MONADIC STYLE CONTROL CONSTRUCTS FOR INFERENCE SYSTEMS

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Abstract. Recent advances in programming languages study and design have established a standard way of grounding computational systems representation in category theory. These formal results led to a better understanding of issues of control and side-effects in functional and imperative languages. Another benefit is a better way of modelling computational effects in logical frameworks. With this analogy in mind, we embark on an investigation of inference systems based on considering inference behaviour as a form of computation. We delineate a categorical formalisation of control constructs in inference systems. This representation emphasises the parallel between the modular articulation of the categorical building blocks (triples) used to account for the inference architecture and the modular composition of cognitive processes.

1. Introduction

Originally grounded in research work in Artificial Intelligence (AI) and, more specifically in AI in medicine and production systems [13], *Nexpert Object* became a commercial success in the late eighties and nineties. Its implementation architecture broke new ground in graphical user interfaces, in portability across operating environments and in distributed object systems. But powerful innovations also went deep into the design of the inference system itself (NXP), which can be traced back to that seminal research work in AI.

In this and a companion paper we offer a formal account of this inference systems, using recent theoretical advances in the understanding of programming language design. In particular, some of the computational effects introduced by the NXP design show a denotational semantics best captured with the current tools in *continuation-passing* or *monadic* styles.

Continuations. The idea of transforming programs to continuation-passing style (CPS) appeared in the mid-sixties [14]. The transformation was formally codified by Fischer and Reynolds in 1972, yielding a standard CPS representation of call-by-value lambda calculus. In the context of denotational semantics [15], described as the theory of meaning for programs, denotations are usually built with the help of functions in some mathematical structures. In CPS these functions are explicitly passed an additional argument κ representing "the rest of the computation", a first step towards full reification of the notion of computation itself. Although

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most of the work on continuations involves a functional language, usually a fragment of typed or untyped lambda calculus, CPS is also useful in understanding imperative languages. Continuations were found to be a major tool for the design of interpreters and compilers for many languages, most prominently Scheme, ML and Haskell.

As such, continuations appear as the raw material of control. Operations on continuations control of the unfolding of a computation—that this translation happens in a standard way is a major result of the theoretical work on continuations. Similarly, in inference systems, of which production systems are a well-studied example [18], designing a proper behaviour relies on the delicate interweaving of *goal-driven* and *data-driven* control. Moreover the importance of the relative data-driven nature of the flow of control sets inference systems apart from both functional and imperative programming languages. In inference systems, both side-effects and procedure calls are mediated through operations on data. This paper shows the relevance of the CPS representation for analysis and assessment of inference systems.

Monads and triples. Originally introduced by Moggi [8] in computer science, monads or triples, a notion from category theory [2], were shown to generalise the continuation-passing style transformation. Monads can model a wide variety of features, including continuations, state, exceptions, input-output, non-determinism, and parallelism [16, 17]. The *monadic style* essentially distinguishes between values and computations ; it sets up a uniform infrastructure for representing and manipulating computations with effects as first-class objects [5]. More specifically, monads introduce a type constructor T through which the type $T\tau$ represents a computation that yields a result of type τ and may have side-effects. Cast in this background, continuations arise as a special case of monad translation.

Inference systems. Inference systems are characterised by a program organization based on data or event-driven operations. Such an organisation can be described as *pattern directed*: patterns occurring in the data select pieces of code in the system to be activated. Systems organised this way have been used primarily for deductive inference [18], and, to a lesser extent, for inductive inference or learning [13]. An inference system has three components: a collection of substructures, called pattern-directed modules, which can be activated or fired by patterns in the data; one or more data structures that may be examined and modified by the pattern-directed modules; and an executive, or interpreter, that controls the selection and activation of the modules. In traditional production systems the pattern-directed modules are called *production rules* and reside in *production memory* while the data structures are stored in the *working memory*. The interpreter uses various strategies in a *recognize-act cycle* to select candidate production rules and execute their action part.

In the *NXP architecture*, this mechanism is supplemented by a goal driven process. Informally, goals are investigated by actively looking for patterns in data, a search usually called *backward chaining* as it involves a recursive descent in the generalized and-or tree representing linked production rules. Such goals are confirmed, rejected or considered "not known" as the data collected during the recursive descent trigger some of the patterns of the production rules, a process dually called *forward chaining*. In addition goals are said to be *evoked* when, hitherto uninvestigated, they also depend on data collected during the backward chaining descent from initial goals. Eventually every goal sharing a triggered data pattern with an

initial goal ends up evoked. These evoked goals are queued for later evaluation once the current agenda of goal investigation or data collection completes, their investigation is suspended or *postponed* until the current one terminates. Considering backward and forward chainings in NXP as computations in the previously mentioned sense, goal evocation can be viewed as an *effect*, comparable, for instance, to the `escape` construct of functional programming languages. Following sections of this paper show how to capture backward and forward chainings as computations and account for goal evocation in continuation-passing and monadic styles.

Logical frameworks. Most of the work we are presenting in this and its companion paper has been designed for validation and checking with Twelf [11], a logical framework developed at Carnegie-Mellon University. In a logical framework such as this, a deductive system is used as a device for establishing semantic properties of the mathematical or computational object under study. Following Martin-Löf introduction of *judgements* as the foundation for constructive mathematics and computer science, logical frameworks represent judgements as types enabling proof-theoretic derivations by type reconstruction. Logical frameworks provide a uniform *judgements-as-types* representation of proofs of semantic properties. In this paper we use the Twelf deductive system to investigate the semantic properties of computations with effects in the NXP inference system (which is itself a deductive system at a different level of abstraction). Obviously terms such as *type*, *deduction* and *inference* have overloaded meanings and we will specify precisely in which context or system we use the latter when there is a risk of confusion.

The rest of this paper is organised in three sections. The first one explores a continuation-passing style representation of the NXP computational architecture. The second presents generalisation to a monadic style description of various effects in the NXP architecture. The last section uses the monadic results of the previous exploration to suggest an abstract implementation of the NXP architecture in the form of the *NXP abstract machine* with implications for building theoretically sound NXP interpreters or compilers.

2. Representing inference control with continuations

2.1. **Terminology.** In the following we will consider simple languages representing various aspects of inference systems, and, more specifically of the NXP architecture. Such languages consist of a syntax, defining the set of well-formed types and well-typed terms, and a semantics, assigning some notion of meaning to terms. We expect a semantics to provide a notion of *program evaluation* or *inference derivation*, usually represented as a (partial) function from a suitable set of terms to some set of observable results, say boolean values.

In *denotational semantics* the semantics is expressed as functions mapping syntactic constructs in the program to mathematical objects in a mathematical domain. Analyses of programs are actually conducted in the mathematical domain, in which properties assessed about denotations of programs translate to properties about programs. In contrast, the *operational semantics* defines an abstract machine with a state, possibly several components, and some set of primitive instructions. The *axiomatic approach* associates an "axiom" with each kind of statement in the programming language stating what we may assert after execution of that statement in terms of what was true beforehand. There is usually a gap between a denotational semantics and an operational semantics for a language and the main

formal work in designing interpreters and compilers is to prove their soundness and faithfulness by proving equivalence between these semantics. Bridging this gap requires to proceed in several stages: at each stage an alternative semantic definition of the language is introduces, embodying successively more and more implementation details. These intermediate stage semantics are called *non-standard semantics* to reflect their instrumental status. With these sequence of steps, denotational techniques often drift into program transformations and non-standard semantics have often been used as intermediate languages in compilers.

2.2. **Continuation-passing style transformations.** Practically all programming languages have some form of control structure or jumping; the more advanced forms of control structures tend to resemble function calls, so much so they are rarely describes as jumps. The library function `exit` in C, for example, may be called with an argument like a function. Its whole purpose, however, is utterly non functional: it jumps out of arbitrarily many surrounding blocks and pending function calls. Such a non-returning function or jump with arguments is an example of why continuations are needed.

