

# A Non-Price-Discrimination Theory of Rebates\*

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## Abstract

According to the conventional theory, rebates are but one of many ways to price discriminate among consumers. The firm sells at the shelf price to those willing to pay more for the product and at a lower, rebated price, to those willing to pay less, achieving the necessary separation by relying on the higher costs of redeeming rebates of high willingness-to-pay consumers.

This theory, however, does not explain several phenomena associated with rebates. Many consumers respond to rebates at the time of purchase, but far fewer redeem them after purchase. Without resorting to irrationality on the part of some consumers, this redemption gap cannot be explained by price discrimination theory. We also see free-after-rebate deals. This practice is also puzzling from the perspective of price discrimination, because a firm will surely lose money from rebate users. Finally, the usage of rebates tends to vary across industries and product categories even when the same consumers participate in each of them.

We propose a non-price-discrimination theory of rebates based on the idea of “utility arbitrage.” Key to our theory is a distinctive property of rebates: unlike other promotion vehicles, they are redeemed after purchase and during consumption. When a product is not consumed instantaneously, the utility that a consumer derives from it will depend on future states of nature. A firm can increase a consumer’s willingness to pay for the product by simultaneously increasing the price she pays in a state of low marginal utility of income and decreasing the price in a state of high marginal utility of income. Rebates help implement utility arbitrage by delivering state-dependent discounts.

Thus, the paper identifies a unique role for rebates in the marketing tool kit. Notwithstanding this role, rebates can still price discriminate among consumers with different willingness to pay. Indeed, their ability to utility arbitrage may enhance their price discrimination ability. This suggests that, abstracting from redemption cost differences, rebates are likely to be superior to coupons.

# 1 Introduction

According to the conventional theory, rebates are a marketing tactic that firms use to price discriminate between high reservation price consumers and low reservation price consumers. The firm sells at the shelf price to those willing to pay more for the product, and at a lower, rebated price to those willing to pay less, achieving the necessary separation by relying on the higher costs of redeeming rebates faced by high reservation price consumers. As a price discrimination device, it would seem, rebates are not fundamentally different from coupons (Narasimhan, 1984; Houston and Howe, 1985; Gerstner and Hess, 1991a; Moraga-Gonzalez and Petrakis, 1999).<sup>1</sup> Indeed, Blattberg and Neslin (1990, p. 13) have described rebates as the “durable goods analog” of coupons.

On closer examination, there are several differences between rebates and coupons. The most obvious one is the one pointed out by Blattberg and Neslin (1990), namely that rebates are used primarily for durable goods whereas coupons are used primarily for frequently purchased packaged goods. Perhaps as a result, rebates involve generally higher discounts than coupons, but are less popular. In a recent survey by *Brand Marketing* magazine, 68% of respondents reported using coupons, while only 50% reported using rebates (<http://www.santella.com/marketing.htm>).

More fundamentally, coupons are redeemed along with the purchase, whereas rebates are redeemed after the purchase. This is a key difference; it leads to some phenomena that are unique to rebates. One such phenomenon is “slippage” (Bulkeley 1998). Many consumers respond to the rebate at the time of purchase, but far fewer redeem them (Dhar and Hoch 1996).<sup>2</sup> Indeed, it is an article of faith in the rebate fulfillment industry that slippage is an important part of the appeal of rebates (Edmonston 2001, Singleton 2001), particularly among companies that specialize in 100% rebates. A number of firms on the Internet offer free-after-rebate deals, and for some—e.g., CyberRebate.com and Free-After-Rebate.net—

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<sup>1</sup>Gerstner and Hess (1991a) discuss an alternative, channel role for rebates. They suggest that rebates can help coordinate a channel by reducing the variation in consumers’ willingness-to-pay. But in their model, rebates and coupons are indistinguishable. See also Gerstner and Hess (1991b).

<sup>2</sup>The redemption rate on rebates is of the order of 10 to 30% on rebates of \$10 to \$30 off \$100 products (<http://www.techbargains.com/rebates.cfm>). According to TCA Fulfillment, even a \$50 rebate on a \$50 item—i.e., free after rebate—gets only a 50% redemption (<http://www.frontlinenow.com/newsletter/redemptionrates.html>).

this type of promotion is their entire business strategy.<sup>3</sup> Needless to say, neither slippage nor 100% discounting are seen with coupons.

It is difficult to explain slippage under price discrimination theory. Slippage can happen if rebate users are “forgetful”: they respond to rebates at the point of purchase, fully intending to redeem them, but then they forget to do so. However, it is difficult to reconcile such behavior with rationality, nor see how firms can benefit from it. Rational rebate users should anticipate their forgetfulness and discount the face value of the rebate based on the probability of their forgetfulness. They must make their purchase decisions based on the price they expect to pay, not the rebated price. The firm must, in turn, increase the face value of its rebates to motivate the marginal consumers to purchase, in effect, compensating them for their forgetfulness. In equilibrium, the amount of compensation would match the amount of forgetfulness, and the firm wouldn’t profit from slippage. Indeed, if rebate users are risk-averse—averse to the risk of forgetfulness—the firm will want to avoid any slippage whatsoever, as it has to raise the face value of its rebates even higher to induce purchases. Slippage is a form of “leakage”—a segment of consumers behaves like another segment instead of behaving distinctively—and leakage is undesirable in price discrimination.

The price discrimination story also cannot explain why firms use free-after-rebate promotions. Such rebates must incur a loss among rebate redeemers, but the theory says that price discriminating firms should profit from anyone they choose to serve.<sup>4</sup>

This paper offers a non-price-discrimination theory of rebates. It is based on the fundamental timing difference between coupons and rebates alluded to earlier. Coupons are redeemed with the purchase; rebates are redeemed after the purchase. We believe that this

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<sup>3</sup>Free-after-rebate promotions are not limited to dotcom companies. A recent Google.com search by the keyword “free after rebate” generated 243,000 results, many of which involve bricks-and-mortar companies selling books, software, electronics, apparel, jewelry.

<sup>4</sup>Conceivably, a firm may use free-after-rebate promotions as a loss-leader—designed to attract customers who will then buy more profitable products—but this story is not entirely convincing either. First, rebates are not an effective and cost-efficient way to generate a loss leader. A simple price markdown would have more impact on traffic and would be less costly. Second, on the Internet, where most of these promotions are found, the one-stop-shopping effect is more muted anyway. It is not clear to what extent a loss-leader can induce additional buying at the same Web site. Finally, the number of products that are free after rebate is sometimes so large that it is difficult to rationalize them all as loss-leaders. As noted earlier, companies like CyberRebate.com put nearly all of their products on 100% rebate.

difference in timing is of critical importance in accounting for the way rebates are used in practice and in establishing a distinct role for rebates in the marketing tool kit. It is the key to explaining phenomena such as slippage and 100% rebates.

