House prices, borrowing constraints and monetary policy in the business cycle

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Abstract

I develop a general equilibrium model with sticky prices, credit constraints, nominal loans and asset prices. Changes in asset prices modify agents' borrowing capacity through collateral value; changes in nominal prices affect real repayments through debt deflation. Monetary policy shocks move asset and nominal prices in the same direction, and are amplified and propagated over time. The "financial accelerator" is not constant across shocks: nominal debt stabilises supply shocks, making the economy less volatile when the central bank controls the interest rate. I discuss the role of equity, debt indexation and household and firm leverage in the propagation mechanism. Finally, I find that monetary policy should not target asset prices as a means of reducing output and inflation volatility.

> *"The population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth or from current income or from any other liquid resources they can command. Typically their indebtedness is rationed by lenders [...] Business borrowers typically have a strong propensity to hold physical capital, producers' durable goods. Their desired portfolios contain more capital than their net worth – they like to take risks with other people's money. Household debtors are frequently young families acquiring homes and furnishings before they earn incomes to pay for them outright; given the difficulty of borrowing against future wages, they are liquidity-constrained and have a high marginal propensity to consume"* (James Tobin, "Asset Accumulation and Economic Activity", 1980)

1 Introduction

Motivations and background. A long-standing tradition in economics, starting at least with Irving Fischer's (1933) debt-deflation explanation of the Great Depression, has recognised the importance of financial factors, such as borrowers' net worth, as key elements of business cycle fluctuations. In this view, deteriorating credit market conditions, such as growing debt burdens and collapsing

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asset prices, are not simply passive reflections of a declining economy, but are themselves a major factor depressing economic activity. More recently, various commentators have expressed concerns that the historically high levels of household and firm indebtedness in many industrialised countries may discourage new borrowing and spending and pose a threat to economic stability.[1]

Although this "credit view" has a long pedigree in macroeconomics,[2] most of the theoretical work on the subject has been partial equilibrium in nature until the late Eighties. Starting with Bernanke and Gertler (1989) though, many authors, including Kiyotaki and Moore (1997 and 2001), Carlstrom and Fuerst (1997 and 2000) Cooley and Quadrini (1999) and Bernanke, Gertler and Gilchrist (2000), have presented dynamic microfounded models where frictions in the financial markets exacerbate output fluctuations in response to aggregate disturbances.[3] All these models feature a "financial accelerator",[4] in that procyclical movements in borrowers' net worth and countercyclical movements in the cost of external funds relative to the internal funds can generate "large" changes in output from "small" changes in productivity or interest rates. Empirically, various authors (e.g., Gertler and Gilchrist, 1994, Hubbard, Kashyap and Whited, 1995, Gilchrist and Himmelberg, 1998) have shown that firms' investment decisions are not only driven by fundamentals, but are also sensitive to various measures of firms' net worth. At the household level, evidence of financing constraints has been documented by Zeldes (1989), Jappelli and Pagano (1989), Campbell and Mankiw (1993) and Bacchetta and Gerlach (1997).

While it is accepted that capital market imperfections matter for the economic activity, there is still controversy on the quantitative relevance of these frictions at the aggregate level.[5] Next, it is unclear in which sectors of the economy these frictions matter most: are those at firm level more important than those at household level? Finally, there is a much debated normative issue: what is the optimal policy response to shocks in an economy with capital market imperfections?

The model set-up. To address these questions, this paper proposes a quantitative model of the interaction between borrowing constraints, asset prices fluctuations and the macroeconomy, by introducing debt and housing market into an otherwise standard monetary business cycle model.[6] The reason for having real estate[7] is practical and substantial. It is practical, because credit market imperfections are conveniently analysed if one requires that borrowers post collateral in order to secure a loan (the collateral being real estate itself). It is also substantial because, although widely recognised that real estate developments play a key role in business fluctuations,[8] few studies have attempted to introduce real estate in a dynamic general equilibrium model.[9] I depart from the traditional monetary business cycle model in three other ways: first, and in order to justify the existence of debt, I introduce in a simple way heterogeneity among agents in the economy. Secondly, since I want a non-trivial role for debt itself, I set up the model so that in equilibrium some agents face binding borrowing constraints. These borrowing constraints apply both to the entrepreneurial and

[1] See e.g. the Economist ("Caught in the jaws", 23 June 2001; Economic Focus, 30 June 2001).

[2] Besides Fischer (1933), early authors to emphasise this view are Gurley and Show (1955), Minsky (1964) and Wojnilower (1980). See Trautwein (2000) for a survey.

[3] See Bernanke, Gertler and Gilchrist (2000) for additional references.

[4] The term has been put forth in the literature by Bernanke, Gertler and Gilchrist (1996).

[5] See for instance Ramey (1993).

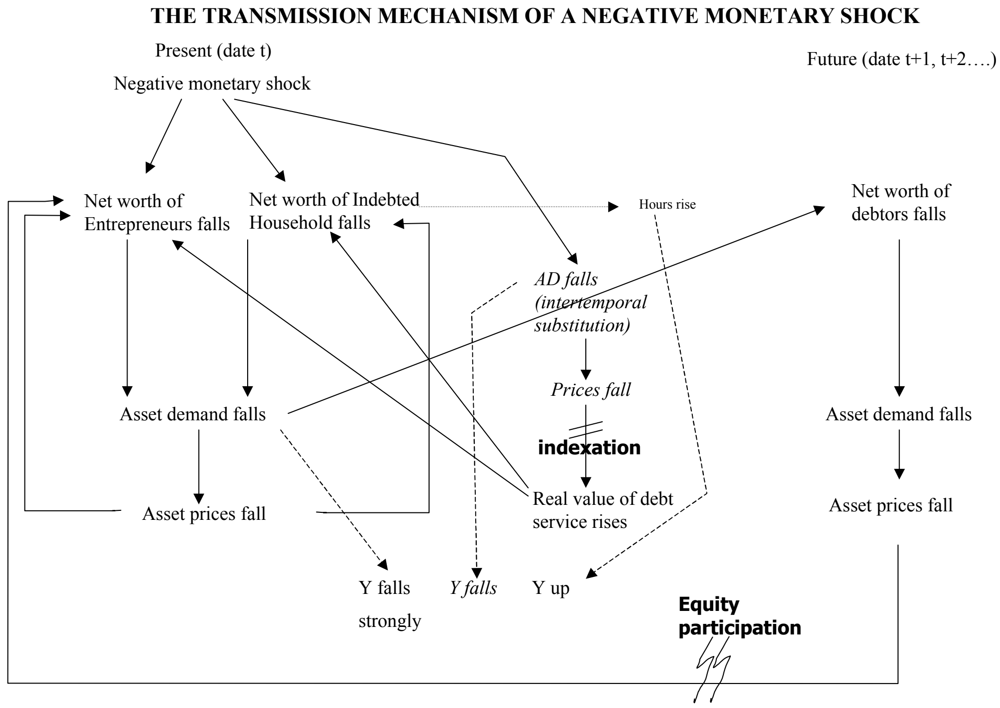
[6] I use the word "standard" since the model builds on the consensus macro model with nominal rigidities which includes aggregate demand, a Phillips curve and a monetary policy rule. See Clarida, Gali and Gertler (1999).

[7] Although houses are a subset of real estate, I use the terms "houses" and "real estate" interchangeably in the paper. I do similarly for real estate prices, house prices and asset prices.

[8] See for instance IMF (2000), Higgins and Osler (1997), Case (2000).

[9] An early attempt was Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992).

the household side: this allows extending to the household sector the kind of frictions that have often been analysed in the literature for firms. Third, I assume that debt obligations are not price level indexed, so that unexpected price changes have an effect on the allocation of real resources (through debt deflation). In this way, I can analyse the independent role of nominal and asset price deflation in the transmission mechanism of monetary policy.[10]



With these elements in mind, the transmission mechanism of monetary policy works as follows (see chart). The central bank has leverage over the real interest rate when prices are sticky. When the interest rate rises, aggregate demand declines; real asset prices and inflation fall. The decline in asset prices affects the borrowing possibilities of the debtors, whose debt capacity is tied to their collateral values. Moreover, the fall in nominal prices makes the real value of debt repayments bigger: thus, debtors are harmed twice. Because of their impact on agents' financial positions and asset holdings, the effects of the shock are amplified and persist over time, even if monetary policy reacts to moderate the economy decline.

While the debt burden effect strongly affects firms, indebted household try to compensate for the fall in net worth by working more. Asset price deflation and consumer price deflation amplify the output fall, and spill over across sectors. The modest increase in hours worked by debtors dampens the output decline. However, the first two effects appear stronger than the third. The model suggests two routes to reduce the economy's sensitivity to disturbances: one is indexation of debt to the price level; the other is equity participation replacing debt. Both agreements reduce the amplification mechanism of monetary disturbances.

[10]Various authors have used a number of shortcuts to analyse the effect of unexpected changes in the price level on economic activity. Typically, this is done by postulating an exogenous "wealth" shock that transfers resources between lenders and borrowers (e.g. Bernanke, Gertler and Gilchrist, 2000, and Carlstrom and Fuerst, 2001). While useful for isolating the consequences of debt redistribution, this approach neglects other consequences of unexpected inflation (e.g. change in aggregate demand) as well as the reasons why inflation moved in the first place (e.g. tight money).

As asset prices are of the main sources of amplification, I investigate whether there is a role for monetary policy to target them as a means of reducing output and inflation volatility. Numerical simulations seem to suggest that the answer is negative: this happens because asset prices have no informational content besides the one already contained in output.

I also discuss whether a move towards real contracting or equity financing can stabilise the economy when the central bank controls the real interest rate. Under real contracting, the answer is negative: this happens because *nominal contracting amplifies* the effects of shocks that the central bank can better offset (demand shocks), while turns out to be *stabilising* for those that induce a negative correlation between output and inflation (supply shocks). Supply shocks are in fact, ceteris paribus, beneficial to borrower's net worth: for instance, when inflation exogenously rises, borrowers' repayments turn out to be lower if obligations are held in nominal rather than real terms. If external financing is in the form of equity, the answer is instead less clear-cut, since it depends on the asset prices response, on the specification of the feedback rule, and on the persistence of the shocks.

Outline of the paper. The outline of the paper is as follows. Section 2 reviews some (own) cross-country VAR evidence on house price developments and the business cycle. Such evidence is meant to highlight the key elements that a model analysing the interaction between asset prices, interest rates, monetary policy and the business cycle has to be consistent with.