In continuation-passing style, a function call is transformed into a jump with arguments to the callee such that one of these is a continuation that enables the callee to jump back to the caller. This idea has been formalised into a standard CPS transformation for lambda calculus.

Definition 2.2.1. The pure untyped lambda calculus Λ is defined by [1]: A set of terms, M, inductively generated over an infinite set of variables $Vars$,

- Terms $M ::= V \mid M\ M$;
- Values $V ::= x \mid \lambda x.M$, x in $Vars$.

The standards CPS translation for Λ in denotational semantics is given by the map $[[\_]]_F : \Lambda \to \Lambda$ originally described by Fisher:

Definition 2.2.2. Let k, m, n in $Vars$ be variables that do not occur in the argument to $[[\_]]_F$.

- $[[V]]_F = \lambda k.k\psi(V)$
- $[[MN]]_F = \lambda k.[[M]]_F(\lambda m.[[N]]_F \lambda n.(mk)n)$
- $\psi(x) = x$
- $\psi(\lambda x.M) = \lambda k.\lambda x.[[M]]_F k$

We will consider a much simpler fragment for the purpose of analysis of effects and control in inference systems and, more specifically, in the NXP architecture. The syntax is made up of simple expressions of boolean arithmetic:

$$E ::= true \mid false \mid E\ or\ E \mid E\ and\ E$$

In this simplified core model of the inference computation, goals are represented by and/or tree expressions, the leaves of which are the working memory elements. Keeping with the essentials, these working memory elements are valued in $\mathbf{B} = \{0, 1\}$ the boolean algebra with boolean operations & (logical and) and | (logical or). This will be relaxed later as we take into account other effects in the NXP architecture. In this toy model, the pattern-directed modules or production rules of inference systems are captured by expressions in the language, and the executive system is naturally a boolean arithmetic evaluator.

Following the denotational semantics approach, the standard semantics for the simple and/or language is defined by:

Definition 2.2.3. Let $B = \{true, false\}$ be the set of constants, Exp the set of expressions inductively defined as in the previous paragraph, and $\mathbf{B}$ the classical boolean algebra, we define the semantic function $\mathcal{B} : B \to \mathbf{B}$ and the evaluation function $[[\_]]_0 : Exp \to \mathbf{B}$ by:

- $[[B]]_0 = \mathcal{B}(B)$
- $[[E_1 \; or \; E_2]]_0 = [[E_1]]_0 \mid [[E_2]]_0$
- $[[E_1 \; and \; E_2]]_0 = [[E_1]]_0 \;\&\; [[E_2]]_0$

This definition does not specify a particular order for evaluation of boolean expressions, nor does it refer directly to continuations. As such it is really denotational and only calls for several steps of refinement to be turned into an operational semantics. It does, however, reflect the basic principles of inference systems introduced earlier and, properly extended, will be an adequate basis on which to build the analysis. The next step towards operational semantics now consists of providing a non-standard semantics for the simple language and proving its denotational equivalence with definition 2.2.3.

2.3. **Two non-standard semantics of computations in the NXP architecture.**

2.3.1. *Continuations to order goal investigations.* The first non-standard semantics introduced here makes explicit reference to continuations. In order to better model the behaviour of inference systems, we add a new operator to our simple and/or tree language to capture sequential investigation of two or more goals:

$$E ::= true \mid false \mid E \; or \; E \mid E \; and \; E \mid E \; ; \; E$$

where E are expressions as in definition 2.2.3, and $E_1 \, ; E_2$ describes investigation of goal E_1 followed by investigation of goal E_2. In imperative programming languages, this is simply the succession of computations of expressions E_1 and E_2. We can now define the following non-standard semantics for this new language as follows:

Definition 2.3.1. Let $\mathbf{K}$ be the set of continuations, in our case the set of functions $\mathbf{B} \to O$ where O is the type of output objects with a distinguished element `exit` : $\mathbf{B} \to O$, and B, Exp and $\mathcal{B}$ as in definition 2.2.3, we define the evaluation function $[[\_]]_1 : Exp \to \mathbf{K} \to \mathbf{K}$ by:

- $[[B]]_1 \; k = k \; \mathcal{B}(B)$
- $[[E_1 \; or \; E_2]]_1 \; k = [[E_1]]_1(\lambda\epsilon_1.[[E_2]]_1(\lambda\epsilon_2.k(\epsilon_1 \mid \epsilon_2)))$
- $[[E_1 \; and \; E_2]]_1 \; k = [[E_1]]_1(\lambda\epsilon_1.[[E_2]]_1(\lambda\epsilon_2.k(\epsilon_1 \;\&\; \epsilon_2)))$
- $[[E_1 \; ; \; E_2]]_1 \; k = [[E_1]]_1(\; [[E_2]]_1 \; k)$

The continuations introduced here follow the convention of mapping results of computations, here values of goals inferred by the system, to a specific output type. With these conventions in place, the top level `read-eval-print` loop of traditional interpreters is captured by the simple semantics

$$[[E]]_1 \; \texttt{exit}$$

which basically executes the computation of the expression E and pass the result to the continuation that yields control back to the user. In addition, definition 2.3.1 introduces a left to right ordering of goal investigation and a left to right ordering

of the data collection process: this appears in the semantics denotation in places where the left subexpression of a composed expression is always evaluated first with a continuation built from the computation of the right subexpression. This of course, is already a form of design choice and alternative semantics are easily produced to represent other options. The embedded design choice then goes one step further away from the standard denotational semantics down the road of an implementation-ready operational semantics.

2.3.2. *Sequence semantics for continuations.* With the refined non-standard semantics introduced in this section, we are setting up a framework for representing the semantics of control of in the NXP architecture. The following definitions refine the operational semantics introduced in 2.3.1:

Definition 2.3.2. We will use the notation $s = \langle B_1, B_2 ... B_n \rangle$ for finite sequences of boolean values, s in $\mathbf{B}^*$ the set of finite sequences of boolean values; we will also use indifferently `nil` and $\langle\,\rangle$ to designate the empty sequence. The finite sequence abstract data type is characterised by the following operations:

$$s \downarrow i = B_i, 0 < i \leq n \; (selection)$$

$$s \uparrow i = \langle B_{i+1}, ... B_n \rangle, 0 < i < n; \texttt{nil}, i = n \; (rest)$$

$$length(s) = n$$

$$s \; + \; s' = \langle B_1, B_2 ... B_n, B'_1, B'_2 ... B'_m \rangle \; (concatenation)$$

$$Or(s) = \langle \; s \downarrow 1 \mid s \downarrow 2 \; \rangle + s \uparrow 2, n \geq 2$$

$$And(s) = \langle \; s \downarrow 1 \; \& \; s \downarrow 2 \; \rangle + s \uparrow 2, n \geq 2$$

The sequence abstract data type is comparable to the stack data type introduced by Stoy [15] for operational semantics of programming languages. In the NXP architecture the sequence semantics captures the succession of goals investigated and evoked by the systems. Based on definition 2.3.2 we are now able to present the (non-standard) sequence semantics for our simple representation of the NXP architecture:

Definition 2.3.3. Let $\mathbf{B}^*$ be the set of finite boolean values sequences with the previously defined abstract data type operations, let Exp and $\mathbf{B}$ be as in definition 2.3.1 let $\mathcal{B}$ be, as before, the semantic function of definition 2.2.3, we introduce the new semantic evaluation function $[[\_]]_2 : Exp \to \mathbf{B}^* \to \mathbf{B}^*$:

- $[[B]]_2 s = \langle \mathcal{B}(B) \rangle + s$
- $[[E_1 \; or \; E_2]]_2 \; s = Or([[E_2]]_2([[E_1]]_2 \; s))$
- $[[E_1 \; and \; E_2]]_2 \; s = And([[E_2]]_2([[E_1]]_2 \; s))$
- $[[E_1 \; ; \; E_2]]_2 \; s = [[E_2]]_2([[E_1]]_2 \; s)$

Remark 2.3.4. Note that, even though the notation is reversed, compared to definition 2.3.1, the previous evaluation function still enforces a left to right order of evaluation on expressions and subexpressions. The left subexpression is always evaluated first and pushed on the sequence before evaluation of the right subexpression. This is particularly important in the specific case of expressions like $E_1 \; ; \; E_2$.