Our theory is based on the idea of “utility arbitrage.” When a product is not instantly consumed—as most durable goods are not—its utility to a consumer is uncertain at the time of purchase. The realized utility may be high or low depending on a host of factors: how the product performs in the future, the consumer’s consumption situation—income level, employment status—and random events like weather, state of the economy, etc (Hauser and Wernerfelt 1990). In other words, a consumer’s utility from a durable, experience good depends on future realizations of states of nature (Fishburn 1974). The opportunity for utility arbitrage arises when a consumer’s marginal utility of income changes with different realizations of nature. When a consumer’s marginal utility of income is high, a price discount will generate a large increase in her utility; when a consumer’s marginal utility of income is low, a price increase of the same magnitude will not reduce her utility as much. This means that a firm can raise its shelf price, and also the consumer’s expected utility, if it simultaneously offers a discount that is redeemable when the consumer’s marginal utility of income is high. The increase in expected utility will raise consumers’ willingness-to-pay, allowing the firm to extract more of their surplus.<sup>5</sup>

However, to conduct utility arbitrage, a firm needs a vehicle that can deliver state-dependent discounts. Rebates are such a vehicle. A consumer has a greater incentive to redeem a rebate when her marginal utility of income is high, than when it is low. By contrast, coupons, because they are redeemed before consumption, cannot deliver state-dependent discounts. Thus, we argue, barring a big disadvantage in redemption costs, rebates are likely to be a better promotion instrument than coupons.

In the next section we present a simple model of a single seller selling to a single consumer to illustrate how utility arbitrage works. The single consumer setting also serves to underscore the difference between our theory and price discrimination. In Section 3 we

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<sup>5</sup>In a recent paper, Xie and Shugan (2001) use uncertain consumption utility to motivate advance selling. Their model is quite different from ours in that no utility arbitrage is present in their model to increase a consumer’s *ex ante* willingness to pay for a product. Advance selling in their model takes advantage of the fact that consumers’ reservation prices become more homogeneous when consumers make their purchase decisions based on expected, rather than realized, utility.

compare a rebate policy with a no rebate policy, and present a set of necessary conditions for an optimal rebate policy to perform utility arbitrage. Section 4 discusses the intuition behind these conditions and their managerial implications. In Section 5, we investigate the robustness of this intuition by showing that the utility arbitrage effect also obtains in models with competition, continuum of states, and multiple market segments. We also compare rebates with coupons in this section. Section 6 concludes the paper.

## 2 The basic model

Consider a market consisting of a monopoly firm and a representative consumer. We assume that the consumer, endowed with an income  $I$ , will purchase at most one unit of the product at any given price  $p$ . The quality of the product is measured by  $q$ . The consumer can also purchase a composite good  $y$ , the price of which is normalized to one. Therefore, the consumer's budget constraint is given by  $p + y \leq I$ . This assumption of a single consumer rules out any motivation for the monopolist to price discriminate in this market.

The utility that the consumer derives from a consumption bundle  $(q, y)$  is state-dependent in that it hinges on future realizations of states of nature. For example, the utility from a pair of hiking boots that a consumer buys for the purpose of hiking in the Himalayas will depend in part on the weather she gets there; the shoes would be more enjoyable under sunny, blue skies than if the Himalayas were cold, damp, and windy. For another example, consider a consumer who has just bought a big-screen TV, only to discover that he has been laid off; the consumer's marginal utility of income will increase under these circumstances. In general, when a product is not consumed instantaneously, the utility that a consumer derives from it will depend on many random events subsequent to the purchase. Some of these are connected to the product itself—how well it performs—but others are related to the consumer—her economic well-being, employment status—and purely random events like the weather.

To capture the state-dependency of consumer utility, let the consumer's utility function be given by  $u_s(q, y)$ , where subscript  $s$  indicates the state of consumption. To develop our basic intuitions in the simplest possible way, we assume for now that there are only two consumption states  $r$  and  $t$ , occurring with probabilities  $\theta$  and  $1 - \theta$ , respectively. We

assume that the utility function increases, at a decreasing rate, with each argument, which is reasonable since marginal utility of quality and marginal utility of income are expected to decrease in quality and income, respectively. In symbols,  $\partial u_s/\partial q > 0$ ,  $\partial u_s/\partial y > 0$ ,  $\partial^2 u_s/\partial q^2 \leq 0$ , and  $\partial^2 u_s/\partial y^2 \leq 0$ . We shall designate state  $r$  as the state in which the consumer's marginal utility of income is high by assuming that  $\partial u_r(q, y)/\partial y > \partial u_t(q, y)/\partial y$  for all  $(q, y)$ . In the sequel we shall abbreviate  $\partial u_s/\partial y$  to  $u'_s(q, y)$ .

### Rational consumer

We assume that the consumer is rational in that she maximizes her expected utility when making purchasing decisions. When no rebate is available, the consumer will purchase the product if and only if

$$\theta u_r(q, I - p) + (1 - \theta)u_t(q, I - p) \geq \bar{u}, \quad (1)$$

where  $\bar{u}$  is the reservation utility that she gets from not consuming the product. Accordingly, the firm's problem of profit maximization, given that it does not offer any rebate and the marginal cost of production is zero, is given by

$$\begin{aligned} \max p & \\ \text{s.t. } & \theta u_r(q, I - p) + (1 - \theta)u_t(q, I - p) \geq \bar{u}, \end{aligned} \quad (2)$$

We denote the optimal solution to this problem as  $p_0$ . Obviously, it must satisfy

$$\theta u_r(q, I - p_0) + (1 - \theta)u_t(q, I - p_0) = \bar{u}. \quad (3)$$

When a rebate is available, the consumer may or may not choose to redeem it. Let  $c_s$  be the consumer's redemption cost for the rebate in state  $s$ . Rebate redemption costs cover a variety of things. They include the cost of remembering to use a rebate before it expires, the cost of filling out the rebate form and complying with the redemption requirements, mailing cost, the cost of waiting for the rebate check to arrive, the cost of depositing the check in the bank, etc. We allow the redemption cost to vary with states of nature to capture the possibility that the consumer's opportunity cost of time may change. For instance, the redemption cost would be higher when the consumer is busy than when she is unemployed. Since opportunity costs of time are correlated with income, it is natural to assume that

$c_t \geq c_r$ . For any given rebate value  $d$ , the consumer will redeem the rebate in state  $s$  if and only if

$$u_s(q, I - p + d) - c_s \geq u_s(q, I - p). \quad (4)$$

### 3 Optimal rebates

The firm will never issue a rebate that the consumer chooses not to redeem in any state. However, the firm can design its rebate such that the consumer is induced to redeem it in both states. In that case, the firm's optimization problem is

$$\begin{aligned} & \max_{(p, d)} p - d & (5) \\ \text{s.t. } & \theta u_r(q, I - p + d) + (1 - \theta)u_t(q, I - p + d) - \theta c_r - (1 - \theta)c_t \geq \bar{u} \\ & u_r(q, I - p + d) - c_r \geq u_r(q, I - p) \\ & u_t(q, I - p + d) - c_t \geq u_t(q, I - p) \end{aligned}$$

Comparing (2) and (5), we can see that the firm's profit is higher in (2). This is because whatever net price the rebate can effect, say  $\tilde{p} = p - d$ , the same can be achieved more directly by setting the firm's shelf price at  $\tilde{p}$ , thereby saving on the need to compensate the consumer for the expected redemption cost. Therefore, the strategy of inducing the consumer to use a rebate regardless of realized state is not an optimal strategy for the firm.

