained for $\Omega_T = 2.62$ as presented in Table 3a. An unexpected result is that *the increase in per-pupil public spending to finance the township school increases integration at the private school.* This is because announcing such a high level of Ω_T presents a threat for white families: if they do not accept a sufficient amount of integration in the private school, then they will have to pay more to finance the township school (the fiscal effect). For white parents, there is a trade-off between the negative human capital externalities of mixing with black children (the school-composition effect) and the tax burden associated with school segregation (the fiscal effect). In our simulation, the government announces a level of per-pupil spending in the township school that is sufficiently high to spur integration in the private school: whites accept to set a low private school fee ($f_C = 1.05$) enabling much integration ($N_{BC}/\overline{N}_B = 77.1\%$) so as to incur a limited tax rate of 4.28%. In comparison with the market equilibrium, the human capital of white children is reduced by 21% while the average human capital of black children increases by almost 7%.

6.1.3 Uniform vouchers

The government now chooses Ω_C so as to maximize the total surplus. With the parameters of our base case, we obtain that *it is preferable not to implement any voucher policy at all* ($\Omega_C = 0$) so that, in this case, the market solution prevails. In fact, with our base case parameters, any uniform-voucher policy proves to be counterproductive. This is because announcing that the same fixed voucher will be given to black students when they attend the private school triggers a strong reaction from whites, which reduces both school integration and the surplus in the economy. To illustrate this point, Table 3b presents the case of a uniform voucher amounting to 0.33 (which is equivalent to only 12.5% of the fee f^*_C charged under the market equilibrium). When the government announces this policy, whites raise the fee at the private school by almost 17%, up to 3.09, which more than offsets the initial gain for voucher recipients. This reduces integration from 45.8% to 42.3% of the black children, so that 3.5% of the black children who previously attended the private school under the market equilibrium switch to the township school. In fact, our simulation shows that *whites have strong incentives to oppose a uniform-voucher policy in order to limit both its human capital effects and its expensive cost in terms of income tax.*[26]

By inspecting (35), it is easy to understand the counterproductive impact of a uniform-voucher program (which triggers a surge in the private-school fee).[27] Because of (*i*) the school-composition effect, a higher f_C will offset the voucher given to black students, ensuring that the private school's composition remains predominantly white. Because of (*ii*) the land rent effect, a higher f_C reduces integration and dampens the pressure exerted on land rents at the vicinity of the private school where whites live. Because of (*iii*) the fiscal effect, a sufficiently higher tuition fee dissuades additional black students from attending the private school, so that vouchers will not have to be paid, and therefore the corresponding fiscal burden will not have to be borne by whites.

Under a busing policy, we have seen that (*ii*) and (*iii*) are quite weak because transpost subsidies have a depressing effect on land rents and because a busing policy is better targeted and thus less costly. In the case of a uniform-voucher policy, all three effects are quite strong, which explains the reaction of whites. Thus, we will now consider a restricted-voucher policy that should limit the effects of (*ii*) and (*iii*).

6.1.4 Restricted vouchers

The government now has two instruments at its disposal ($\underline{\omega}$ and $\overline{\omega}$). In this context, the optimal restricted-voucher policy can be defined as the choice of both $\overline{\omega}$ and $\underline{\omega}$ that maximize the total surplus. To do that, the government can calculate for each value of $\underline{\omega}$, the value of $\overline{\omega}$ that maximizes the total surplus, and then choose the value of $\underline{\omega}$ that corresponds to the greatest surplus.

To solve this program, a reasonable restriction would be to impose $\overline{\omega} > \underline{\omega}$ so that the families with the lower incomes would be offered greater vouchers. Surprisingly, in all the simulations we made, the surplus is always reduced when $\overline{\omega} > \underline{\omega}$. Indeed, we find that the optimal restricted-voucher policy with $\overline{\omega} \geqslant \underline{\omega}$ is $\underline{\omega} = \overline{\omega} = 0$, which comes down not to implement any policy at all (see Table 3a). The explanation is quite simple: even though the fiscal effect should be reduced with restricted vouchers (since the policy is now better targeted to those who need it), the school-composition effect and the land-rent effect remain sufficiently strong to cause an increase in f_C that reduces both integration and the total surplus. In Table 3b, we provide the example of $\underline{\omega} = 0$ and $\overline{\omega} = .16$ which causes f_C to increase by 82% (reaching

[26] In this respect, the increase in the private school's fee is not surprising. In Sweden, for instance, it has been observed that private schools raised their fees by 9 percent following the introduction of a voucher system (Miron, 1993).

[27] It should be noted that these three effects are predominant in all the simulations we have carried out. The inefficiency of uniform vouchers can only be removed at the cost of very improbable assumptions on the model's parameters.

$\overline{f} = 4.84$) so as to dissuade all black families to send their children to the private school. In this example the total surplus is reduced by more than 6%.

Now, if we allow $\underline{\omega}$ to be greater than $\overline{\omega}$, a restricted-voucher policy becomes efficient again. Our simulations show that the optimal busing policy with $\underline{\omega} \geqslant \overline{\omega}$ is $\overline{\omega} = 0$ and $\underline{\omega} = 1.96$ (see Table 2a). This means that vouchers are now increasing with parental income, the richest black family receiving a voucher equal to 74% of the private school fee under the market solution while the poorest among black families would be entitled to a voucher of zero value. Even though this policy may seem *a priori* unfair, it has very positive effects: not only is private schooling subsidized for black children but the private school fee decreases from 2.65 to 2 so that an additional 21.9% of black families are able to send their children to the private school, which raises the average human capital of black children by more than 10%. The total surplus is increased by 3.3%.

