Labor Market Search and Real Business Cycles: Reconciling Nash Bargaining with the Real Wage Dynamics*

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Abstract

This paper modifies the standard one-sector stochastic growth model in an effort to explain the observed low procyclicality of the aggregate real wage in the US. The modifications include labor market matching with Nash-bargaining of wages and preferences as introduced in the literature by Rogerson and Wright [1988]. These preferences are non-separable in consumption and leisure. They imply that in an equilibrium with efficient risk-sharing, the utility of employed agents exceeds that of unemployed agents. The simulation results suggest that our modified model overcomes one important weakness of the standard model, namely the predicted high contemporaneous correlation of the aggregate real wage with both output and labor input.

Keywords: search, wage, bargaining, welfare, business cycle

JEL Classification: E24, E31

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Introduction

Models of the business cycle with search frictions and wage bargaining improve our understanding of the US labor market stylized facts (see Merz [1995] and Andolfatto [1996]). These models are capable of replicating that:

(i) the correlation between unemployment and job vacancies is negative,

(ii) employment is more volatile than labor productivity and the real wage,

(iii) labor's share of output behaves countercyclically,

(iv) labor productivity is a leading indicator of labor input,

(v) GNP growth is characterized by positive serial correlations.[1]

However, this framework does not allow for a weak procyclical real wage[2], whereas the US data shows a weak correlation between the real wage and the business cycle (see Cooley and Prescott [1995], Kydland [1995] and Stock and Watson [2000]). This casts some doubts on the labor market search explanation of the business cycle.[3]

This failure in the model's predictions is all the more puzzling that implicit contracts and efficiency wage theories have been found to succeed in replicating the real wage rigidity. For instance, Boldrin and Horvath [1995] successfully illustrated the explanatory power of the implicit contracts theory. Providing workers with an income insurance that smoothes the real wage dynamics, entrepreneurs take advantage of a more flexible labor supply. This entails the desired low procyclicality of the real wage. A similar sluggishness in the real wage has been obtained by Collard and De Lacroix [2000], by extending the gift exchange model of Danthine and Donaldson [1990] to include social or personal norms, which allow the reference wage to

[1] See Cogley and Nason [1995].

[2] In the standard Real Business Cycle (RBC) model driven by a single productivity shock the aggregate real wage is highly procyclical (see, *e.g.*, King and Rebelo [2000]).

[3] Extensions of the labor market search model to allow for endogenous firing decisions do not overcome this empirical shortcoming (see Merz [1999] and Den Haan, Ramey, and Watson [2000]); so does also for the introduction of nominal rigidities and monetary shocks (see Chéron and Langot [2000]).

include various measurements of past wages.

Against this background, the main objective of this paper is to show that the wage bargaining theory can also provide some foundations to real wage rigidity. We pursue this objective in the context of the matching framework, since it gives some support to the Nash-bargaining process as mean to share the rents created by job-matches.

In a search environment, the direct link between productivity and real wage is weakened by bilateral bargaining considerations. In particular, the real wage appears to be not only driven by the labor productivity, but also by the worker's outside options should the negotiations fail. Using conventional specifications for preferences and technology and a complete set of insurance contracts, these outside options behave in a procyclical manner, which contributes to the upward pressure on wages during cyclical upturns. Conversely, if these outside options were to behave in a countercyclical manner, the procyclicality of real wage could be weakened significantly. The question is then to determine the factors that account for the procyclical or countercyclical nature of these outside options?

In the standard RBC setup, preferences are such that individuals use insurance markets to equate consumption across states of employment and unemployment. In equilibrium, employed agents work harder than unemployed agents (who are only engaged in search activity), so that from an *ex post* perspective, employed agents are actually worse off than unemployed agents. During an economic boom, wealth increases for everyone, but the unemployed have more leisure time to enjoy it. This puts upward pressure on the outside options and the real wage.

We show that this procyclical nature of outside options can be reversed by postulating a particular set of non-separable preferences, as earlier used by Rogerson and Wright [1988] (RW hereafter).[4] While remaining within

[4]Such non-separability has also proven useful in a variety of environments; see, for instance, Greenwood, Hercowitz, and Huffman [1988] (standard RBC model), Benhabib, Rogerson, and Wright [1991] (household production and aggregate fluctuations), Christiano, Eichenbaum, and Evans [1997] (monetary business cycle model), Hairault [2002] (international business cycle model) and Gomes, Greenwood, and Rebelo [1998] (heterogeneous agents model with idiosyncratic risks and aggregate shocks). Furthermore, the

the framework of a well-functioning perfect insurance market, these preferences have the property that employed agents consume more than unemployed agents. Furthermore, with these preferences, the market-based good (consumption) is always viewed, at the margin, as more valuable than the home-based good (leisure). Accordingly, from an *ex post* perspective, employed agents (insiders) are better off than unemployed agents (outsiders), and can take more advantage of the economic boom. This depresses the outside options, in putting a downward pressure on the real wage.

The simulation results of the calibrated model suggest that the combination of labor market matching, Nash bargaining of wages and RW preferences provides a robust explanation to the observed low contemporaneous correlation of the real wage with both output and labor input.

The remainder of the paper elaborates on the quantitative properties of the hypothesis outlined above. Section 2 presents the model and investigates the implications of RW preferences for the wage bargaining. Section 3 shows the empirical results. Section 4 concludes the paper.

1 The Model

1.1 Labor market flows

It is assumed that the law of motion for aggregate employment depends on the number of hirings, M_t, which is determined by a conventional constant returns-to-scale matching technology (see Pissarides [1990]). Let N_t and V_t respectively be the number of workers and the total number of new jobs made

home production theory provides some foundations to this non-separability: "for any model with home production there is a model without home production, but with different preferences, that generates the same outcome for equilibrium quantities, and so there is a sense in which models with home production are observationally equivalent to those without" (Benhabib, Rogerson, and Wright [1991]). Actually, the underlying RW preferences are such that: *(i)* market and non-market work are perfect substitutes, *(ii)* home production technology is linear *(iii)* market goods are preferred to non-market goods. It is straightforward to show that the functional form $\log(C_m+C_h)+A\log(1-H_m-H_h)+aC_m$ (with m and h standing for "market" and "non-market") can be the separable preferences underlying RW preferences. The condition *(iii)* is consistent with $a>0$.

available by firms, the employment evolves according to:

$$N_{t+1} = (1-s)N_t + M_t, \quad \text{with} \quad M_t = \Upsilon V_t^{\psi} \left[e(1-N_t)\right]^{1-\psi}, \quad \Upsilon > 0, 0 < \psi < 1$$

where $e > 0$ and $0 < s < 1$ are the constant search effort and the exogenous separation rate of job-worker pairs.