Consider, finally, a strategy in which the consumer redeems the rebate in only one state, state  $r$ , where her marginal utility of income is higher.<sup>6</sup> We call this the "targeted state." Then, the firm's optimization problem becomes

$$\max_{(p, d)} p - \theta d \quad (6)$$

$$\text{s.t. } \theta u_r(q, I - p + d) + (1 - \theta)u_t(q, I - p) - \theta c_r \geq \bar{u} \quad (7)$$

$$u_r(q, I - p + d) - c_r \geq u_r(q, I - p) \quad (8)$$

$$u_t(q, I - p + d) - c_t \leq u_t(q, I - p). \quad (9)$$

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<sup>6</sup>It is impossible for the consumer to redeem in state  $t$  and not in state  $r$  because  $u_t(q, I - p + d) - c_t \geq u_t(q, I - p)$  implies  $u_t(q, I - p + d) - u_t(q, I - p) \geq c_t \geq c_r$ , and  $u_r(q, I - p + d) - u_r(q, I - p) > u_t(q, I - p + d) - u_t(q, I - p)$ , from the definition of state  $r$  as the state with higher marginal utility of income.

Denote the optimal solution to this problem by  $p^* = p_1$  and  $d^* = p_1 - p_2$ . Then, the firm's payoff under rebating can be expressed as a probability-weighted sum of the full and discounted prices, *i.e.*  $p^* - \theta d^* = \theta p_2 + (1 - \theta)p_1$ . The firm will use a rebate if and only if rebating is more profitable than no rebating, *i.e.*,

$$\theta p_2 + (1 - \theta)p_1 \geq p_0. \quad (10)$$

We now analyze the optimization problem (6)-(9) in four steps. First, note that whenever rebating is more profitable, we must have  $p_2 < p_0 < p_1$ . This is because if both  $p_1$  and  $p_2$  were less than  $p_0$ , then the convex combination of  $p_1$  and  $p_2$  on the left of (10) would also be below  $p_0$ , violating (10). If both  $p_1$  and  $p_2$  were higher than  $p_0$ , then (7) would not be satisfied, given (3). Thus, for any  $d^* > 0$ , we must have  $p_2 < p_0 < p_1$ . In other words, when a firm uses rebates, its discounted price is lower, and its shelf price higher, than the price it would charge in the benchmark case of no rebating.

Second, (8) cannot be binding at the optimal rebate solution. This is because if (8) were binding, then offering rebates cannot be optimal. To see this, note that if  $u_r(q, I - p_1 + d^*) - c_r = u_r(q, I - p_1)$ , then  $\theta u_r(q, I - p_1) + (1 - \theta)u_t(q, I - p_1) = \theta u_r(q, I - p_1 + d^*) + (1 - \theta)u_t(q, I - p_1) - \theta c_r \geq \bar{u}$  (by (7)). That is, pricing at  $p_1$  in both states and not offering rebates is feasible, and, since  $p_1 > p_2$ , also more profitable. What this means is that the firm will always design its rebate program, when rebating is profitable, such that the consumer in the targeted state is happier using it than not using it.

Third, at the optimal solution, (7) must be binding. The argument is as follows. Suppose (7) is not binding. There are two cases: either (9) is not binding or it is. In the former case, the firm can increase its profit by raising  $p_1$ , as both (7) and (9) provide room. In the latter case, the firm can increase its profit by raising both  $p_1$  and  $p_2$ , while satisfying (9) as an equality, taking advantage of the room provided by (7). Therefore, at the optimal solution, either (7) and (9) are both binding or only (7) is binding.

Fourth, in the Appendix we show that when (9) is not binding, an optimal rebate program must equalize the consumer's marginal utility of income in the two states of consumption. That is,  $p_1$  and  $p_2$  must satisfy  $u'_r(q, I - p_2) = u'_t(q, I - p_1)$ .

The following proposition summarizes the necessary conditions for a rebate program to be optimally designed and to be more profitable than not offering rebates.

**Proposition 1** *An optimal rebate program  $(p_1, p_2)$ , more profitable than the best no rebate program  $(p_0)$ , must satisfy the following conditions:*

$$p_2 < p_0 < p_1, \quad (11)$$

$$\theta u_r(q, I - p_2) + (1 - \theta)u_t(q, I - p_1) - \theta c_r = \bar{u}, \quad (12)$$

$$u_r(q, I - p_2) - c_r > u_r(q, I - p_1), \quad (13)$$

$$u_t(q, I - p_2) - c_t = u_t(q, I - p_1) \text{ or} \quad (14)$$

$$u_t(q, I - p_2) - c_t < u_t(q, I - p_1) \text{ and } u'_r(q, I - p_2) = u'_t(q, I - p_1). \quad (15)$$

## 4 Rebates and utility arbitrage

Our analysis establishes a unique role for rebates in markets where consumers face uncertain consumption experiences. In such markets, consumers' marginal utility for income is likely to differ across different states of consumption, creating a demand for state-specific prices. Rebates respond to this demand by letting consumers to self-select whether to redeem or not. In the process, the consumer's up-front willingness to pay rises, allowing the firm to extract more consumer surplus, and increase its profits.

Figure 1 illustrates the intuition. The figure shows two downward-sloping utility curves, corresponding to the two consumption states. When the firm is not offering any rebate, its optimal price is  $p_0$ , and the consumer pays this price in each state. The optimal  $p_0$  makes the consumer indifferent between purchasing and not purchasing the product ((3)).