We now have a better understanding of why a uniform-voucher policy or a restricted-voucher policy decreasing with income are not implementable and why a busing policy or a restricted-voucher policy increasing with income improve the total surplus. In the first two cases, the marginal cost of increasing the fee in the private school is not too high compared to the large positive effects on (*i*) school composition, (*ii*) the land rent, and (*iii*) the tax rate. In this context, whites choose a fee at a level that makes it preferable for the government not to implement any of these policies. Indeed, by giving all black families that attend the private school the same voucher or by giving more to the poorest black families, the government strongly amplifies all three effects (*i*), (*ii*) and (*iii*) because, without any reaction from whites, these policies would strongly enhance integration, competition in the land market and increase the tax burden. On the other hand, when the government proposes a voucher program that gives more money to rich black families than to poor ones, the effect on (*i*) school composition and on (*iii*) the tax rate are more acceptable for whites in the sense that it would be more costly for them to make this policy not implementable than to accept it. Concerning the busing policy, even though the government proposes to give more to poor black families than to rich ones (since the rich live closer to the private-school, they face lower transport costs and are eligible for lower subsidies), we do not obtain an inefficient result as with a uniform voucher or with a restrictive voucher decreasing with income. This is because the land rent effect is very different since subsidizing transport costs decreases the slope of the bid rents (see (12) and (13)). As a result, the land rent effect is much stronger under a voucher policy than under a busing policy, and this is what triggers the very strong reaction from whites which makes some voucher policies inefficient.

To sum up, our simulations suggest that *a uniform-voucher policy or a progressive voucher scheme decreasing with income will be countered by whites because such policies would have stronger effects not only on private school composition but also on the tax rate and the land rents paid by white families.* With a view to maximizing social surplus, our results also suggest that *a voucher scheme increasing with income should be preferred to direct township-school subsidies or to busing* as indicated by the ranking in Table 3a. However, these optimal policies involve very different costs and it is not clear whether such optimal policies can be implemented in a country where there exist limits to taxation and redistribution.[28] We will now look at an alternative way to rank our policies and present the effect of each one of these policies for a given outlay.

6.2 Comparing policy measures at a given cost

Table 3b presents the effect of the different educational policies all financed by a 1% taxation of parental income.[29]

6.2.1 Busing

The government budget only permits to subsidize 48.5% of the transport costs of the black children who attend the private school, leading to an increase in integration so that an additional 6.4% of all black children now attend the private school. This policy thus leads to an increase in the human capital of black children (from 8.8 to 9.2) whereas whites are harmed by the negative peer group effects in their school and their surplus is reduced by 1%. Interestingly, the surplus of landlords decreases because the upward pressure on land rents associated with greater integration is more than offset by the depressing effect of taxation and transport subsidies on the price of land. In this context, the surplus of black families increases by 3% so that the overall effect is a small increase in the total surplus reaching 965.

6.2.2 Township improvement

Using the same level of taxation now makes it possible to spend .29 on the improvement of the township school. As already discussed, this policy causes integration to rise (from 45.8% to 51.4% of black families) because it makes segregation costly for whites who react by lowering the private school fee by 11.3% . Contrary to the previous policy, the surplus of landlords now rises because the depressing effect of taxation does not compensate the upward pressure on land rents caused by school integration. In this context, since whites pay higher land rents, have a lower human capital and pay taxes, their surplus can only decrease whereas the higher taxes and land rents paid by blacks are more than compensated by the higher average human capital of their children. The overall effect is an increase in the total surplus which reaches 971.

6.2.3 Uniform vouchers

As discussed in the previous subsection, a uniform-voucher policy is inefficient since it triggers a strong reaction from whites who raise the private-school fee so as to decrease integration. A uniform-voucher policy financed by a 1% tax rate corresponds to a uniform voucher of .29 and a 16.6% increase in the private school fee so that

[28] In particular, there is a risk of white flight out of the country.

[29] This means that all the policies we consider cost 8.4 units of income since the sum of all incomes amounts to 840 in our base case assumptions.

3.5% of black children switch from the private school to the township school. All in all, the total surplus decreases from 963 to 948.

6.2.4 Restricted vouchers

As discussed previously, a restricted-voucher program decreasing with income is always countered by a rise in the private school fee, which reduces both school integration and the total surplus. Furthermore, a restricted-voucher policy financed by a 1% tax rate cannot be implemented with the parameters of our base case. In Table 3b we present the effects of a policy announcing $\underline{\omega} = 0$ and $\overline{\omega} = .16$ which is sufficient to cause complete school segregation and a 6.3% reduction in the total surplus which decreases from 963 to 902.

To the contrary, a restricted-voucher program increasing with income can be implemented and increases the total surplus. Table 3b shows that a policy financed by a 1% tax rate can be implement by announcing $\underline{\omega} = .36$ and $\overline{\omega} = 0$. Whites react by lowering the private school fee by 6.8% so that integration is spurred and 52.5% of black families now have access to the private school. The surplus of whites decreases while the surplus of blacks and that of landlords increase. As a net effect, the total surplus increases and reaches 972.

To conclude, comparing the effects of the above policies for an equal outlay would suggest that restricted vouchers increasing with income and township-improvement policies are more efficient than busing while uniform vouchers are inefficient.

[*Insert Tables* 3*a and* 3*b here*]

7 Conclusion

This paper has developed a model that captures some of the recent events experienced by South Africa: the emergence of private schools and soaring school fees. In our model, these two facts present a major threat to the racial and spatial integration between white and non-white communities. This suggests that white families who control private schools tend to overprice education in order to limit non-white attendance, protecting themselves from negative human capital externalities. This market solution only permits the partial and limited school integration of the communities who were relatively less discriminated against under Apartheid (Indians and Coloureds to a lesser extent). However, this is not socially optimal because whites do not take into account the impact of their fee-setting policy on the welfare of non-whites. Therefore, we have contemplated different governmental policies publicly financed by an income tax and which take into account the possibility of being countered by the fee-setting behavior of whites. We proceed in two different ways. First, we compare the optimal level of government intervention for each policy. Then, we compare the different policies for a given cost so as to determine the most cost-effective intervention. Our simulation results suggest that uniform school vouchers or restricted vouchers decreasing with income are not efficient because they may trigger a very strong negative reaction from whites (in order to avoid the harmful peer group effects and the financial burden of the policy). To the contrary, restricted vouchers increasing with income, transportation subsidies (busing) and public spending in township schools seem to be more efficient and to improve the racial integration between communities. We found however that the restricted-voucher policy that increases with parental black income is the policy that is the most efficient and the less costly since it triggers the weakest reaction from whites.