1.2 Households

Remaining within the confines of complete insurance markets allows to derive the optimal workers' decision rules by solving the program of a representative household. This agent has the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [N_t U\left(C_t^n, 1-h_t\right) + (1-N_t) U\left(C_t^u, 1-e\right)] \tag{1}$$

where E denotes the expectation operator and $0 < \beta < 1$ is the discount factor. C_t^n and C_t^u stand for the consumption of employed and unemployed agents. At this stage, the contemporaneous utility function U is simply assumed to be increasing and concave in both arguments. h_t denotes the hours per employed worker.

The capital stock K_t is rented to the firm at price r_t and depreciates at rate $0 < \delta < 1$. Each household aims at choosing a contingency plan $\{C_t^u, C_t^n, K_{t+1} \mid t \geq 0\}$ that maximizes (1) subject to the constraints:

$$\begin{aligned} N_t C_t^n + (1-N_t) C_t^u + K_{t+1} &= (1-\delta+r_t)K_t + w_t h_t N_t + \pi_t \\ N_{t+1} &= (1-s)N_t + \Psi_t(1-N_t) \end{aligned}$$

given a stochastic process for $\{w_t, r_t, h_t, \pi_t, \Psi_t \mid t \geq 0\}$ and some initial conditions (N_0, K_0), where $\Psi_t \equiv \frac{M_t}{1-N_t}$. w_t and π_t are the real wage and lump-sum dividends remitted by firms.

1.3 Firms

The model has also a representative firm with a constant returns-to-scale Cobb-Douglas production function that uses capital and labor hours to produce output:

$$Y_t = A_t K_t^{\alpha} \left(N_t h_t\right)^{1-\alpha} \tag{2}$$

where $0 < \alpha < 1$, and A_t is a stochastic term representing random technological progress. Let ω be the unitary cost of a vacancy job, each firm chooses a contingency plan $\{N_{t+1}, K_t, V_t \mid t \geq 0\}$ that maximizes the expected discounted value of the dividend flow:

$$E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \{Y_t - w_t h_t N_t - r_t K_t - \omega V_t\}$$

subject to the constraints, (2), and:

$$N_{t+1} = (1 - s) N_t + \Phi_t V_t$$

given a stochastic process for $\{w_t, r_t, h_t, p_t, \Phi_t | \geq 0\}$ and an initial condition for N_0, where $\Phi_t \equiv \frac{M_t}{V_t}$. λ_t gives firms' valuation of profits.[5]

1.4 *Nash*-bargaining whenever unemployed agents are better off

Real wage and hours worked are derived from the standard *Nash*-bargaining model. We allow for two specifications of the bargaining process depending on the momentary utility function. Let start by considering conventional additively separable preferences between consumption (C_t^z) and leisure (L_t^z):

$$U\left(C_t^z, L_t^z\right) = \log C_t^z + \Gamma_t^z \equiv U_t^z \qquad z = n, u \tag{3}$$

where $\Gamma_t^n = \gamma \frac{(1-h_t)^{1-\eta}}{1-\eta}$ with $\gamma, \eta > 0$ and $\Gamma_t^u = \Gamma^u \quad \forall t$. It is worth stressing that optimal households' decisions rules imply:

$$C_t^n = C_t^u \equiv C_t \tag{4}$$

$$U_t^u = U_t^n + \Gamma_t \tag{5}$$

where $\Gamma_t = \Gamma^u - \Gamma_t^n$. Assuming that near the steady state, the value of leisure is greater for unemployed agents than for employed agents ($\Gamma^u > \Gamma_t^n \Rightarrow \Gamma_t > 0 \quad \forall t$), this entails that *the unemployed workers are better off than the employed workers.*

[5] Due to the efficient risk-sharing assumption, at the equilibrium, λ_t equates the marginal utility of consumption for the representative household.

Let $0 < \xi < 1$ be the firm's bargaining power, the solution of the *Nash* maximization problem is given by:[6]

$$w_t h_t = (1-\xi)\left[(1-\alpha)\frac{Y_t}{N_t} + \frac{\omega V_t}{1-N_t}\right] + \xi\left(\frac{U_t^u - U_t^n}{\lambda_t}\right) \quad (6)$$

$$(1-\alpha)\frac{Y_t}{N_t} = \frac{\gamma(1-h_t)^{-\eta}}{\Gamma_t} h_t \left(\frac{U_t^u - U_t^n}{\lambda_t}\right) \quad (7)$$

where λ_t denotes the *Lagrange* multiplier associated with the representative household's budget constraint. This multiplier equates the marginal utility of consumption, which is the same for employed and unemployed agents (from the efficient risk-sharing property). The representative worker's wage bill turns out to be some weighted average of, *(i)* the worker's contribution to output plus hiring costs per unemployed worker, and *(ii)* the worker's endogenous outside options. The latter refer to the positive utility gap between unemployed and employed agents expressed in units of the consumption good.

The dynamics of these outside options depends on income and intertemporal substitution effects, through variations in λ_t. Let us think to the impact of an economic boom relying on a positive temporary technological shock:

- The income effect states that, as wealth increases, wealthier workers grant *more* value to leisure and the unemployed position as well.
- Conversely, the intertemporal effect states that, as the increase in real wages is temporary, it is employed workers interest to take advantage of their current labor market position, granting *less* value to the unemployed position and working harder.

Equation (6) shows that the income effect exceeds the intertemporal substitution effect: since $U_t^u > U_t^n$, a decrease in λ_t (which arises at economic boom) entails a higher increase in U_t^u/λ_t than in U_t^n/λ_t. In plain words, at cyclical upturns, the outsiders (unemployed workers) take more advantage of the economic boom than the insiders (employed workers), which contributes to increasing the outside options. This suggests that the dynamics of the real wage is a combination of two procyclical components, *(i)* labor productivity plus hiring costs, and *(ii)* the outside options.

