Predicting Bidders' Willingness to Pay in Online Multi-Unit Ascending Auctions:

Analytical and Empirical Insights

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Abstract

A large fraction of online auction activity deals with selling multiple identical units of an item, using progressive discriminatory or uniform pricing approaches. Our research objective is to gain insights into the price formation process of such auctions, focusing primarily on the progressive bid information that is revealed to infer bidders' willingness to pay (WTP) in real time. We derive *a priori* estimates of bidders' maximum willingness to pay based on the assumption of a myopic best response relationship between the bids made and their underlying valuations. Unlike prior econometric studies that utilize aggregate data to understand underlying demand curves of products, we use an automated agent to capture observable micro-data from bidding activity in several hundred Internet auctions. We present an analytical model for myopic best response bidding in multi-unit progressive discriminatory (a.k.a. Yankee) and uniform pricing options, and use the inverse function to estimate the underlying willingness to pay from the *joint consideration* of the revealed bid values and the bid strategies adopted. We test the WTP prediction model against thousands of bids made on hundreds of real online auctions from Samsclub.com and Ubid.com. Our analysis is able to accurately "type" the bidding strategy based on observable variables using a Logit classification model. Results indicate that approximately 2/3 of online bidders can be considered as using a myopic best response strategy. Our prediction results indicate that we are able to estimate, on average, within 3% of revealed willingness to pay for Yankee auctions, and within 4.7% for uniform multi-unit auctions. In addition, we are able to estimate, on average, within 7% of the final auction price by the 30th time percentile of the uniform price auctions, and within 10% of the final auction price by the 40th time percentile of a discriminatory Yankee auction.

1. Introduction and Background

Online auctions exemplify the Internet's ability to become a temporally and spatially unconstrained market maker. A large fraction of online auction activity deals with selling multiple identical units of an item, such as aging computer hardware, using either a discriminatory (Ubid.com) or a uniform (Samsclub.com) pricing approach. Our research objective is to gain deeper insights into the price formation process of auctions that progressively reveal more information about bidders' willingness to pay, and use those insights into deriving *a priori* estimates of the expected bid revisions.

We should point out the subtle distinction between the traditional notion of the *valuation* of a product and our use of the phrase *willingness to pay*. Both reflect private information that the bidder possesses, but given the online context, *willingness to pay* bounds *valuations* from below. This is primarily driven by the use of a suggested retail price signal that effectively caps the willingness to pay, irrespective of the valuations. In no case, in our extensive dataset of consisting of 78,014 bids from over 900 online auctions, did the final bid exceed 90% of the suggested retail price. Section 3 describes this in more detail. For this reason, in the rest of this paper we use the phrase *Willingness To Pay (WTP)*.

While demand estimation has been the focus of many econometric papers, it has typically been done using aggregate data, with results hinging on some tenuous assumptions about consumers' preferences. In contrast, we rely on Internet enabled multi-unit auctions to undertake demand estimation using micro-data, making empirically established, and also less demanding assumptions about bidder behavior.

Progressive ascending multi-unit auctions have received only limited attention in the literature, usually under a set of assumptions that do not hold up in the online context. For instance, bidders are assumed to be homogeneous, typically typed as being symmetric, risk-neutral, and adopting Bayesian-Nash equilibrium strategies. While tenable in the context of face-to-face single item auctions, this set of assumptions readily breaks down in the vast majority of multi-unit online auctions. For such auctions, it is well known that the computation of equilibrium bidding strategies is intractable (Nautz & Wolfsetter, 1997). ***A key distinguishing feature of our work is a minimalist myopic best response bidding assumption that ties bidders revealed bids to their underlying willingness to pays***. Our empirical results, described in detail in Section 5, indicate that approximately 2/3 of online bidders conform to this strategy.

In this research, we wish to fill this gap in the literature by creating an analytical model that capitalizes on the online environment's enhanced information acquisition capabilities. We begin by tackling the tricky issue of myopically predicting a consumer's willingness to pay for a product, based on the joint consideration of the bidding strategy pursued and the bid values revealed, both of which are observable on the Internet. The key ingredients here are an understanding of the bidding strategies pursued by the bidders, developing a real-time ability to detect such strategies as the auction progresses (by means of a logistic classification model) and finally, an analytical model that imputes a bidder's wiliness to pay from the bids made. We present an analytical relationship¸ for both discriminatory (a.k.a. Yankee auctions) and uniform pricing auctions, between the bids made and the private underlying willingness to pay, assuming a myopic best response strategy that is adopted by a significant percentage of online bidders.

In order to test the analytical model for predicting bidders' willingness to pay, we use an automated agent based data collection tool, and measure the predictions against the revealed preferences observed in the thousands of bids made on hundreds of real online auctions from Samsclub.com. Our data analysis indicates that the classification model is able to accurately “type” the bidding strategy based on observable variables, and the prediction model comes close to predicting the bidder’s wiliness to pay, as estimated by their final bids.

Bapna, Goes and Gupta (2003a) and Engelbrecht-Wiggans (1999) demonstrate that progressive multi-unit auctions have multiple equilibria, some of which are more desirable than others, from a revenue perspective. Further, Bapna, Goes and Gupta (2003b) have also shown that online auctioneers are often far away from optimal mechanism design choices that could increase their likelihoods of obtaining the desirable equilibria. In this context, real-time value discovery tools, such as the one demonstrated in this paper, will provide the foundation for dynamically calibrating the online auction mechanism, so as to maximize their likelihood of obtaining the desirable equilibria. They can also serve as building blocks for designing the next generation of smart bidding agents whose incentives are aligned with bidders.

In this paper, our primary research goal is to demonstrate how we can assess bidders’ willingness to pay in progressive multi-unit auctions. This predicted WTP also allows us to estimate, with high accuracy, the expected closing price of these auctions, during early stages of the auction. *To the best of our knowledge, no other study has used the enhanced information acquisition and processing capabilities of the online environment, where observance of the price formation process can be used to infer bidder willingness to pay in real time.*

The rest of this paper is organized as follows. In section 2, we provide an overview of the literature on online auctions and value prediction. In section 3, we provide insights into the market mechanism that we are investigating. We describe its basic design and provide some performance indicators. In section 4, we develop the analytical model for predicting bidder’s willingness to pay. The bidder strategy classification and prediction accuracy are tested empirically in sections 5 and 6 respectively. Lastly, in section 7 we conclude and present directions for future work.

2. Relevant Literature

Given the vast body of auction literature [see, for example, McAfee and McMillan (1987), Milgrom and Weber (1982), Milgrom (1989), Rothkopf and Harstad (1994) and Menezes (1996) for a detailed literature review and analysis] it is instructive to begin by briefly examining what, if anything, is new about online auctions. Arguably, online auctions have expanded scope and scale, compared to their traditional counterparts. There is early evidence that participation in online auctions is endogenously influenced ([Bajari and Hortascu (2001)] have shown, with data from eBay, that modifying the mechanism affects the entry decisions), while the traditional assumption in the literature takes the number of bidders at an auction as exogenously given [Paarsch (1992), Laffont, Ossard and Vuong (1995)]. The expanded scale and scope of the auction has made the participation of online auctions non-captive of its audience. The bidders in online auctions come and go at will, while in traditional environments bidders are captivated through the close of the auction. Particularly relevant to our work are the enhanced computational and networking resources that have made multi-unit auctions more feasible. Multiple units can be sold simultaneously, not as a single lot, but to multiple buyers who exhaust the lot.

Another significant difference between the two auction environments is that the online environment does not benefit from the skills and experience of human auctioneers. The online auctions are propelled by static rules that govern the constitution and submission of valid bids, while traditional auctions benefit from the experience of the auctioneer to pit bidders against each other by skillfully assessing the bidders utility and pacing the auction bidding accordingly. Put succinctly, our research is aimed at marrying the best of both worlds. Our goal is to commoditize the human expert who can run a single auction expertly, and substitute it with a computationally intensive real-time decision making tool that could, armed with the prediction information which is the focus of this paper, potentially support the simultaneous conduction of hundreds of online auctions in a more efficient manner.