Even though our model is quite stylized, we believe that it gives an interesting insight into the efficiency of different education policies that could be implemented in post-Apartheid South Africa. In particular, it shows that publicly financed transportation policies or restrictive vouchers scheme targeted to black families, often put forward in the public debate, could be efficient if implemented in South African cities. It also shows the limits of a uniform-voucher policy.

The critical aspect of our model (but also of the new South Africa) is the reaction of whites who form a minority group but, nevertheless, have the power to manipulate fees in private schools. In this context, the political aspect of education in South Africa is the next step to study and we leave it for future research.

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A Appendix

A.1 Appendix 1: Existence, uniqueness and properties of the residential equilibrium

Lemma 1 *Whites always outbid all blacks for central locations and thus reside at the vicinity of the city-center.*

Proof. As standard in urban economics (Fujita, 1989), it is necessary to rank bid rents in order of relative steepness so as to determine an equilibrium configuration with heterogenous workers. Thus, to prove the result, we need to show that whites always have steeper bid rents than blacks. Let us thus determine the bid rent slopes. From equations (11)-(13), we obtain:

$$\frac{\partial \Psi_W}{\partial x} = -(m + ty_W + \mu) < 0$$

$$\frac{\partial \Psi_{BC}}{\partial x} = -(m + ty_B + \mu) < 0 \tag{36}$$

$$\frac{\partial \Psi_{BT}}{\partial x} = -(m + ty_B - \mu) < 0 \tag{37}$$

Using $m > \mu$, it is clear that all bid rents are decreasing in x. We want to show that whites have the steepest bid rent so that they always live closer to the city center. Since $y_W > y_B$ for any value of $y_B \in \left[\underline{y}_B, \overline{y}_B\right]$, it is straightforward to see that

$$\left|\frac{\partial \Psi_W}{\partial x}\right| > \left|\frac{\partial \Psi_{BT}}{\partial x}\right|$$

and

$$\left|\frac{\partial \Psi_W}{\partial x}\right| > \left|\frac{\partial \Psi_{BC}}{\partial x}\right|$$

The bid rent of a white family is thus always steeper than that of any black family. This implies that whites reside close to the city center. ■

Lemma 2 *In equilibrium,*

(i) white families reside at the vicinity of the city-center and black families locate next to whites;

(ii) any black family which child attends the central school reside closer to the city-center than any black family which sends its child to the township school;

(iii) within each black group (either type BC or BT), families locate in order of decreasing income, i.e. the richer the family, the closer it resides to the city-center.

Proof.

(i) This is a direct consequence of Lemma 1 since whites always bid away blacks at the outskirts of the city.

(ii) When we will solve the school attendance problem (see section 4.1.2 and Appendix 2) we will show that richer families among blacks send their children to the private school. This means that black families which child attends the central school have a higher income than black families which send their child to the township school. Thus, using (36) and (37), we have:

$$\left|\frac{\partial \Psi_{BC}}{\partial x}\right| > \left|\frac{\partial \Psi_{BT}}{\partial x}\right|$$

which implies (ii).

(iii) We have:

$$\frac{\partial^2 \Psi_{BC}}{\partial x \partial y_B} = \frac{\partial^2 \Psi_{BT}}{\partial x \partial y_B} = -t < 0$$

which means that, within each black group, families locate in order of decreasing income. ■

Lemma 3 *The mapping $y_B(\cdot)$ that assigns a black family with income $y_B(x)$ to a location x is given by:*

$$y_B(x) = \overline{y}_B - \frac{(x - \overline{N}_W)}{\overline{N}_B}(\overline{y}_B - \underline{y}_B) \qquad \textit{for} \quad \overline{N}_W \leq x \leq \overline{N}$$

Proof. It is quite easy to determine the mapping described by the expression above since it is a direct consequence of Lemma 2. Indeed, since income is uniformly distributed among blacks and since black families are ranked in order of decreasing income, we easily obtain the mapping which expresses the income of each black family as a function of its location. It is easily checked that for the black family residing in $\overline{N}_W$, we have $y(\overline{N}_W) = \overline{y}_B$ whereas for the black family located at the city edge, we have $y(\overline{N}) = \underline{y}_B$. ■

Proof of Proposition 1

After having defined the mapping between the location and the income of blacks (see (18)), we will now determine the different equilibrium utilities. For that, since we have a continuum of bid rents, we use a similar technique as in Brueckner, Thisse and Zenou (2002). Since there are three categories of families (whites, blacks who send their children to the central school and blacks who send their children to the peripheral school), we proceed in three steps.

First step:

Let us start with the region $[\overline{N}_W + N_{BC}, \overline{N}]$ which exclusively hosts black families of type BT (i.e. black families which child attends the township school). We want to express the relation between equilibrium utility and location. Since land is allocated to the highest bidder, the equilibrium land rent for any $x \in [\overline{N}_W + N_{BC}, \overline{N}]$ is defined by:

$$R(x) = \max_{y_B} \Psi_{BT}(x, v_{BT}(y_B)) \tag{38}$$

We have shown that the only possible urban configuration is by order of decreasing income according to (18). We assume that Ψ_{BT} is concave in y. In equilibrium, it must hold that:

$$\frac{\partial \Psi_{BT}(y_B, v_{BT}(y_B))}{\partial y_B} \mid_{y_B = y_B(x)} = 0$$

which, using (13), implies:

$$\frac{\partial v_{BT}(y)}{\partial y_B} = 1 - tx(y_B) \tag{39}$$

and consequently:

$$v_{BT}(y_B) = \frac{t\overline{N}_B (y_B)^2}{2(\overline{y}_B - \underline{y}_B)} + \left[1 - t\overline{N}_W - \frac{t\overline{N}_B \overline{y}_B}{\overline{y}_B - \underline{y}_B}\right] y_B + K \tag{40}$$

where K is a constant. At this stage, we can check that Ψ_{BT} is concave in y as assumed. Indeed, we have:

$$\frac{\partial^2 \Psi_{BT}}{\partial y_B^2} = -\frac{\partial^2 v_{BT}(y_B)}{\partial y_B^2} = -\frac{t\overline{N}_B}{\overline{y}_B - \underline{y}_B} < 0$$