The opportunity for profitable rebating arises when the consumer's marginal utility of income differs across the two states in the absence of rebates, *i.e.*  $u'_r(q, I - p_0) > u'_t(q, I - p_0)$ . Then the firm can use rebates to "utility arbitrage": decreasing the price the consumer pays in the state of high marginal utility of income and simultaneously increasing the price paid in the state of low marginal utility of income. Figure 1 shows that as the firm reduces the price charged in state  $r$  from  $p_0$  to  $p_2$ , the consumer's utility in that state will increase by  $\Delta u_+ = u_r(q, I - p_2) - u_r(q, I - p_0)$ . The price increase in state  $t$  from  $p_0$  to  $p_1$  will cause a decrease in the consumer's utility in that state by  $\Delta u_- = u_t(q, I - p_0) - u_t(q, I - p_1)$ . For a sufficiently large difference in the marginal utility of income, and a sufficiently small

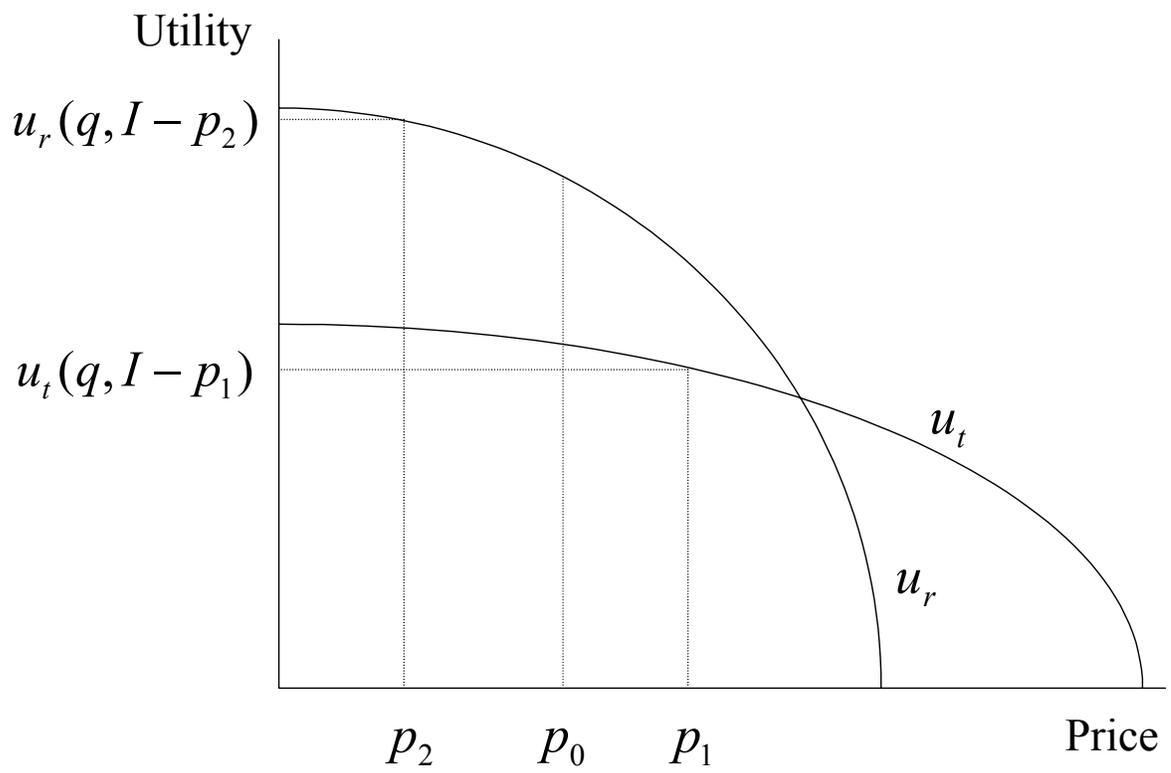


Figure 1: Utility arbitrage with rebates

redemption cost  $c_r$ , it is possible to choose  $p_1$  and  $p_2$  such that  $\theta\Delta u_+ - (1-\theta)\Delta u_- \geq \theta c_r$ , i.e., a rebate program can be designed in such a way that the expected utility gain compensates for the additional redemption cost imposed on the consumer.

The firm benefits from this utility arbitrage because it increases the consumer's ex ante willingness to pay and allows more surplus to be extracted. Indeed, the firm will always make sure that the consumer's net utility gain exactly compensates for the cost— $\theta\Delta u_+ - (1-\theta)\Delta u_- = \theta c_r$ —in order to extract the maximum surplus. This explains why (7) is always binding. Arbitrage opportunities exist as long as the difference in marginal utility of income persists (provided (9) is not binding). That is why,  $u'_r(q, I - p_2) = u'_t(q, I - p_1)$  must hold at the optimal solution when (9) is not binding. Rebates can deliver state-dependent discounts via consumer self-selection of the decision to redeem. All else equal, the consumer is more likely to redeem a rebate when her marginal utility of income is high than when it is low.

It is important to note that the rationale for rebates established in Proposition 1 is quite different from the conventional wisdom. In our theory, rebates allow a firm to charge an individual consumer different prices in different states of nature. The profitability of rebating does not hinge on the existence of multiple segments of consumers with different price sensitivities, as in the traditional price discrimination story. Furthermore, rebates deliver state-dependent discounts through *post-purchase* consumer self-selection, a function that no other promotional vehicle can perform. Because of this functionality, it is quite possible that a consumer will not redeem a rebate after the purchase, even though it was the reason the consumer bought in the first place. In this theory, then, “slippage” is an integral part of how rebates work, and essential to their profitability.

Nevertheless, an interesting analogy exists between traditional price discrimination and utility arbitrage. Whereas price discrimination requires a firm to charge a high price to consumers of low price sensitivity and a low price to consumers of high price sensitivity, utility arbitrage calls for a high price when a consumer is in a state of low marginal utility of income, hence low price sensitivity, and a low price when she is in a state of high marginal utility for income, hence high price sensitivity. To what extent changes in marginal utility of income translate into changes in price sensitivity depends on, among other things, the consumer's income level and the product's price level (Allenby and Rossi 1991). Never-

theless, this analogy suggests that the principle of price discrimination can guide a firm in designing a utility arbitraging pricing structure.

Finally, note that Proposition 1 establishes necessary conditions for rebates to be more profitable than no rebates, not sufficient conditions. A simple way to see this is that it is possible to satisfy all these conditions and still not have a profitable rebate program. This can happen because the redemption cost  $c_r$  is so high that the  $(p_1, p_2)$  that satisfy (11)-(15) are too small. However, for a sufficiently small  $c_r$ , the utility arbitrage argument above shows that a profitable rebate program exists.

**Proposition 2** *For a sufficiently small redemption cost  $c_r$ , rebates are more profitable than no rebates.*

## Managerial implications

The utility arbitrage perspective on rebates provides many fresh managerial implications. First, the profitability of rebates hinges on the difference in a consumer's marginal utility of income across states of nature. The larger this difference, the more profitable it is for the firm to use rebates. Furthermore, if  $u'_s$  and  $c_s$  are negatively correlated, rebates are more likely to be profitable. In fact, if  $c_t$  approaches  $\infty$ ,  $c_r$  approaches 0, and  $u'_r - u'_t \neq 0$ , rebates are always better than no rebates. This suggests that rebates are better suited for price promotions targeted at certain types of consumers. These are the consumers whose marginal utility of income is variable. High or low income consumers are probably *not* the right targets. For these consumers, the marginal utility of income is not likely to vary very much, either because their consumption expenditure is too small a fraction of their total income to matter, or because it is too large a fraction for them not to care all the time. Medium income consumers are the best customers for rebates.