Section 3 presents a basic model of the interplay between debt, monetary policy and economic activity. The model considers the independent role of price rigidities, debt deflation and house prices fluctuations in transmitting a monetary shock. The main agents are entrepreneurs and households. Both agents are infinitely lived and consume a final good. *Entrepreneurs* use labour and real estate to produce a wholesale good. They have limited borrowing possibilities: the amount they can borrow cannot exceed a fraction of the value of their real estate holdings. Entrepreneurial impatience (assumed throughout) guarantees that the borrowing constraint is binding in equilibrium.[11] *Households* work, consume and demand the residual supply of real estate. Notice the variety of roles that real assets (e.g. houses) play: input for production and collateral for the entrepreneur, durable good for the household. The model is completed by *retail outlets* (owned by households), that provide the source of nominal rigidity in the economy; and a *central bank* conducting monetary policy in the form of an interest rate rule.

Section 4 relaxes the assumption that firms are the sole borrowers. It builds an extended version that captures the following facts: (a) in the aggregate, the household sector is a net lender to firms; (b) firms are likely to face borrowing constraints; (c) some households are themselves borrowing constrained. This setup captures some "great ratios" relative to credit aggregates and housing market, such as the values of residential and commercial housing stock and mortgage debt over total output.

In Section 5, I calibrate the model and use it to address a variety of issues, not all of them related to monetary policy. How do inflation and technology disturbances propagate in an economy where lending and borrowing occur in equilibrium? How do movements in asset prices influence consumption and investment? What is the different role of households and firms financial conditions in amplifying the effects of shocks?

Section 6 addresses the role of house price targeting and debt indexation in the formulation of systematic monetary policy. Concluding remarks, links to the literature and extensions for future research are in Section 7.

[11] That is, they would like to borrow more given the equilibrium interest rate but are unable to do so.

2 Some VAR evidence on house prices and the business cycle

In Iacoviello (2000), I analyse the empirics of house prices and the macroeconomy in 6 European countries (France, Germany, Italy, Spain, Sweden and the UK) over the period 1970-1998. For each country, I specify a five dimensional VAR with $X_t = \begin{bmatrix} y_t & mp_t & q_t & R_t & \pi_t \end{bmatrix}'$, where X_t is a vector comprising logged real income (y_t), a measure of logged real money balances (mp_t), a logged real house price index (q_t), a short-term nominal interest rate (R_t), and consumer price inflation (π_t). Using the King, Plosser, Stock and Watson (1991) common trends procedure, I specify their dynamic interaction by means of a vector-error-correction (VECM) model.[12] I identify two permanent disturbances and three transitory ones, the latter having no long-run effect on the level of variables. Within the transitory shocks, I use a recursive identification scheme to isolate a monetary shock (zero impact effect on output and inflation), a demand shock (zero impact effect on inflation) and an inflation shock (potential impact effect on all variables). The results can be summarised as follows:

1. A contractionary monetary policy shock leads to a decrease in prices, with house prices falling more than consumer prices. Real house prices and output display a hump-shaped response, bottoming down together after about six quarters, and reverting back in the long run to their pre-shock values.

2. A positive demand shock leads to a transitory increase in output, to a slight increase in consumer price inflation and a strong increase in real house prices. Nominal rates temporarily increase: this can represent the endogenous response of the central bank to inflationary fears.

3. Monetary and demand disturbances are both consistent with procyclicality of house prices. Importantly, the timing of the response of house prices and output is similar.

4. The historical decompositions of the time series for house prices support the hypothesis that the major cycles in house prices have been caused by the concentration of a variety of factors pushing all in the same direction.

The VAR results suggest that house prices can be effectively embedded in a simple macroeconometric model that can give quantitative estimates of the sensitivity of asset prices to economic conditions. In addition, they provide evidence that house prices, as expected, are very sensitive to the stance of monetary policy and to other macroeconomic disturbances. They also show that understanding house price dynamics can shed some light over several macroeconomic episodes of the last quarter of century in Europe.

Figure 1 summarises the evidence regarding the effects of a monetary tightening,[13] thus suggesting the basic ingredients that are needed in a model of house prices, monetary policy and business fluctuations. Real and nominal rates rise; output, inflation and real house prices fall. Yet, such evidence is only part of the story. In a comparative study (across Finland, Germany, Norway and

[12] King, Plosser, Stock and Watson (1991) distinguish between structural shocks with permanent effects on the level of the variables (say, a positive supply shock, raising output in the long-run) from those with only temporary effects (say, a demand shock that can be thought to have zero long-run effect on output and other real variables). The permanent shocks are the sources of the "common stochastic trends" across the series, and the number of these shocks is equal to the number of variables in the system less the number of cointegrating relationships between them. The (remaining) transitory innovations equal the number of cointegrating relationships (intuitively, a cointegrating vector identifies a linear combination of the variables that is stationary thus eliminating the trend, so that shocks to it do not eliminate the steady state in such a system).

[13] For more evidence about the effects of all shocks and other findings, see Iacoviello (2000).

the UK, chosen for their wide degree of heterogeneity in the housing finance system) of the housing market and credit channel of monetary policy, Iacoviello and Minetti (2000) find evidence of a bank lending channel for Finland and the UK and of a possible balance sheet channel for Germany. In addition, the findings show across countries a clear-cut relationship between evidence and strength of a credit channel, efficiency of the housing finance system[14] and type of institutions active in mortgage provision. From that evidence, we conclude that housing markets play an important role in the business cycle, not only because housing is a very volatile component of aggregate demand but also because, especially since firms and households have limited borrowing possibilities, changes in house prices can have important wealth effects on consumption and investment choices. Therefore in order to understand the linkages between house prices and the business cycle modelling real estate *per se* is not sufficient: one has also to include borrowing constraints and wealth effects in the framework.

3 The basic model

Consider a discrete time, infinite horizon economy. The main actors are *entrepreneurs* (below I will use "entrepreneur" and "firm" interchangeably) and *patient households*, both infinitely lived and of measure one. The term "patient" reflects the assumption that households have lower discount rates than firms and helps distinguish this group from the impatient households that I will introduce in the extended model.

Entrepreneurs produce a wholesale good, hiring household labour and combining it with collateralisable real estate. Households consume, supply labour, demand real estate and money balances. Besides them, there are *retailers* and a *central bank*. As in Bernanke, Gertler and Gilchrist (2000), retailers are needed to introduce price inertia in the model in a tractable manner: what retailers do is simply to transform the firm's output into a composite good, over which each retailer has some pricing power. The central bank simply adjusts the mix of money supply and transfers to support an interest rate rule.

As their activities are somewhat conventional, I start in the next subsection with the patient households problem. Throughout the paper, their choice variables are denoted with a prime.

3.1 Patient households

The household sector is reasonably conventional, with the exception that I add real estate demand in the utility function.[15] The idea is, as we will see below, to have housing market investment across sectors (although the aggregate supply of real estate is fixed), so that shifts in asset holdings have first-order effects on economy activity, as in Kiyotaki and Moore (1997). Households maximise a lifetime separable utility function given by:

$$\max_{B'_t, h'_t, L'_t, M'_t/P_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c'_t)^{1-\rho'}}{1-\rho'} + j \ln h'_t - \frac{\tau'}{\eta'} (L'_t)^{\eta'} + \chi \ln \frac{M'_t}{P_t} \right)$$

[14] We consider three aspects of the efficiency that are relevant for the presence of a credit channel: (a) the depth of the funding system for housing finance institutions; (b) presence of a diversified range of mortgage lenders and (c) the degree of sharing of credit risk; see Iacoviello and Minetti (2000) for more details.

[15] I am not alone in this modelling choice. Diaz-Gimenez et al. (1992) and Miles (1992), among the others, use a similar device, introducing houses in the utility function. Unlike national statistics, I do not include imputed rents in my model definition of output. Doing so does not affect the results of the paper in any significant way. In addition, I assume that housing and consumption are separable, following Bernanke (1984) who studies the joint behaviour of the consumption of durable and non-durable goods and finds that separability across goods is a good approximation.

where E_0 denotes the expectation operator conditional on time 0 information, $\beta \in (0, 1)$ is the discount factor, c'_t is time t consumption, h'_t represents the services of the house held by the household at period t (assumed proportional to the stock of housing held), L'_t is time t labour and M'_t/P_t are real money balances.[16]

Every period households derive utility from c'_t and h'_t (priced at $q_t \equiv Q_t/P_t$), rent their labour input to entrepreneurs at a real wage of $w_t \equiv W_t/P_t$, borrow in real terms an amount $b'_t \equiv B'_t/P_t$ (or lend $-b'_t$), pay (or receive) back $R_{t-1}B'_{t-1}/P_t$, where R_{t-1} is the predetermined nominal interest rate paid on loans made between $t-1$ and t. Obligations are therefore set in money terms, an issue that I will return to below when I consider the entrepreneur's problem.

Let $\Pi_t \equiv P_t/P_{t-1}$ denote the gross rate of inflation from period $t-1$ to period t. In real terms, the household budget constraint can be summarised as follows:

$$c'_t + q_t\left(h'_t - h'_{t-1}\right) + \frac{R_{t-1}}{\Pi_t} b'_{t-1} = b'_t + w'_t L'_t + F_t\left[-\frac{M'_t - M'_{t-1}}{P_t} + T'_t\right]$$

where F_t denotes lump-sum dividends received from ownership of retail firms (described below)[17] and the terms in square brackets represent net transfers from the central bank that are financed by printing money. Solving this problem yields traditional first order conditions for consumption/saving (3.1) and labour supply (3.2); the dynamic real estate h' demand relationship (3.3) is a standard equation for the demand of a durable good. By usual arguments, the budget constraint holds with equality:

$$\frac{1}{(c'_t)^{\rho'}} = \beta E_t\left(\frac{R_t}{\Pi_{t+1}\left(c'_{t+1}\right)^{\rho'}}\right) \tag{3.1}$$

$$\frac{w'_t}{(c'_t)^{\rho'}} = \tau'\left(L'_t\right)^{\eta'-1} \tag{3.2}$$

$$\frac{q_t}{(c'_t)^{\rho'}} = \frac{j}{h'_t} + \beta E_t\left(\frac{q_{t+1}}{\left(c'_{t+1}\right)^{\rho'}}\right) \tag{3.3}$$

By taking the first-order condition with respect to money holdings, I would obtain a standard money demand equation in which money holdings are related positively to consumption and negatively to the nominal interest rate. Since I focus in what follows on monetary rules that target interest rates, money supply will always respond to meet money demand at the desired equilibrium nominal interest rate. As utility is separable in money balances, the actual quantity of money has no implications for the rest of the model, and can therefore be ignored.