[6]See the appendix for the derivation.

Equation (7) governs the intratemporal allocation of consumption and leisure. The number of hours worked is set so that the marginal product of labor equates the elasticity of the outside options with respect to hours per worker. Importantly, as the income effect exceeds the substitution one, this suggests that the increase in hours worked at business cycle peaks -relying on the rise of labor productivity- is actually smoothed.[7]

1.5 *Nash*-bargaining whenever unemployed agents are worse off

Let us now consider non-separable preferences of the type assumed by Rogerson and Wright [1988]:[8]

$$U\left(C_t^z, L_t^z\right) \;=\; \log\left(C_t^z + \tilde{\Gamma}_t^z\right) + aC_t^z \equiv \tilde{U}_t^z \qquad z = n, u \tag{8}$$

where $\tilde{\Gamma}_t^n = \tilde{\gamma}\frac{(1-h_t)^{1-\eta}}{1-\eta}$ with $\tilde{\gamma}, \eta > 0$, $\tilde{\Gamma}_t^u = \tilde{\Gamma}^u \;\; \forall t$ and $a \geq 0$. In this new environment, optimal households' decision rules imply:

$$C_t^n \;=\; C_t^u + \tilde{\Gamma}_t \tag{9}$$

$$\tilde{U}_t^n \;=\; \tilde{U}_t^u + a\tilde{\Gamma}_t \tag{10}$$

where $\tilde{\Gamma}_t = \tilde{\Gamma}^u - \tilde{\Gamma}_t^n$. Still assuming that near the steady state $\tilde{\Gamma}^u > \tilde{\Gamma}_t^n \; \forall t$, and $a > 0$, it comes that *unemployed workers consume less and are worse off than employed workers*.[9]

Wage and hours worked equations are now given by:

$$w_t h_t \;=\; (1-\xi)\left[(1-\alpha)\frac{Y_t}{N_t} + \frac{\omega V_t}{1-N_t}\right] + \xi\left[\tilde{\Gamma}_t - \frac{\tilde{U}_t^n - \tilde{U}_t^u}{\lambda_t}\right] \tag{11}$$

$$(1-\alpha)\frac{Y_t}{N_t} \;=\; \frac{\tilde{\gamma}(1-h_t)^{-\eta}}{\tilde{\Gamma}_t} h_t \left[\tilde{\Gamma}_t - \frac{\tilde{U}_t^n - \tilde{U}_t^u}{\lambda_t}\right] \tag{12}$$

[7]Technically, this means that the decrease in λ_t implies an increase in h_t.

[8]It should be noticed that, without strong restrictions on structural parameters, these preferences are not consistent with balanced growth.

[9]Technically, for $a > 0$, RW preferences imply that the marginal utility of market-based consumption is always higher than the marginal utility of home-based consumption (leisure). This means that the market-based good is always viewed as more valuable than the home-based good.

The outside options for the employed agent include the gain in leisure $\tilde{\Gamma}_t$ minus the utility loss expressed in units of the consumption good should the worker becomes unemployed.[10]

Since $\tilde{U}_t^n > \tilde{U}_t^u$, a decrease in λ_t (at economic boom) entails a higher increase in $\tilde{U}_t^n/\lambda_t$ than in $\tilde{U}_t^u/\lambda_t$. This suggests that the intertemporal substitution effect now exceeds the income effect, implying that the real wage is positively correlated with λ_t.

In plain words, employed agents take more advantage of a cyclical upturn than unemployed agents, which gives them incentives to temperate their wage claims. The dynamics of the real wage is therefore a combination of, *(i)* a procyclical component, labor productivity plus hiring costs, and *(ii)* the outside options which are as much countercyclical as the positive utility gap between employed and unemployed workers, $\tilde{U}_t^n - \tilde{U}_t^u$, is large (as a is high).

In turn, equation (12) indicates that, since the substitution effect now exceeds the income one, this contributes to enhancing the increase in hours worked following the rise of labor productivity at business cycle peaks.[11]

2 Empirical Results

2.1 Parameterization

To begin, a first subset of parameters and restrictions on average values rely on Andolfatto [1996]. The quarterly rate of transition from employment to non-employment is set equal to $s = 0.15$, the elasticity of the matching function with respect to vacancies and, as well, the firms' bargaining power, to $\xi = \psi = 0.6$.[12] The averages for the probability that a vacant position becomes a productive job, the employment ratio, the fraction of time spent

[10] The utility gap is given by $U_t^n - U_t^u = a\,(C_t^n - C_t^u)$. This gap refers to the imputed value by employed workers to the consumption gain.

[11] In order to guarantee the existence of a unique equilibrium path, it is well-known that labor supply and demand curves must have standard slopes. In our context, positive slope for the labor supply is equivalent of saying that outside options dynamics has less incidence on the real wage than labor productivity and hiring costs dynamics (see equations (11) and (12)). For plausible calibrations of the model, uniqueness and existence of equilibrium are found to be guaranted.

[12] This implies that the equilibrium unemployment is socially-efficient (see Hosios [1990]).

working and the fraction of time spent searching are computed to be $\Phi^\star = 0.9$,[13] $N^\star = 0.57$, $h^\star = 1/3$ and $e = \frac{1}{2}h^\star$ (stars stand for steady state values).