Note that a recurring theme in this study is the use of the information available on hand to the online auctioneer. Thus, critical to our work is the open auction format in which value signals are iteratively broadcasted to the participating agents. If understood correctly, these signals can explain the underlying bidder valuations or reserve prices, and can form a vital input in enhancing a mechanisms capability to equitably allocate resources. Carare (2003) demonstrates the utility of working with micro-data, observable in the online auction environment, to derive marginal valuations of bidders' for CPU specific variables. The goal of Carare (2003) is to recover distributions of valuations for a specific product, namely computer processors, and it uses data solely from discriminatory (pay-your-bid) online auctions. Our work, in contrast, attempts at modeling bidding behavior for real-time predictive purposes, for a broad spectrum of products sold through, both uniform and discriminatory, multi-unit online auctions[4].

Crampton (1998) identifies the benefits of progressive open multi-unit auctions over their sealed bid counterparts; (i) efficiency of the price discovery process; (ii) revenue maximization; (iii) reduction of the winners curse; and (iv) privacy and implementation. On the other hand, Engelbrecht-Wiggans and Kahn (1997) and Engelbrecht-Wiggans et.al (1999), show that multi-unit auctions, especially those that use a uniform pricing scheme, give bidders an incentive to reduce their demand, resulting in inefficient allocations. Ausubel (1997) proposed an ascending-

[4] Both are widely used in the B2C online market. Ubid.com and Samsclub.com are representative popular sites.

bid auction for multiple-units, that ameliorates the demand reduction incentive in multi-unit auction by progressively and iteratively increasing the "asking" price with each iteration of the auction. However, Ausubel (1997) does not show how auctioneers should determine the increments of the "ask" price. The price increment aspect has implications on auction efficiency and revenue. We posit that accurate prediction of bidders' willingness to pay can form the basis of dynamically determining optimal asks. This remains a promising area of future research.

The utility of valuation prediction has been recognized in the Artificial Intelligence field, where automated agents employ value discovery models as components of bidding agents. Parkes and Ungar (2000), use the notion of myopic best-response bidding strategies among agents to illustrate how proxy bidders that embrace this strategy can be shielded from manipulation. In their paper, myopic best response is described as a bidding strategy where bidders submit bids that maximize their utility, given the prevailing prices. An initial research challenge in adopting the Parkes and Ungar (2000) approach is determining whether such bidders exist in the online environment we are considering. This argument connotates that bidders are perhaps non-homogenous in their bidding approaches in the online environment, and is supported by the early work in this area of Bapna, Goes and Gupta (2001) who identify at least three different bidding strategies adopted in Yankee auctions. For a real-time prediction and calibration approach to be applicable it is first necessary to understand the bidding strategy space adopted by the bidders, and then have the ability to use the information available during the course of the auction to accurately type the bidders into the strategic space. We demonstrate which categories of bidder's fall under the myopic best response bidding strategy bucket, and how to detect them in real time.

In a related study, Plott and Salmon (2001), use a surplus maximization strategy to describe bidding behavior in simultaneous ascending auctions. Although the auction mechanism is different from the one studied by Parkes and Ungar (2000), the notion of myopic best response is used as a way of tying bidders' iterative type revelation, to their wiliness to pay.

Attempts to predict wiliness to pay among bidders are done as an effort to increase the efficiency of resource allocation. Even for fairly well developed markets, such as the exchange markets for financial instruments, predictions of agents' valuations has been attempted through the establishment of market pre-opening games that solicit bidder demands without actual commitments. Using the Paris Bourse as a test bed, Biais et al (1999), examined the accuracy of valuation information derived from pre-opening market trade games. Their study shows that although the information derived from such games is noisy in the early stages of the game, there is some convergence to true market values as the market opening time approaches. The approach of the price formation study by Biais et al (1999), depicts environmental similarity to our approach of predicting bidder wiliness to pay in open ascending price auctions. The initial phase of such auctions is equivalent to the pre-opening game at the Paris Bourse, and the later stages of the auction can expect to witness more concerted and accurate revelations. Consequently, we expect our prediction results to improve as the auction progresses, a result we demonstrate.

Other studies have approached value prediction as a learning activity, where the predictor seeks to know the actual bidder valuation that is masked behind observed bids. Economic game theory literature provides two dominantly used models of agent learning: the fictitious play and reinforced learning model. Dekel, Fudenberg, and Levin (2001) provide insights of these two learning models in the context of playing Bayesian games. Their results show the conditions necessary for Nash equilibrium play in repeated games. A limitation of their study is the very restrictive assumptions necessary to justify the concept of Nash equilibrium. Additionally, the study underscores the complexity of the problem with growing number of bidders and bidder strategies. The study takes the context of a repeated game. Although an iterative auction provides multiple opportunities for bidders to revise their bids, each bid revision occurs in a different context from the previous one. The prices, the bidders, and essentially the auction environment are different.

In the next section we develop our wiliness to pay prediction model.

3. Progressive Online Multi-unit Auctions

Our research deals with a popular online auction mechanism in the wider B2C category of auctions. This mechanism offers consumers multiple units of the same item. Bidders compete for the items, with each bidder submitting a bid indicating the quantity they desire and the per unit price they are willing to pay. These auctions are conducted in an open format and bidders can see the bids of competing bidders. Bidder participation in these auctions increases over time. Bidders join the auction at anytime during the auction duration. Thus although the auctions share some similarities to the traditional auctions, bidders are not captives of the auction process, as is the case in the latter mechanism.

The auctioneer spells out auction rules that govern the bidding activity. The main sets of rules guide the constitution and submission of bids are as follows:

The minimum required bid: All bidders are expected to submit bids that are at least as high as the minimum required bid. This rule is important as long as the units supplied are fewer than demand. When demand quantities exceed the lot size, subsequent bidding is guided only by the bid increment. The bid increment is the minimum increment by which a bidder must exceed the minimum winning bid in order to win an item in the auction. If a bidder exists that is willing to bid at new this level, the minimum winning bid is displaced from the winning list and replace by the new bidder. Bidders are not bound to bid in increments of the bid increment, and as noted by Easley and Tenorio (1999), jump bidding is often observed in Yankee auctions.

The auction sites give the auction closing time. Some auctions sites extend the auction duration if bidding activity is observed at last few minutes of the auction. Samsclub.com auctions refer to this design as *Popcorn* auctions.

Another feature of online auctions is the suggested retail price, or a buy–out price. With the suggested price, the auctioneer gives bidders an indicative price at which they can acquire the same product. The buy-out price has a similar effect, but also affords the bidders the chance of buying the product at the suggested price instead of participating in the bidding process. Essentially, these variables cap the performance of the auction to the suggested values, as rational bidders will not exceed bidding beyond the suggested retail price, and where a buyout price exists, rational bidders will seize this opportunity once it becomes eminent that the bidding will exceed the buyout price. A summary of final auction prices relative to the suggested retail prices is given in Table 1. Note that at the time that we collected the data, Ubid.com did not feature a suggested retail price; hence we do not include those auctions in the results presented in Table 1. The data is classified by the size of deviations and also by the intensity of bidding activity, which is given as a ratio of the number of bidders to the auction lot size. ***In no case, in our extensive dataset of 787 uniform price online auctions, did the final bid exceed 90% of the suggested retail price***. Additionally, this holds regardless of the intensity of bidding as given by the ratio of bidders to auction lot size. The *p*-values support a hypothesis that the final auction bids are smaller than the suggested retail prices.

Bidders to lot size ratio		< -50%	-50% - < -20%	-20% - < -10%
1	Number of bidders	153	117	2
	Average deviations	-63.49%	-37.37%	-18.46%
	P- value	0.000	0.000	0.005
>1 – 2	Number of bidders	732	178	3
	Average deviations	-63.42%	-40.41%	-15.39%
	P- value	0.000	0.000	0.000
>2 -3	Number of bidders	44	9	1
	Average deviations	-61.80%	-34.73%	-13.33%
	P- value	0.000	0.000	0.000
> 3	Number of bidders	5	1	1
	Average deviations	-66.25%	-47.37%	-13.33%
	P- value	0.000	0.000	0.000

Table 1 : Percent Deviation of Finale Bids to Suggested Retail Prices

We can surmise from these results that the bidders willingness to pay is capped by the suggested retail prices. In the next section we provide a method for iteratively predicting the bidders' willingness to pay.

4. Prediction Model - Myopic Best Response Strategy

Our prediction is based on the assumption that there exists a myopic best response strategy that defines the relationship between the underlying value of a product and the bid made by the bidder in a competitive exchange. We will later develop a classification scheme that will be able to detect, in real time, bidders who do and do not adopt this strategy.