3.2 Entrepreneurs

Entrepreneurs produce a wholesale good utilising a standard Cobb-Douglas constant returns to scale technology, which uses real estate and labour as inputs. The production function is:

$$Y_t = A_t\left(h_{t-1}\right)^{\nu}\left(L_t\right)^{1-\nu} \tag{3.4}$$

where A_t is the technology parameter, h is real estate demand, L is the labour input.

Wholesale output cannot be immediately transformed into consumption. Following Bernanke, Gertler and Gilchrist (2000), I assume that a continuum of *retailers* transform wholesale output into

[16] It can be shown that a utility function of this form is not consistent with balanced growth if $\rho \neq 1$, since it implies growing (if $\rho < 1$) or falling (if $\rho > 1$) labour supply over time as productivity grows.

[17] Experiencing with non lump-sum dividends did not affect the main results of the paper.

a composite final good through a unique instantaneous technology. Entrepreneurs sell their wholesale output Y_t to retailers at a nominal price of P_t^w, or, equivalently, at a real price (in terms of final goods) of $P_t^w/P_t \equiv 1/X_t$, where P is the aggregate price level and $X > 1$ is the markup of retail over wholesale goods. In other words, the market value of the entrepreneurs output in terms of consumption goods is Y_t/X_t.

As stated in the introduction, there is a limit on the net obligations of the Entrepreneur. I assume that entrepreneurs borrowing capacity is limited to a fraction $m \leq 1$ of next period's expected value of real estate holdings discounted by the nominal rate of interest to be paid. Nominal obligations of the entrepreneur B_t are thus bound by:

$$B_t \leq mE_t\left(Q_{t+1}\right)h_t/R_t$$

and in real terms, dividing both sides by P_t:

$$b_t \leq \frac{mE_t\left(q_{t+1}\Pi_{t+1}\right)h_t}{R_t}$$

In other words, lenders impose a margin requirement on entrepreneurs of $1-m$. I do not attempt here to derive this constraint endogenously. However, this kind of borrowing limit could arise for instance due to liquidation costs, if – in case of default – legal and other cost amount to a fraction $1-m$ of the real estate value.

Absent borrowing constraints, the economy would behave like the standard representative agent economy in which the returns to each input and across agents would be equated. To make matters interesting, one wants to obtain a steady state in which the entrepreneurial return to savings R^E (to be calculated below) is greater than the equilibrium interest rate R, which in turn implies that entrepreneurs will be borrowing constrained in equilibrium. At the same time, one has to ensure that entrepreneurs will not postpone consumption and quickly accumulate wealth so that they are completely self-financed and the borrowing constraint is not binding in the steady state equilibrium.

There are several ways of dealing with this problem. Here, I take the easiest route and assume that entrepreneurs discount the future more heavily than households.[18] Formally, they maximize the intertemporal objective c_t:[19]

$$\max_{B_t,h_t,L_t} E_0 \sum\nolimits_{t=0}^{\infty} \gamma^t \frac{c_t^{1-\rho}}{1-\rho}$$

[18]Another possibility would be an OLG structure, where in each period a fraction of entrepreneurs dies. Even if each surviving entrepreneur gets richer over time, aggregate entrepreneurial net worth is constant since old wealthy entrepreneurs who die are replaced by young poor ones.

[19]Assuming that entrepreneurs derive utility services from housing h or money balances would not change the results of the paper. The key insight is that entrepreneurs' savings are more productive than households' savings in equilibrium.

where[20] I assume $\gamma < \beta$, subject to the following constraints, expressed in real units:

$$\frac{Y_t}{X_t} + b_t = c_t + q_t \left(h_t - h_{t-1}\right) + \frac{R_{t-1}}{\Pi_t} b_{t-1} + w_t' L_t \tag{3.5}$$
$$Y_t = A_t h_{t-1}^{\nu} L_t^{(1-\nu)}$$
$$R_t b_t \leq m E_t \left(q_{t+1} h_t \Pi_{t+1}\right)$$

where the presence of $R_{t-1} b_{t-1}/\Pi_t$ in the flow of funds (3.5) incorporates the assumption that debt obligations are not indexed to the price level. That is, each period the entrepreneur repays the nominal amount borrowed in the previous period B_{t-1} times the predetermined nominal interest rate R_{t-1}. Changes in the price level between $t-1$ and t imply that the realised real interest rate paid, R_{t-1}/Π_t, can be different from the one expected in $t-1$, $R_{t-1}/\left(E_{t-1}\Pi_t\right)$. I use the assumption that debt is not indexed to the price level on empirical grounds: in low-inflation countries, almost all debt contracts are set in nominal terms.[21] Likewise, I also assume that markets for insurance against aggregate shocks do not exist.

Define λ_t as the time t shadow value of the borrowing constraint. The first-order conditions for an optimum are the consumption Euler equation, housing (real estate) demand and labour demand:

$$\frac{1}{c_t^{\rho}} = E_t \left(\frac{\gamma R_t}{\Pi_{t+1} c_{t+1}^{\rho}} \right) + \lambda_t R_t \tag{3.6}$$
$$\frac{1}{c_t^{\rho}} q_t = E_t \left(\frac{\gamma}{c_{t+1}^{\rho}} \left(\nu \frac{Y_{t+1}}{X_{t+1} h_t} + q_{t+1} \right) + \lambda_t m \Pi_{t+1} q_{t+1} \right) \tag{3.7}$$
$$w_t' = \frac{(1-\nu) Y_t}{X_t L_t} \tag{3.8}$$

Both the Euler and the housing demand equations differ from the usual formulations because of the presence of λ_t, the Lagrange multiplier on the borrowing constraint. λ_t equals the increase in lifetime utility that would stem from borrowing R_t dollars, consuming (equation 3.6) or investing (equation 3.7) the proceeds, and reducing consumption by an appropriate amount next period. A subtler point is that borrowing constraints not only affect the intertemporal allocation of resources, but, as real estate can be used as collateral, they also affect the within-period one.

The assumption that $\gamma < \beta$ ensures that entrepreneurs will be constrained in steady state (or in a neighbourhood of it). In fact, the Euler equation in the household problem guarantees that, in a steady-state with zero inflation, the gross nominal interest rate is equal to $R = 1/\beta$, the household time preference rate. Combining this result with the steady state Entrepreneurial Euler equation for consumption yields:

$$\beta = \gamma + \lambda c^{\rho} \Longrightarrow \lambda = \left(\beta - \gamma\right)/c^{\rho}$$

[20] So long as $\rho > 0$, entrepreneurs are not risk neutral and therefore maximise a convex function of lifetime profits. Models of business cycle and agency costs in the creation of new capital typically assume risk neutral entrepreneurs, since that yields in equilibrium an "optimal" intra-period debt contract. Carlstrom and Fuerst (2001) discuss this issue further. In modelling firms' behaviour in their model of monetary shocks, agency costs and business cycle, they consider two alternatives. In one, entrepreneurs are infinitely lived, risk neutral and more impatient than households: net worth sharply responds to shocks, as the elasticity of entrepreneurial savings to changes in the real rate of interest is, to a first approximation, $1/\rho \longrightarrow \infty$. In the other, a constant fraction of entrepreneurs die each period, so that net worth responds passively and slowly to changes in the real rate: in the aggregate, this is equivalent to a formulation in which entrepreneurs are extremely risk averse. Selecting ρ equal to, say, 1, as I do in the paper, can be considered as a convenient shorthand between these two extremes.

[21] Explaining why contracts are not indexed would be beyond the scope of the paper. See Jovanovic and Ueda (1997) for a possible microfoundation.

Therefore it follows from $\beta > \gamma$ that $\lambda > 0$ and the borrowing constraint will always hold with equality. Alternatively, define R_t^E the Entrepreneurial gross return on her savings. Optimality implies, combining 3.6 and 3.7 in order to eliminate λ_t:

$$R_t^E = E_t \left(\frac{q_{t+1}(1-m) + \nu Y_{t+1}/(X_{t+1}h_t)}{q_t - m\Pi_{t+1}q_{t+1}/R_t} \right)$$

In steady state:

$$R^E = 1/\gamma > R = 1/\beta$$

therefore the borrowing constraint will be holding with equality:

$$R_t b_t = m E_t (q_{t+1}\Pi_{t+1}) h_t \tag{3.9}$$

Equation (3.9) shows that entrepreneurial net worth and financial conditions will therefore depend on the economy's conditions: the stance of monetary policy and fluctuations in the price of real estate will in fact have important consequences on the amount of funds that the entrepreneur can borrow. The model therefore captures in a simple way the links between monetary policy, asset prices and borrowing possibilities first formalised in Kiyotaki and Moore (1997) theory of credit cycles.

3.3 Other agents

3.3.1 Retailers

As standard in the literature, to motivate sticky prices I assume monopolistic competition and implicit costs of adjusting nominal prices. I follow Bernanke, Gertler and Gilchrist (2000) in assuming that monopolistic competition occurs at the retail level. Retailers do nothing in the model other than buying goods from the entrepreneurs, differentiating them costlessly, re-selling them in the market and transferring the profits from their activity to the households.