A second subset of parameters is set in a fairly standard way. The elasticity of output with respect to capital, the depreciation rate of capital and the discount factor are assumed to be $\alpha = 0.4$, $\delta = 0.012$, and $\beta = 0.985$ (see Cooley and Prescott [1995]). We choose $\eta = 4$, which implies that the average individual labor supply elasticity is equal to $\eta^{-1}\left(\frac{1}{h^\star} - 1\right) = 1/2$, a value consistent with the bulk of empirical estimates (see MaCurdy [1981]). Parameters describing the stochastic process for the technological level are provided by Prescott [1986], *i.e.*, $\rho_A = 0.95$ and $\sigma_{\epsilon_A} = 0.007$.[14] Concerning the magnitude of aggregate expenditures in the course of search activity, we follow Abowd and Kramarz [1998] by setting ω and the ratio of recruiting expenditures to output to $\frac{\omega V^\star}{Y^\star} = 0.5\%$, so that these expenditures are equal to 2.5% of the annual labor cost.[15]

Lastly, with standard preferences[16], parameters γ, and Γ^u are computed to be consistent with steady-state restrictions.[17]. With RW preferences, we have one additional condition (equation 9) and one additional parameter, a, as well. By using CEX micro-data in 1990, we calculate the ratio of the average unemployed consumption to that of the employed (calculation detailed in the appendix). The latter is thought of as corresponding to the related steady-state ratio in our model, by computing $\frac{C^{u\star}}{C^{n\star}} = 16\%$. Then, it turns out that a is derived from the steady-state restriction, simultaneously with the parameters $\tilde{\gamma}$ and $\tilde{\Gamma}^u$. [18] This calibration strategy entails that the consumption of the unemployed workers would have to be augmented by 24% to enjoy the same contemporaneous utility as the employed workers.

The equilibrium can now be computed numerically. Following Uhlig

[13]The search technology parameter Υ is chosen to be consistent with the steady state restriction imposed by the computed value for $\Phi^\star$.

[14]The productivity shock is assumed to be governed by the stochastic process, $\log A_{t+1} = \rho_A \log A_t + (1-\rho_A)\log A + \epsilon_{A,t+1}$ with $\epsilon_{A,t} \rightsquigarrow \mathcal{N}(0, \sigma_{\epsilon_A})$.

[15]With this calibration, the labor's share of output remains approximately equal to 60%.

[16]In that case, remember that $\frac{C_t^u}{C_t^n} = 1 \forall t$.

[17]In the appendix describing the general equilibrium, these restrictions rely on equations (27) and (28).

[18]In the appendix describing the general equilibrium, these restrictions rely on equations (29), (30) and (31).

[1999], the computation of second order moments relies on a frequency–domain technique.

2.2 Models Evaluation

We use US quarterly data for the sample period 1964:1-2002:1 (see appendix B.2). Table 1 reports statistics summarizing the cyclical properties of the US and model economies. Models include the standard RBC model (RBC), the labor market search model with standard preferences ($LMS1$) and the labor market search economy with RW preferences ($LMS2$). Simulation results are completed by Impulse Response Functions (IRF) reported in figure 1.[19]

With Prescott's [1986] calibration of the technological random process, the $LMS2$ economy accounts for 91% of the US standard deviation of real per-capita output, instead of 56% and 76% for the RBC and $LMS1$ economies, respectively. The first thing to note is how the combination of trading frictions and RW preferences allows the labor market search model to match well the volatility of the business cycle. In addition, it appears that RW preferences also account for a higher volatility of aggregate consumption and a lower volatility of the investment spending.

Let us now turn to the models' ability to replicate stylized facts featuring the US labor market: *(i)* unemployment is negatively correlated with vacancies (the Beveridge curve), *(ii)* aggregate labor input varies almost as much as the output and lags the business cycle by one quarter, *(iii)* labor's share of output behaves countercyclically and lags the cycle by four quarters, *(iv)* labor productivity and real wage are weakly correlated with both hours worked and output, and lead the business cycle by one quarter.[20]

The standard RBC model cannot account for anyone of these stylized facts (see table 1). The labor market search economy with standard preferences implies some noticeable improvements along both these lines. The

[19]For each variable, an IRF expresses the percentage deviations from steady state in response to a one percent positive technological shock.

[20]It is well-known that labor productivity leads employment by two quarters. This is consistent with the facts that output leads employment by one quarter and labor productivity leads output by one quarter (see Burnside, Eichenbaum, and Rebelo [1993] and Fairise and Langot [1994]).

$LMS1$ model is able to replicate that the variability of the labor input is greater than that of labor productivity and the real wage. Since the equilibrium wage takes into account of the outside options, which are smoother than the marginal product of labor (figure 1), the model predicts that labor productivity is more volatile than the real wage, and that the labor's share strongly lags the cycle and behaves countercyclically.[21] Lastly, trading frictions, by allowing for large hump-shaped responses of both employment and output to productivity shocks (figure 1), imply that the procyclical feature of labor productivity and the real wage is lower in the $LMS1$ economy than in the RBC model. These results show the empirical relevance of the labor market search assumption. However, the $LMS1$ model understates the volatility of both hours worked and labor's share. It also fails to replicate the leading role of the return to working on the business cycle. More importantly, the $LMS1$ model still generates grossly procyclical movements of the real wage. The contemporaneous correlations of the real wage with output and the aggregate labor input are highly overestimated, 0.89 and 0.69 in $LMS1$, instead of 0.28 and 0.03 in the data.

With the incorporation of RW preferences, most of these empirical shortcomings are overcome. The $LMS2$ model is capable of generating a weak contemporaneous correlation of the real wage with output, 0.46, and a correlation of the real wage with the aggregate labor input close to zero, 0.17. As it can be seen, the $LMS2$ model also exhibits a lower procyclical feature of labor productivity. In particular, the correlation of labor productivity with output falls to 0.31. It shall be added that RW preferences improve the performance of the labor-market search model along several other lines: variability of the labor's share is increased (despite it is still underestimated), the relative standard deviation of labor input to output is now consistent with data, labor productivity and the real wage lead by one quarter the output.

Impulse Response Functions to a technological shock in the $LMS2$ model (figure 1) give the basic intuitions for these results. At the impact period of the shock, the increase in labor productivity and hiring costs implies a rise of

[21] The fact that the labor's share of output lags the cycle has been documented for instance by Kydland and Prescott [1990].

the real wage. The decrease in the outside options then enhances the impact of the reversion of labor productivity and hiring costs to steady-state. This real wage dynamics accounts for a long-lasting increase in employment and output. This allows to match the cyclical feature of the real wage, together with the volatility of the labor input and the lead of the return to working on the business cycle.

Lastly, in line with Cogley and Nason [1995], we report the autocorrelation function of the output growth, to assess models' ability to replicate positive serial correlations of GNP growth. Figure 2 shows that the $LMS2$ model embodies the most important propagation mechanism.