We begin with the model for the myopic best response bidding strategy. This strategy can be interpreted as a surplus-maximizing bid calculated by a bidder in a given round of the auctions assuming that all his competitors bids remain unchanged from the previous round. As new arrivals come in, and bidders get displaced from the winning list, we allow for belief revision (in a Bayesian sense) by the same bidder, to account for the additional information that is available. This results in a revision of the bidder's willingness to pay each time a bidder revises her bid.

Consider an auction for N units of an item. Let the current winning bids be denoted by $x_1, x_2, ..., x_N$, ordered by magnitude and within magnitude, by time of submission. Bidders submitting these bids are assumed to have private willingness to pay (WTP) values equal to $W_1, W_2, ..., W_N$ respectively. These WTP values are bounded below by the bids already submitted, that is $W_1 \geq x_1, W_2 \geq x_2, ..., W_N \geq x_N$. When a new bid z is received, it must be greater than x_1, which is displaced from the winning list. We assume that the new bid z was determined to myopically optimize the expected gain that the bidder will derive from the auction. The myopic best response (hereafter referred to as MBR) strategy forms the basis of our WTP prediction approach. Much like the myopic best-response bidding strategy of Parkes and Ungar (2000), our strategy maximizes a bidders expected surplus, given the already submitted bids and a belief on the actual WTP values of bidders who submitted the earlier bids. The belief regarding other bidders WTP values is a probability distribution with support in the range of the lowest winning bid and an upper bound, which can be set to a publicly known price for the item being auctioned, such as the suggested retail price for the auction. Recall, that our empirical analysis of the suggested retail price (Table 1) indicated that it was indeed an effective cap on the support of the distribution The myopic approach allows for belief revision as the auction progresses, as bidders who resubmit bids revise their initial beliefs about others' WTP values. Implicit in the above discussion is the condition, that a significant percentage of bidders do indeed use this strategy. We empirically test for this condition in section 5.

Let the bidder who submitted the new bid z have a WTP value denoted by W. Suppose that the new bid z is greater than k of the current winning bids. Therefore, the new sequence of winning bids is $x_2, x_3, ..., x_k, z, x_{k+1}, ..., x_N$. For the new bidder, assuming a MBR strategy, to win given this state of the auction, at least one of the k bidders whose bids are smaller than z must have a WTP value that is less than z , assuming no new bidders join the auction. This search is conducted across the known all potential bidders, those that have revealed some preference. As the approach allows for bid revisions, the information signals of the new arrivals are, by design, captured ex post.

Let the new bidder's belief about the WTP values of any of the current winners, be an independent random variable with a density function f_i , and a distribution function F_i , with support in the range $\lfloor x_1, m \rfloor$, where x_1 is the smallest winning bid, and m is an indicative fixed price for the item. The indicative fixed price could be assessed using price comparison agents that are available on the Internet. Also, a number of online auction sites provide indicative retail prices for items being auctioned[5]. Figure 1 below illustrates a generic belief function for a specific bidder's WTP value, conditional on the submitted bid.

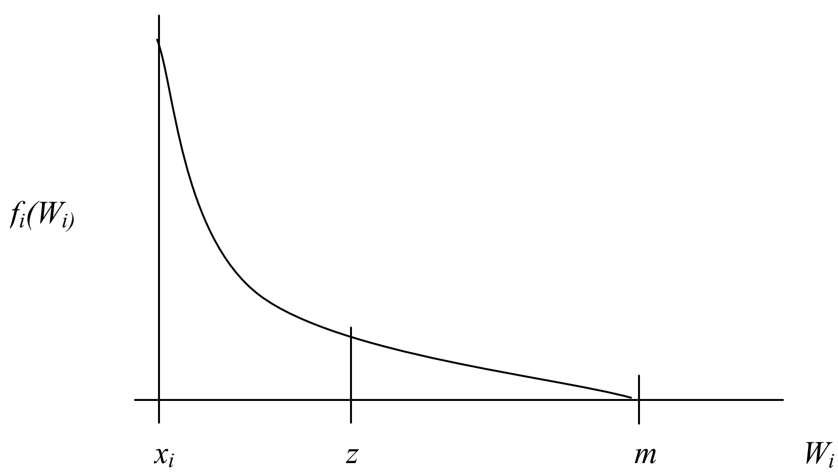


Figure 1: Willingness to Pay Value Belief Function

The probability that a current winner's WTP is greater that z is given as;

[5] Ebay has what is called a 'buy-it-now' price, and Ubid suggest a 'maximum bid price.'

$$P[W_i > z] = 1 - \int_z^m f_i(W_i)dW_i = 1 - F_i(z) \qquad (1)$$

Assuming that the WTP value for the bidders are independently distributed, the probability that at least one of the currently winning bidders has a WTP value that is less than z is,

$$1 - \prod_{i=1}^{k} (1 - F_i(z)) \qquad (2)$$

Observe that this is an asymmetric model in the support of the distribution of the individual bidders, hence the use of the $\prod$ notation and the *i* subscript to the WTP distribution. Let the price paid by the new bidder equal to P_d and P_u in Yankee and uniform price auction. It is obvious that $P_d = z$, as each bidder pays a price equal to their bids in yankee auction. On the other hand $x_1 < P_u \leq z$, and will be a function of the willingness to pay of the *k* bidders who are outbid by the new bid and by the value of the bid new z itself. Thus $P_u \equiv \varphi(W_1, W_2, ..., W_k, z)$.

If the auction uses a discriminatory pricing scheme, where bidders pay a price equal to their bids, the new bidder will enjoy an expected gain equal to:

$$E(G) = (W - z) * \left(1 - \prod_{i=1}^{k} (1 - F_i(z))\right) \qquad (3)$$

And the expected gain in a uniform price auction will be given by:

$$E(G) = (W - P_u) * \left(1 - \prod_{i=1}^{k} (1 - F_i(z))\right) \qquad (4)$$

We assume that the observed bid z optimizes the expected gain expressions given above at equations 3 or 4, depending on the pricing scheme. Therefore, the observed bid should satisfy the first and second order conditions for a maximum expected gain. Equations (5) and (6) show the first order conditions for maximum expected gain under a Yankee and uniform pricing scheme respectively.

$$\frac{\partial(E(G))}{\partial z} = -\left(1-\prod_{i=1}^{k}(1-F_i(z))\right) + (W-z)\left(\sum_{j=1}^{K} f_j(z)\prod_{i=1-\{j\}}^{k}(1-F_i(z))\right) = 0 \qquad (5)$$

$$\frac{\partial(E(G))}{\partial z} = -P_u^{'}\left(1-\prod_{i=1}^{k}(1-F_i(z))\right) + (W-P_u)\left(\sum_{j=1}^{K} f_j(z)\prod_{i=1-\{j\}}^{k}(1-F_i(z))\right) = 0 \qquad (6)$$

Where $P_u^{'} = \frac{\partial P_u}{\partial z} = \frac{\partial \varphi(W_1, W_2, ..., W_k, z)}{\partial z}$

After observing the bid *z*, and assuming that it was determined by the bidder to maximize his expected gain, we can make inferences about the corresponding WTP value of the new bidder. By solving equations 5 and 6 for W, we get the predicted WTP value of the bidder under the respective pricing scheme. The expressions for WTP value prediction are given in equations 7 and 8.

$$\hat{W}_{Yankee} = z + \frac{\left(1-\prod_{i=1}^{k}(1-F_i(z))\right)}{\left(\sum_{j=1}^{K} f_j(z)\prod_{i=1-\{j\}}^{k}(1-F_i(z))\right)} \qquad (7)$$

$$\hat{W}_{Uniform} = P_u + \left(\frac{P_u^{'}\left(1-\prod_{i=1}^{k}(1-F_i(z))\right)}{\left(\sum_{j=1}^{K} f_j(z)\prod_{i=1-\{j\}}^{k}(1-F_i(z))\right)}\right) \qquad (8)$$

In the next subsection we present the prediction model assuming a Triangular distribution of bidders WTP values. This distribution closely resembles the belief function of Figure 1, and is analytically tractable.

4. 1 Distribution Specific WTP Prediction Model

For expositional clarity, we first present our prediction model for an auction with *N=2* items on sale. Subsequently, we generalize the model for any *N*. Let the current winning bids be x_1 and x_2 with $x_1 \leq x_2$ and their corresponding WTP values of W_1 and W_2 respectively. Recall, that the actual distribution of the bidder's WTP is bounded above by m, a known fixed price for the item being auctioned. Bidders' WTP values are assumed to be independent and following a triangular distribution, with support in the range $[x_1, m]$. It follows with certainty that the actual WTP is greater or equal to the bid submitted. It is also realistic to expect that the chance of the actual WTP value being greater than any point between the distribution support range, decreases as the point of reference increases.