Assume a continuum of retailers of mass 1, each indexed by z, buy wholesale goods Y_t from entrepreneurs at P_t^w in a competitive market, slightly differentiate the goods at no cost to transform them into $Y_t(z)$ and sell $Y_t(z)$ at a nominal price of $P_t(z)$. Final goods are the CES aggregate:

$$Y_t' = \left(\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$. Given the aggregate output index,[22] the corresponding price index is given by:

$$P_t = \left(\int_0^1 P_t(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}}$$

so that each retailer faces a downward sloping demand curve for his good of:

$$Y_t(z) = (P_t(z)/P_t)^{-\varepsilon} Y_t'$$

Each retailer chooses a sale price $P_t(z)$ taking as given the price of wholesale goods P_t^w and the demand curve. The sale price can be changed in every period only with probability $1-\theta$. Denote with P_t^* the "reset" price, i.e. the new price set by retailers who are able to change, and with $Y_t^*(z)$

[22] In general the CES aggregate production function given above makes exact aggregation difficult. However it is possible to show (see e.g. Gertler, 2000) that a linear aggregator of the form $Y_t' = \int_0^1 Y_t(z)\ dz$ is equal to Y within a local region of the steady state. In what follows, I will therefore consider total output as Y_t.

the corresponding demand. Each "resetting" retailer chooses P^* to maximise expected discounted profits given by:

$$\sum\nolimits_{k=0}^{\infty} \theta^k E_t \left(\Lambda_{t,k} \frac{P_t^* - P_{t+k}^w}{P_{t+k}} Y_{t+k}^* (z) \right), \text{ where } \Lambda_{t,k} = \beta \left(C_t'/C_{t+k}' \right)^{\rho'}, \ P_t^w = P_t/X_t$$

where Λ is the household relevant discount rate (retailers act as agents for the household), P_t^w is the nominal price of wholesale goods, P_{t+k} is the aggregate price level, and $Y_{t+k}^* (z) = (P_t^* (z) / P_{t+k})^{-\varepsilon} Y_{t+k}$ is the demand the shareholder faces when choosing P_t^*. Differentiating the objective with respect to P_t^* gives the first order condition that the reset price must satisfy:

$$\sum\nolimits_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,k} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \left[\frac{P_t^*}{P_{t+k}} - \frac{X}{X_{t+k}} \right] \right\} = 0 \tag{3.10}$$

where the steady state markup equals $X = (1 - 1/\varepsilon)^{-1}$. In words, this conditions says that the optimal price is set so that expected discounted marginal revenue equals expected discounted marginal cost, where the marginal cost is the inverse of the markup. Finally, total profits $F_t = (1 - 1/X_t) / Y_t$ are rebated lump-sum to unconstrained households.

As a fraction θ of prices stays unchanged, the aggregate price level evolution is given by:

$$P_t = \left(\theta P_{t-1}^{\varepsilon} + (1 - \theta) (P_t^*)^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \tag{3.11}$$

Plugging (3.10) into (3.11) and log-linearising yields a (forward looking) Phillips curve for the model, which states that current inflation depends positively on expected inflation and negatively on the markup $X_t \equiv P_t/P_t^w$ of final over wholesale goods. As Bernanke, Gertler and Gilchrist (2000) emphasise, this markup varies inversely with the state of demand. With nominal price rigidities, a contraction in demand (following, e.g., tight money) forces retailers who cannot change their prices to sell less. They therefore reduce purchases of wholesale goods from the entrepreneurs who in turn produce less. The fall in the flexible wholesale price P^w implies that the markup (P/P^w) rises.

3.3.2 Central bank policy and the interest rate rule

The only role of central bank is to make lump sum transfers of money to the real sector (this introduces a means of changing the money stock). In practice, it is not necessary to track money supply. Each money demand equation[23] in fact implies a unique mapping between nominal interest rates, real money demand and consumption. Under interest rate targeting, these relations simply determine the path of the endogenous money supply stock. To implement its choice of the nominal interest rate, the central bank simply adjusts the money stock (i.e., transfers to each agent) to satisfy the money demand equations. One can therefore drop the money demand schedules and the central bank budget constraint and replace them with an interest rate rule of the form (following Taylor, 1993):

$$R_t = (R_{t-1})^{r_r} \left[(P_t/P_{t-1})^{1+r_\pi} (X_t)^{-r_x} \right]^{1-r_r} \varepsilon_{r,t} \tag{3.12}$$

This rule describes the behaviour of the central bank in the following way: monetary policy responds systematically to the contemporaneous values of inflation (with a positive and typically larger than one coefficient, i.e. $r_\pi > 0$) and markup (with a negative coefficient, i.e. $r_x < 0$). The markup is in fact the inverse of the output gap (ratio of actual output to output that would obtain in the flexible price equilibrium), and signals excess or under-capacity in the economy. From the Phillips

[23] "Each" is used because in the extended model of the next section impatient households demand money too.

curve below (L.9), we will see that relative price distortions are eliminated only when the markup is constant. Monetary policy should lower interest rates when the markup rises since signals that the economy is operating below its optimal capacity. Finally, so long as $r_r > 0$, the rule also accounts for some degree of interest rate smoothing, because of the observed central banks tendency to smooth interest rate adjustments. $\varepsilon_{r,t}$ is a white noise shock process with zero mean and variance $\sigma^2_{\varepsilon^r}$.

3.4 Market clearing

There are four markets in the economy: the labour market, the real estate market, the goods market, the bond market. The respective market clearing conditions are given by:

$$L_t = L'_t \tag{3.13}$$

$$h_t + h'_t = H = 1 \tag{3.14}$$

$$Y_t = c_t + c'_t \tag{3.15}$$

$$b_t + b'_t = 0 \tag{3.16}$$

The model has a unique steady state equilibrium. This occurs because entrepreneurs always hit the borrowing constraint and borrow up to the limit, paying back in the steady state the interest rate on debt and rolling the equilibrium level of debt over forever.

3.5 Equilibrium

Equilibrium is an allocation $\{h_t, h'_t, L_t, L'_t, Y_t, c_t, c'_t, b_t, b'_t\}_{t=0}^{\infty}$ together with a sequence of values $\{w'_t, R_t, P_t, P^*_t, X_t, \lambda_t, q_t\}_{t=0}^{\infty}$ satisfying equations (3.1) to (3.16), given $\{h_{-1}, R_{-1}, b_{-1}, P_{-1}\}$ and the sequence of monetary shocks $\{\varepsilon_{r,t}\}$.

3.6 Steady state

Before turning to the log-linearised model, it is useful to look at the steady state properties of the equilibrium:[24]

$$\Pi = 1,\ R = 1/\beta,\ \lambda = (\beta - \gamma)/c^{\rho} \tag{SS1}$$

$$\frac{h}{h+h'} = \frac{\gamma\nu(1-\beta)}{\gamma\nu(1-\beta) + j\left[(X-\nu)(1-\gamma-(\beta-\gamma)m) + \gamma\nu(1-\beta)m\right]} \tag{SS2}$$

$$\frac{qh}{Y} = \frac{\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X} \tag{SS3}$$

$$\frac{b}{Y} = \frac{\beta m\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X} \tag{SS4}$$

$$c = \frac{\nu}{X}Y - (1-\beta)mqh = \nu\left(\frac{1-\gamma-(1-\gamma)\beta m}{1-\gamma-(\beta-\gamma)m}\right)\frac{Y}{X} \tag{SS5}$$

$$c' = \frac{X-\nu}{X}Y + (1-\beta)mqh = \left(X-\nu+\frac{\gamma\nu(1-\beta)m}{1-\gamma-(\beta-\gamma)m}\right)\frac{Y}{X} \tag{SS6}$$

In the nonstochastic steady state with zero inflation, the nominal and real interest rate equal the time preference rate, and the multiplier on the borrowing constraint is positive. The share of housing accruing to the entrepreneurial sector is increasing in ν, the elasticity of output to real estate, and

[24] I assume zero inflation and technological progress in steady state. Modifying the results to account for positive inflation or growth is straightforward, and does not affect the results.

decreasing in j, the weight on housing in the household utility function. Likewise, entrepreneurial borrowing and household consumption (entrepreneurial consumption) are increasing (decreasing) in m and ν. In particular, a higher m pushes the steady state debt to output ratio up in two ways: firstly, in increases the numerator in (SS4), trivially allowing more debt for given asset prices. Secondly, a higher m is tantamount in steady state to a transfer of resources from "unproductive" households to "productive" entrepreneurs, therefore it raises equilibrium house prices from (SS3) and debt as well. I will return to these formulas in Section 4.4 when I analyse the steady state of the extended model.

3.7 The model in linearised form

Define the constants $\iota \equiv (1-\beta)\, h/(1-h)$, $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$ and $\gamma_e \equiv \gamma + m(\beta-\gamma)$, $\gamma_c \equiv (1-m\beta)\,\rho$, denoting respectively: the price elasticity of household (residual) housing demand; the elasticity of inflation to markup; the entrepreneurial effective discount rate; the entrepreneurial elasticity of real estate demand to consumption. Let variables with a hat denote percent deviations from the steady state, and let those without a time subscript denote the steady state values. The model can be reduced to a log-linearised system given by the following blocks of 10 equations in the variables: $Y, \;\; c, \;\; c', \;\; q, \;\; h, \;\; b, \;\; \pi, \;\; X, \;\; L, \;\; R$;

1. aggregate demand

$$\widehat{Y}_t = \frac{c}{Y}\widehat{c}_t + \frac{c'}{Y}\widehat{c'}_t \tag{L.1}$$

$$\rho'\widehat{c'}_t = \rho' E_t\widehat{c'}_{t+1} - \left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{L.2}$$

$$\frac{c}{Y}\widehat{c}_t = \frac{b}{Y}\widehat{b}_t - \frac{Rb}{Y}\left(\widehat{\pi}_t - \widehat{R}_{t-1} - \widehat{b}_{t-1}\right) + \frac{\nu}{X}\left(\widehat{Y}_t - \widehat{X}_t\right) - \frac{qh}{Y}\left(\widehat{h}_t - \widehat{h}_{t-1}\right) \tag{L.3}$$

the first equation is the market clearing condition in absence of investment. The second equation is the Euler equation for household consumption, while the third is the flow of fund/consumption function for the entrepreneurs.

2. housing market dynamics

$$\widehat{q}_t = \gamma_e E_t\widehat{q}_{t+1} + (1-\gamma_e)\, E_t\left(\widehat{Y}_{t+1} - \widehat{h}_t - \widehat{X}_{t+1}\right) - m\beta\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) + \gamma_c\left(\widehat{c}_t - E_t\widehat{c}_{t+1}\right) \tag{L.4}$$

$$\widehat{q}_t = \beta E_t\widehat{q}_{t+1} + \iota\widehat{h}_t + \rho'\widehat{c'}_t - \beta\rho' E_t\widehat{c'}_{t+1} \tag{L.5}$$

the first equation is housing demand for entrepreneurs, while the second is housing demand for the households.