Second, the variability of marginal utility of income across different states of nature can be caused by product-specific consumption experiences. In our model, variability in marginal utility of income can be caused by variability in quality  $q$  (provided  $\partial^2 u_s / \partial q \partial I \neq 0$ ). Quality may vary either because quality is inherently variable, or because the quality perceived by the consumer varies, or simply because the consumer's consumption experience is susceptible to the influence of random factors. This means that rebates are better suited

for promoting certain product categories than others. The key is to identify products for which a varied consumption experience can translate to a varied marginal utility of income. Low involvement, commodity products, such as toothpaste, batteries, gasoline, frozen vegetables, etc., may not be suited for rebate promotions, as consumers' consumption experiences with these products vary little, or the variations matter little. Emotionally involving products, new products, infrequently purchased products, and complex products such as electronics are good candidates to provide varied consumption experiences. In turn, these variations are likely to be passed on as changes in marginal utility of income in categories that are neither too high-priced nor too low-priced for the average consumer. However, what is "low-priced" and what is "high-priced" is hard to pin down. One would think that margarine is a low-priced category, but Allenby and Rossi (1991) find significant income effects there.

Third, because of the variability in the consumer's marginal utility of income, the consumer in our model does not always redeem a rebate. Our representative consumer sometimes "feels like" sending in the rebate and at other times she does not feel like it. Therefore, free-after-rebate promotions can be profitable. The profit comes from the high regular price that the consumer ends up paying when she does not redeem the rebate.

However, our analysis casts some doubt on the wisdom of free-after-rebate promotions as a long-term strategy, especially in an environment where consumers are becoming more vigilant about rebate redemptions. Free-after-rebate promotions, by definition, fix the discounted price to be zero, requiring the shelf price to be high for profitability. Such a practice is not sustainable in an environment of increasing consumer redemptions. As the redemption rate increases, slippage decreases, and the firm will have to increase its shelf price to maintain profitability, which will only increase redemptions even more. Ultimately this could lead to losses. CyberRebate.com is a case in point. This company recently filed for bankruptcy. The creditors include customers to whom the company owes tens of thousands of dollars. These customers are professional rebate redeemers who only purchase from the company the free-after-rebate items and then auction them off at ebay.com or donate them to charities (*The Wall Street Journal*, May 18, 2001, B1).

Finally, in our model, the redemption cost plays an important role in the profitability of the firm's rebate program. Rebates are more profitable if  $c_r$  is lower and  $c_t$  is higher. A

lower  $c_r$  will allow the firm to extract more surplus because it does not need to compensate the consumer as much to induce participation. A higher  $c_t$  will benefit the firm by relaxing constraint (9). But if the redemption cost is the same in both states of consumption—either because the rules of redemption are so strict that they are binding in both states of consumption or because they are so lenient that they don’t matter—then the firm’s profit has an inverted- $U$  relationship with this cost. The firm must therefore set the redemption cost carefully. Making rebate redemption “costless,” say through the Internet, is not advisable. Nor is it advisable to burden consumers with onerous redemption requirements.

## 5 Extensions

Propositions 1 and 2 were based on an analysis of a monopoly firm selling to a representative consumer with two states of nature. However, the idea of utility arbitrage seems robust enough to transcend these simplifying assumptions. In this section, we extend our analysis to incorporate competition, continuous states of nature, and multiple segments of consumers.

### Rebates and competition

In a perfectly competitive market, firms are price takers, and make zero expected profits. Rebates can provide utility arbitrage even in this environment.

In a competitive equilibrium, a firm should never offer a rebate that the consumer redeems in both states. To see this, let  $U$  denote the consumer’s expected utility purchasing from a firm offering such a rebate. In equilibrium, the firm must make zero profit because of free entry. This means that the firm must be setting  $p_1 - d = 0$ , since the consumer redeems its rebate in both states. Then, the expected utility for the consumer who purchases from the firm is  $U = \theta u_r(q, I) + (1 - \theta)u_t(q, I) - \theta c_r - (1 - \theta)c_t$ . However, this cannot be an equilibrium, because a firm can improve its profit by abandoning the rebate promotion and setting  $p_0 > 0$  such that  $\theta u_r(q, I - p_0) + (1 - \theta)u_t(q, I - p_0) = U$ . With this deviation, the consumer will still buy from the firm, but the firm makes a positive profit. Rebates are an inefficient way to implement a shelf price reduction. The more efficient way is to simply lower the shelf price so that the consumer does not have to incur a redemption cost.

Competing firms may offer rebates such that the consumer will redeem them only in state  $r$ . Let  $u^R = \theta u_r(q, I - p_1 + d) + (1 - \theta)u_t(q, I - p_1) - \theta c_r$  and  $u^N(p_0) = \theta u_r(q, I - p_0) + (1 - \theta)u_t(q, I - p_0)$ .  $u^R$  and  $u^N(p_0)$  denote, respectively, the consumer's expected utility with a rebate redeemable in state  $r$  and the consumer's expected utility without any rebate. In any competitive equilibrium, each firm must solve the following optimization problem:

$$\max_{(p_1, d)} u^R \tag{16}$$

$$s.t. \quad p_1 - \theta d = 0 \tag{17}$$

$$u_r(q, I - p_1 + d) - c_r \geq u_r(q, I - p_1) \tag{18}$$

$$u_t(q, I - p_1 + d) - c_t \leq u_t(q, I - p_1). \tag{19}$$

The conditions that characterize the optimal solutions to this problem are the necessary and sufficient conditions for the competitive equilibrium with rebates if  $u^R > u^N(0)$ . It is easy to see that the solution to the optimization problem is the same as that to (6)-(9) because of the following duality: maximizing  $p_1 - \theta d$  while keeping the consumer's utility constant gives the same first-order conditions as maximizing the consumer's utility while keeping  $p_1 - \theta d$  constant (at zero).

Thus, our conclusions from the basic model remain valid under competition. This is not surprising. Fundamentally, rebates as a tool for utility arbitrage benefit a firm by taking advantage of a consumer's changing demand for discounts. Therefore, any optimizing firm will use them regardless of whether the market is competitive or not, as long as the opportunity for utility arbitrage exists. As noted earlier, the opportunity exists if  $c_r$  is sufficiently small.

## Continuous states

As a consumer's consumption utility is subject to many influences, the state of nature in which a consumer may find herself could potentially be many. With many more states of nature than two, is utility arbitrage through rebates still possible? We address this question here by assuming that states of nature are a continuum.