For the 2 unit case, Appendix 1 exhaustively enumerates the feasible auction outcomes as a consequence of the third bid submission, as well as the likelihood of each outcome. Consider the case, where $z > x_2$. Since $x_2 > x_1$, by the transitivity axiom of real numbers, it follows that $z > x_1$. Under this scenario Appendix 1 (under the case $z > x_2$) lists the six possible price outcome, along with their respective likelihood, derived from the Triangular distribution. Aggregating the feasible outcomes and their likelihood, the new bidder can expect to pay a price equal to x_2 with probability $\left(1-\frac{(m-x_2)^2}{(m-x_1)^2}\right)$ or pay $\frac{(x_2+z)}{2}$ with probability $\left(\frac{(m-x_2)^4-(m-z)^4}{\left((m-x_2)^2(m-x_1)^2\right)}\right)$. The expected gain for the bidder would thus be as shown in equation 9.

$$E(G)=(W-x_1)\left(1-\frac{(m-x_2)^2}{(m-x_1)^2}\right)+\left(V-\frac{(x_2+z)}{2}\right)\left(\frac{(m-x_2)^4-(m-z)^4}{\left((m-x_2)^2(m-x_1)^2\right)}\right) \qquad (9)$$

A bid that maximizes the expected surplus should satisfy the expression given in equation 10.

$$\frac{\partial(E(G))}{\partial z} = -\frac{1}{2}\left(\frac{(m-x_2)^4-(m-z)^4}{\left((m-x_2)^2(m-x_1)^2\right)}\right)+\left(W-\frac{(x_2+z)}{2}\right)\left(\frac{4(m-z)^3}{\left((m-x_2)^2(m-x_1)^2\right)}\right)=0 \qquad (10)$$

Assuming that the received bid z maximizes the computed surplus, we solve for the inferred WTP value W, and present the predicted WTP expression in equation 11.

$$\hat{W} = \frac{1}{2}\left(x+z+\frac{\left((m-x_2)^4-(m-x_1)^4\right)}{4(m-z)^3}\right) \qquad (11)$$

In the same spirit, the expression for expected surplus and predicted WTP values are presented in Table 2 for all possible out comes of a 2-unit auction.

Pricing Scheme	Bid Range	Expected Gain	Predicted WTP Values
Uniform Pricing	$x_1 < z \le x_2$ **(i)**	$(W-z)\left(1-\frac{(m-z)^2}{(m-x_1)^2}\right)$	$\frac{1}{2}\left(3z-m+\frac{(m-x_1)^2}{(m-z)}\right)$
	$z > x_2$ **(ii)**	$(W-x_2)\left(1-\frac{(m-x_2)^2}{(m-x_1)^2}\right)+\left(W-\frac{(x_2+z)}{2}\right)\left(\frac{(m-x_2)^4-(m-z)^4}{\left((m-x_2)^2(m-x_1)^2\right)}\right)$	$\frac{1}{2}\left(x_2+z+\frac{(m-x_2)^4-(m-z)^4}{4(m-z)^3}\right)$
Yankee Pricing	$x_1 < z \le x_2$ **(iii)**	$(W-z)\left(1-\frac{(m-z)^2}{(m-x_1)^2}\right)$	$\frac{1}{2}\left(3z-m+\frac{(m-x_1)^2}{(m-z)}\right)$
	$z > x_2$ **(iv)**	$(W-z)\left(1-\frac{(m-z)^4}{(m-x_2)^2(m-x_1)^2}\right)$	$z+\left(\frac{(m-x_2)^2(m-x_1)^2-(m-z)^4}{4(m-z)^3}\right)$

Table 2: Prediction of WTP Values for a 2 Unit Auction (Triangular Distribution)

Next we generalize the WTP prediction for model for any N.

4.2 Generalized WTP Prediction Model

Given a lot size of *N* items and an ordered list of currently winning bids, $x_1, x_2, ..., x_N$, a new bidder will displace bid x_1. The probability of the new bid z winning when it is greater than k of the *N* currently winning bids is equal to the probability that at least one of the *k* bidders has a WTP value that is less than z. Note that the results provided above for the two item auctions are valid for the generalized prediction model with a lot size when *k =1 or 2*. Table 3 contains the summarized results for the expected surplus and bidder's WTP value prediction, assuming that bidders' actual WTP follow independent triangular distributions, with support in $\lfloor x_1, m \rfloor$.

Pricing Scheme	Expected Gain	Predicted WTP Value
Uniform Pricing	$\left(W - \frac{x_1 + z}{2}\right)\left(1 - \frac{(m-z)^{2k}}{\prod_{i=1}^{k}(m - x_i)^2}\right)$	$\frac{1}{2}\left(z + x_2 + \frac{(m-z)}{2k}\left(\frac{\prod_{i=1}^{k}(m - x_i)^2}{(m-z)^{2k}} - 1\right)\right)$
Yankee Pricing	$(W - z)\left(1 - \frac{(m-z)^k}{\prod_{i=1}^{k}(m - x_i)}\right)$	$z + \left(\frac{\prod_{i=1}^{k}(m - x_i) - (m-z)^k}{k(m-z)^{k-1}}\right)$

Table 3: Generalized Prediction of Bidder's WTP (Triangular Distribution)

As *N* increases, it becomes intractable to exhaustively enumerate the state space, as we did for the 2 unit case in Appendix 1. Hence the inferred WTP prediction in Table 3, represent a first order approximation. For the uniform price auction, the price that a bidder pays is set at the average price between the displaced and the incoming bid, while for Yankee auctions, the price paid by bidders is equal to the bid itself.

For the prediction model to be of practical use, we need to test the robustness of our assumption regarding the usage MBR bidding strategy.

5. <u>Bidding Strategy Classification</u>

To evaluate the accuracy of our WTP prediction model, we used micro-level bid data from real online auctions. We begin by explaining the details of the dataset.

5.1 Online Auction Data Collection

Our analysis uses data from two multi-unit online auctions[6], for which we were able to deploy automatic auction-tracking agents to observe and collect data on entire auction proceedings. One auction site uses a uniform price auction mechanism, while the other uses a Yankee auction mechanism. Our automated agent was able to collect bidding data from 787 uniform price auctions and 205 Yankee auctions, recording in a database a total of 78,014 bids or bid revisions. The auction-tracking agent was programmed to visit the identified online auction's web pages in intervals of 5-15 minutes, take snapshots of the auction, and record the bidding history of the auction site. The auction-tracking agent then compared the newly downloaded auction history with previously recorded history. If differences in the history files were observed, the new activity of the auction was added to the history file. With this technique, we were able to maintain a complete history of the auction, noting the bids submitted and revisions made by each bidder in the auctions we tracked. Appendix 3 provides a list of auction variables that our tracking agent collected data on.

After completing the data collection exercise, we investigated the data for completeness. This required streaming the bidder arrival process through an auction programmed that replicated

[6] Samsclub.com and UBid.com

the online auction, and making sure that the auction concluded with the same winners as the actual action. Some of the auctions had significant chunks of missing data, in which case the simulated auctions did not converge to the same equilibria as the actual auctions. In such cases, we opted to drop them from our data set.

In addition to investigating the data, we pre-processed the data to compute values for the parameters needed in the WTP model. For example, for each bidding instance, we computed the number of bids that were lower than the new bid. This is one of the input parameters into the WTP prediction models. In addition, to facilitate the computation of the WTP prediction accuracy, we recorded the final bids of each bidder. This value was used as an indicator of the bidders actual WTP, and was compared to the predicted WTP to measure our models accuracy.

A major assumption of our WTP prediction model is that bidders are practicing a MBR strategy when bidding in these auctions. In practice, and as observed in past research, the strategies adopted by bidders in multi-unit online auctions are varied. Our WTP methods are thus applicable for a sub- class of bidders who use a MBR strategy.

5.2 Logit Model for Classifying Myopic Best Response Bidders

The model for bidder WTP prediction presented in section 3 above assumes a single bidding strategy – the MBR strategy. Bapna et.al (2001), indicate that there are potentially several categories of bidding strategies in discriminatory online multi-unit auctions. It is likely that similar categories exist in uniform price multi-unit auctions. However, no research has explicitly analyzed this. Table 4 reviews the bidder classification by Bapna et al (2001), and relates it to the effect of predicting bidder WTP based on the MBR assumption.