3. borrowing constraint

$$\widehat{b}_t = E_t\widehat{q}_{t+1} + \widehat{h}_t - \left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{L.6}$$

4. aggregate supply

$$\widehat{Y}_t = \nu\widehat{h}_{t-1} + (1-\nu)\,\widehat{L}_t \tag{L.7}$$

$$\widehat{Y}_t = \eta'\widehat{L}_t + \widehat{X}_t + \rho'\widehat{c}'_t \tag{L.8}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} - \kappa\widehat{X}_t \tag{L.9}$$

the aggregate supply side of the economy includes respectively the log-linear version of the production function, the labour market equilibrium condition and the Phillips curve.

5. monetary policy rule

$$\widehat{R}_t = (1 - r_r)\left((1 + r_\pi)\,\widehat{\pi}_t - r_x \widehat{X}_t\right) + r_r \widehat{R}_{t-1} + \widehat{\varepsilon}_{r,t} \quad \text{(L.10)}$$

This is the log-linear version of the monetary policy rule, in which the nominal rate responds systematically to inflation (with a coefficient larger than 1 whenever r_π is positive) and to the markup, which can be interpreted as a measure of the shortfall of output Y_t from its natural level that would prevail under zero inflation.

One concern regarding policy rules is that they may lead to indeterminacy (multiple equilibria) or explosiveness (no equilibrium). However, reasonable values of r_x and a positive value for r_π were enough to guarantee a unique rational expectations solution in the model and its extension below.

3.8 The transmission mechanism: indexation, equity and collateral effects

The simple model highlights the main mechanism at the work in the model, namely the interaction between the real interest rate channel, the house price channel, and the debt deflation channel in the transmission of shocks to the economy. For now, I focus only on monetary shocks; later, when I examine the full model, I will also look at the effects of other types of disturbances.

Absent investment, the real interest rate channel works by discouraging current consumption and signalling agents to consume more in the future. With sticky prices, the monetary authority can change the short-term real rate, gaining leverage over the near-term course of the economy. A positive impulse to $\widehat{\varepsilon}$ in L.10 leads – via intertemporal substitution – to a fall in household consumption (from L.2) and consequently in output. The effect is reinforced through the fall in house prices (L.4 and L.5), which leads to lower borrowing (L.6) and lower productive housing investment (since entrepreneurs steady state return is higher than household return on savings). Debt deflation plays a role too: since obligations are not indexed, the fall in inflation raises the cost of debt services, further depressing entrepreneurial consumption. This can be seen from equation (L.3), that illustrates how entrepreneurs (debtors) consumption is negatively affected by unanticipated deflation.[25]

How strong are the various mechanisms driving the credit channel? To give some quantitative flavour, I calibrate the model at quarterly frequencies and simulate the effects of a 1% rise in the residual of the Taylor rule (I defer most of the calibration details to the extended model). m, the commercial LTV ratio, is given a value of 60%. The risk aversion parameters are $\rho = 1$ and $\rho' = 2$. θ, the probability of not resetting prices for the individual retailer, is given the standard value of 0.75. β is set equal to 0.99 and $\gamma = 0.98$, implying a steady state annual return on entrepreneurs assets of 8%. I set ν, the elasticity of output to real estate, equal to 10% (implying a steady state value for h, the real estate holdings of the firm of 44% of the total stock) and the households labour supply elasticity to 1, i.e. $\eta' = 2$. As for the policy rule, I assume for now pure inflation targeting, that is $r_x = 0$, and $r_\pi = 0.5$, so that a permanent rise in inflation of 1% leads to a long run rise in the real rate of 0.5% (r_r is set to 0.8). This overstates the magnitude of the credit channel so long as policy does not intervene on the nominal rate when output (markup) falls (rises). However, it has the virtue of isolating the exogenous component of the reaction function from its endogenous component.

Figure 2 shows the severe amplifying power of (1) collateral and (2) debt deflation channel.

It is instructive to start from the bottom row, which shows the effects of a monetary contraction when both effects are shut off.[26] In this case, (1) current borrowing possibilities are not affected by

[25] To gain insight into the independent role of debt deflation, recall that equation L.3 is the only equation in which the realised (as opposed to the expected one) real interest rate $\widehat{\pi}_t - \widehat{R}_{t-1}$ appears.

[26] The technical details of these alternative assumptions are spelled in the Appendix.

changes in asset prices or the interest rate; and (2) debt repayments are indexed, that is the repayment in every period is adjusted ex post on the basis of the realised price level (so that borrowers and lenders do not run the risk of unanticipated inflation or deflation). A rise in the real rate of about 1% (4% on an annual basis) causes output to fall by 1.3%. The fall in output results from intertemporal substitution in consumption and from complementarity between leisure and consumption, that drives labour effort down.

The fourth row shows the results when debt is no longer indexed to the price level, but the collateral channel is still shut off: output falls by 2%. The additional output drop stems from the fact that the fall in prices increases the debt service burden of entrepreneurs and reduces their net worth. Weakened in their financial positions, entrepreneurs have thus to cut back more on consumption (1st column) and real estate spending.

Compare this with the benchmark case in which both frictions are on (top row): output falls by about 2.3%. This further decline is the effect of the fall in the collateral value, which directly chokes off current entrepreneurial borrowing possibilities. It takes a small departure from the standard monetary business cycle model to generate a debt to income ratio which is consistent with a sizeable amplification mechanism.[27]

The second and third rows of Figure 2 consider the scenario in which the composition of entrepreneurial external finance is a mix of debt and equity.[28] To give an example, in the context of the residential housing market, Caplin, Chan, Freeman, and Tracy (1997) have proposed the formation of so-called "housing partnerships", a financing arrangement that would allow a household to share ownership of its home with outside investors. Such partnerships would significantly reduce the up-front costs and the carrying costs of owning a house, enabling debtors to collateralise a smaller part of their real estate holdings issuing equity for the remaining part. More in general, it has been observed[29] that it is desirable to have the lenders to share in the gains and the losses due to economy-wide factors, linking payments to macroeconomic performance. Although the microeconomic foundations and the consequences of such arrangements are beyond the scope of the paper, it is easy to explore the debt-equity mix effects. The key difference with the model above is simple (the Appendix discusses the technical details): if liabilities issued to outsiders are divided between equity and debt, nominal repayments at time t equal $R_{t-1}B_{t-1}$, weighted by the debt share of external finance, $1-n$; plus the dividend, equal to the realised value of real estate at time t, mQ_th_{t-1}, weighted by the equity share of external finance, n. The second (50% equity) and third row (100% equity) show how, the more is external finance equity financed, the more is the severity of a monetary shock reduced. With equity finance, the fall in net worth of the firm makes now external finance less expensive than in the 100% debt case. This mitigates the fall in housing investment, and the consequent output contraction. Thus, there are two important distinctions regarding the importance of leverage: first, m dictates how much of the asset is usable as collateral; second, n dictates how much of the collateralisable wealth is divided between debt and equity. Both matter for the real economy, with the amplification

[27] The puzzling aspect of the debt deflation here is that while unanticipated deflation clearly makes debtors worse off, it also makes creditors better off, and so is only a redistribution. Here, though, debtors are relatively more impatient (which also explains why they became debtors in the first place) and thus have a higher propensity to consume out of lifetime wealth. There is empirical evidence on this (discussed in Section 5.1), see e.g. Lawrance (1991).

[28] There is a long literature on this argument, most of which weights the benefits of debt as an incentive mechanism for the firm against the costs of financial distress. Here I am clearly biased in favour of the high costs argument. See Gertler and Hubbard (1993) for a discussion.

[29] See e.g. Gertler and Hubbard, 1993, and Bernanke, 1993.

effects of monetary policy being increasing in m and $1 - n$.[30]

Price rigidities are at the heart of the transmission mechanism, since they enable the monetary authority control over the short-term real rate. It must be stressed, though, that the same mechanisms would be at work even if prices were fully flexible and monetary policy were cast in the form of a money supply rule instead of the Taylor rule L.10. A shock to the quantity of money would have zero or extremely limited effects (see e.g. Fischer, 1979) in a representative agent economy or in an economy when debt is indexed. When, like in the setting of this paper, debt is non-indexed, an unexpected decrease in money supply produces a contractionary effect on the economy.

4 The full model: household and entrepreneurial debt

The basic model assumes that all mortgaged real estate is used by firms. In reality, financial frictions apply to both firms and households. While the previous section models entrepreneurial consumption, it lacks the kind of descriptive realism emphasised, for example, in the quote from Tobin at the beginning of the paper. In addition, while "investment" occurs in the form of transfers of real estate between agents, aggregate investment is zero. In addition, the absence of adjustment costs is probably not compelling from a theoretical viewpoint.

This section extends the basic model to capture the following facts:

1. Households are net lenders to firms in the economy;

2. Firms are likely to face borrowing constraints;

3. Some households are in debt and face borrowing constraints; typically, the collateral they use to borrow is the property they own together with a multiple of their current income.

This extension has several merits. It can capture some great ratios relative to credit aggregates and the housing market, such as the values of residential and commercial housing stock and mortgage debt over output. It allows a more realistic analysis of the impact of a various range of disturbances such as inflation, technology and preference shocks. It renders possible an analysis of the different roles played in the economy by household and firm's debt. It can shed light on the redistributive effects of monetary policy, of the kind stressed, for instance, by Romer and Romer (1999): unanticipated deflation increases the real value of nominal assets and liabilities. It therefore causes real capital losses for nominal debtors, and real capital gains for nominal creditors (most of the poor are net nominal debtors with medium and long-term debts, the two most important categories being typically real estate and installment debt).

I assume that households fall into two groups, "impatient" and "patient" ones. Like in the model of the previous section, the steady state outcome will be such that the patient household will not face binding borrowing constraints whereas both entrepreneurs and impatient households will.[31]

Despite featuring infinitely lived agents, the model framework has also an overlapping generations interpretation. One can think of impatient consumers (low wealth, high marginal propensity to consume, and debtors in equilibrium as will we see) as young in the saving phase of their life cycle

[30] See also the discussion in Gertler and Hubbard (1988).

[31] This is a corollary of a theorem stated in Becker (1980) and also analysed in Becker and Foias (1987). In Becker (1980), for instance, all the steady state capital stock in an economy with a single capital good is held by the most patient household. The other households hit the zero constraint on capital holdings and earn only wage income. For a recent discussion, see also Browning, Hansen and Heckman (2000).

who anticipate a rapid profile of income growth, while the model patient households (high wealth, low marginal propensity to consume and creditors) represent consumers in the latter phase of the life-cycle who expect slow or no income growth.