To conclude this empirical evaluation, it should be emphasized that in Boldrin and Horvath [1995], contemporaneous correlations of the output with labor productivity and the real wage are 0.94 and 0.41, respectively. In Collard and De Lacroix [2000], the contemporaneous correlation of output with the real wage goes up to 0.98 if the unknown structural parameters are fixed using over-identifying restrictions based on output and real wage second order moments. In that respect, this suggests that our "bargaining explanation" of real rigidity is, at least, as plausible as their own explanations. More generally, our model allows to match an important number of aspects of the US labor market dynamics.

3 Concluding Remarks

This paper reconciles the Nash-bargaining of wages with the observed real wage dynamics. This outcome is obtained by postulating in a labor-market search model preferences who imply that, in an equilibrium with efficient risk-sharing, the utility of employed agents exceeds that of unemployed agents. Insiders agents (employed workers) are found to take more advantage of a cyclical upturn than outsiders (unemployed workers), which entails a decrease in the workers' outside options and smoothes the rise of the real wage. The calibrated model is then found to account for the weakly procyclical behavior of the aggregate real wage.

This suggests that the real wage rigidity can be explained not only by implicit contracts and efficiency wage theories but also by Nash-bargaining. The current explanation relies on insiders-outsiders conflicts instead of either firms-employees insurance relationship for Boldrin and Horvath [1995] or interpersonal employees comparisons for Collard and De Lacroix [2000]. Furthermore, it is the outcome of a non-cooperative game and does not dependent on the introduction of either institutional rigidities (labor contract being signed at least one period before shocks in Boldrin and Horvath [1995]) or reduced-form in preferences (effort function being conjectured in Collard and De Lacroix [2000]).

References

Abowd, J., and F. Kramarz (1998): "The Cost of Hiring and Separations," working paper 6110, NBER.

Andolfatto, D. (1996): "Business Cycle and Labor Market Search," *American Economic Review*, 86(1), 112–132.

Benhabib, J., R. Rogerson, and R. Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99(6), 1166–1187.

Boldrin, M., and M. Horvath (1995): "Labor contracts and business cycles," *Journal of Political Economy*, 103(5), 972–1004.

Burnside, C., M. Eichenbaum, and S. Rebelo (1993): "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, 101(2), 245–273.

Chéron, A., and F. Langot (2000): "The Phillips and Beveridge Curves Revisited," *Economics Letters*, 69(3), 371–376.

Christiano, L., M. Eichenbaum, and C. Evans (1997): "Sticky Price and Limited Participation Models of Money," *European Economic Review*, 41, 1201–1249.

Cogley, T., and J. Nason (1995): "Outputs Dynamics in Real Business Cycle Models," *American Economic Review*, 85, 492–511.

Collard, F., and D. De Lacroix (2000): "Gift Exchange and the Business Cycle: the Fair Wage Strike Back," *Review of Economic Dynamics*, 3(1), 166–193.

Cooley, T., and E. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, pp. 1–38. Princeton University Press, Princeton.

Danthine, J., and J. Donaldson (1990): "Efficiency Wages and the Business Cycles Puzzle," *European Economic Review*, 34(7), 1275–301.

Den Haan, W., G. Ramey, and J. Watson (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(2).

Fairise, X., and F. Langot (1994): "Labor Productivity and the Business Cycle: Can RBC Models be saved?," *European Economic Review*, 38(8), 1581–1594.

Gomes, J., J. Greenwood, and S. Rebelo (1998): "Equilibrium Unemployment," working paper, University of Rochester.

Greenwood, J., Z. Hercowitz, and G. Huffman (1988): "Investment, Capacity Utilisation, and the Real Business Cycle," *American Economic Review*, 78, 402–417.

Hairault, J. (2002): "Labor-Market Search in a Two-Good Two-Country Model," *Review of Economic Dynamics*, (5), 535–558.

Hosios, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57(3), 279–298.

King, R., and S. Rebelo (2000): "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics*, ed. by J. Taylor, and M. Woodford. North-Holland.

Kydland, F. (1995): "Business Cycles and Aggregate Labor Market Fluctuations," in *Frontiers of Business Cycle Research*, ed. by T. Cooley. Princeton University Press, Princeton.

Kydland, F., and E. Prescott (1990): "Real facts and a monetary myth," *Federal Reserve Bank of Mineapolis Quarterly Review*, 14, 3–18.

MaCurdy, T. (1981): "An Empirical Model of Labour Supply in a Life Cycle Setting," *Journal of Political Economy*, 89(61), 1059–1085.

Merz, M. (1995): "Search in the Labor Market and Real Business Cycle," *Journal of Monetary Economics*, 36, 269–300.

——— (1999): "Heterogeneous job-matches and the cyclical behavior of labor turnover," *Journal of Monetary Economics*, 43, 91–124.

Pissarides, C. (1990): *Equilibrium Unemployment Theory*. Basil Blackwell.

Prescott, E. (1986): "Theory Ahead of Business Cycle Measurement," *Carnegie-Rochester Conference Series on Public Policy*, 25, 11–44.

Rogerson, R., and R. Wright (1988): "Involuntary Unemployment in Economies with Efficient Risk Sharing," *Journal of Monetary Economics*, 22(3), 501–516.

Stock, J., and M. Watson (2000): "Business Cycle Fluctuations in U.S. Macroeconomic Time Series," in *Handbook of Macroeconomics*, ed. by J. Taylor, and M. Woodford. North-Holland.

Uhlig, H. (1999): "A Toolkit for Analysing Nonlinear Dynamic stochastic Models Easily," in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, and A. Scott. Oxford University Press.