Remark 2.3.5. In the sequence semantics of the simple NXP language, continuations are actually "implemented" as finite sequences, initially with type operations similar to stacks (sequence and rest, for instance). In the NXP architecture, the

boolean values leaves of and/or trees are determined from examination of the working memory or from interaction with the user (asking questions). This operation is not captured in the simple language we have presented so far but will be discussed later. It is important however it somehow constrains when operations are actually performed on a finite sequence s of $\mathbf{B}^*$. Namely, we consider that boolean values in a sequence s are valued *just in time* when they are passed as arguments to the sequence operations *And*, *Or*, and the like. (This can be compared to lazy evaluation in languages like Scheme or Haskell.) Boolean values in sequences of definition 2.3.3 are then representations of *delayed* accesses to the working memory. The reifications of those delayed executions, usually called *thunks*, are triggered by the goal investigation process as needed. The lazy evaluation convention though does not affect the inference computation nor the control of its unfolding.

We ground the previous definition with the following result:

Proposition 2.3.6. *The semantics 2.2.3 and 2.3.3 are* congruent *i.e.*

$$[[E]]_2\ s = \langle [[E]]_0 \rangle +\ s$$

for all E and for all s.

Proof. The proof is by induction on expressions. Obviously :

$$[[B]]_2\ s = \langle \mathcal{B}(B) \rangle +\ s$$

by definition. For the other syntactic forms of expressions :

$$\begin{aligned}
[[E_1\ or\ E_2]]_2\ s &= Or([[E_2]]_2([[E_1]]_2\ s)) \\
&= Or([[E_2]]_2(\langle [[E_1]]_0 \rangle +\ s)) \\
&= Or(\langle [[E_2]]_0 \rangle +\ (\langle [[E_1]]_0 \rangle +\ s)) \\
&= Or(\langle [[E_2]]_0, [[E_1]]_0 \rangle +\ s) \\
&= \langle [[E_2]]_0 \mid [[E_1]]_0 \rangle +\ s \\
&= \langle [[E_1\ or\ E_2]]_0 \rangle +\ s
\end{aligned}$$

and similarly for the *and* expression. □

2.3.3. *Formalisation in a Logical Framework (Twelf).* The previous approach lends itself to formalisation in a Logical Framework. In this section, we use the codification of formalisation techniques into a meta-language offered by Twelf, a logical framework developed at Carnegie-Mellon University [11], to specify proofs of inference systems properties. This is done in three stages : The first one is the representation of the abstract syntax of the simple language under investigation; the second stage is the representation of the language semantics (both notions of values and types, and notions of computations, i.e. operational semantics); the last stage is the representation of the properties of the language (for instance, congruence of semantics and type preservation).

We base the representation of the simple language expressions on abstract syntax (rather than concrete) in order to expose the essential structure and focus on the semantics and its properties rather than on lexical analysis and parsing—although the Logical Framework could also help here. The representation technique is called *higher-order abstract syntax* and basically uses types in Twelf to capture all syntax.

Definition 2.3.7. Going back to definition 2.2.3 of the simple NXP language, we then declare a new Twelf type:

```
exp : type.
```

to represent expressions of *Exp*, intending that every Twelf object M of type `exp` represents a simple language expression and vice-versa. Similarly, we declare boolean values as a new Twelf type:

```
bool : type.
```

and declare two distinguished instances of this newly defined `bool` type:

```
true : bool.
```

```
false : bool.
```

With these basic types in place we now turn to boolean values expressions built from & and | operators. Twelf captures these operators as *expression constructors* which translate into constant functional types:

```
& : bool -> bool -> bool.
```

```
| : bool -> bool -> bool.
```

for which we will use infix mode for better readability.

In a second stage we introduce a representation of deductions following the idea of *judgements-as-types*. In Twelf, deductions are objects and judgements are types, so that proofs in the semantics space are in fact type reconstructions in Twelf. In order to do so, we introduce a type family `eval0` indexed by representations of expressions (*Exp*) and boolean values (**B**):

```
eval0 : exp -> bool -> type.
```

such that we have types such as `eval0 e b` which depend on objects.

Axioms are simply represented as types such as these, while semantics operations can be viewed as constructors which, given deductions of their arguments, yield a deduction of their result.

Definition 2.3.8. For the simple NXP language we introduce:

```
constant : bool -> exp.
```

```
and : exp -> exp -> exp.
```

```
or : exp -> exp -> exp.
```

and the following representation of their respective evaluations:

```
eval0cst : eval0 (constantB) B.
```

```
eval0or : eval0 (E2 or E1) (V1 | V2) <- eval0 E1 V1 <- eval0 E2 V2.
```

```
eval0and : eval0 (E2 and E1) (V1 & V2) <- eval0 E1 V1 <- eval0 E2 V2.
```

which capture the standard semantics of definition 2.2.3.

Remark 2.3.9. Using Twelf built-in deductive capabilities we can now explicit deductions such as in the following example:

```
eval0 (constant true and constant true or constant false) V.
---------- Solution 1 ----------
 V = (false | true) & false.
 A = eval0_and eval0_cst (eval0_or eval0_cst eval0_cst).
```

which simply shows the result of the boolean operation as `V` and the deduction as `A`. Note that the deduction `A` may be considered as a logical proof of the value `V` or alternatively as the logical program implementing `eval0` viewed as a function call.

The non-standard semantics of the NXP architecture are captured in the same way, with the introduction of an additional Twelf type, `bool_seq` for sequences of boolean values.

Definition 2.3.10. The Twelf representation of the sequence semantics for NXP consists firstly of an evaluation function which builds a functional programming style representation of expressions of the simple language :

```
%%% The sequence abstract data type
nil    : bool_seq.
atom   : bool -> bool_seq.

select : bool_seq -> int -> bool.
tail   : bool_seq -> int -> bool_seq.
length : bool_seq -> int.
+      : bool_seq -> bool_seq -> bool_seq. %infix right 10 +.

%%% The functional programming style representation
eval2 : exp -> bool_seq -> bool_seq -> type.

eval2_cst : eval2 (constant B) nil (atom B).
eval2_cst2: eval2 (constant B) S (atom B + S).
eval2_or_s : eval2 (E1 or E2) S (or_s S2)
                <- eval2 E1 S S1 <- eval2 E2 S1 S2.
eval2_and_s : eval2 (E1 and E2) S (and_s S2)
                <- eval2 E1 S S1 <- eval2 E2 S1 S2.
```

and secondly of an evaluation function that performs the functionally specified operations on the input sequence :

```
eval2a : bool_seq -> bool_seq -> type.

eval2a_atom : eval2a (atom B) (atom B).
eval2a_atom_2 : eval2a (atom B + S) (atom B + S')
         <- eval2a S S'.
eval2a_or_s1 : eval2a (or_s (S'')) (atom(B1 | B2) + S')
           <- eval2a S'' (atom B1 + atom B2 + S)
           <- eval2a S S'.
eval2a_or_s2 : eval2a (or_s (S'')) (atom(B1 | B2))
           <- eval2a S'' (atom B1 + atom B2).
eval2a_or_s_nil : eval2a (or_s (A1 + A2 )) (atom(B1 | B2) )
           <- eval2a A1 (atom B1) <- eval2a A2 (atom B2).
eval2a_and_s1 : eval2a (and_s (S'')) (atom(B1 & B2) + S')
           <- eval2a S'' (atom B1 + atom B2 + S)
           <- eval2a S S'.
eval2a_and_s2 : eval2a (and_s (S'')) (atom(B1 & B2))
           <- eval2a S'' (atom B1 + atom B2).
eval2a_and_s_nil : eval2a (and_s (A1 + A2 )) (atom(B1 & B2) )
```

```
          <- eval2a A1 (atom B1) <- eval2a A2 (atom B2).
```

The Twelf representation of the sequence semantics function is simply the composition of the previously shown functions :

```
eval2b : exp -> bool_seq -> bool_seq -> type.
eval2b_1 : eval2b E S S' <- eval2 E S S'' <- eval2a S'' S'.
```

Remark 2.3.11. There are many ways the non-standard semantics of the NXP architecture could have been mapped to Twelf's metalanguage. Definition 2.3.10 is in fact more "operational" than really needed. It separates a functional programming equivalent of the expression parsed from the execution of this representation. The previous definition emphasises the distinction between *instructions* and a *machine* executing instructions. The impact of this formal distinction will be discussed in the implementation section of this paper.