Plugging (40) into (17) and solving the equation for K yields the following equilibrium utility:

$$\begin{aligned} v_{BT}(y_B) &= \left[\overline{y}_B - \frac{(x - \overline{N}_W)}{\overline{N}_B}(\overline{y}_B - \underline{y}_B)\right] \left[1 - \frac{t}{2}\left(x + \overline{N}_W + \frac{\overline{N}_B \overline{y}_B}{(\overline{y}_B - \underline{y}_B)}\right)\right] \\ &\quad + \frac{t\overline{N}_B (\underline{y}_B)^2}{2(\overline{y}_B - \underline{y}_B)} - m\overline{N} - f_T + \alpha h_T \end{aligned} \tag{41}$$

Moreover, it is easily obtained from (41) that:

$$\frac{\partial v_{BT}(x)}{\partial x} = -\frac{(\overline{y}_B - \underline{y}_B)}{\overline{N}_B}(1 - tx) \tag{42}$$

which is always negative under the assumption that $(1 - t\overline{N} > 0)$. This assumption is clearly satisfied in our model since, multiplying it by $\underline{y}_B$, it is tantamount to $\underline{y}_B > \underline{y}_B t\overline{N}$, which just means that the family living at the edge of the city earns more than its time commuting costs. The equilibrium utility of a black family which sends its child to the suburban school is thus a decreasing function of that family's distance to the city center.

Second step:

Let us now consider the region $[\overline{N}_W, \overline{N}_W + N_{BC}]$ which exclusively hosts black families of type BC (i.e. black families which child attends the central school). Using the same procedure and (16), we easily obtain:

$$\begin{aligned} v_{BC}(y_B) &= \left[\overline{y}_B - \frac{(x - \overline{N}_W)}{\overline{N}_B}(\overline{y}_B - \underline{y}_B)\right]\left[1 - \frac{t}{2}\left(x + \overline{N}_W + \frac{\overline{N}_B \overline{y}_B}{(\overline{y}_B - \underline{y}_B)}\right)\right] \\ &\quad + \frac{t\overline{N}_B(\underline{y}_B)^2}{2(\overline{y}_B - \underline{y}_B)} - m\overline{N} - f_C + \alpha h_C - \mu(\overline{N}_W - \overline{N}_B + 2N_{BC}) \end{aligned} \tag{43}$$

and

$$\frac{\partial v_{BC}(x)}{\partial x} = -\frac{(\overline{y}_B - \underline{y}_B)}{\overline{N}_B}(1 - tx) \tag{44}$$

which is always negative since $1 - tx$ must always be positive.

Third step:

Let us finally consider the region $[0, \overline{N}_W]$ occupied by whites families. Using (15), we obtain:

$$v_W = y_W(1 - t\overline{N}_W) - \frac{(\overline{y}_B + \underline{y}_B)}{2} t\overline{N}_B - m\overline{N} - \mu\left(\overline{N}_W - \overline{N}_B\right) - f_C + \alpha h_C - 2\mu N_{BC} \tag{45}$$

A.2 Appendix 2: School choice

Lemma 4 *The disposable income of black families of type BC (respectively BT) is a decreasing function of the distance to the city center.*

Proof. The disposable income is the income net of land rent, fee and transport costs (and is entirely spent to consume the composite good). It is given by inverting the budget constraint (2) in z_{BC} or z_{BS}. If we denote $\Phi_{BC}(x)$ (respectively $\Phi_{BT}(x)$)

the disposable income of a black family of type BC (respectively BT) located in x, we have:

$$\Phi_{BC}(x, N_{BC}) = y_B(x) - [m + y_B(x)t]\, x - \mu x - f_C - R(x) \quad \text{for } x \in [\overline{N}_W, \overline{N}_W + N_{BC}] \tag{46}$$

and

$$\Phi_{BT}(x, N_{BC}) = y_B(x) - [m + y_B(x)t]\, x - \mu(\overline{N} - x) - f_T - R(x) \quad \text{for } x \in [\overline{N}_W + N_{BC}, \overline{N}] \tag{47}$$

where $R(x)$ is the equilibrium land rent. It is then easy to verify that:

$$\frac{\partial \Phi_{BC}}{\partial x} = \frac{\partial \Phi_{BT}}{\partial x} = -\frac{(\overline{y}_B - \underline{y}_B)}{\overline{N}_B}(1 - tx)$$

which is always negative. This means that disposable income is decreasing with distance to the city center. ■

Proof of Proposition 2

In this appendix, we show that, for $\underline{f} < f_C < \overline{f}$, there exists a unique corresponding value for $N_{BC} \in]0, \overline{N}_B[$ and that this value is decreasing in f_C. Let us start with the first statement. Since we have assumed that families are constrained by their budget and have shown that disposable income is a decreasing function of distance to the CBD (see Lemma 4), N_{BC} is determined by:

$$\Phi_{BC}(\overline{N}_W + N_{BC}, N_{BC}) = 0 \tag{48}$$

which means that any family leaving beyond $\overline{N}_W + N_{BC}$ cannot afford to send its child to the central school. Simple calculations show that Equation (48) comes down to the following second degree equation:

$$\Gamma(N_{BC}) = E N_{BC}^2 + F N_{BC} + G = 0 \tag{49}$$

with

$$E \equiv \left[\frac{t(\overline{y}_B - \underline{y}_B)}{2\overline{N}_B}\right] > 0$$

$$F = -\left[2\mu + \frac{(\overline{y}_B - \underline{y}_B)}{\overline{N}_B}(1 - t\overline{N}_W)\right] < 0$$

and

$$G = \left[\overline{y}_B(1 - t\overline{N}) + \frac{(\overline{y}_B - \underline{y}_B)}{2} t\overline{N}_B - m\overline{N} + \mu(\overline{N}_B - \overline{N}_W) - f_C\right]$$

which can be shown to be positive using $f_C < \overline{f}$.