4.1 The setup

Time is discrete. The model economy consists of the following agents, each infinitely lived and of measure one , impatient households (double-prime $''$), entrepreneurs.

- Households inherit real estate and bonds from $t-1$; derive utility from consumption c and real estate h (residential housing); supply labour to the entrepreneur. Each household is allowed to borrow up to a fraction of real estate holdings and to a multiple of the wage income. Households differ in their time preference rates. *Impatient households* (denoted with a double prime $''$) borrow in equilibrium and face a binding borrowing constraint. *Patient households* (denoted with a prime $'$) lend in equilibrium and do not hit the borrowing limit: therefore, I assume ex ante for simplicity that they do not face borrowing constraints.

- *Entrepreneurs* inherit variable capital, real estate and bonds from the previous period; combine capital, real estate and household labour to produce a homogeneous intermediate (wholesale) good. They can borrow up to a fraction of the value of real estate used in the production and face adjustment costs in changing the stock of variable capital.

- A *central bank* runs monetary policy and *retail outlets* transform the wholesale good into differentiated final goods. Each retailer can adjust prices only with some probability.

The problems of patient households, retailers and central bank are unchanged. I consider therefore the (slightly modified) entrepreneurs problem and then move to impatient households behaviour.

4.2 Entrepreneurs

Entrepreneurs produce a wholesale good using a Cobb-Douglas constant returns to scale technology with real estate, variable capital and two types of labour as inputs. The production function is:

$$Y_t = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} L_t'^{\alpha(1-\mu-\nu)} L_t''^{(1-\alpha)(1-\mu-\nu)} \tag{4.1}$$

where A is the technology parameter, h is real estate, L' and L'' are the patient and impatient household labour inputs and K is new capital (which depreciates at rate δ over time) created at the end of each period using consumption goods. The installation of new capital entails a convex adjustment cost of $\psi\left(I_t/K_t-\delta\right)^2 K_t/2$, paid in units of the final good.[32]

After production, entrepreneurs sell their output Y_t to retail outlets at a real price of $P_t^w/P_t \equiv 1/X_t$, where P is the aggregate price index and $X>1$ is the markup of retail over wholesale goods.

Entrepreneurs maximize the intertemporal objective c_t:

$$\max_{B_t, I_t, K_t, h_t, L_t', L_t''} E_0 \sum\nolimits_{t=0}^{\infty} \gamma^t \frac{c_t^{1-\rho}}{1-\rho}$$

[32]The formulation of the convex adjustment cost is widespread in the literature, see for instance the discussion in Kiley (2000). The value of ψ dictates the elasticity of investment to the price of capital. It is easy to show that in a neighborhood of the steady state this elasticity is equal $1/\psi\delta$.

where $\gamma < \beta$, subject to flow of funds, capital accumulation, technology and borrowing constraints:

$$\frac{Y_t}{X_t} + b_t = c_t + q_t \left(h_t - h_{t-1}\right) + \frac{R_{t-1}}{\Pi_t} b_{t-1} + w'_t L'_t + w''_t L''_t + I_t + \frac{\psi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \tag{4.2}$$

$$I_t = K_t - (1-\delta) K_{t-1} \tag{4.3}$$

$$Y_t = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} L_t'^{\alpha(1-\mu-\nu)} L_t''^{(1-\alpha)(1-\mu-\nu)}$$

$$R_t b_t \leq m E_t \left(q_{t+1} h_t \Pi_{t+1}\right)$$

where I assume that only real estate is collateralisable and that entrepreneurs cannot borrow more than a fraction m of the expected market value of the real estate at $t+1$. Denoting with λ_t and u_t the Lagrange multipliers respectively on borrowing constraint and capital accumulation equation, the first order conditions with respect to B_t, I_t, K_t, h_t, L'_t, L''_t are:

$$\frac{1}{c_t^{\rho}} = E_t\left(\frac{\gamma R_t}{\Pi_{t+1} c_{t+1}^{\rho}}\right) + \lambda_t R_t$$

$$u_t = \frac{1}{c_t^{\rho}}\left(1 + \psi\left(\frac{I_t}{K_t} - \delta\right)\right) \tag{4.4}$$

$$u_t = \frac{1}{c_t^{\rho}}\left(\psi\left(\frac{I_t}{K_t} - \delta\right)\frac{I_t}{K_t} - \frac{\psi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2\right) + \gamma E_t\left(\frac{\mu Y_{t+1}}{c_{t+1}^{\rho} X_{t+1} K_t} + u_{t+1}(1-\delta)\right) \tag{4.5}$$

$$\frac{1}{c_t^{\rho}} q_t = E_t\left(\frac{\gamma}{c_{t+1}^{\rho}}\left(\nu \frac{Y_{t+1}}{X_{t+1} h_t} + q_{t+1}\right) + \lambda_t m \Pi_{t+1} q_{t+1}\right)$$

$$w'_t = \frac{\alpha(1-\mu-\nu) Y_t}{X_t L'_t} \tag{4.6}$$

$$w''_t = \frac{(1-\alpha)(1-\mu-\nu) Y_t}{X_t L''_t} \tag{4.7}$$

The consumption/savings choice is the same as the previous section's. Equations 4.4 and 4.5 are standard in models of investment with adjustment costs. Equation 4.4 defines the shadow value of one unit of investment today, which equals the marginal cost of investment. Equation 4.5 states that this value must be equated across time. At the optimum, it says that the shadow price of capital must equal the capital's marginal product next period plus the capital contribution to lower installation costs plus the shadow value of capital in the next period. Two labour demand equations, 4.6 and 4.7, one for each household group, complete the first order conditions.

As before, the assumption $\gamma < \beta$ ensures that the borrowing constraint will be binding in steady state. The details of the proof are identical to those of section 3.2 and are therefore omitted.

4.3 Impatient households

Impatient households face a problem similar to that of patient ones, with the only difference that they will face a binding borrowing constraint in equilibrium. They choose consumption c''_t, housing h''_t, labour supply L''_t (and real money holdings M''_t/P_t) in order to solve:

$$\max_{B''_t, h''_t, L''_t, M''_t/P_t} E_0 \sum\nolimits_{t=0}^{\infty} \left(\beta''\right)^t \left(\frac{\left(c''_t\right)^{1-\rho''}}{1-\rho''} + j \ln h''_t - \frac{\tau''}{\eta''}\left(L''_t\right)^{\eta''} + \chi \ln \frac{M''_t}{P_t}\right)$$

where I assume $\beta'' < \beta$ (this ensures an equilibrium in which impatient households will be borrowing constrained). Their budget and borrowing constraints in real terms are:

$$c_t'' + q_t\left(h_t'' - h_{t-1}''\right) + \frac{R_{t-1}}{\Pi_t} b_{t-1}'' = b_t'' + w_t'' L_t'' \left[-\frac{M_t'' - M_{t-1}''}{P_t} + T_t''\right] \tag{4.8}$$

$$b_t'' = E_t\left(\frac{m'' q_{t+1} h_t'' \Pi_{t+1}}{R_t}\right) + m^y w_t'' L_t'' \tag{4.9}$$

The first constraint is the agents' flow of fund. The second constraint states that impatient households cannot borrow more than the sum of m'' times housing wealth and m^y times current wage income. As for the Entrepreneurs, I do not attempt to derive this constraint endogenously, although the constraint is consistent with standard lending criteria used in mortgage and consumer loans: it seems reasonable to assume that consumers can borrow against the value of the housing wealth and a multiple of current labour income (which act as a proxy for lifetime income).

Denoting with λ'' the Kuhn-Tucker multiplier on the borrowing constraint, the first order conditions with respect to b_t'', h_t'', L_t'' are:

$$\frac{1}{\left(c_t''\right)^{\rho'}} = E_t\left(\frac{\beta'' R_t}{\Pi_{t+1}\left(c_{t+1}''\right)^{\rho''}}\right) + \lambda_t'' R_t \tag{4.10}$$

$$\frac{q_t}{\left(c_t''\right)^{\rho''}} = \frac{j}{h_t''} + E_t\left(\frac{\beta'' q_{t+1}}{\left(c_{t+1}''\right)^{\rho''}} + \lambda_t'' m'' q_{t+1}\Pi_{t+1}\right) \tag{4.11}$$

$$\tau''\left(L_t''\right)^{\eta''-1} = \frac{w_t''}{\left(c_t''\right)^{\rho''}} + \lambda_t'' m^y w_t'' R_t \tag{4.12}$$

Like for the entrepreneurs, the equations for consumption and housing choices hold with the addition of the multiplier associated to the borrowing restriction. By combining the two equations, one can also show that the borrowing limit distorts the intratemporal allocation of resources between consumption and housing. Similar considerations carry over to the labour supply equation, which shows the extra-incentive to work besides wage, given by the fact that more hours of labour enhance the household's ability to borrow.

An additional equilibrium condition links money balances demand and nominal interest rates. This condition turns out to be redundant under interest rate targeting, so long as the central bank respects for each group the equality between money injections and transfers, keeping the term in square brackets in the budget constraint (equation 4.8) equal to zero.

4.4 Steady state

I skip the definition of equilibrium, which goes along the same lines as the definition of equilibrium of the basic model. It is interesting to examine some of the steady state results under the assumption of zero inflation. Define $s' \equiv (\alpha(1-\mu-\nu) + X - 1)/X$ and $s'' \equiv (1-\alpha)(1-\mu-\nu)/X$ respectively the shares of total income accruing to patient and impatient households. It is easy to show that the real estate shares accruing to each sector respectively satisfy (for ρ, ρ' and ρ'' equal to 1):

$$\frac{qh}{Y} = \frac{\gamma\nu}{1-(\gamma+(\beta-\gamma)m)}\frac{1}{X} \tag{4.13}$$

$$\frac{qh'}{Y} = \frac{j}{1-\beta}\left(s' + \frac{(1-\beta)m\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X} + \left(\frac{m''\left(1-(1-\beta)\frac{m^y}{\beta}\right)(1-\beta)}{1-\beta''-m''\left(\beta-\beta''-j(1-\beta)\right)} + \frac{1-\beta}{\beta}m^y\right)s''\right) \tag{4.14}$$

$$\frac{qh''}{Y} = \frac{j\left(1-(1-\beta)\frac{m^y}{\beta}\right)}{1-\beta''-m''\left(\beta-\beta''-j(1-\beta)\right)}s'' \tag{4.15}$$

Equation (4.13) shows the value of commercial real estate over output, which is proportional to ν, the elasticity of output to real estate. The denominator captures the present value formula for the value of an asset. The term $\gamma + (\beta - \gamma)m$ can be in fact interpreted as the entrepreneur's effective discount rate, given the presence of borrowing constraints. A tighter borrowing constraint (lower m), everything else constant, lowers such rate, thus pushing down the entrepreneurial returns and the ratio itself.