Remark 2.3.12. Theorem-proving capabilities of the Twelf environment are also extremely useful in assessing properties of the simple language. For instance, as a verification of the congruence of semantics we can query Twelf as follows :

```
%query 1 1
eval2b ((constant true and constant false)
    or constant true
    or constant false) nil
   S.
---------- Solution 1 ----------
S = atom ((false | true) | false & true).
A =
   eval2b_1
      (eval2a_or_s2
          (eval2a_or_s1 eval2a_atom
              (eval2a_atom_2
                  (eval2a_atom_2
            (eval2a_and_s2
                (eval2a_atom_2 eval2a_atom))))))
      (eval2_or_s (eval2_or_s eval2_constant2 eval2_constant2)
          (eval2_and_s eval2_constant2 eval2_constant)).

%query 1 1
eval0 ((constant true and constant false)
    or constant true
    or constant false) V.
---------- Solution 1 ----------
V = (false | true) | false & true.
A = eval0_or
    (eval0_and eval0_cst eval0_cst)
        (eval0_or eval0_cst eval0_cst).
```

which basically shows deductions of proposition 2.3.6 for a particular expression.

Within this framework we are now ready to set forth the semantics of some of the major inference features of the NXP architecture, and, with the Logical Framework

mapping introduced in this section, ground some of its behavioural properties into mathematical logic.

2.3.4. *Sequence semantics for goal evocation.* The sequence semantics previously presented was designed to make it relatively natural and simple to express goal evocation in the NXP inference architecture. Informally goal evocation, which is triggered by the data collection process, *queues* a goal for postponed investigation, once the current threads of investigation complete. In turn, we will see that this sequence semantics is suggestive of an operational semantics closer to implementation.

Definition 2.3.13. Let us refine the simple NXP language with the following: Exp, expressions:

$$E ::= b \mid E \; or \; E \mid E \; and \; E \mid E \; ; \; E \mid b \; post \; E$$

with b in $\mathbf{B}$ and where the new expression $b \; post \; E$ collects data value b and evokes goal E (which, as an expression, is itself an arbitrarily long sequence of expressions). The other expressions are as in definition 2.3.3.

The semantics of goal evocation is captured by extending the previous definition 2.3.3 of sequence semantics with the additional mapping:

Definition 2.3.14. Let $\mathbf{B}^*$ be the set of finite boolean values sequences with the previously defined selection, rest and concatenation operations, let Exp and $\mathbf{B}$ be as in definition 2.3.3 let $\mathcal{B}$ be, as before, the semantic function of definition 2.2.3, we extend the semantic evaluation function $[[\_]]_2 : Exp \to \mathbf{B}^* \to \mathbf{B}^*$ as follows:

- $[[B]]_2 s = \langle \mathcal{B}(B) \rangle + s$
- $[[E_1 \; or \; E_2]]_2 \; s = Or([[E_2]]_2([[E_1]]_2 \; s))$
- $[[E_1 \; and \; E_2]]_2 \; s = And([[E_2]]_2([[E_1]]_2 \; s))$
- $[[E_1 \; ; \; E_2]]_2 \; s = [[E_2]]_2([[E_1]]_2 \; s)$
- $[[B \; post \; E]]_2 \; s = \langle \mathcal{B}(B) \rangle + \; s \; + [[E]]_2 \langle \rangle$

Remark 2.3.15. The definition of the *post* semantic function captures the postponed execution of the goal investigation. It effectively replaces the current continuation with the same *followed* by the evaluation of a goal or a sequence of goals. The effect is of completing first the current goal evaluation process and then pursue with the postponed goals evoked during that initial process.

Remark 2.3.16. Although it is implicit in the definition of sequences, the previous definition relies on a left-to-right ordering of operation on sequences as in remark 2.3.4. If we required an operational semantics closer to implementation, we would enforce this by letting sequences hold unevaluated expressions, or thunks, as well as instructions of a virtual machine, transforming the sequence abstract data type into a conventional stack abstract data type.

Remark 2.3.17. In a refined operational semantics, we would need to be a bit more specific than the previous definition 2.3.14. More formally, we already mentioned that boolean values in goal expressions are actually evaluated by accessing the working memory of the inference system (possibly causing further interaction with the user). The working memory is represented as an environment ρ, a map from a set of variables $Vars$ to boolean values $\mathbf{B}$. Goal expressions then refer to variables and the semantic function is expressed in term of ρ. The map ρ is partial in the

sense that its value is not necessarily defined on all variables at all times. In the occurrence of one of these unvalued variables, the goal evaluation process triggers an atomic transaction with the user, requesting the value which is then assigned in ρ. In programming languages this caching effect is also known as *memoizing* and can be captured in different ways in an operational semantics. The postponed evaluation of goals then proceeds in the environment as resulting from the preceding evaluation cycle. In this, postponed evaluation differs from re-evaluation that would, in contrast, proceed from scratch in a new environment ρ_0 totally undefined.

With this semantics we can review some elementary results formalising the intuitions underlying goal evocation in the NXP architecture.

Proposition 2.3.18. *Sequencing and postponing goals are commutative operations in the following restricted sense:*

$$[[B_1 \; post \; E_1 \; ; \; B_2 \; post \; E_2]]s = [[B_1 \; post \; (E_2 \; ; \; E_1) \; ; \; B_2]]s$$

and

$$[[B_1 \; post \; E_1 \; ; \; B_2 \; post \; E_2]]s = [[B_1 \; ; \; B_2 \; post; (E_2 \; ; \; E_1)]]s$$

Proof. By simple application of the concatenation operation on sequences. □

This simply states that a sequence of goals can be locally evoked either as a group or individually without affecting the overall behaviour. In the accompanying paper, this result will be set in the context of knowledge acquisition mechanisms concurrently running with the evaluation process. More specifically, this restricted commutativity will be cast in the context of the chunking processes described by Newell in the Soar architecture [9]. From an implementation perspective, proposition 2.3.18 may be used for optimisation of the execution by preprocessing goal expressions, gathering groups of evoked goals rather than processing them individually. This would seem particularly useful when building a lazy interpreter for the NXP architecture.

Remark 2.3.19. Goal evocation in the NXP architecture is reminiscent of *exceptions* in functional of imperative programming languages. In the latter the current continuation is replaced immediately with another one. The behaviour is then to abandon the current process and switch to another one. In the former the continuation is just *added* after the current one; the resulting behaviour being to successively go through the processes to their completion. The analogy will be better illustrated in the categorical investigation of the NXP architecture in the next section.

Goal evocation as exemplified by the NXP architecture is an instance of associative cognitive mechanisms. Theses associative processes are at work in the background of the evaluation process itself interacting with it by driving the *focus of attention* of the inference system. While the backward and forward chaining of the goal evaluation process could be described as *strong* or *local* focus of attention, goal evocation represents a form of *global*, *weaker* forward chaining. More generally if *learning* processes have traditionally been studied separately from the performance component of inference systems, in recent resaerch work emphasis has been put on accounting for interactions between performance and learning [9]. In this perspective understanding and representing of inference control are critical as control is the operational juncture between the two subsystems. Furthermore the representation semantics is of direct service in specifying the implementation of inference systems.