Let us now show that there exists a unique $N_{BC} \in]0, \overline{N}_B[$ such that $\Gamma(N_{BC}) = 0$. It is straightforward that: $\Gamma(0) = C > 0$ while, using the fact that $f_C > \underline{f}$, it is easy to show that: $\Gamma(\overline{N}_B) < 0$. Furthermore, inspection of (49) shows that it is a convex parabola. Now, since $\Gamma(\cdot)$ is a continuous function and a convex parabola, with $\Gamma(0) > 0$ and $\Gamma(\overline{N}_B) < 0$, then there exists a unique value N_{BC} comprised between 0 and $\overline{N}_B$, such that $\Gamma(N_{BC}) = 0$. It is given by:

$$N_{BC} \equiv \frac{-F - \sqrt{F^2 - 4EG}}{2E} \tag{50}$$

Let us now show that N_{BC} decreases with f_C. By plugging (43) into (12), and the resulting (12) into (46), we get an analytical expression for $\Phi_{BC}(x, N_{BC})$. Using the fact that $\Phi_{BC}(\overline{N}_W + N_{BC}, N_{BC}) = 0$ by definition of N_{BC}, and applying the implicit function theorem, we easily obtain:

$$\frac{\partial N_{BC}}{\partial f_C} = \frac{-\overline{N}_B}{2\mu\overline{N}_B + (\overline{y}_B - \underline{y}_B)\left[1 - t(\overline{N}_W + N_{BC})\right]} < 0 \tag{51}$$

A.3 Appendix 3: Existence and efficiency of the market equilibrium

Proof of Proposition 3

We will show that the solution to the maximization problem of whites, i.e. $\max\limits_{f_C} v_W$, exists. To study the continuity of the utility function v_W, we need to know the values of v_W when f_C tends towards $\underline{f}$ and when it tends towards $\overline{f}$. To do that, let us first express v_W, which is given by (45), for all possible values of f_C. Using $N_{BC} = \overline{N}_B$ when $f_C \leqslant \underline{f}$, and $N_{BC} = 0$ when $f_C \geqslant \overline{f}$ (see Selod and Zenou, 2002), we have:

$$v_W = \begin{cases} v_W = D - 2\mu\overline{N}_B - f_C + \alpha h_C(\overline{N}_B, f_C) & \text{for} \quad f_C \leqslant \underline{f} \\ v_W = D - f_C + \alpha h_C(N_{BC}(f_C), f_C) - 2\mu N_{BC}(f_C) & \text{for} \quad \underline{f} < f_C < \overline{f} \\ v_W = D - f_C + \alpha h_C(0, f_C) & \text{for} \quad f_C \geqslant \overline{f} \end{cases} \tag{52}$$

where $D = y_W(1 - t\overline{N}_W) - (\overline{y}_B + \underline{y}_B)t\overline{N}_B/2 - m\overline{N} - \mu(\overline{N}_W - \overline{N}_B)$ and where $h_C(N_{BC}(f_C), f_C)$ is defined by (6) and $N_{BC}(f_C)$ by (50). This function is continuous on each of the three intervals.

To show that v_W has a maximum, we will use the Weierstrass theorem that states that any continuous real valued function defined on a compact set is bounded and reaches its boundaries on that compact set.

In our framework, we have $v_W : [0, f_C^{\max}] \to \Re_+$, where $f_C^{\max}$ is given by:

$$f_C^{\max} = y_W(1 - t\overline{N}_W) - (\overline{y}_B + \underline{y}_B)t\overline{N}_B/2 - m\overline{N} - \mu(\overline{N}_W - \overline{N}_B) > 0$$

$f_C^{\max}$ is the maximum fee that whites can impose. It is such that their budget constraint is binding when there is complete school segregation and $N_{BC} = 0$. $f_C^{\max}$ is greater than any other fee that may be binding the budget constraint of whites when N_{BC} is greater than zero since we have shown that $\frac{\partial N_{BC}}{\partial f_C} < 0$. Since $f_C^{\max}$ is finite, then $[0, f_C^{\max}]$ is a compact set.

To apply the Weierstrass theorem, it remains to show that the function $v_W : [0, f_C^{\max}] \to \Re_+$ is continuous. For that, we have to check that:

$$\lim_{f_C \to \underline{f}} v_W(f_C) = v_W(\underline{f})$$

$$\lim_{f_C \to \overline{f}} v_W(f_C) = v_W(\overline{f})$$

By using (52), these two relations are immediately verified and the function $v_W(\cdot)$ is continuous on a compact set, which proves the existence of a maximum, and thus of an equilibrium.

Observe that, because of the complexity of the function $v_W(\cdot)$, we have not been able to prove that the equilibrium is always unique. In fact, using the fact that $\eta < 1$, it can only be proven that $v_W(\cdot)$ is strictly concave on $[0, \underline{f}]$ and on $[\overline{f}, f_C^{\max}]$, so that there always exists a unique maximum on each one of these closed subsets. Unfortunately, nothing can be said analytically about $v_W(\cdot)$ on $]\underline{f}, \overline{f}[$ and it cannot be ruled out that the utility function of whites has several global maxima on that open subset. It should be noted, however, that all our simulations always yielded a (graphically obvious) unique equilibrium.

Proof of Proposition 4

Let us compare the market and the government solutions with respect to the optimal fee f_C. The market solution is obtained when whites choose f_C in order to maximize their utility. Using (45), we easily obtain:

$$\frac{\partial v_W}{\partial f_C} = -1 + \alpha \frac{\partial h_C}{\partial f_C} - 2\mu \frac{\partial N_{BC}}{\partial f_C} = 0 \tag{53}$$

which must hold for any interior solution. The solution of (53) is denoted by f_C^*.