Table 1: Cyclical Properties

	US			*RBC*			*LMS1*			*LMS2*		
$\sigma(Y)$	1.91			1.08			1.46			1.80		
Variable X	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Consumption	0.40	0.83	0	0.26	0.90	0	0.25	0.89	0	0.60	0.86	+1
Investment	3.07	0.97	0	4.59	0.99	0	4.60	0.99	0	3.44	0.93	0
Hours	0.86	0.82	+1	0.26	0.99	0	0.70	0.94	+1	0.93	0.95	+1
Labor productivity	0.57	0.51	-1	0.75	0.99	0	0.42	0.82	0	0.31	0.38	-1
Real wage	0.45	0.28	-1	0.75	0.99	0	0.32	0.89	0	0.16	0.46	-1
Labor's share	0.55	-0.30	+4	0	0	0	0.13	-0.41	+3	0.16	-0.30	+3
$corr$(Hours,Labor productivity)	-0.07			0.99			0.57			0.08		
$corr$(Hours,Real wage)	0.03			0.99			0.69			0.17		

Notes: $\sigma(Y)$ is the percentage deviation of output
column (1) is $\sigma(X)/\sigma(Y)$, column (2) is $corr(X,Y)$
column (3) is the phase shift in X relative to Y: $-j$ or $+j$ corresponds to a lead or lag of j quarters.

Figure 1: IRF

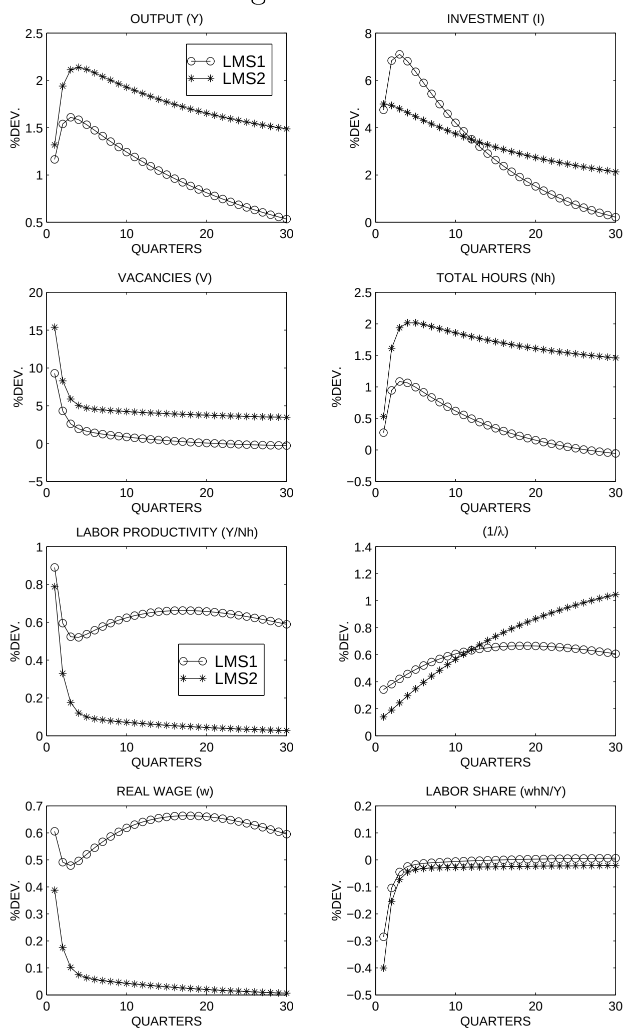
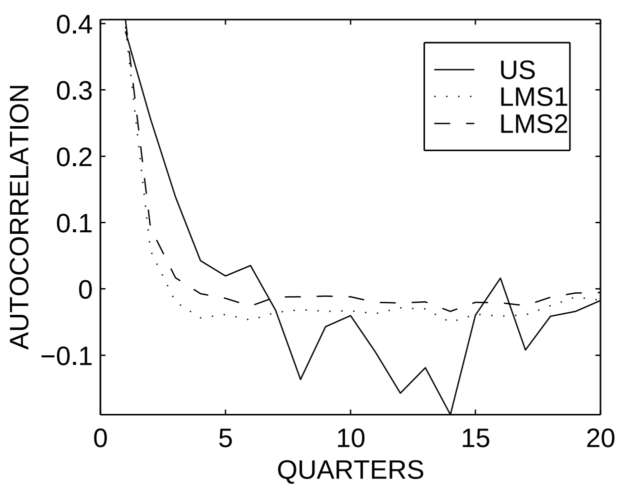

OUTPUT (Y)
LMS1
LMS2
%DEV.
QUARTERS
INVESTMENT (I)
%DEV.
QUARTERS
VACANCIES (V)
%DEV.
QUARTERS
TOTAL HOURS (Nh)
%DEV.
QUARTERS
LABOR PRODUCTIVITY (Y/Nh)
LMS1
LMS2
%DEV.
QUARTERS
(1/λ)
%DEV.
QUARTERS
REAL WAGE (w)
%DEV.
QUARTERS
LABOR SHARE (whN/Y)
%DEV.
QUARTERS

Figure 2: ACF of output growth



A Technical Appendix: Solving for the Search Equilibrium

A.1 Households

As in Andolfatto [1996], it is assumed that job flows are governed by the matching/separation process, whereas worker flows are determined exogenously by a game of "musical chairs". At the beginning of each period, the entire work force is "shuffled" randomly across the given set of jobs. Hence, N_t becomes the probability of employment for each household in any period. Given this idiosyncratic risk, a risk-averse worker may choose to purchase B_t units of unemployment insurance at a price τ_t, which delivers B_t units of the consumption good whenever he became unemployed within the period. The cash flow for an insurance company is given by $\Pi_t^A \equiv \tau_t B_t - (1 - N_t) B_t$. Free entry on the unemployment insurance market entails that a zero profit condition holds, which yields $\tau_t = 1 - N_t$.