Bidder Type	Characteristics	Predicted WTP
Evaluators	Early one time high bidders; clear idea on their willingness to pay; bids higher than minimum required	Over prediction; actual bid equal to WTP
Participators	Makes low initial bid, progressively monitor auction and make revisions	Initially under prediction; accuracy increase with revisions.
Opportunists	Place minimum required bid just before auction closes	Accurate prediction of WTP

Table 4: Bidder Categories and Predicted WTP

From the classification presented above, our model can be applied to the class of bidders who exhibit the participatory and opportunistic bidding behavior. The constitution of bids in these two classes of bidders is the same, based on the straightforward bidding approach, and differing only in the auction joining times. However, applying the model to the evaluators would yield erroneous results, as the bid of an evaluator is equal to the actual willingness to pay of that particular bidder. *It is therefore necessary to identify evaluators and isolate them before applying the WTP prediction model.*

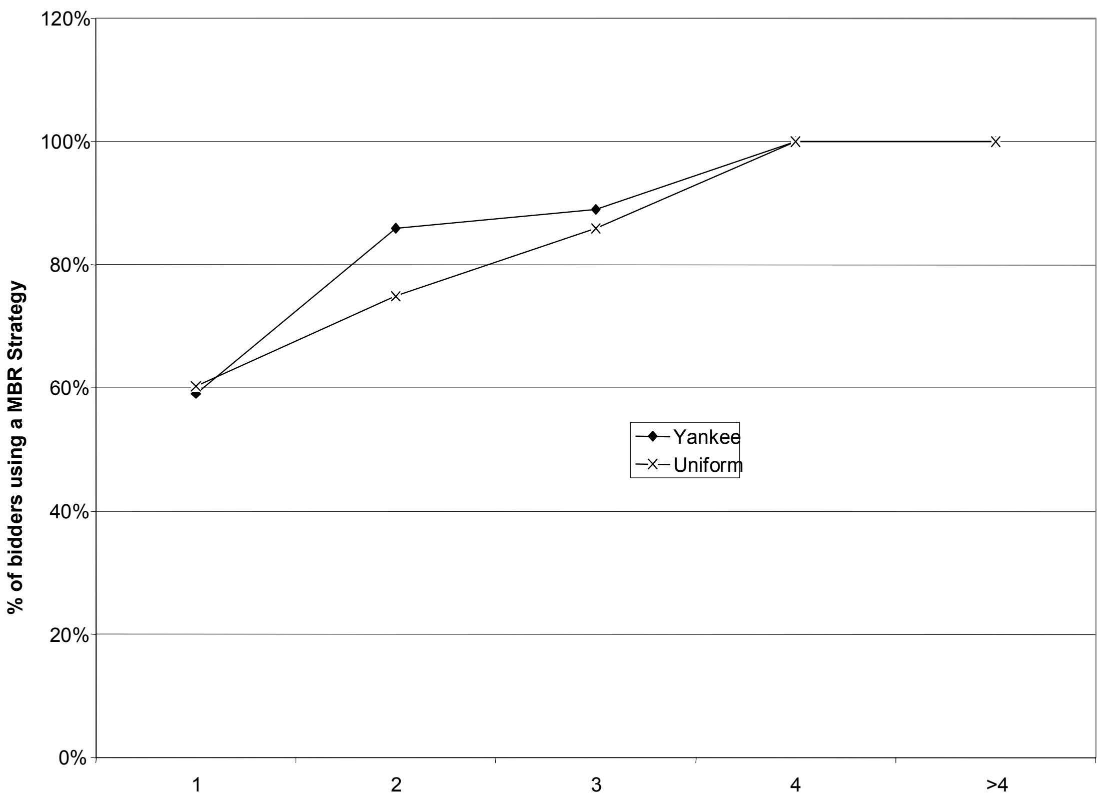
We developed a logistic regression model to classify bidding instances where bidders applied a MBR strategy in the constitution of bids. *A key consideration in developing the model was that it should rely on information that is available to the auctioneer, in real time, when a bidding instance occurs.* This consideration will guarantee that the auctioneer will have all the requisite data to classify a bidder's strategy and subsequently predict his WTP.

5.2.1 Dependent Variable: At each bidding instance our prediction model estimates the consumer's willingness to pay for a product. By definition, the final bid made by bidder adopting a MBR strategy represents a conservative estimate of their actual willingness to pay. So if our predicted WTP at a particular bidding instance is no more that 10% greater than the final bid of a specific bidder, we assume that the bid that led to the prediction was constituted using a MBR strategy. The above operationalization would be sufficient if bidders themselves did not revise their willingness to pay as the auction progresses. To accommodate revisions in willingness to pay, we further classify as MBR, bidders whose predicted WTP value lies below 110 % of the final bid value. This classification scheme yields a value for the dependent variable, which is the strategy classification. It is important to note that it does not really matter whether bidders explicitly carry out such an optimization. Under a broader umbrella, such behavior could be viewed simply as being rational. Another way of looking at this is that it indicates that the submitted bid implicitly conforms to a gain maximizing bid calculation. Table 5 below provides a summary of the proportion of bidders in our data set classified as using a MBR bidding strategy.

	% MBR Bidders	% Non-MBR Bidders
Uniform Price Auctions	62%	38%
Yankee Auctions	63%	37%

Table 5: Distribution of Bidders' by Bidding Strategy

5.2.2 Explanatory Variables: The next challenge was to identify explanatory variables that would explain the likelihood of a bidder adopting the MBR strategy. We identified auction variables and the relationship they have to the classification results shown in Table 5. We consider variables for which an auctioneer can acquire data on at the time of making a prediction on the bidders WTP. *This means that at any bidding instance, an auctioneer will be able to determine a bidders bidding strategy before applying our WTP prediction model*. A quick investigation of some key auction variables yields consistent patterns between their values and the strategy used by bidders. Figure 2a shows the trend of the proportion of bidders who use a MBR strategy as a function of the number of revisions that a bidder has made. The proportion of MBR bidders increases with the number of revisions that bidders have made. The trends are similar in Yankee auctions and uniform price auctions. Figure 2b reveals the proportion of bidders using the MBR strategy as the auction progresses. The trend generally declines over the auction duration for the Yankee auction. That is, the proportion of bidders using a MBR strategy declines as the auction progresses. On the other hand, the uniform price auctions exhibit a decline in the proportion of MBR bidders for the first half part of the auction, after which the proportion of bidders using a MBR strategy starts to increase.



(a) Number of Bid Revisions

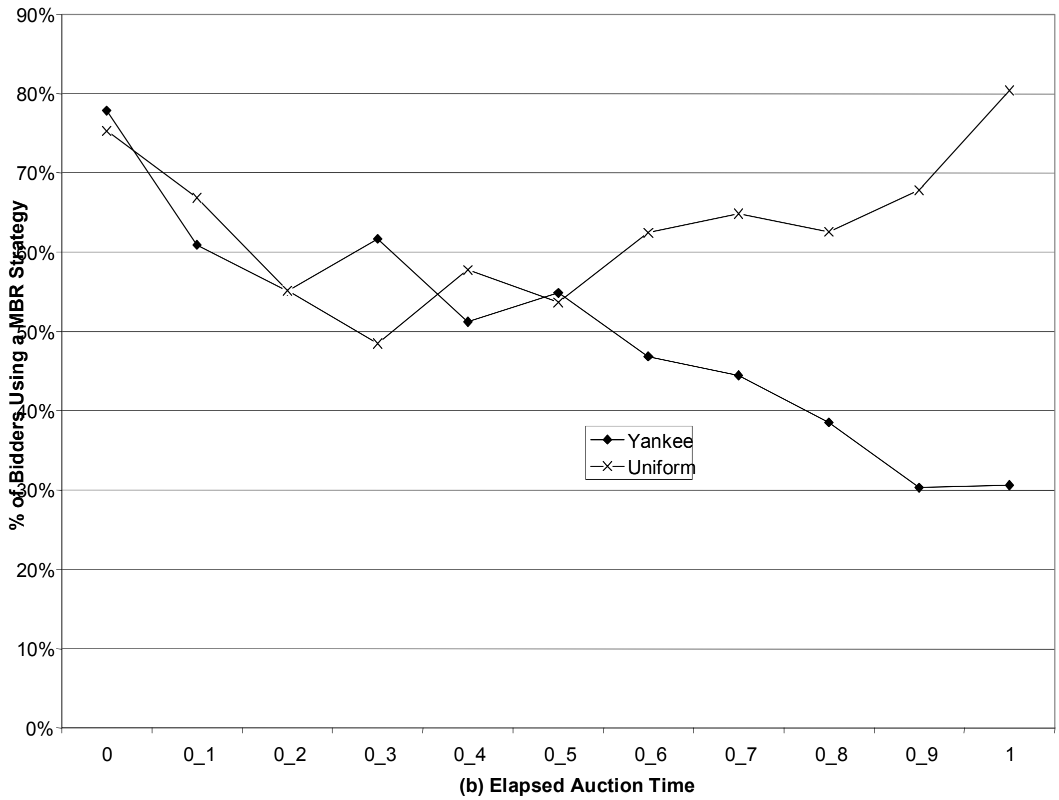


Figure 2: Percent of Bidders Using MBR (a) Over the Auction Duration and (b) by the Number of Bid Revisions