The government solution is obtained when the government chooses f_C so as to maximize the total surplus (26). Since the land rent $R(x)$ is a pure transfer between agents, it cancels out when computing the total surplus. Thus, using (8), (9) and (10), the latter writes:

$$\begin{aligned} S_{Total} &= \int_0^{\overline{N}_W} \left[y_W - (m + t\, y_W)\, x - \mu\, x - f_C + \alpha h_C \right] dx \\ &+ \int_{\overline{N}_W}^{\overline{N}_W + N_{BC}} \left[y_B - (m + t\, y_B)\, x - \mu\, x - f_C + \alpha h_C \right] dx \\ &+ \int_{\overline{N}_W + N_{BC}}^{\overline{N}} \left[y_B - (m + t y_B)\, x - \mu \left(\overline{N} - x \right) - f_T + \alpha h_T \right] dx \end{aligned}$$

We have:

$$\begin{aligned}\frac{\partial S_{Total}}{\partial f_C} &= \left(-1+\alpha\frac{\partial h_C}{\partial f_C}\right)\overline{N}_W + \left(-1+\alpha\frac{\partial h_C}{\partial f_C}\right)N_{BC} \\ &+ \left[y_B - (m+t\,y_B)(\overline{N}_W + N_{BC}) - \mu(\overline{N}_W + N_{BC}) - f_C + \alpha h_C\right]\frac{\partial N_{BC}}{\partial f_C} \\ &- \left[y_B - (m+ty_B)(\overline{N}_W + N_{BC}) - \mu(\overline{N}_B - N_{BC}) - f_T + \alpha h_T\right]\frac{\partial N_{BC}}{\partial f_C} \\ &+\alpha\frac{\partial h_T}{\partial f_C}(\overline{N}_B - N_{BC})\end{aligned}$$

This is equivalent to

$$\begin{aligned}\frac{\partial S_{Total}}{\partial f_C} &= \left(-1+\alpha\frac{\partial h_C}{\partial f_C}\right)(\overline{N}_W + N_{BC}) \\ &+ \left[\alpha(h_C - h_T) - (f_C - f_T) - \mu\left(\overline{N}_W - \overline{N}_B + 2N_{BC}\right)\right]\frac{\partial N_{BC}}{\partial f_C} \\ &+\alpha\frac{\partial h_T}{\partial f_C}(\overline{N}_B - N_{BC}) \qquad (54)\end{aligned}$$

The solution of (54) is denoted by f_C^{**}. Now, comparing (53) and (54), it is clear that the two solutions do not coincide so that, in general, the market equilibrium is not efficient since the goverment solution would lead to a higher surplus.

Now, in order to compare these two solutions, we evaluate (54) at f_C^*, i.e. at the solution of (53). Using (53), we have:

$$\begin{aligned}\left.\frac{\partial S_{Total}}{\partial f_C}\right|_{f_C=f_C^*} &= \left(2\mu\left.\frac{\partial N_{BC}}{\partial f_C}\right|_{f_C=f_C^*}\right)(\overline{N}_W + N_{BC}^*) \\ &+ \left[\alpha(h_C^* - h_T^*) - (f_C^* - f_T) - \mu\left(\overline{N}_W - \overline{N}_B + 2N_{BC}^*\right)\right]\left.\frac{\partial N_{BC}}{\partial f_C}\right|_{f_C=f_C^*} \\ &+\alpha(\overline{N}_B - N_{BC})\left.\frac{\partial h_T}{\partial f_C}\right|_{f_C=f_C^*}\end{aligned}$$

Now, differentiating (7) with respect to f_C, we finally have

$$\begin{aligned}&\left.\frac{\partial S_{Total}}{\partial f_C}\right|_{f_C=f_C^*} \\ &= \left.\frac{\partial N_{BC}}{\partial f_C}\right|_{f_C=f_C^*}\left[\alpha(h_C^* - h_T^*) - (f_C^* - f_T) + \mu\overline{N} - \alpha\frac{(\overline{h}_B - \underline{h}_B)}{2}\frac{(\overline{N}_B - N_{BC}^*)}{\overline{N}_B}\ f_T^{\eta}\right]\end{aligned}$$

Since $\left.\frac{\partial N_{BC}}{\partial f_C}\right|_{f_C=f_C^*} < 0$, whites *locally* overprice tuition fees in the private solution, if and only if the term in brackets is positive. We know however from (22) that

$$\alpha(h_C^* - h_T) > (f_C^* - f_T^*) + \mu\left(\overline{N}_W - \overline{N}_B + 2N_{BC}^*\right)$$

Using that property, a sufficient condition for $\left.\frac{\partial S_{Total}}{\partial f_C}\right|_{f_C=f_C^*}$ to be negative is

$$2\mu\left(\overline{N}_W + N_{BC}^*\right) > \alpha\frac{\left(\overline{h}_B - \underline{h}_B\right)}{2}\frac{\left(\overline{N}_B - N_{BC}^*\right)}{\overline{N}_B}\ f_T^\eta$$

which is always verified when

$$f_T < \left[\frac{4\mu\overline{N}_W}{\alpha(\overline{h}_B - \underline{h}_B)}\right]^{1/\eta}$$

When this condition is satisfied, a marginal decrease in the equilibrium fee f_C^* always increases the social surplus. Finally, observe that if the total surplus is concave in f_C, then whites globally overprice education since the slope of the total surplus is negative for $f_C = f_C^*$ (which means that the total surplus maximum is attained for $f_C^{**} < f_C^*$).