In this environment, each household aims at choosing the contingency plan $\mathcal{C}_t = \{B_t, C_t^n, C_t^u, K_{t+1}^n, K_{t+1}^u\}$ that solves the following problem:

$$\begin{aligned} \mathcal{W}(\Omega_t^H) &= \max_{\mathcal{C}_t} \Big\{ N_t \left[U\left(C_t^n, 1 - h_t\right) + \beta E_t \mathcal{W}(\Omega_{t+1}^{H,n}) \right] \\ &\quad + (1 - N_t) \left[U\left(C_t^u, 1 - e\right) + \beta E_t \mathcal{W}(\Omega_{t+1}^{H,u}) \right] \Big\} \end{aligned}$$

$$\text{s.t.} \quad C_t^n + \tau_t B_t + K_{t+1}^n \leq (1 - \delta + r_t) K_t + \pi_t + w_t h_t \qquad (13)$$

$$C_t^u + \tau_t B_t + K_{t+1}^n \leq (1 - \delta + r_t) K_t + \pi_t + B_t \qquad (14)$$

where $\Omega_t^{H,z}$ summarizes the state of a worker of type $z = n, u$. FOC imply:

$$\begin{aligned} \lambda_t^n &\equiv U_1\left(C_t^n, 1 - h_t\right) \\ \lambda_t^u &\equiv U_1\left(C_t^u, 1 - e\right) \\ \lambda_t^u &= \lambda_t^n \\ \beta E_t \left[\frac{\partial \mathcal{W}(\Omega_{t+1}^{H,z})}{\partial K_{t+1}^z} \right] &= \lambda_t^z \qquad \text{for } z = n, u \end{aligned}$$

This implies:

$$U_1\left(C_t^n, 1 - h_t\right) = U_1\left(C_t^u, 1 - e\right)$$

Concavity and continuity of the value function imply that:

$$K_{t+1}^n = K_{t+1}^u \equiv K_{t+1}$$

The difference between the two budget constraints (equations (13) and (14)) gives the optimal choice of insurance:

$$B_t = w_t h_t - (C_t^n - C_t^u)$$

It is straightforward to show that the dynamic problem of a typical household can now be written as follows:

$$\begin{aligned}
\mathcal{W}(\Omega_t^H) &= \max_{\{C_t^n, C_t^u, K_{t+1}\}} \left\{ N_t U\left(C_t^n, 1-h_t\right) + (1-N_t) U\left(C_t^u, 1-e\right) + \beta E_t \left[\mathcal{W}(\Omega_{t+1}^H)\right] \right\} \\
\text{s.t.} \quad K_{t+1} &= (1-\delta+r_t)K_t + \pi_t + N_t w_t h_t - N_t C_t^n - (1-N_t)C_t^u \\
N_{t+1} &= (1-s)N_t + \Psi_t(1-N_t)
\end{aligned}$$

Optimal decisions for the households are thus fully summarized by:

$$\begin{aligned}
\lambda_t &= \beta E_t \left[\lambda_{t+1}(1-\delta+r_{t+1})\right] \\
\lambda_t &= U_1\left(C_t^n, 1-h_t\right) = U_1\left(C_t^u, 1-e\right)
\end{aligned}$$

A.2 Firms

Let $\mathcal{V}(\Omega_t^F)$ be the maximum expected value of the firm in state Ω_t^F. This value function must satisfy the following recursive relationship:

$$\mathcal{V}(\Omega_t^F) = \max_{\{V_t, K_t\}} \left\{ A_t K_t^{\alpha} (N_t h_t)^{1-\alpha} - w_t h_t N_t - r_t K_t - \omega V_t + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \mathcal{V}(\Omega_{t+1}^F) \right] \right\}$$

$$\text{s.t.} \quad N_{t+1} = (1-s)N_t + \Phi_t V_t \tag{15}$$

where $\Phi_t \equiv \frac{M_t}{V_t}$. Optimal decisions for the firm are:

$$\begin{aligned}
r_t &= \alpha \frac{Y_t}{K_t} \\
\frac{\omega}{\Phi_t} &= \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - w_{t+1} h_{t+1} + \frac{(1-s)\omega}{\Phi_{t+1}} \right) \right]
\end{aligned}$$

A.3 Derivation of wage and hours worked equations

The marginal value of employment for a worker is defined by:

$$\begin{aligned}\frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_t} &= \lambda_t \left(w_t h_t - C_t^n + C_t^u \right) + U\left(C_t^n, 1-h_t\right) - U\left(C_t^u, 1-e\right) \\ &\quad + \left(1 - s - \Psi_t\right) \beta E_t \left[\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_{t+1}} \right] \end{aligned} \tag{16}$$

For the firm we have:

$$\frac{\partial \mathcal{V}(\Omega_t^F)}{\partial N_t} = F_{2,t} h_t - w_t h_t + (1-s)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}(\Omega_{t+1}^F)}{\partial N_{t+1}} \right] \tag{17}$$

where $F_{2,t} h_t$ is the output for a person that works h_t hours[22]. Then, we shall define the total surplus associated with the formation of a job-worker pair (measured in units of the consumption good) as follows:

$$S_t = \frac{\partial \mathcal{V}(\Omega_t^F)}{\partial N_t} + \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_t} / \lambda_t$$

Let $0 < \xi < 1$ be the firm's share of this surplus, the sharing rule implies:

$$\xi \frac{\partial \mathcal{V}(\Omega_t^H)}{\partial N_t} = (1-\xi)\lambda_t \frac{\partial \mathcal{V}(\Omega_t^F)}{\partial N_t} \tag{18}$$

From the firm's side, we have the following first-order condition:

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}(\Omega_{t+1}^F)}{\partial N_{t+1}} \right] = \frac{\omega}{\Phi_t} \tag{19}$$

Equations (18) and (19) yield:

$$\xi E_t \left[\frac{\beta}{\lambda_t} \frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_{t+1}} \right] = (1-\xi)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}(\Omega_{t+1}^F)}{\partial N_{t+1}} \right] = (1-\xi)\frac{\omega}{\Phi_t} \tag{20}$$

By combining conditions (16), (17), (18) and (20), and given that $F_{2,t} \equiv (1-\alpha)\frac{Y_t}{N_t}$, we derive the following equilibrium wage rule:

$$w_t h_t = (1-\xi) \left[(1-\alpha)\frac{Y_t}{N_t} + \frac{\omega V_t}{1-N_t} \right] + \xi \left[C_t^n - C_t^u - \frac{U\left(C_t^n, 1-h_t\right) - U\left(C_t^u, 1-e\right)}{\lambda_t} \right] \tag{21}$$

[22] Given the production function (2), we know that $F_{2,t} \equiv (1-\alpha)[Y_t/N_t h_t]$. Since the ratio of the capital stock to the labor input $K_t/N_t h_t \equiv k_t$ is determined by the rental cost, $r_t = \alpha A_t k_t^{\alpha-1}$, it comes that $F_{2,t} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} r_t^{\frac{\alpha}{\alpha-1}}$. Following Pissarides [1990] and Andolfatto [1996], it is assumed that the weight of a worker is 'small' so that $F_{2,t}$ is taken as given by the agents during the bargaining process.