3. A categorical investigation of inference control

3.1. **Motivations.** Category theory has in the past few years become a tool of choice for to investigate properties of programming languages. Because it focuses on very high-level properties abstracted from a number of subdomains of mathematics (domain theory, topology, set theory, etc.) its results are characterized by a wide range of applicability.

Composition if of the essence of categories. It is only natural that sequencing and its variations, which are the basic building blocks of inference systems control constructs—as seen in the previous section—, find a well-fit mathematical expression in categorical terms. In this section, we suggest such a categorical construction for analyses of control in inference systems. Starting from the original insight of Moggi, we use triples, a categorical construction with its origins in algebra, to define effects and controls in the NXP architecture. Following Wadler's lines of exploration we produce a category to account for goal evaluation and goal evocation in the NXP architecture. Finally, other results in *monadic style* representation of computations offer interesting analogies for the representation of behaviours of inference systems, particularly in *layering* effects around the core categorical representation of the architecture. The categorical study of knowledge acquisition in inference systems—in its multi-faceted forms—constitutes the major part of the companion paper covering the NXP architecture.

3.2. **Categorical background.** This section defines the basic notions from category theory that we need in the formalisation of the NXP architecture in monadic style. Readers are referred to [2] for a comprehensive presentation of categories and triples. Let C be a category, we denote by $Obj(C)$ the objects of C and by $Hom(A, B)$ the set of arrows with source object A and target object B.

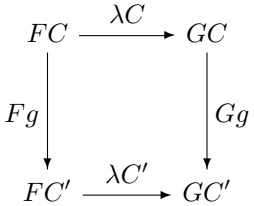
Definition 3.2.1. If $\mathbf{C}$ and $\mathbf{D}$ are categories, a functor $F : \mathbf{C} \to \mathbf{D}$ is a map for which:

- If $f : A \to B$ is an arrow of $\mathbf{C}$, then $Ff : FA \to FB$ is an arrow of $\mathbf{D}$;
- $F(id_A) = id_{FA}$; and
- If $g : A \to B$, then $F(g \circ f) = Fg \circ Ff$.

A functor is a morphism of categories, a map which takes objects to objects, arrows to arrows, and preserves source, target, identities and composition. More generally F *preserves* a property P that an arrow f may have if $F(f)$ has property P whenever f has. It *reflects* property P if f has the property whenever $F(f)$ has.

A natural transformation is defined as a "deformation" of one functor to another.

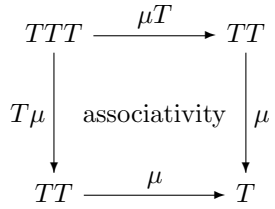
Definition 3.2.2. If $F : \mathbf{C} \to \mathbf{D}$ and $G : \mathbf{C} \to \mathbf{D}$ are two functors, $\lambda : F \to G$ is a natural transformation from F to G if λ is a collection of arrows $\lambda C \to \lambda D$, one for each object C of $\mathbf{C}$, such that for each arrow $g : C \to C'$ of $\mathbf{C}$ the following diagram commutes:



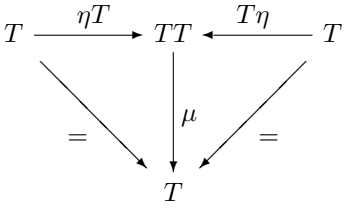
The arrows λC are the components of λ. The natural transformation λ is a *natural equivalence* if each component of λ is an isomorphism in $\mathbf{D}$.

3.3. **Triples, monads and categories for computations.** Triple or monads are, from one point of view, abstraction of certain properties of algebraic structures, namely monoids. They are categorical constructs that originally arose in homotopy theory and were used in algebraic theory. Moggi [8] was the first to discover the connection between triples and semantics of effects in programming language design. Since then the monadic style has pervaded theoretical research on denotational and operational semantics.

Definition 3.3.1. A triple $\mathbf{T} = (T, \eta, \mu)$ on a category $\mathbf{C}$ is an endofunctor $T : \mathbf{C} \to \mathbf{C}$ together with two natural transformations $\eta : id_{\mathbf{C}} \to T$, $\mu : TT \to T$ subject to the following commutative diagrams:



expressing associative identity, and:



expressing left and right unitary identities. The component of μT at an object X is the component of μ at TX, while the component of $T\mu$ at X is $T(\mu X)$; similar descriptions apply to η.

Remark 3.3.2. There is an alternate way of defining a triple based on a result due to Manes.

Let $\mathbf{C}$ be a category with:

- A function $T : Obj(C) \to Obj(C)$;
- for each pair of objects C and D, a function $Hom(C, TD) \to Hom(TC, TD)$, denoted $f \to f^*$;
- for each object C of $\mathbf{C}$ a morphism $\eta C : C \to TC$;

subject to the following conditions:

- For $f : C \to TD$, $f = \eta TD \circ f^*$;
- for any object C, $(\eta C)^* = id_{TC}$;
- for $f : C \to TD$ and $g : D \to TE$, $(g^* \circ f)^* = g^* \circ f^*$;

is equivalent to a triple on $\mathbf{C}$.

The equivalence results from constructions of triples from adjoint pairs separately discovered by Eilenberg-More and by Kleisli. The function $(\_)^* : Hom(C, TD) \to$

$Hom(TC, TD)$ is also known as the Kleisli star. This alternate definition emphasises the connection between a triple and a monoid, an algebraic structure with an associative operation and a unit element. Wadler suggested a straightforward interpretation of the Kleisli star in programming language semantics. In this context, the purpose of the star operation is to combine two computations, where the second computation may depend on a value yielded by the first [17]. More precisely if m is a computation of type $T\tau_1$ and k a function from values to computations (such as a continuation) $\tau_1 \to T\tau_2$, then $k^*(m)$, or $m * k$, is of type $T\tau_2$ and represents the computation that performs computation m, applies k to the value yielded by the computation, and then performs the computations that results. It binds the result of computation m in computation k. Different definitions for the triple T and the star operation then give rise to different monads to represent different control operators such as `escape/exit`, `call/cc`, `prompt/control` or `shift/reset` [16, 12].

As noted by Wadler and others, monadic and continuation-passing styles appear closely related [17]. The actual correspondence, however, is formally quite involved. Filinski has shown the remarkable result that *any* monadic effect whose definition is itself expressible in a functional language can be synthetised from just two constructs: first-class continuations and a storage cell [5, 6]. This has direct consequences on the feasibility of various implementations of the NXP architecture as investigated in the last section of this paper.

3.4. **Monadic style semantics for capturing side-effects.** In most programming languages, evaluation may have implicit side-effects that are not predicted by the type of the expression. This is typically the case with goal evocation in the NXP architecture where collecting data may trigger goals for further, delayed evaluation. From their first introduction in the world of programming languages triples, or monads, were precisely used to distinguish between *values*, and *computations* whose evaluation may have side-effects. The semantic separation leads to a stratified style where a pure functional language, for instance, is used to to express the manipulation of values, and one or several monadic sublanguages are used to express manipulation of computations [17]. Layering several monadic sublanguages is formalised using *triple morphisms* as in Filinski's [6]. Interactions between the core functional language and the monadic extensions are mediated by the type system which keeps track of the computational effects and their propagation.

Each side-effect introduces a new type of computation associated to a particular triple T. For instance, triples have been defined for effects such as exceptions, state, and input/output. Generally speaking, in monadic style semantics the type Ta designates a computation yielding a value of type a with possible side-effects. The semantics of these side-effects is captured by proper definition of the natural transformations of triple T (its unit and Kleisli star). Formally and following Manes' construction, the unit of the triple turns a value into a computation that returns that value without side-effect:

$$\eta : a \to Ta.$$

The Kleisli star applies a function of type $a \to Tb$ to a computation of type Ta, basically chaining two computations in succession. Following Wadler's popular notation—of using $*$ as an infix operator—the star operation:

$$(*) : Ta \to (a \to Tb) \to Tb$$

represents application of a continuation to a computation of the given type. In the following paragraphs we will make the analogy even more explicit by using the notation:

$$m * \lambda x.k$$

where m and k are expressions and x is a variable. The above can be read as follows: perform the computation m, bind x to the resulting value and perform computation k. In so-called impure languages, the above notation is similar to:

$$\texttt{let x = m in k}.$$

Note that, however, that the latter does not properly distinguishes pure types (no side-effect) from computation types (with possible side-effects).