Table 1a: Base Case and variations from the Base Case

	$\overline{f}$	f_C^*	$\frac{N_{BC}^*}{\overline{N}_B}$	h_C^*	h_T^*	h_{black}^{child*}	S_W^*	S_B^*	S_L^*
Base Case	4.84	2.65	45.8%	13.3	5.1	8.8	478	322	163
$\alpha = 1/3$	4.84	1.14	78.9%	10.6	4.4	9.3	399	257	194
$\alpha = 2/3$	4.84	4.84	0%	16.4	6	6.0	586	353	133
$\rho \to +\infty$	4.84	1.84	63.5%	13.6	4.7	10.3	508	369	178
$\rho \to -\infty$	4.84	4.84	0%	16.4	6	6.0	476	293	133
$\eta = .3$	4.84	4.84	0%	19.3	6	6.0	532	293	133

Base case: $\alpha = \frac{1}{2},\ h_W = y_W = 12,\ \overline{h}_B = \overline{y}_B = 8,\ \underline{h}_B = \underline{y}_B = 4,\ f_T = 1,$
$\rho = 5,\ \eta = .2,\ m = \frac{20}{1000},\ \mu = \frac{10}{1000},\ t = \frac{2}{1000},\ \overline{N}_W = 40,\ \overline{N}_B = 60.$

Table 1b: Changing population and human capital distributions (Comparative statics of the Base Case)

$\overline{N}_W$	$\overline{N}_B$	f_C^*	$\frac{N_{BC}^*}{\overline{N}_B}$	Average child's human capital: Whites	Average child's human capital: Blacks
50	50	2.24	52.4%	13.1	9.2
40 (Base Case)	60 (Base Case)	2.65	45.8%	13.3	8.9
25	75	5.2	0%	16.7	6
$\frac{y_W}{\overline{y}_B} = \frac{h_W}{\overline{h}_B}$	$\frac{y_W}{\underline{y}_B} = \frac{h_W}{\underline{h}_B}$	f_C^*	$\frac{N_{BC}^*}{\overline{N}_B}$	Average child's human capital: Whites	Average child's human capital: Blacks
1.5 (Base Case)	3 (Base Case)	2.65	45.8%	13.3	8.9
1.25	2.5	2.04	76.6%	12.2	10.6
1	2	1.01	100%	10.8	10.8

Base case: $\alpha = \frac{1}{2},\ h_W = y_W = 12,\ \overline{h}_B = \overline{y}_B = 8,\ \underline{h}_B = \underline{y}_B = 4,\ f_T = 1,$
$\rho = 5,\ \eta = .2,\ m = \frac{20}{1000},\ \mu = \frac{10}{1000},\ t = \frac{2}{1000},\ \overline{N}_W = 40,\ \overline{N}_B = 60.$

Table 2a: Population, human capital and income in major South African metropolitan areas in 1996

	Pretoria	Cape Town	Port Elizabeth	Johannesburg	Durban
Total Population (m):	1.2	2.5	0.8	3.3	2.5
Population distribution:					
Non-white	62%	78%	82%	83%	88%
-African/Black	58%	26%	57%	76%	61%
-Coloured	2%	51%	24%	4%	3%
-Indian/Asian	2%	1%	1%	3%	24%
White	38%	22%	18%	17%	12%
Percentage population aged 19-24 with matric or higher:					
Non-white	46%	41%	29%	36%	41%
-African/Black	45%	32%	25%	35%	31%
-Coloured	51%	46%	36%	45%	53%
-Indian/Asian	78%	80%	69%	77%	72%
White	88%	93%	79%	82%	84%
Percentage population aged 45-64 with matric or higher					
Non-white	10%	9%	7%	9%	10%
-African/Black	9%	7%	6%	7%	7%
-Coloured	16%	9%	7%	15%	12%
-Indian/Asian	26%	30%	27%	31%	15%
White	66%	80%	51%	63%	60%
Median income class:	$R1001-R1500$	$R501-R1000$	$R201-R500$	$R501-R1000$	$R501-R1000$
Percentage population aged 45-64 above median income class:					
Non-white	23%	36%	35%	34%	34%
-African/Black	22%	28%	32%	33%	28%
-Coloured	34%	39%	40%	39%	43%
-Indian/Asian	42%	46%	54%	50%	42%
White	68%	71%	71%	73%	73%

Source: Computed by the authors from Statistics South Africa's Census 1996

Table 2b: Summary data for eight schools in greater Cape Town in 1998

Name of the school	Number of pupils	Fees in Rand	% Racial make-up
Usasazo	900	R30	A: 100%
Elda Mahlente	760	R60	A: 100%
Rylands P S	650	R120	I: 80-85% A: 15-20%
Windermere	580	R120	C: 85-90% A: 10-15%
Rylands H S	1076	R200	I/C: 97% A: 3%
Kenmere	886	R250	C: 95% A: 5%
Buren	540	R700	W: 50% C: 40% A: 10%
Rustenberg	783	R4400	W: 62% A/C/I: 38%

Source: Lemon (1999)
A=African; W=White; C=Coloured; I=Indian

Table 3a: Comparing optimal policy measures

	f_C	$\frac{N_{BC}}{\overline{N}_B}$	h_C	h_T	h_{black}^{child}	S_W	S_B	S_L	S_{Total}	τ	Policy Ranking
Market solution (base case)	2.65	45.8%	13.3	5.1	8.8	478	322	163	963	-	
Social optimum	.87	84.9%	10.0	4.3	9.1	465	352	200	1017	-	
Busing $\boldsymbol{\theta = 58.25\%}$	2.66	53.4%	13.1	4.9	9.3	473	334	158	965	1.25%	Third
Township improvement $\mathbf{\Omega_T = 2.62}$	1.05	77.1%	10.5	5.8	9.4	454	347	189	990	4.28%	Second
Uniform vouchers $\mathbf{\Omega_C = 0}$	2.65	45.8%	13.3	5.1	8.8	478	322	163	963	0%	Inefficient
Restricted vouchers (with $\underline{\omega} = 0$) $\boldsymbol{\overline{\omega} = 0}$	2.65	45.8%	13.3	5.1	8.8	478	322	163	963	0%	Inefficient
Restricted vouchers (with $\overline{\omega} = 0$) $\boldsymbol{\underline{\omega} = 1.96}$	2.00	67.9%	12.1	4.6	9.7	444	373	178	995	6.28%	First

Base case: $\alpha = \frac{1}{2}$, $h_W = y_W = 12$, $\overline{h}_B = \overline{y}_B = 8$, $\underline{h}_B = \underline{y}_B = 4$, $f_T = 1$, $\rho = 5$, $\eta = .2$, $m = \frac{20}{1000}$, $\mu = \frac{10}{1000}$, $t = \frac{2}{1000}$, $\overline{N}_W = 40$, $\overline{N}_B = 60$.