Let  $\theta \in \mathfrak{R}$  be a real number indexing continuous states of consumption. Then,  $u_\theta(q, \cdot)$  is the consumer's utility function in state  $\theta$  and  $c_\theta$  is the corresponding cost of rebate

redemption. Define  $\theta^* \in \mathfrak{R}$  such that

$$u_{\theta^*}(q, I - p_2) - c_{\theta^*} = u_{\theta^*}(q, I - p_1).$$

In state  $\theta^*$ , the consumer is indifferent between redeeming the rebate and not redeeming it. Let  $\theta$  be ordered such that  $\theta > \theta^*$  implies  $u_{\theta}(q, I - p_2) - c_{\theta} > u_{\theta}(q, I - p_1)$  and  $\theta < \theta^*$  implies  $u_{\theta}(q, I - p_2) - c_{\theta} < u_{\theta}(q, I - p_1)$ . The density function of  $\theta$  is denoted by  $f(\theta)$ .

Then, if the firm does not use any rebate, its optimal price in this market with a single consumer will simply be determined by the consumer's participation constraint, *i.e.*,

$$\int_{\theta \in \mathfrak{R}} u_{\theta}(q, I - p_0) f(\theta) d\theta = u_0. \quad (20)$$

If the firm uses a rebate, then the firm's optimization problem becomes

$$\max_{p_1, p_2} \int_{\theta^*} p_2 f(\theta) d\theta + \int_{\theta^*}^{\theta^*} p_1 f(\theta) d\theta \quad (21)$$

$$s.t. \int_{\theta^*} u_{\theta}(q, I - p_2) f(\theta) d\theta + \int_{\theta^*}^{\theta^*} u_{\theta}(q, I - p_1) f(\theta) d\theta - \int_{\theta^*} c_{\theta} f(\theta) d\theta \geq u_0 \quad (22)$$

Obviously, the participation constraint (22) must be binding so that the firm can extract the maximum surplus from the consumer. By the same argument as in Section 3, we must also have  $p_1 > p_0 > p_2$  for the rebate program to beat the optimal no-rebate program. Also, from (20) and (22), we have

$$\int_{\theta^*} u_{\theta}(q, I - p_2) f(\theta) d\theta + \int_{\theta^*}^{\theta^*} u_{\theta}(q, I - p_1) f(\theta) d\theta - \int_{\theta^*} c_{\theta} f(\theta) d\theta - \int_{\theta^*} u_{\theta}(q, I - p_0) f(\theta) d\theta = 0$$

Collecting terms,

$$\int_{\theta^*} [u_{\theta}(q, I - p_2) - u_{\theta}(q, I - p_0)] f(\theta) d\theta + \int_{\theta^*}^{\theta^*} [u_{\theta}(q, I - p_1) - u_{\theta}(q, I - p_0)] f(\theta) d\theta = \int_{\theta^*} c_{\theta} f(\theta) d\theta > 0.$$

This implies, using the mean value theorem,

$$\int_{\theta^*} u'_{\theta}(q, I - \xi_{\theta})(p_0 - p_2) f(\theta) d\theta > \int_{\theta^*}^{\theta^*} u'_{\theta}(q, I - \varsigma_{\theta})(p_1 - p_0) f(\theta) d\theta,$$

where  $p_2 \leq \xi_{\theta} \leq p_0$  and  $p_0 \leq \varsigma_{\theta} \leq p_1$ , and, in turn,

$$\frac{p_1 - p_0}{p_0 - p_2} < \frac{\int_{\theta^*} u'_{\theta}(q, I - \xi_{\theta}) f(\theta) d\theta}{\int_{\theta^*}^{\theta^*} u'_{\theta}(q, I - \varsigma_{\theta}) f(\theta) d\theta}. \quad (23)$$

In short, if the firm is profit-maximizing in designing its rebate program, condition (23) must hold.

However, a firm will not use rebate promotions unless doing so makes it better off, or unless

$$\int^{\theta^*} p_1 f(\theta) d\theta + \int_{\theta^*} p_2 f(\theta) d\theta - p_0 \geq 0.$$

A slight manipulation yields

$$\int^{\theta^*} (p_1 - p_0) f(\theta) d\theta \geq \int_{\theta^*} (p_0 - p_2) f(\theta) d\theta.$$

This means that rebating is better than not rebating if

$$\frac{p_1 - p_0}{p_0 - p_2} \geq \frac{\int_{\theta^*} f(\theta) d\theta}{\int^{\theta^*} f(\theta) d\theta}. \quad (24)$$

Combining (23) and (24),

$$\int_{\theta^*} \frac{f(\theta)}{\int_{\theta^*} f(\theta) d\theta} u'_\theta(q, I - \xi_\theta) d\theta > \int^{\theta^*} \frac{f(\theta)}{\int_{\theta^*} f(\theta) d\theta} u'_\theta(q, I - \varsigma_\theta) d\theta. \quad (25)$$

Since  $p_2 \leq \xi_\theta \leq p_0$ ,  $p_0 \leq \varsigma_\theta \leq p_1$ , and  $u_\theta$  is concave in income, we must have  $u'_\theta(q, I - \xi_\theta) < u'_\theta(q, I - p_0)$  and  $u'_\theta(q, I - \varsigma_\theta) > u'_\theta(q, I - p_0)$ . Then, (25) implies

$$\int_{\theta^*} \frac{f(\theta)}{\int_{\theta^*} f(\theta) d\theta} u'_\theta(q, I - p_0) d\theta > \int^{\theta^*} \frac{f(\theta)}{\int_{\theta^*} f(\theta) d\theta} u'_\theta(q, I - p_0) d\theta.$$

This condition can be written more compactly as

$$E[u'_\theta(q, I - p_0) | \theta > \theta^*] > E[u'_\theta(q, I - p_0) | \theta < \theta^*]. \quad (26)$$

Condition (26) simply says that the necessary condition for rebating to be more profitable than no rebating is that the *mean* marginal utility of income in the states of nature where the rebate is redeemed is higher than the mean marginal utility of income in the states where it is not. This condition is what drives the utility arbitrage we saw earlier in Section 3. In fact, if we take  $\theta > \theta^*$  as state  $r$  and  $\theta < \theta^*$  as state  $t$ , this condition is precisely the condition  $u'_r > u'_t$ . However, the additional insight we gain from this analysis is that the opportunity for utility arbitrage depends on variations in average consumer marginal utility of income across states of nature, not the number of states of nature *per se*.

## Consumer heterogeneity, and coupons versus rebates

The models analyzed so far assumes a single consumer. In reality, there are many consumers, heterogenous in their marginal utilities of income in different consumption states, and likely

also in their product preferences and redemption costs. Could consumer heterogeneity render rebates inoperative as a vehicle for utility arbitrage?