The choice of explanatory variables used in the logistic regression model was based on the analytical model, on the insights from Figure 2, as well as on some observable strategic behavior or bidding aggressiveness measures. From the analytical model we hypothesized that the upper bound on the expected bid price for an item *m* and the lot size of the auctions *N* are likely to influence the bidding strategy adopted. These are captured in X_{1i} and X_{3i} respectively in equation 12, the best-fit Logit regression model:

$$Log\left(\frac{P_i}{1-P_i}\right) = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \varepsilon_i \qquad (12)$$

Where:

P_i - Probability that the bidder is using the MBR strategy

X_{1i} - Ratio of current bid to a suggested market price

$X_{2i} = \frac{K}{N}$ - Ratio of number of current winning bids that are smaller than the current bid, to lot size

X_{3i} - Normalize elapsed auction time.

$X_{4i} =$ Number of bid revisions that the bidders has made up to the bidding instance being considered (not the same as the ex-post total number of revisions)

X_{2i} and X_{4i} capture the strategic behavior of the bidders. These variables together capture the range of strategic behavior presented in Table 4. High values of X_{4i} would indicate that the bidder was either a participator or an opportunist. A ratio close to 1 for X_{1i} would suggest an evaluatory, non MBR, type of bidder.

Using a statistical analysis software package (SPSS), we regressed the data on the model proposed at equation 12 above. The estimates of the model coefficients and their statistical significance are shown in Table 6a and 6b for Yankee auction and Uniform price auctions respectively. The difference in coefficient estimates between the two data set reveals distinctions in the way bidders bid in the two auctions, not surprising given the two contrasting pricing rules. All the model variables are significant in explaining the predicted classification. In Appendix 3, we present correlation matrices for the variables used in the model. The values support independence among the predictor variables.

Coefficient	Estimate	S.E.	Wald	Df	Sig.
β_1	-0.456	.213	4.6	1	.032
β_2	-7.879	.315	623.8	1	.000
β_3	0	0	12.095	1	.001
β_4	1.952	.140	193.395	1	.000
Constant	2.992	.248	143.192	1	.000

Table 6a: Strategy Classification Model's Coefficient estimates – Yankee Auctions

Coefficient	Estimate	S.E.	Wald	Df	Sig.
β_1	1.781	0.271	43.29	1	.000
β_2	-4.517	0.185	593.992	1	.000
β_3	1.361	0.141	93.273	1	.000
β_4	0.417	0.101	16.894	1	.000
Constant	0.862	0.152	31.967	1	.000

Table 6b: Strategy Classification Model's Coefficient estimates – Uniform Price Auctions

As indicated in Table 7, the fitted model yielded, on average, a 79.5% and 72.4% classification accuracy on the Yankee and uniform price auctions respectively. A bidder is classified as using a MBR strategy if the logistic regression model yields a probability $P_i > 0.5$ on the model given at equation 12. Note that all the variables are available to the auctioneer at the time a bid is submitted. Hence, the model can be used to classify bidders in real time.

Actual Categories	**Classification Accuracy**	
	Yankee Auctions	Uniform Price Auctions
Non-MBR	81.9%	72.1%
MBR	76.9%	72.7%
Overall	79.5%	72.4%

Table 7: Logit Model Classification Accuracy

The strategy classification model presented here is an integral part of the WTP prediction. It verifies the strategy bidders are using before applying our WTP model. Next, we show the accuracy of our WTP predictions, along with our ability to predict the auctions' expected price during the early stages of the auction.

6. Empirical Validation Of Prediction Model

Using data collected from the online auctions as described in section 4.1, we preceded to validate the accuracy of the predicted willingness to pay. We created a program that can replicate and manage the stream of the bid arrival process as it occurred in the actual auction. As each bid was recorded, we used the WTP prediction model given in Tables 3 and 4 to predict the bidders' willingness to pay. Note, that at the time that we collected the data, Ubid.com did not feature a suggested retail price, hence, for demonstrative purposes, we used the value of the maximum bid in the Yankee auctions for the parameter *m,* that caps the support of the WTP distribution. Alternatively, *m* could be computed as the average retail price from using a price comparison agent, such as MySimon.com. We compared the predicted WTP to the actual WTP as given by the final bid of a specific bidder. We use the bidders' final bids as proxies for their actual WTP. Note, that the final bid is a realistic estimate of the actual WTP for the losers of both uniform and discriminatory multi-unit auctions, and a conservative estimate of the winners of such auctions.

6.1 Accuracy of the WTP Prediction Model

Table 8 shows the percentage difference between WTP as predicted by our model and the actual WTP. The results show that on average, our predictions are 3 percent and 4.7 percent lower than bidders' actual willingness to pay in Yankee and uniform price auctions respectively. The table also shows a 95 percent confidence interval for the observed deviation between the predicted and actual WTP.

	Yankee Auctions	Uniform Price Auctions
Average Deviation	-3%	-4.7%
Standard Deviation	22%	13.67%
95% confidence interval on Deviation	[-5%,-1%]	[-7.2%,-2.33%]

Table 8: Predictions Accuracy of Bidders' WTP

In order to get further insights into the workings of our prediction model we classify our prediction errors according to auctions' duration as well as the number of bid revisions made by bidders. These results are displayed in Figures 3a and 3b respectively.