We are now ready to present the monadic style definition of the previous standard and non-standard evaluators for the NXP architecture.

3.5. **A monadic style presentation of the NXP architecture.**

3.5.1. *The standard evaluation triple.* The new presentation is simply a rework of the semantic evaluation functions of definition 2.2.3 in terms of the appropriate triple, unit, and Kleisli star.

Definition 3.5.1. The standard monadic style semantic of the simple and/or expression language is captured by the map *eval*:

$$\begin{aligned} eval : Exp &\to T\mathbf{B} \\ eval(b) &= \eta b \\ eval(E_1 \ or \ E_2) &= eval(E_1) * \lambda x.eval(E_2) * \lambda y.\eta(x \mid y) \\ eval(E_1 \ and \ E_2) &= eval(E_1) * \lambda x.eval(E_2) * \lambda y.\eta(x \ \& \ y) \end{aligned}$$

Remark 3.5.2. In the definition above lambda abstraction binds less tightly and the application star binds more tightly, so that we can get rid of parentheses.

Remark 3.5.3. The above evaluation is much more flexible in that is separates values from computations. Values are evaluated through the unit semantic function η of the triple, while the sequencing of computations is specified by the application star. The variations in denotational semantics we explored in the successive definitions of the previous section are investigated here by simply changing definitions of unit and application.

3.5.2. *The non-standard evaluation triple.* In the sequence triple, a computation accepts an initial sequence and returns a value paired with the final sequence.

Definition 3.5.4. The sequence triple $T : \tau \to \tau \times \mathbf{B}^*$ with unit η defined by

$$\eta(b) = \lambda x.(b, \langle b \rangle + x)$$

where sequence operations are as in definition 2.3.2; and with application star

$$(*) : T\tau_1 \to (\tau_1 \to T\tau_2) \to T\tau_2$$

such that:

$$m * k = \lambda x.\texttt{let}\ (a, S_1) \ = \ m\ x\ \texttt{in let}\ (b, S_2) \ = \ k\ a\ x\ \texttt{in}\ (b, S_2)$$

and the evaluator as in definition 3.5.1 captures the non-standard sequence semantics of the NXP architecture.

Remark 3.5.5. The sequence triple is similar to the *state triple* introduced by Moggi and by Wadler to represent programming languages with instructions operating on a global state. The seemingly awkward definition of the application star simply expresses the intermediary sequences and results of first computing m and then applying k to the result.

Remark 3.5.6. For evaluation purposes, the *eval* semantic function of the generic interpreter of definition 3.5.1 has $T\ \mathbf{B}$ as its domain, meaning that computations with the sequence triple are of type $\mathbf{B} \times \mathbf{B}^*$ as expected.

In order to introduce goal evocation as a last touch to the sequence triple, we note that goal evocation is *only* a side-effect and does not interfere with values. This critical characteristic which was somewhat implicit in the previous continuation passing style presentation can now be made explicit in the monadic style. We formalise this separation by defining an additional semantic function in the sequence triple T: *post* of type $T\ ()$, i.e. $() \to () \times \mathbf{B}^*$. The $()$ type signals that the *post* function is only concerned with effects on sequences and basically ignores pure values.

Definition 3.5.7. The evocation function for the sequence triple is defined by:

$$post : T\ ()$$

and

$$post\ E = \lambda s.((), s + \mathtt{inr}(E))$$

where `inr` is the right injection $\mathbf{B} \times \mathbf{B}^* \to \mathbf{B}^*$.

With the evocation function thus defined, we extend the definition of the standard interpreter to accommodate the goal evocation expression in the following final definition of the sequence triple.

Definition 3.5.8. The monadic style sequence semantic of the simple and/or expression language is captured by the map *eval* in the sequence triple T:

$$\begin{aligned} eval &: Exp \to T\mathbf{B} \\ eval(b) &= \eta b \\ eval(E_1\ or\ E_2) &= eval(E_1) * \lambda x.eval(E_2) * \lambda y.\eta(x \mid y) \\ eval(E_1\ and\ E_2) &= eval(E_1) * \lambda x.eval(E_2) * \lambda y.\eta(x\ \&\ y) \\ eval(b\ post\ E) &= postE * \eta b \end{aligned}$$

In that definition we simply replaced ηb with $post\ E * \eta b$ in the computation. Properties of natural transformations assure that the call to *post* indeed percolates through higher-level goal evaluations. In the definition 3.5.8, as in the previous section definitions we insist on a left-to-right evaluation ordering of sequences.

Remark 3.5.9. Note that the definition of *post* could be viewed as a family of semantic functions indexed by goals, E in the representation. A better notation in this case would be $post_E$ to capture goal indices. Although equivalent in the monadic style presentation suggested above for the NXP architecture, this alternative notation will help formalise the knowledge acquisition part of the NXP inference architecture.

4. Operational semantics and implementation issues

4.1. **Layering effects in the NXP architecture.** Filinski's and Wadler's work [6, 17] have emphasised the compositional nature of triples in representing computations. Filinski's results in particular are geared towards presenting a framework for computational effects which makes it possible to describe effects in a *modular* way—an idea which permeates the design of the Haskell functional programming language, to mention only one. Drawing on the latter framework, we are able to add *inference effects*, so critical to the NXP architecture and to inference systems at large, *incrementally*. Following this approach, the NXP architecture is actually specified by a sequence of definitional translations, each one of which "translates away" one level of inference effects. For example, we can refine the description of the NXP architecture with goal evocation and dynamic working memory by specifying it as a composition of a goal evocation and a working memory translation. Informally we will also talk of *composing* a goal evocation and a working memory triple.

The monadic composition relies on the following general idea: assume we have two triples T and U over a base language L, where U is in a certain sense "more general" than T. There is a standard translation of effects, L^M into the base language given their monadic representation. Using this monadic translation we can give two different translations from L^T to L: the original monadic translation for T and a variant translation using U-representations of T-effects, inducing the same evaluation semantics. The reader is referred to [6] for a formal proof of these results—interestingly enough the crux of the demonstration relies on the result that continuations are in a precise sense a *universal* effect: any definable triple can be simulated by a continuation triple on a language with only first-class continuations (a la Scheme) and typed states.

We present here other control constructs of the NXP architecture and propose candidates for their formal monadic representation. The working memory, a major element of the NXP architecture, is detailed by itself in the next section.

4.1.1. *Associative links, context.* In the NXP architecture, the data-triggered goal evocation mechanism is complemented by second goal-triggered goal evocation process based on associative links between goals and goals or subgoals. Associative links are defined by a binary relation: the *contextual* relation.

Definition 4.1.1. The simple base language for inference systems expression is extended with the `context` expression. Let us define Exp, the set of expressions:

$$E ::= b \mid E \textit{ or } E \mid E \textit{ and } E \mid E \; ; \; E \mid b \textit{ post } E \mid E \textit{ context } E$$

with b in $\mathbf{B}$ and where the new expression E *context* E evaluates the first expression and evokes the second (which, as an expression, is itself an arbitrarily long sequence of expressions). The other expressions are as in definition 2.3.13.

Remark 4.1.2. The goal-triggered goal evocation has lower priority than the data-triggered one. In the concrete expression E_1 `context` E_2, any goal evoked by the evaluation of E_1 through `post` subexpressions will be evaluated before actual evaluation of E_2. Priorities aside `context` and `post` have the same semantics.

We reuse the sequence triple to capture the semantics of the context links in the NXP architecture.

Definition 4.1.3. Associative links semantics in the NXP architecture extend the *eval* semantic map in the sequence triple as follows:

$$eval(E_1\ context\ E_2) = postE_2 * eval(E_1)$$

Remark 4.1.4. Note that the *post* primitive is now used twice: in the triple sequence used to capture goal evocation, and in the second triple sequence used to capture context links.