This wage equation reflects upon the specification of preferences (equations (3) and (8)), by satisfying either conditions (4)-(5) or conditions (9)-(10), respectively. It is then straightforward to obtain the equilibrium wage rules reported in the main text.

Lastly, the optimal level of hours per worker solves the maximization problem of S_t with respect to h_t. This leads to the following relationship:

$$\lambda_t F_{2,t} = (1-h_t)^{-\eta} \times \begin{cases} \tilde{\gamma}(\lambda_t - a) & \text{if non-separable preferences} \\ \gamma & \text{if separable preferences} \end{cases}$$

A.4 The Search Equilibrium

The search equilibrium is characterized by the following system of equations:

$$N_{t+1} = (1-s)N_t + \Upsilon V_t^{\psi}(e(1-N_t))^{1-\psi} \tag{22}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1}\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)\right] \tag{23}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{24}$$

$$Y_t = I_t + C_t + \omega V_t \tag{25}$$

$$\frac{\omega}{\Phi_t} = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - w_{t+1}h_{t+1} + (1-s)\frac{\omega}{\Phi_{t+1}}\right]\right\} \tag{26}$$

$$\Phi_t = \Upsilon\left(\frac{V_t}{e(1-N_t)}\right)^{\psi-1}$$

$$Y_t = A_t K_t^{\alpha}(hN_t)^{1-\alpha}$$

and, either the following conditions if preferences are defined by equation (3),

$$\lambda_t = C_t^{-1}$$

$$w_t h_t = (1-\xi)\left[(1-\alpha)\frac{Y_t}{N_t} + \frac{\omega V_t}{1-N_t}\right] + \xi\left(\frac{\Gamma_t}{\lambda_t}\right) \tag{27}$$

$$(1-\alpha)\frac{Y_t}{h_t N_t} = \frac{\gamma(1-h_t)^{-\eta}}{\lambda_t} \tag{28}$$

or the following ones if preferences are defined by equation (8),

$$\lambda_t = \left[C_t^u + \tilde{\Gamma}^u\right]^{-1} + a$$

$$
\begin{aligned}
C_t^n - C_t^u &= \tilde{\Gamma}_t & (29)\\
w_t h_t &= (1-\xi)\left[(1-\alpha)\frac{Y_t}{N_t} + \frac{\omega V_t}{1-N_t}\right] + \xi\left(\frac{\lambda_t - a}{\lambda_t}\right)\tilde{\Gamma}_t & (30)\\
(1-\alpha)\frac{Y_t}{h_t N_t} &= \tilde{\gamma}(1-h_t)^{-\eta}\left(\frac{\lambda_t - a}{\lambda_t}\right) & (31)
\end{aligned}
$$

B Data

B.1 Micro-Data

In order to compute the ratio of the unemployed consumption to that of the employed we use micro-data from the Consumer Expenditure Survey (CEX) in 1990.

Table 2: Consumer units of two or more persons

	No earner	One earner	Two earners	Three or more
NCU	9183	18442	31918	10162
AAE	18960$	28010$	35898$	43443$

NCU: Number of consumer units (in thousands)
AAE: Average total annual expenditures (in dollars)

A no-earner household[23] is a household in which none of the members worked more that one week during the last year. Since there is on average 3.1 agents in the household (with two or more agents), to be consistent with our model we define: $C^{no} = \frac{18960}{3.1}$. This value is thought of as corresponding to the average consumption of a representative no-earner agent. The latter is either non-employed or retired. Then, we can get the average employed consumption (of an agent who worked more than one week during the last year) by imputing the average consumption of no-earners in each type of households:

[23] A household is defined as a consumer unit following the definition adopted in the CEX survey.

$$C^n = \frac{18842 \times (28010\$ - 2.1 \times C^{no}) + 31918 \times (35898\$ - 1.1 \times C^{no}) + 10162 \times (43443\$ - 0.1 \times C^{no})}{18442 + 31918 \times 2 + 10162 \times 3}$$

This implies that $\frac{C^{no}}{C^n} = 0.42$. In addition, the sample of persons (P), so that $P = N + U + R$, includes employed workers (in number N) non-employed workers (U) and retired agents (R). Our construction of the average employed consumption entails $N/P = 0.52$. In addition, since in 1990 the number of retired household requires $R/P = 0.15$, we find $U/P = 0.33$ to be the percentage of non-employed workers in the population, and $N/[U+N] = 0.61$ to be the employment rate. Finally, the ratio of the unemployed consumption to that of the employed is found by solving for the following condition:[24]

$$C^{no} = \frac{UC^u + RC^r}{U+R} \Longleftrightarrow \frac{C^u}{C^n} = \frac{C^{no}}{C^n} - \frac{R}{U}\left(\frac{C^r}{C^n} - \frac{C^{no}}{C^n}\right)$$

Let assume that the consumption of a retired person equals that of an employed person, $C^r = C^n$, we obtain $\frac{C^u}{C^n} = 16\%$. This number is thought of as corresponding to the steady-state ratio of the unemployed consumption to the employed one, in our model.

B.2 Macro-Data

The macro-data used in this study is real aggregate data of the United States for the sample period 1964:Q1-2002:Q1; the source is the Federal Reserve Economic Data (FRED) bank at the Federal Reserve Bank of Saint Louis (HTTP://RESEARCH.STLOUISFED.ORG/FRED/).

- Consumption (C) = real consumption of nondurable goods + real consumption of services (National Income and Product Account)
- Investment (I) = real consumption of durable goods + real fixed private investment (National Income and Product Account)
- Output (Y) = $C + I$

[24] Since the distinction between unemployed and *not in the labor force* is ignored in our model, as in Andolfatto [1996], we improperly refer to non-employed workers as unemployed workers.

- Hours (Nh) = Aggregate Weekly Hours: Private Nonfarm Payrolls (Establishment survey)
- Real wage (w) = Real Compensation Per Hour: Nonfarm Business Sector (Establishment survey)
- Labor productivity ($Y/(Nh)$) = Y/Nh
- Labor's share of output = wNh/Y