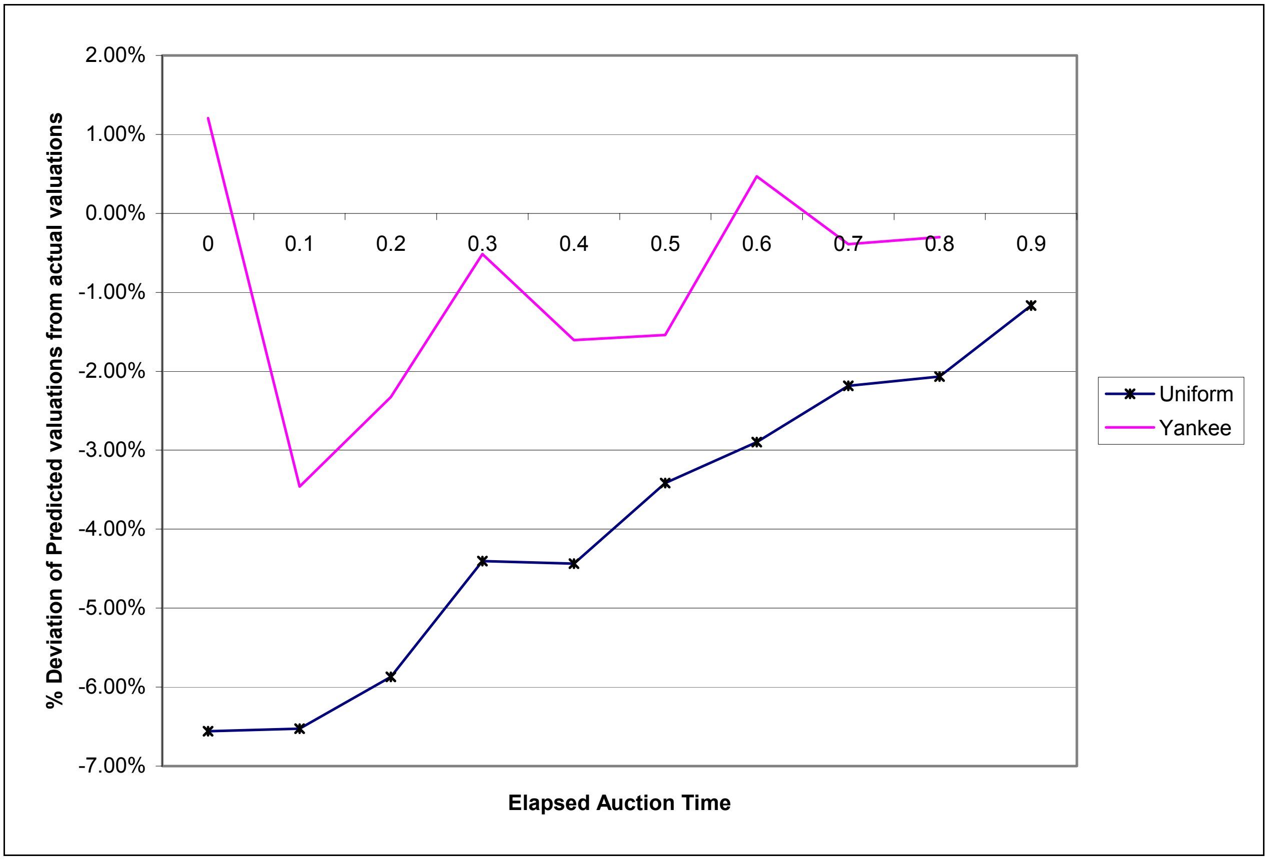


Figure 3a: Accuracy of the WTP model over the Auction Duration

Over the auction duration, the model for Yankee price auctions over-predicts bidders willingness to pay in the initial part of the auction, and the prediction accuracy generally improves as the auction proceeds. The predictions for the uniform auction are consistently below the actual bidders' willingness to pay and the prediction accuracy increases as the auction progresses. Towards the latter stages of the auction, our model over predicts bidders willingness to pay for the Yankee auctions. These patters of prediction accuracy seem consistent with the underlying incentives that each auction format provides to the bidders, as well as some differences induced by the mechanism designs.

Figure 3b reveals the patterns of the prediction accuracy of our model with the number of bid revisions the bidders made in the auctions.

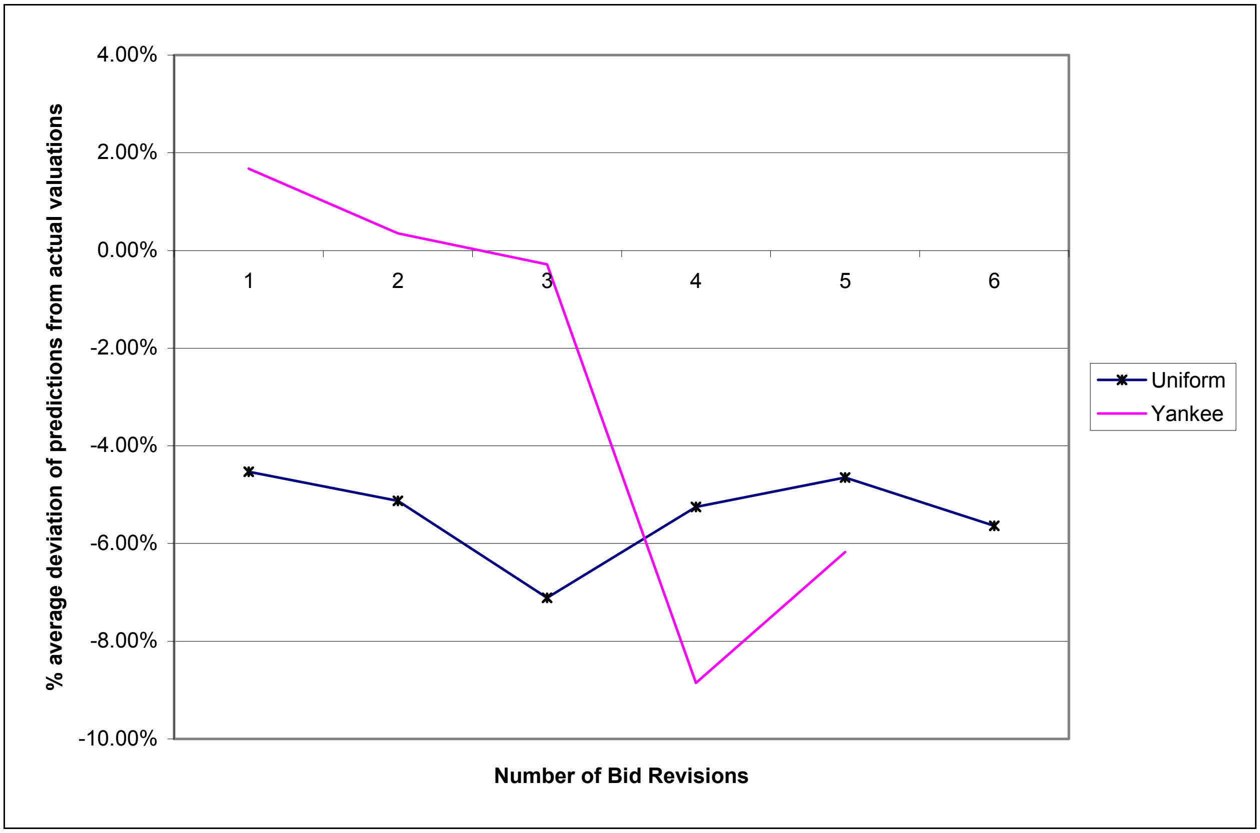


Figure 3b: Accuracy of the WTP model as a function of Bid Revisions

The highest prediction accuracy for Yankee auctions is witnessed among the class of bidders who submit between two and three bid revisions before the close of the auction, whereas the prediction accuracy is more or less consistently just under the 5% mark, except for the 3 bid revision case, for the uniform multi-unit auction, irrespective of the number of bid revisions.

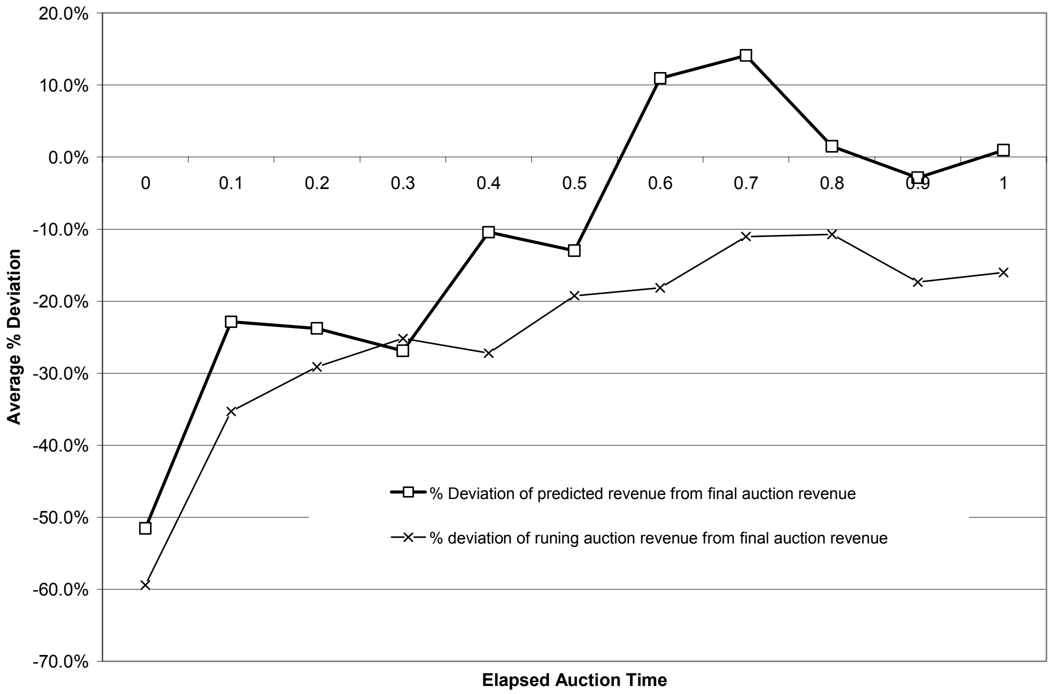
6.2 Inference on Final Auction Revenue

Another aspect of our prediction model that we investigated was its capability to infer the final auction revenue. We estimate such a revenue curve by considering the outcome, say from a hypothetical mechanism, which induces the bidders were to offer prices that are equal to their predicted willingness to pay. If the predictions are accurate, the auctioneer should be able to infer the final auction revenue from the predicted willingness to pay. Such inference can be done in the early stages of the auctions and can be used a strategic tool by the auctioneers to dynamically adjust the mechanism's design. These adjustments could be made through dynamic buy-it-now prices, bid increments or lot sizes. A detailed analysis of these is beyond the scope of this paper, but we do present our estimates of the auction's closing price, estimated as the auction progresses. The earlier such an estimate can be made, with accuracy, the more utility it has for the auctioneer to use it to dynamically calibrate the mechanism. At any stage of the auction, the revenue for the uniform and the discriminatory auctions, say R_u and R_D respectively, auction can be estimated as:

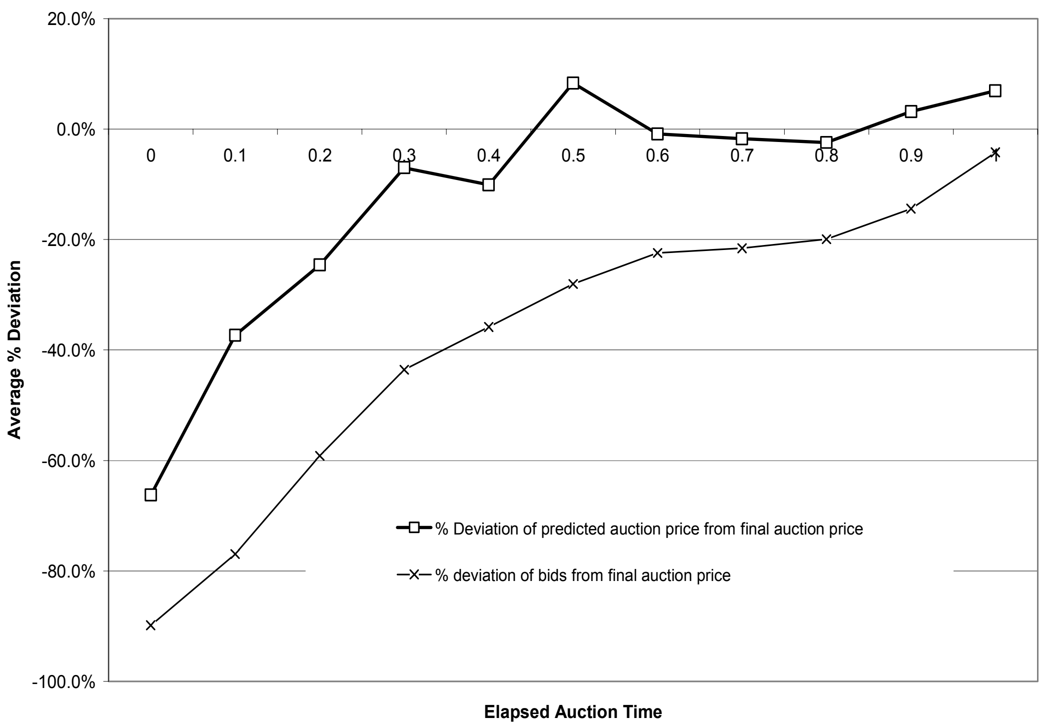
$$R_u = N * W_{(N)} \tag{13}$$

$$R_D = \sum_{i=1}^{N} W_{(i)} \tag{14}$$

Where $W_{(i)}$ represents the i^{th} highest estimated WTP, at that time. Figure 4 presents a comparison of the progression of the actual auction revenue to the predicted revenue (as per equations 13 and 14), over time.



(a) Yankee Auctions



(b) Uniform Price Auctions

Figure 4: Deviation of Predicted Auction Revenue from the Final auction Revenue (a) Yankee auctions; (b) Uniform price auction

7. Conclusions and Future Research

This work illustrates how the enhanced information acquisition and processing capabilities of the online environment can be used to understand the micro-level details of the price formation process in two popular types of progressive multi-unit online auctions. Making a minimalist myopic best response bidding assumption to ties bidders observed bids to their underlying willingness to pay, together with a Logit classification model that can be implemented with information available at run-time, we are able to derive tight estimates of bidder's willingness to pay in real time. Our estimation and strategy classification procedure, applied to 987 online auctions with over 78,000 bids, allows us to come, on average within 3% of revealed willingness to pay for Yankee auctions, and within 4.7% for uniform multi-unit auctions.

Particularly interesting is the fact that there exists an opportunity to design mechanisms that can significantly shorten the duration of such auctions. For all practical purposes, an auctioneer using our estimation procedure can expect predicting up to 90% of the auction's underlying revenue by the 40^{th} time percentile for two of the most popular types of multi-unit online auctions. We have recently observed the multi-unit auctioneer Ubid.com to be using a *static* "ubuy-it" price in parallel with Yankee bidding. Our intuition, based on the insights from the prediction results we have presented in this paper, is that a *dynamic* ubuy-it price is going to lead to a more allocatively efficient mechanism. The details of determining the dynamic buy-it-now prices in the multi-unit setting, using the information acquisition and processing capabilities that are at our disposal in the online auction environment, remains a promising area of future research.

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(Appendices are to be treated as online supplements)

Appendix 1.: Exhaustive Summary of Feasible Auction Outcomes

When $x_1 < z < x_2$	**Price**		**Probability**
	Uniform	Yankee	$1*\left(1-\frac{(m-z)^2}{(m-x_1)^2}\right)$
$W_1 < z, W_2 > z$	z	z	
When $z > x_2$	**Price**		**Probability**
Case	Uniform	Yankee	
$W_1 < x_2, x_2 < W_2 < z$	x_2	z	$\left(1-\frac{(m-z)^2}{(m-x_2)^2}\right)\left(1-\frac{(m-x_2)^2}{(m-x_1)^2}\right)$
$x_1 < w_1 < z, x_2 < W_2 < z$	$\frac{(x_2+z)}{2}$	z	$\left(1-\frac{(m-z)^2}{(m-x_2)^2}\right)\left(\frac{(m-x_2)^2-(m-z)^2}{(m-x_1)^2}\right)$
$W_1 > z, x_2 < W_2 < z$	$\frac{(x_2+z)}{2}$	z	$\left(1-\frac{(m-z)^2}{(m-x_2)^2}\right)\left(\frac{(m-z)^2}{(m-x_1)^2}\right)$
$W_1 < x_2, W_2 > z$	x_2	z	$\left(\frac{(m-z)^2}{(m-x_2)^2}\right)\left(1-\frac{(m-x_2)^2}{(m-x_1)^2}\right)$
$x_2 < W_1 < z, W_2 > z$	$\frac{(x_1+z)}{2}$	z	$\left(\frac{(m-z)^2}{(m-x_2)^2}\right)\left(\frac{(m-x_2)^2-(m-z)^2}{(m-x_1)^2}\right)$
$W_1 > z, W_2 > z$	W	W	$\frac{(m-z)^4}{(m-x_2)^2(m-x_1)^2}$

Appendix 2. Online Auction Data Variables Monitored by Tracking Agent:

Variables	Description
Auction variables	
Lot Number	A unique ID that identifies each auction
Product Description	Product description details
Current Bid	The current minimum winning bid level
Bid Increment	Auction's pre-set bid increment
Number of Bids	Number of bidders who have already submitted bids.
Quantity	Number of items being sold
Opening Bid	Pre-set minimum starting bid
Retail Price	Displayed retail price
Open Date	The time the auction begun
Close Date	Pricing method (Yankee or Uniform)
Auction Type	
Bidder variables	
Member ID	A unique ID that identifies each bidder
Bid Amount	The amount a bidder tendered
Quantity	The quantity bid for
Won	The quantity allocated to the bidder
Bid Date	The time the bid was submitted
Status	Winning or losing status.

Appendix 3: Corrleation data

A: Yankee Auctions Dataset

	X1	X2	X3	X4
X1	1			
X2	-0.146	1		
X3	-0.277	-0.13	1	
X4	-0.929	-0.106	0.248	1

B: Uniform Price Auctions Dataset

	X1	X2	X3	X4
X1	1			
X2	-0.394	1		
X3	-0.368	-0.13	1	
X4	-0.052	-0.082	0.222	1

Variables:

X1- Ratio of current bid to a suggested market price

X2- Ratio of number of current winning bids that are smaller than the current bid, to lot size

X3- Normalize elapsed auction time.

X4 - Number of bid revisions.