Table 3b: Comparing policy measures at given cost

	f_C	$\frac{N_{BC}}{\overline{N}_B}$	h_C	h_T	h^{child}_{black}	S_W	S_B	S_L	S_{Total}	τ	Policy Ranking
Market solution (base case)	2.65	45.8%	13.3	5.1	8.8	478	322	163	963	-	
Social optimum	.87	84.9%	10.0	4.3	9.1	465	352	200	1017	-	
Busing $\boldsymbol{\theta} = \mathbf{48.5\%}$	2.66	52.2%	13.1	5.0	9.2	474	332	159	965	1%	Third
Township improvement $\mathbf{\Omega_T = .29}$	2.35	51.4%	12.8	5.2	9.1	475	329	167	971	1%	Second
Uniform vouchers $\mathbf{\Omega_C = .33}$	3.09	42.3%	13.8	5.2	8.8	468	321	159	948	1%	Inefficient
Restricted vouchers $\overline{\boldsymbol{\omega}} = \mathbf{0}$ and $\underline{\boldsymbol{\omega}} = \mathbf{.36}$	2.47	52.5%	12.9	4.9	9.1	471	333	168	972	1%	First
Restricted vouchers $\overline{\boldsymbol{\omega}} = \mathbf{.16}$ and $\underline{\boldsymbol{\omega}} = \mathbf{0}$	4.84	0%	16.4	6.0	6.0	476	293	133	902	0%	Inefficient

Base case: $\alpha = \frac{1}{2},\ h_W = y_W = 12,\ \overline{h}_B = \overline{y}_B = 8,\ \underline{h}_B = \underline{y}_B = 4,\ f_T = 1,$
$\rho = 5,\ \eta = .2,\ m = \frac{20}{1000},\ \mu = \frac{10}{1000},\ t = \frac{2}{1000},\ \overline{N}_W = 40,\ \overline{N}_B = 60.$

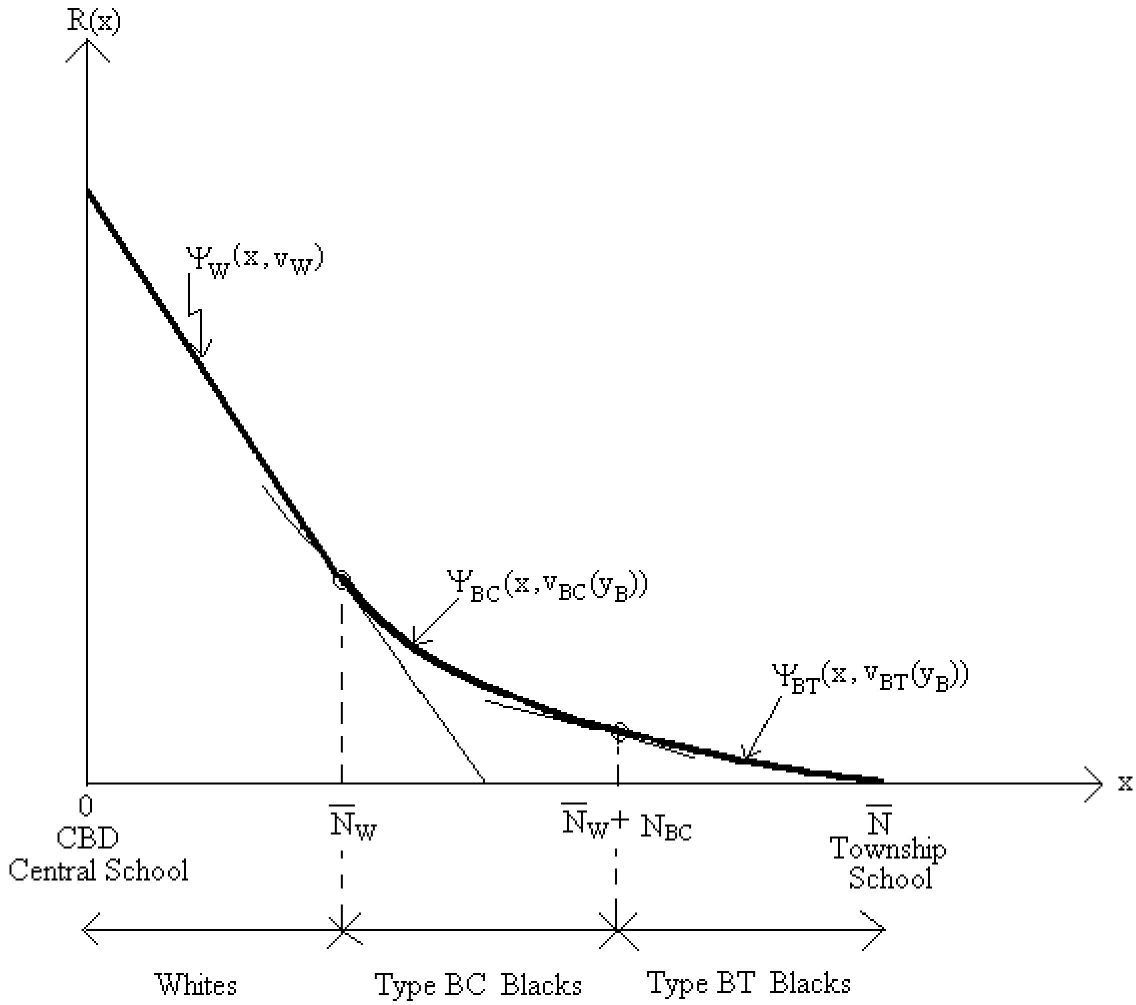

Figure 1: The Urban Equilibrium