The answer is no because all the different kinds of consumer heterogeneity effectively boil down to one basic heterogeneity: some consumers would like to redeem rebates in one state, others in another state, and still others either never redeem rebates or redeem them all the time. This is because rebates as a vehicle for utility arbitrage work through consumer self-selection, at the individual level, and not through synchronization of different consumers' marginal utilities of income. Different consumers may experience states of high or low marginal utility of income at different times, and the rebate offer is available to all for use when they see fit.

However, consumer heterogeneity in terms of product preferences or redemption costs can weaken the effectiveness of rebates as a vehicle for utility arbitrage. With diverse types of consumers in the market, rebates, like any other promotional vehicle, can generate many effects, not all of which are beneficial to a firm. Although rebates can still benefit a firm through slippage and a higher shelf price, they could also choke off some demand and offer unnecessary discounts to some consumers. Of course, the same can be said about other promotional vehicles such as coupons.

Indeed, we will argue that rebates can perform better than coupons, on demand-side considerations alone, even in a market with consumer heterogeneity. By “demand-side considerations alone” we mean that we do not take into account any advantages or disadvantages these promotion vehicles may possess because of redemption costs. That is, we eliminate redemption costs as a differentiating factor between these promotion vehicles. The natural way to do this is to assume that, for every consumer, coupon redemption cost  $C$ , is nothing but her average rebate redemption cost  $\theta c_r + (1 - \theta)c_t$ .

To illustrate how rebates can perform better than coupons, start with a coupon program  $(p_1, p_2)$ , with  $p_1$  as the shelf-price and  $p_2$  as the discounted price;  $d_0 = p_1 - p_2$  is the face value of the coupon. Given this program, the market may divide itself into three segments:

$$\begin{aligned}
X &= \{\text{consumers who buy at the shelf-price}\}, \\
Y &= \{\text{consumers who buy with the coupon}\}, \\
Z &= \{\text{consumers who do not buy the product}\}.
\end{aligned}$$

The firm will determine  $p_1$  and  $d_0$  to make the marginal consumer in  $Y$  is indifferent between buying and not buying the product. That is,

$$\theta u_r^y(q, I^y - p_1 + d_0) + (1 - \theta)u_t^y(q, I^y - p_1 + d_0) - C^y = u_0^y. \quad (27)$$

Now suppose the coupon is relabelled as a rebate. In this rebate program,  $p_1$  is the shelf-price and  $p_2$  is the rebated price. The coupon and rebate offers may seem similar, but, in fact, they are quite different. With the rebate offer, consumers can decide after state  $r$  or state  $t$  is realized, whether or not to redeem the rebate. With the coupon, they have to decide up-front, before state  $r$  or state  $t$  is realized. Thus, in one case, they can make the redemption decision based on realized utility, whereas in the other; they have to decide based on expected utility.

Given the additional flexibility built into a rebate offer, segments  $X$ ,  $Y$  and  $Z$  may split further, as follows:

$$\begin{aligned}
X_1 &= \{\text{consumers in } X \text{ who buy and redeem the rebate in state } r \text{ but not in state } t\}, \\
X_2 &= \{\text{consumers who buy but never redeem the rebate}\}, \\
Y_1 &= \{\text{consumers in } Y \text{ who buy and redeem the rebate in state } r \text{ but not in state } t\}, \\
Y_2 &= \{\text{consumers who buy and always redeem the rebate}\}, \\
Z_1 &= \{\text{consumers in } Z \text{ who buy and redeem the rebate in state } r \text{ but not in state } t\}, \\
Z_2 &= \{\text{consumers who do not buy the product}\}.
\end{aligned}$$

Consumers in  $X_2$ ,  $Y_2$ , and  $Z_2$  do not change their behavior as a result of the change from coupon to rebate, so they do not have any profit implications.  $Y_1$  and  $Z_1$  consumers, however, are clearly profit-increasing: earlier all consumers in  $Y$  were redeeming the promotion in both states, now some of them they do so in state  $r$  only; earlier no consumer in  $Z$  was buying the product, now some of them do. The creation of  $X_1$  is the only negative outcome of relabelling the coupon as a rebate: they buy at a lower price in state  $r$  now than before.

Rebates can be more profitable than coupons if  $X_1$  is small relative to  $Y_1$  and  $Z_1$ . To see this, suppose  $X_1$  is so small relative to  $Y_1$  that the marginal consumer lies in  $Y_1$ . (If the marginal consumer lies in  $Y_2$ , then the relabelled rebate yields the same profit as the coupon.) This consumer's expected utility is higher with a rebate than with an equivalent coupon because she bears the cost of redemption only in state  $r$ —where the redemption cost is lower, and marginal utility of income higher—whereas earlier she was bearing the redemption cost in both states. So the firm can set the rebate  $d$  so that

$$\theta u_r^y(q, I^y - p_1 + d) + (1 - \theta)u_t^y(q, I^y - p_1) - \theta c_r^y = u_0^y. \quad (28)$$

Since  $u_t^y(q, I^y - p_1 + d) - c_t^y < u_t^y(q, I^y - p_1)$  for consumers in  $Y_1$ ,

$$\begin{aligned} \theta u_r^y(q, I^y - p_1 + d) + (1 - \theta)u_t^y(q, I^y - p_1) - \theta c_r^y - (1 - \theta)c_t^y &< u_0^y \\ &= \theta u_r^y(q, I^y - p_1 + d_0) + (1 - \theta)u_t^y(q, I^y - p_1 + d_0) - C^y. \end{aligned}$$

Therefore,  $d < d_0$ . The firm is strictly better off with the rebate promotion than with the coupon promotion not only because of fewer redemptions (slippage) from consumers in  $Y_1$ , but also because the rebate is smaller.

This analysis underscores the advantages rebates have over coupons on the demand side. Coupons, because they are redeemed before the product is consumed, can only price discriminate among consumers based on their expected value for the product. Rebates can also price discriminate like this, but in addition they can provide utility arbitrage benefits *within* segments of consumers. Indeed, the utility arbitrage function of rebates can be seen as enhancing its price discrimination function. Rebates allow consumers to create additional variety in the product line. Think of the coupon offer as really two “products,” a “higher quality” offering, “shelf-price, no redemption costs,” and a “lower quality” offering, “discounted price, redemption costs  $C$ .” With rebates, a third “product,” intermediate in “quality” between the other two, is added to the line. This “product” is “shelf-price in state  $r$ , discounted price in state  $t$ , redemption cost  $c_r$  in state  $r$ .” The added variety in the firm's offerings helps enhance price discrimination. The firm can extract more surplus in the “middle” of the market—by creating a subsegment  $Y_1$  out of  $Y$ , leading to a decrease in the discount—and, simultaneously, increase market coverage by expanding the market at the “bottom” end (by creating a subsegment  $Z_1$  out of  $Z$ ). In addition, of course, the utility arbitrage effect leads to slippage which is also beneficial.