Equations (4.14) and (4.15) are the two components of the value of residential housing stock relative to GDP. h' and h'' are respectively the holdings of unconstrained and constrained agents. These ratios depend on the relative productivity of each of the two household groups, weighted by j, and discounted appropriately, taking into account the redistribution of income from debtors to creditors that takes place in steady state. Were the bond market closed, $m = m'' = m^y = 0$, (4.13), (4.14) and (4.15) in fact would collapse to the standard present value formulas for the price of an infinitely lasting asset:

$$\frac{qh}{Y}|_{m=0} = \frac{\gamma\nu}{1-\gamma}\frac{1}{X},\ \frac{qh'}{Y}|_{m=m''=m^y=0} = \frac{j}{1-\beta}s',\ \frac{qh''}{Y}|_{m''=m^y=0} = \frac{j}{1-\beta''}s''$$

Analogous considerations apply for the debt to output ratios in steady state:

$$\frac{b}{Y} = \frac{\beta m\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X} \tag{4.16}$$

$$\frac{b''}{Y} = \left[\frac{jm''(\beta-(1-\beta)m^y)}{1-\beta''-\left(\beta-\beta''-j(1-\beta)\right)m''}+m^y\right]s'' \tag{4.17}$$

(4.16) is the value of commercial mortgage debt over output, which is increasing in m, the LTV ratio. (4.17) is the equation for household debt over output, where the first addend in square brackets is debt directly tied to the value of the house, while the second denotes the income multiple component: the ratio is clearly increasing in the loan-to-value, m'', and loan-to-income, m^y.

Given the assumption that only entrepreneurs can create new capital, the steady state economy-wide saving rate is given by:

$$\frac{S}{Y} = \frac{\delta K}{Y} = \frac{\delta\gamma\mu}{1-\gamma(1-\delta)}\frac{1}{X}$$

4.5 The complete log-linearised model

To study the model dynamics, I log-linearise the equilibrium conditions around the steady state. To save on notation, I drop the expectation operator before variables dated $t+1$, which must be intended as in expected value conditional on the information available at time t. All variables indexed with a $t-1$ are predetermined at time t.

It is convenient express the log-linearised model in terms of the following six blocks of equations:

1. aggregate demand

$$0 = \frac{c}{Y}\widehat{c}_t + \frac{c'}{Y}\widehat{c'}_t + \frac{c''}{Y}\widehat{c''}_t + \frac{I}{Y}\widehat{I}_t - \widehat{Y}_t \tag{E.1}$$

$$\rho'\widehat{c'}_t = \rho'\widehat{c'}_{t+1} - \widehat{R}_t + \widehat{\pi}_{t+1} \tag{E.2}$$

$$\beta\rho''\widehat{c}''_t = \beta''\rho''\widehat{c}''_{t+1} - \left(\beta-\beta''\right)\widehat{\lambda}''_t - \beta\widehat{R}_t + \beta''\widehat{\pi}_{t+1} \tag{E.3}$$

$$\beta\rho\widehat{c}_t = \gamma\rho\widehat{c}_{t+1} - (\beta-\gamma)\widehat{\lambda}_t - \beta\widehat{R}_t + \gamma\widehat{\pi}_{t+1} \tag{E.4}$$

$$\rho\widehat{c}_t = \rho\widehat{c}_{t+1} - \zeta\left(\widehat{Y}_{t+1} - \widehat{X}_{t+1} - \widehat{K}_t\right) + \psi\delta\left(\widehat{I}_t - \widehat{K}_t - \gamma(1-\delta)\left(\widehat{I}_{t+1} - \widehat{K}_{t+1}\right)\right) \tag{E.5}$$

2. housing market

$$\widehat{q}_t = \gamma_e \widehat{q}_{t+1} + (1-\gamma_e)\left(\widehat{Y}_{t+1} - \widehat{h}_t - \widehat{X}_{t+1}\right) + m_e\left(\widehat{\lambda}_t + \widehat{\pi}_{t+1} + \widehat{c}_{t+1}\right) + \rho\left(\widehat{c}_t - \widehat{c}_{t+1}\right) \tag{E.6}$$

$$\widehat{q}_t = \gamma_h \widehat{q}_{t+1} + \frac{jc''^{\rho''}}{qh''}\left(\widehat{j}_t - \widehat{h}''_t\right) + m_h\left(\widehat{\lambda}''_t + \widehat{\pi}_{t+1}\right) + \rho''\widehat{c}''_t - \beta''\rho''\widehat{c}''_{t+1} \tag{E.7}$$

$$\widehat{q}_t = \beta\widehat{q}_{t+1} + \frac{jc'^{\rho'}}{qh'}\left(\widehat{j}_t - \widehat{h}'_t\right) + \rho'\widehat{c'}_t - \beta\rho'\widehat{c'}_{t+1} \tag{E.8}$$

$$0 = h\widehat{h}_t + h'\widehat{h}'_t + h''\widehat{h''_t} \tag{E.9}$$

3. borrowing constraints

$$\widehat{b}_t = \widehat{q}_{t+1} + \widehat{h}_t + \widehat{\pi}_{t+1} - \widehat{R}_t \tag{E.10}$$

$$\widehat{b}''_t = s_q\left(\widehat{h}''_t + \widehat{q}_{t+1} + \widehat{\pi}_{t+1}\right) + s_y\left(\widehat{Y}_t - \widehat{X_t}\right) - (1-s_y)\widehat{R}_t \tag{E.11}$$

4. aggregate supply

$$\widehat{Y}_t = \widehat{A}_t + \nu\widehat{h}_{t-1} + \mu\widehat{K}_{t-1} + \alpha(1-\nu-\mu)\widehat{L'}_t + (1-\alpha)(1-\nu-\mu)\widehat{L''}_t \tag{E.12}$$

$$\widehat{Y}_t = \widehat{X}_t + \eta''\widehat{L}''_t + s_l\rho''\widehat{c}''_t - (1-s_l)\left(\widehat{\lambda}''_t + \widehat{R_t}\right) \tag{E.13}$$

$$\widehat{Y}_t = \widehat{X}_t + \eta'\widehat{L}'_t + \rho'\widehat{c}'_t \tag{E.14}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} - \kappa\widehat{X}_t + \widehat{u}_t \tag{E.15}$$

5. Flows of funds / Evolution of state variables

$$\widehat{K}_t = \delta\widehat{I}_t + (1-\delta)\widehat{K}_{t-1} \tag{E.16}$$

$$\frac{b}{Y}\widehat{b}_t = \frac{c}{Y}\widehat{c}_t + \frac{qh}{Y}\left(\widehat{h}_t - \widehat{h}_{t-1}\right) + \frac{I}{Y}\widehat{I}_t + \frac{Rb}{Y}\left(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_t\right) - (1-s'-s'')\left(\widehat{Y}_t - \widehat{X}_t\right) \tag{E.17}$$

$$\frac{b''}{Y}\widehat{b}''_t = \frac{c''}{Y}\widehat{c}''_t + \frac{qh''}{Y}\left(\widehat{h}''_t - \widehat{h}''_{t-1}\right) + \frac{Rb''}{Y}\left(\widehat{b}''_{t-1} + \widehat{R}_{t-1} - \widehat{\pi}_t\right) - s''\left(\widehat{Y}_t - \widehat{X}_t\right) \tag{E.18}$$

6. Monetary policy rule and shock processes

$$\widehat{R}_t = (1-r_r)\left[(1+r_\pi)\widehat{\pi}_t - r_x\widehat{X}_t\right] + r_r\widehat{R}_{t-1} + \widehat{\varepsilon}_{r,t} \tag{E.19}$$

$$\widehat{j}_t = \rho_j\widehat{j}_{t-1} + \widehat{\varepsilon}_{j,t}$$

$$\widehat{u}_t = \rho_u\widehat{u}_{t-1} + \widehat{\varepsilon}_{u,t}$$

$$\widehat{A}_t = \rho_A\widehat{A}_{t-1} + \widehat{\varepsilon}_{A,t}$$

where I have defined the following constants: $\gamma_e \equiv \gamma + m(\beta-\gamma)$, $\gamma_h \equiv \beta'' + m''\left(\beta-\beta''\right)$, $m_e \equiv m(\beta-\gamma)$, $m_h \equiv m''\left(\beta-\beta''\right)$, $\zeta \equiv 1-\gamma(1-\delta)$, $s_q \equiv m''\beta qh''/b''$, $s_y \equiv m^y w'' L''/b''$, $s_l \equiv 1/\left(1+\left(1-\beta''/\beta\right)m^y\right)$, $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$.

Equation E.1 is the goods market clearing condition that defines total output.

Unlike the standard DNK model, here the traditional, forward-looking, household IS curve E.2 is supplemented by two more Euler equations; while patient households behaviour prices real bonds, the presence of borrowing constraints in the constrained households and entrepreneurs Euler equations (respectively E.3 and E.4) breaks the exclusive link between real interest rates and changes in consumption. If some consumers are unable to smooth consumption because of liquidity constraints, factors besides the real interest rate are an additional factor in explaining consumption dynamics. E.5 is the firm capital demand equation.

The housing market comprises the three asset demand equations: E.6 is the firm demand; E.7 and E.8 are the housing demand equations of, respectively, impatient and patient households. E.9 is the market clearing condition.

Borrowing constraints for respectively firms (equation E.10) and households (E.11) appear in block 3. For constrained households, the terms s_q and s_y indicate the financing shares of housing wealth and human wealth.

The aggregate supply block comprises the linearised version of the production function (E.12) and the two labour market equilibrium conditions, E.13 and E.14. The last equation of the block, E.15, is a short-run aggregate supply curve, as implied by Calvo-type price-setting behaviour: the residual $\widehat{u}_t$ is meant to capture any factor outside the model that might affect marginal costs (e.g. an oil-price shock), while introducing a short-run trade-off between inflation and inverse of markup. See also Giannoni (2000) and Clarida, Gali and Gertler (1999).