In some respect, associative links are higher level control constructs in inference systems. Compared to pattern-directed modules of the canonical architecture, they do not rely on actual patterns in the working memory elements but rather on the behaviour of the inference process itself, posting goals for later evaluation when the inference, seen as computation, reaches some particular goal or subgoal. Alternatively the composition of identical sequence triples could also be viewed as the more general triple defined by $T : \tau \to \tau \times \mathbf{B}^* \times \mathbf{B}^*$ where the second sequence is in fact used for contextually linked goals.

4.1.2. *The* `reset` *action.* Independently of the various actions operating on the working memory, calling for a special treatment as explained in the next section, the NXP production rules support a `reset` action on a variable with the immediate effect of setting its value to "unknown" regardless of its previous assignment.

From the behavioural standpoint, the representation of the computation of such an expression is simply to *unevaluate* it. When `reset` operates on a data, the semantics is simply to reevaluate it should the computation need it at some later time. When `reset` operates on a goal (an expression in the simple NXP language) however, the situation is quite different: indeed the reset goal, considered as data in a later computation, should be reevaluated but what of its antecedents? And what if further computations do not explicitly need the reset goal?

In the NXP architecture the effect of the `reset` action propagates back from the goal to its antecedents but does not affect posted goals from evocation or contextual links. This propagation only interferes with the computation process by forcing a reevaluation, i.e. a new expansion of the reset expression. Its semantics is handled by the working memory executive which mediates all accesses to values.

4.2. **Working memory and user interaction.** In the semantics definition of the previous sections we oversimplified an essential element of an inference system, namely the *working memory*. The working memory is holding the data structures on which the pattern-directed modules operate. In the traditional view of inference systems, data is often said to *enter* or *leave* working memory as it is added or deleted by the action part of those modules. In the cognitive sciences framework, the working memory is the representation in inference systems of the *short-term memory*. Inflow and outflow of data in working memory capture the essentially local characteristic of short term memory which can only hold a limited number of items ("the magic number seven") and only for a short period of time.

In most inference systems, and particularly in traditional production systems where pattern-directed modules are production rules, the working memory is usually structured as a database of records. Working memory elements are instances of pre-defined or user-declared data structures with typed attributes. The pattern parts of the pattern-directed modules simply express boolean conditions over

these attributes or, in some cases such as in the OPS series of production system languages [7], the presence or absence of particular records.

As such the basic structure is similar to the concept of an *environment* in denotational semantics. An environment tells what the identifiers mean in an expression: it says what values the identifiers denote. In order to capture the level of indirections, it it usual to introduce Ide a set of identifiers (variable names) and the domain U of environments as $U : Ide \rightarrow \mathbf{B}$. With this the simple NXP language complies with the following definition.

Definition 4.2.1. Exp is now the set of expressions :

$$E ::= x \mid E \textit{ or } E \mid E \textit{ and } E \mid E \; ; \; E \mid x \textit{ post } E \mid E \textit{ context } E$$

with x in Ide.

In the NXP architecture, the working memory additionally plays a much more dynamic role. In a major departure from the *closed world* assumption of former inference systems, the working memory of the NXP architecture is also the locus of interaction with the external environment, whether data is captured through interactions with users or from other applications and systems. Indeed the historical implementation, *Nexpert Object*, is famous for its pioneering use of API (Application Programming Interface) and middleware to integrate inference capabilities to entreprise applications and information systems in heterogeneous environments. In order to fulfil this role, the NXP working memory has its own low-level simplified executive module whose responsibility is to plan for the acquisition of the value of a particular data required by the inference process. In most cases, the plan is a simple look-up of the value for the requested data in memory. If this fails, either because the value is unknown yet or because a previous inference operation has reset that particular data item, the value is sought outside the NXP system either by asking the user (this in the form of a popup *question* window) or by sending the request to the middleware layer to distant databases and applications (*Nexpert Object* has a wide variety of foreign request mechanisms ranging from file loading and SQL queries up to COM or Corba-based dynamic queries to application servers). Beyond the marshalling of the returned value, its type conversion, the working memory executive is also able to synchronise the request for value with the higher-level chaining and goal-driven processes. It may pre-process group of requests and use caching in order to optimise usage of costly data access channels, for instance.

A second dimension of dynamic behaviour that sets the NXP working memory apart from former production systems architecture is the evolutive nature of the working memory structure. This aspect is fully developed in the companion paper, but let us simply state here that the NXP architecture allows for pre-processing of the working memory elements based on previous runs of the inference systems. This pre-processing helps determine the initial *focus of attention* of the system which, in turn, assigns priorities in the goal-driven process. This experience-based *skill acquisition* process differs from historically studied machine learning processes concerned with alteration of the collection of pattern-directed modules, or production memory, e.g. chunking [9].

Operations of the working memory mini-executive are transparent to the semantic definitions of previous sections. An operational semantics that would be closer to actual implementation would replace the simple set of boolean constants, $\mathbf{B}$, with

a family of semantic functions get_B, indexed on a set of variables, representing working memory elements and capturing the simple semantics of request-response interactions with the external environment. Those functions could be further refined either in an asynchronous framework, accounting for message-based communication channels for instance, or in a lazy evaluation framework in which the mini-executive is implemented as a lazy interpreter.

The monadic style representation of the semantics of NXP architecture's working memory follows the layered approach discussed in the previous subsection. By analogy with Haskell's I/O monads [4], we introduce the following definition.

Definition 4.2.2. The working memory monad W describes a computation as a series of channels paired with the value returned.

$$\begin{aligned}
\textbf{type}\ W\ b =& \langle C_1, \cdots, C_n, b\rangle \\
\eta :& b \to Wb \\
\eta x =& \langle \star_1, \cdots, \star_n, x\rangle \\
(*) :& Wb_1 \to (b_1 \to WM\ b_2) \to W\ b_2 \\
m * k =& \texttt{let}\ \langle csx_1, \cdots, csx_n, bx\rangle = m\ \texttt{in} \\
& \texttt{let}\ \langle csy_1, \cdots, csy_n, by\rangle = k\ bx\ \texttt{in} \\
& \langle csx_1; csy_1, \cdots, csx_n; csy_n, by\rangle
\end{aligned}$$

where channels, C_i, represent distinct request-response communication channels; csx_i denotes state x of channel C_i; $\star_i$ denotes the undefined (in the strict sense) state of channel C_i ; bx and by denote boolean values from $\mathbf{B}$, and $csx_i; csy_i$ denotes the state of channel C_i after stepping through state csx_i then csy_i.

Remark 4.2.3. The previous definition does not specify the operational semantics of communication channels. In an operational semantics, channels would use the state monad which simply captures that requests and responses flowing through the channel change its state. The definition also leaves the number of channels unspecified. In the NXP architecture, at least one of these channels is identified as the user interaction channel through which question-answer based interactive transactions can be set up.

Remark 4.2.4. This definition also leaves the choice of channel state semantics open. In particular, different semantics are called for synchronous and asynchronous communication patterns. Similarly, definition 4.2.2 excludes interaction between communication channels oversimplifying some of the capabilities of the mini-executive in the *Nexpert Object* reference implementation of the NXP architecture.

The advantage of the monadic style is in this instance twofold. Firstly, in the layered approach outlined in the previous paragraphs, the composition of monads adequately reflects the juncture of the inference processes, chainings and goal-driven, with interactive processes. Here this articulation is naturally represented by the composition of the NXP monad from definition 3.5.8 with the working memory monad from definition 4.2.2. This architectural pattern will be reused in the companion paper to present the composition of inference with associative and dynamic memory processes.

Secondly, it brings forward the nature of the working memory mini-executive itself and suggests various operational implementations. In particular, lazy interpretation could be useful in the case of continuous data streams channels, for instance, or for combined management of interacting communication channels—both situations quite frequent in real-time enterprise application integration.