This may suggest that rebates dominate coupons, but there are two caveats. First, we have alluded to the difficulty segment  $X_1$  poses. This is a cannibalization issue: rebates have a greater potential than coupons to cannibalize full-price sales. Second, we have assumed that both promotion vehicles have equivalent redemption costs. In reality, one would expect redemption costs to be generally lower for coupons than for rebates—at least in the bricks-and-mortar world. This is because rebates add additional redemption costs on top of what coupons already have. The redemption costs associated with coupons are: search costs—the costs incurred in finding a coupon—coupon clipping costs, organizing costs, and costs of remembering to use them before their expiry dates. With rebates, we have all of these, plus the additional costs involved in fulfilling rebate requirements, filling out the redemption form, mailing costs, waiting-for-the-check-to-arrive costs, etc. As a result, it is possible that a rebate offer is simply not attractive enough for a large part of the market. In the Internet world, however, this argument is much weaker. Redemption costs may not be all that different between online coupons and online rebates. In this world, therefore, rebates are likely to be preferred to coupons.

## 6 Conclusion

This paper develops a new theory of rebates based on the idea of “utility arbitrage.” It is motivated by the weaknesses in the existing price discrimination theory. The current theory does not distinguish between rebates and coupons. It views both as price discrimination devices, useful for discriminating between consumers with different willingness to pay. However, this overlooks a key difference between the two promotion instruments: rebates are redeemed after the purchase whereas coupons are redeemed with the purchase. As a result, this theory cannot shed light on the phenomena that are unique to rebates. For instance, many consumers respond to rebate offers, but far fewer redeem them. In the industry this is known as slippage, and it is an open secret that many marketers count on slippage as an important source of revenue. On the Internet, in particular, we see an extreme manifestation of slippage: free-after-rebate promotions. Several firms have made such promotions an essential part of their business strategy, and, some, like CyberRebate.com, have come to grief (perhaps as a result). Price discrimination theory does not provide any guidance on

slippage or the viability of 100% rebates.

In this paper we explore the implications of the unique timing of rebate redemptions. We argue that the fact that rebates are redeemed after purchase is of critical importance in understanding how they work. When consumers purchase a durable good, and consider subsequently whether or not to redeem a rebate, their redemption decision will depend on a number of things that are uncertain at the time of purchase. In particular, the redemption decision will depend on the consumer's future marginal utility of income and her future redemption cost. A firm can increase a consumer's up-front willingness to pay by conducting utility arbitrage: giving discounts when they are most valuable and withholding discounts when they matter the least. Rebates are uniquely suited to this task because their redemption takes place after purchase, when consumers can self-select whether or not to redeem a rebate based on their realized state. Coupons, because they are redeemed before consumption, cannot do utility arbitrage. The dual functionality of rebates suggests that, abstracting from redemption cost differences, rebates are likely to be superior to coupons. They can expand the market, extract more consumer surplus (smaller discount and/or a higher shelf-price), and create slippage.

The utility arbitrage theory of rebates offers a fresh perspective on the practice of rebate promotions. Rebates benefit firms not because they provide "phantom discounts" or because they take advantage of consumer forgetfulness, but because they enhance consumers' willingness to pay for a product by providing a way to utility arbitrage between different states of consumption. From this perspective, although slippage is important for a firm's profitability, it should not be the primary focus of rebate promotions. The primary focus should be on enhancing consumers' willingness to pay for the product and on capturing some part of the added value by appropriate rebate design. If the procedural requirements for redeeming rebates are excessive, then consumers will not expect to redeem them, and their willingness to pay the shelf price up front will decrease. Free-after-rebate promotions, on the other hand, may be too much of a good thing. While they can be optimal under certain circumstances, they are too rigid to be generally optimal. In general, optimal rebate design must engage both aspects of rebates—shelf price and rebated price—not just the shelf price. Certain product categories are better suited for rebates than others, and the theory points to moderately priced experience goods as the best candidates. Similarly, certain types of

consumers are better targets for rebates than others. The key test is whether consumers' marginal utility of income is likely to vary across future consumption states, making them more or less inclined to redeem rebates. Very rich, or very poor consumers, will likely fail the test, the former because they will never redeem, and the latter because they will always redeem.

## Appendix

### Proof of $u'_r(q, I - p_2) = u'_t(q, I - p_1)$ when (9) is not binding

Suppose, to the contrary, that  $u'_r(q, I - p_2) > u'_t(q, I - p_1)$  in the optimal rebate program. Consider small deviations in the rebate program  $\hat{p}_2 = p_2 - \Delta_2$ ,  $\hat{p}_1 = p_1 + \Delta_1$  ( $\Delta_1 > 0$ ,  $\Delta_2 > 0$ ), that keep the consumer's expected utility the same, and satisfy the constraints (8) and (9). That is,  $\theta u_r(q, I - p_2 + \Delta_2) + (1 - \theta)u_t(q, I - p_1 - \Delta_1) - \theta c_r = \theta u_r(q, I - p_2) + (1 - \theta)u_t(q, I - p_1) - \theta c_r = \bar{u}$ ,  $u_r(q, I - p_2 + \Delta_2) - c_r \geq u_r(q, I - p_1 - \Delta_1)$ , and  $u_t(q, I - p_2 + \Delta_2) - c_t \leq u_t(q, I - p_1 - \Delta_1)$ . The first of these implies

$$\theta[u_r(q, I - p_2 + \Delta_2) - u_r(q, I - p_2)] - (1 - \theta)[u_t(q, I - p_1) - u_t(q, I - p_1 - \Delta_1)] = 0,$$

and, in turn,

$$\theta \Delta_2 u'_r(q, I - \xi) - (1 - \theta) \Delta_1 u'_t(q, I - \zeta) = 0 \quad (p_2 - \Delta_2 \leq \xi \leq p_2, p_1 \leq \zeta \leq p_1 + \Delta_1).$$

by the intermediate-value theorem. Since  $u'_r(q, I - p_2) > u'_t(q, I - p_1)$ , and  $u'_r$  and  $u'_t$  are continuous,  $u'_r(q, I - \xi) > u'_t(q, I - \zeta)$ . Therefore,  $(1 - \theta) \Delta_1 > \theta \Delta_2$ . In other words, the new rebate program is feasible and profit-increasing, which is not possible. Hence  $u'_r(q, I - p_2)$  cannot be greater than  $u'_t(q, I - p_1)$ . A similar argument, but now with deviations  $\hat{p}_2 = p_2 + \Delta_2$ ,  $\hat{p}_1 = p_1 - \Delta_1$  ( $\Delta_1 > 0$ ,  $\Delta_2 > 0$ ), shows that  $u'_r(q, I - p_2)$  cannot be less than  $u'_t(q, I - p_1)$ . ■

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