4.3. **Operational semantics: an NXP machine.** As a last step towards an operational semantic for the NXP architecture, we will think of the computation as described by the operation of a state-based machine:

$$\texttt{until}\ term(\sigma)\ \texttt{do}\ \sigma = step(\sigma)$$

At each stage of the machine's operation the state, σ, is modified to a new state as specified by the single step state transformation function *step*; this is repeated until a terminal state, recognised by the predicate *term*, is reached. The operation is captured by a formal definition.

Definition 4.3.1. With S a set of states, $step : S \to S$ a function from states to states, and $term : S \to \mathbf{B}$ a function from states to truth values, we define the general `machine` function :

$$\texttt{machine}(step, term) = fix(\lambda f \lambda \sigma.\texttt{if}\ term(\sigma)\ \texttt{then}\ \sigma\ \texttt{else}\ f \circ step(\sigma))$$

where the expression `if ... then ... else` has the usual meaning and where fix is the fixed point operator (here acting on f).

Remark 4.3.2. With this very general definition a machine is simply identified by the couple of functions *step* and *term*.

In order to exhibit an example of an NXP machine, we will transform the sequences of the previously described non standard semantics into operational sequences. Here the denotational semantics naturally suggests an operational semantics: instead of queuing up denoted values in sequences, we now queue *instructions* for the NXP machine sequences. Each expression of the simple language is *compiled* into a sequence of mixed instructions and arguments, constituting a program then passed to an abstract machine (with proper *term* and *step* definitions).

Definition 4.3.3. We introduce the syntactic domains: $\mathbf{B}$ of boolean values, Ins of instructions, $Prg = Ins^*$ of finite sequences of instructions, or programs.

The syntax for instructions is:

$$I ::= \texttt{get}\ x\ |\texttt{and}\ |\texttt{or}\ |\texttt{reset}\ x$$

The semantics of this machine code will basically arrange that, as might be expected, `get` accesses the working memory to retrieve the value of a particular identifier—possibly triggering side-effects in the working memory executive—and place it on top of the sequence; `or` will form the logical or of the top two elements on the sequence; similarly for `and`; and `reset` which causes the working memory executive to reset values used in the evaluation of a variable–possibly triggering side-effects.

Remark 4.3.4. With this definition in place we have shifted emphasis from a semantic function mapping expressions to denotations to a compiling function mapping expressions to syntactic values, namely sequences of instructions.

The compiling function transforms expression into programs fit for the NXP machine.

Definition 4.3.5. Following the lines of the non standard semantics, we introduce the compiling function $C : Exp \times Prg \times Prg \to Prg \times Prg$.

$$\begin{aligned}
C(x, s, p) &= s + \langle \texttt{get}\ x \rangle, p \\
C(E_1\ or\ E_2, s, p) &= \texttt{let}\ s', p' = C(E_2, C(E_1, s, p))\ \texttt{in}\ s' + \langle \texttt{or} \rangle, p' \\
C(E_1\ and\ E_2, s, p) &= \texttt{let}\ s', p' = C(E_2, C(E_1, s, p))\ \texttt{in}\ s' + \langle \texttt{and} \rangle, p' \\
C(E_1\ ;\ E_2, s, p) &= C(E_2, C(E_1, s, p)) \\
C(x\ post\ E, s, p) &= \texttt{let}\ s', p' = C(E, \langle\rangle, \langle\rangle)\ \texttt{in}\ s' + \langle \texttt{get}\ x \rangle, p' + p \\
C(E_1\ context\ E_2, s, p) &= \texttt{let}\ s', p' = C(E_2, \langle\rangle, \langle\rangle)\ \texttt{in}\ C(E_1, s, p), p' + s' + p
\end{aligned}$$

Remark 4.3.6. In this definition we need two operational sequences s and p. The first one represents direct continuations as processed by evaluation and evocation of goals. The second one is used to process contextual links and goal associations.

Remark 4.3.7. As remarked in the previous section, the compilation process could be captured by a specific triple then blurring the formal distinction between compilation and semantic evaluatiation of NXP simple language expressions.

We now complete the definition of an operational semantics for the NXP architecture by displaying a *term* and a *step* function for the compiled code definitions 4.3.5 and 4.3.3.

Definition 4.3.8. Let $S = Prg \times \mathbf{N} \times \mathbf{B}^*$ be a set of states, and $\sigma = (\Pi, n, s)$ be a typical member of S. Simply enough, we specify the effect of any single instruction on the stack by defining a function $I : Ins \to \mathbf{B}^* \to \mathbf{B}^*$ as follows:

$$\begin{aligned}
I(\texttt{get}\ x, s) &= \langle get(x) \rangle + s \\
I(\texttt{reset}\ x, s) &= s \\
I(\texttt{or}\ , s) &= Or(s) \\
I(\texttt{and}\ , s) &= And(s)
\end{aligned}$$

Remark 4.3.9. In this definition *get* and *reset* are invocations of the working memory executive. In fact $reset : Ide \to \langle\rangle$ always returns the empty stack while $get : Ide \to \mathbf{B}$ returns the boolean value denoted by identifier x—both calls having possible side-effects. The semantics of these calls is represented by the working memory triple.

Finally we define an instance of the abstract machine.

Definition 4.3.10. The NXP virtual machine is defined by the set S of states, and the *term* and *step* functions:

$$step(\Pi, n, s) = (\Pi, n + 1, I(\Pi \downarrow i, s))$$

and

$$term(\Pi, n, s) = n \geq length(\Pi)$$

Remark 4.3.11. Thus when the machine executes a program Π with the starting stack s–usually the empty stack $\langle\rangle$–the final stack is obtained as specified by the semantic function M, defined by:

$$M : Prg \to \mathbf{B}^* \to \mathbf{B}^*$$
$$M(\Pi, s) = in_3(\texttt{machine}(step, term)(\Pi, 1, s))$$

where in_3 is the canonical projection of the third element of the triplet.

Remark 4.3.12. Note that the final program passed to the NXP machine is the concatenation of the two compiled operational sequences, accounting for the lower priority of the associative links. This is captured in the following congruence proposition.

Proposition 4.3.13. *The effect of running a compiled expression is the one given by the sequence semantics, more specifically:*

$$M(\Pi, s) = [[E]]_2 s$$

where

$$\Pi = \texttt{ let } s', p' = C(E, s, \langle\rangle) \texttt{ in } p' + s'$$

for all E, s.

Interestingly enough the compiler is not the only operational semantics suggested by the non standard semantics studied above. Building an interpreter for the NXP architecture is also possible along the same lines as the compiler, but instructions are then executed on the fly. More generally several implementations can still be chosen at this stage, somehow independently of the operational semantics: eager or lazy evaluation, for instance, is applicable to both operational semantics.

5. Conclusions and further research

Inference may be considered as a special form of computing. This basic tenet of the whole Artificial Intelligence theoretical field has been comprehensively captured by Newell's *knowledge level* hypothesis [10], and if the distinction between behaviour and beliefs is well established, the nature of behaviour as computation is still much debated.

Inspired by the analogy between the latter distinction and the one usually drawn between computations and values in the study and design of programming languages, we have presented an architectural view of control constructs in inference systems and, more specifically, of *Nexpert Object* as a canonical inference system. The resulting NXP architecture has grounded semantics expressed in categorical terms using the monadic style of presentation.

The benefits of this formal representation is twofold. On the one hand, the categorical foundation of this representation helps study of the NXP architecture in a logical framework, such as Twelf, and formally assess its architectural properties. On the other hand, the denotational (and derived non standard) semantics strongly suggest operational semantics for the implementation of the NXP architecture. Formally provable congruence relation henceforth ascertain the soundness of the resulting implementation.

Finally the categorical foundation of the NXP architecture extends to the formal representation of other cognitive processes beyond inference. In particular the

monadic style can be used to articulate the working memory process to inference in a modular way. It can also be used to express relationship between inference and learning or adaptative processes reflecting the long term effect of experience on inference.

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