The flow-of-funds block includes the dynamic law of motion for capital (E.16) and the two equations describing evolution of net worth of entrepreneurs, E.17, and constrained households, E.18 (the one for unconstrained households being satisfied by Walras' law). At least two points are worth emphasising: price level surprises affect agents' net worth, since they affect the real value of the obligations which are fixed in nominal terms; secondly, house price changes affect net worth too, through their effect on investment, consumption and housing demand.

Equation E.19 is the monetary policy rule. The last set of equations impose that the exogenous disturbances to preferences, inflation and technology follow stationary AR(1) processes.

5 Model simulation

5.1 Calibration

I need to assign values to the following parameters:

Preferences	$\beta, \gamma, \beta'', j, \eta', \eta'', \rho, \rho', \rho''$
Production	$\alpha, \mu, \nu, \delta, \psi$
Finance	m, m'', m^y
Aggregate supply	X, θ
Monetary policy rule	r_r, r_x, r_π
Shocks autocorrelation	$\rho_A, \rho_\varepsilon, \rho_u, \rho_j$
Shocks variances	$\sigma_A^2, \sigma_\varepsilon^2, \sigma_u^2, \sigma_j^2$

The model is calibrated at quarterly frequencies at the nonstochastic steady state in order to roughly match empirical counterparts.

Some parameters do not require much discussion as they just pin down commonly used values in business cycle analysis. I set β, the unconstrained households discount rate, to be equal to 0.99, so that the annualised real interest rate is 4%. I select δ, the depreciation rate, to 3%, and μ, the elasticity of output to variable capital, to 0.3; Together with X, the steady state markup, and γ, the entrepreneurial discount rate (discussed below) the last two values pin down the steady state saving rate of the economy to 17%.

Next, I calibrate the time preference rates of entrepreneurs and impatient households. I begin with γ. I treat it as unobservable, but try to match its reciprocal, which corresponds to the entrepreneurial internal rate of return. I assume that this is twice as big as the equilibrium real rate, and set $\gamma = 0.98$. The model results were similar choosing alternative values. Then I pick a value for β'', the impatient households time discount rate. Empirical studies (e.g. Lawrance, 1991) find that subjective rate of time preference are higher for poor than for rich households. Using the Panel Study of Income Dynamics, Lawrance estimates subjective rates of time preference three to five percentage points higher, on an annual basis, for households with low permanent incomes. This suggests two facts: first, a value for β'' equal to about 0.98, in the middle range of the Lawrance's estimates; secondly, that impatient households have low permanent incomes too.[33] This is useful information for the calibration of α, the relative productivity of patient with respect to impatient households. It seems appropriate to pick a value of greater than one half: I choose $\alpha = 0.6$.

Now consider the parameters governing the real estate (commercial and residential) market in the model. I have to choose j, ν, m, m'', m^y. For the US, Case (2000, Table 5) gives some rough estimates of the nation's commercial real estate portfolio in 1999: excluding apartments, the value of commercial real estate is 39% of GDP, whereas the value of commercial mortgage debt is around 17%. There are no detailed figures for the commercial real estate in the Euro area: the European Mortgage Federation (EMF)[34] indicates that commercial mortgage loans are lower than in the US, around 8% of GDP. For the residential real estate market, Case (2000, Table 1) reports that the US residential real estate portfolio is worth 125% of GDP; the figure for residential mortgage debt is (Table 4) about 65% of GDP. For Europe, the EMF reports that residential mortgage debt is lower than in the US, around 36% of GDP. From Miles (1992), it is possible to calculate that the residential housing stock in European countries is a bigger share of GDP than in US , in the 160% range. There are also a sizeable component of personal loans not secured on a property: in the Euro-area the total household debt over GDP is about 48%[35], hence one can conjecture that "unsecured" debt is 12% of GDP.

Roughly, one would like the calibrations to match these values. I start with ν, the elasticity of output to entrepreneurial real estate holdings of entrepreneurs. From equation (4.13), ν affects the steady state value of commercial real estate over GDP. I set $\nu = 0.025$, implying that the ratio is 40% of GDP. I then pick j to obtain a plausible value for residential real estate over GDP. A value for j equal to 0.10 implies that the residential housing stock is worth 155% of annual GDP.

I then calibrate debt in order get plausible values for the debt to income and debt to asset values steady state ratios. I choose $m = 0.6$, which implies a steady state commercial debt to GDP ratio of 25%. Next, I move to household debt: some households borrow against the value of house, some against some multiple of wage income. Actual lending criteria are more complex than those modelled in the paper, with lending being probably a strictly convex function of household income and household assets.[36] As I model the two sources of funds as perfect substitutes, I pick lower values

[33] Zeldes (1989) finds that the "traditional" Euler equation holds for "high wealth" individuals but is rejected for "low wealth" individuals, for whom current resources help to predict consumption. In a similar vein, Attanasio, Goldberg and Kyriazidou (2000) also find that poor households are more likely to be borrowing constrained.

[34] See EMF, "Position paper on the commission's consultation document on regulatory capital requirements for EU credit institutions and investment firms", page 3, 2000.

[35] See for instance chart 55 in the Bank of England Financial Stability Review, June 2001.

[36] Caplin, Chan, Freeman and Tracy (1997, ch.3) describe the lending criteria in the US mortgage market.

than those probably adopted in practice, and I choose $m'' = 0.5$ and $m^y = 4$. This yields steady state values of "mortgaged" debt of 20% and of "uncollateralised" personal sector debt of 26%. This in turn affects the steady state consumption shares in total output of unconstrained households, constrained households and entrepreneurs, which are respectively 43%, 26% and 14% of total output. As the quantitative results of the model are clearly affected by different choices for m, m'' and m^y, I will experiment below (Section 5.3) with different values.

To conclude, I need a range of parameters that are not directly observable from the first moment of data. I assume zero adjustment costs for housing, to convey the empirically grounded idea that housing responds quickly to a monetary contraction (see Bernanke and Gertler, 1995, and McCarthy and Peach, 2001). For the capital adjustment cost, I set ψ equal to $1/\delta$, implying a steady state unit elasticity of investment to the capital shadow price, about the value found in Cummins, Hassett and Oliner (1999). As for the risk aversion, I choose standard logarithmic preferences, and set $\rho = \rho' = \rho'' = 1$. Making borrowers risk neutral (i.e., $\rho = \rho'' = 0$) would result in fact in implausibly volatile aggregate entrepreneurial consumption and net worth. I then have to choose a value for the parameters governing the disutility of labour, η' and η''; I set both of them equal to 1.1, implying a Frisch wage elasticity of 10. This is certainly higher than what microeconometric studies would suggest, but has the virtue of rationalising the weak observed response of real wages to monetary disturbances (see also the discussion in Rotemberg and Woodford, 1997 and in subsection 5.3).

Finally, I calibrate the parameters relating to the price adjustment, to the markup and to the Taylor rule. I follow convention and set θ, the probability of not changing prices for each retailer, equal to 0.75, so that the average period between price adjustments is 1 year. I choose the elasticity of demand so that the steady state markup is 5%, as suggested by evidence from Basu and Fernald (1997) for US data. For the Taylor rule, I choose $r_\pi = 0.5$ and $r_x = 0.5$, as in Taylor (1993) original formulation, and I set the inertial coefficient r_r equal to 0.8, consistent with most empirical estimates of Taylor rules which incorporate some form of partial adjustment of the short-term interest-rate instrument. I obtained similar results by casting the interest rate rule as a function of actual output Y rather than the markup X.

The variance of the shock processes is calibrated to get house price volatility bigger than output volatility, a fact consistent with the data. To obtain this, it suffices to give a relatively high variance to the housing preference shock, i.e. an exogenous change in j. I also assume that the technology, credit and preference shocks very persistent.

The baseline parameters associated with the model are summarised in Table 5.1.

5.2 Model dynamics

This subsection describes the behaviour of the full model in response to monetary, productivity and inflation and preference shocks. My objective is to assess the properties of the model and its consistency with the empirical evidence. To the extent that the model generates plausible responses of the variables to a wide range of disturbances, and given that its structural relations are derived from individual optimisation problems, I can also use the model for purposes of welfare analysis. For the parameters of Table 5.1, Figure 3 displays the impulse response functions of the key model variables, comparing the benchmark responses (solid lines) with the responses of the model version when all credit market frictions and wealth effects are shut off (dashed lines).[37]

[37] This is done by assuming a steady state equilibrium with zero debt ($m = m^y = m'' = 0$), together with 100% equity financing and indexation of debt contracts.

Table 5.1: Calibrated parameters in the benchmark model

Parameter	Description	Value	Parameter	Description	Value
Preferences: discount rates			*Borrowing limits*		
Patient households	β	.99	Entrepreneur LTV	m	.6
Entrepreneurs	γ	.98	Household LTV	m'', m^y	.5,.4
Impatient households	β''	.98			
other preference parameters			*Sticky prices*		
housing services	j	.1	Markup	X	1.05
labour supply aversion	$\eta' = \eta''$	1.1	prob. fixed price	θ	.75
risk aversion	ρ, ρ', ρ''	1,1,1			
Technology: factor productivity			*Monetary Policy*		
Patient hh relative prod.	α	.6	rule parameters	r_r, r_π, r_x	.8,.5,.5
variable capital share	μ	.3	targets	X_t and π_t	
housing share	ν	.025			
other technology parameters			*Shock processes*		
adjustment cost	ψ	33	autocorrelation	$\rho_A, \rho_\varepsilon, \rho_u, \rho_j$	.98,0,.8,.99
depreciation rate	δ	.03	variances:	$\sigma_A^2, \sigma_\varepsilon^2, \sigma_u^2, \sigma_j^2$	.01,.005,.01,.03

5.2.1 Responses to a monetary shock

The bottom row of Figure 3 shows the response of the benchmark economy to a monetary tightening. An increase in the real interest rate is associated with a fall in output and inflation, and a fall in real house prices. The major differences between the model and the data is that the peak response of output is immediate in the model, while occurs with some delay in the data. As the VAR evidence in Figure 1 shows, house prices also show a delayed response in the data, so the problem of lag discrepancy applies virtually to all model variables. In addition, the response of house prices in the data is of a bigger magnitude than the GDP response (about three times as big), while the model responses are of similar sizes.

Some interesting features of the impulse responses are worth mentioning. First, expected inflation falls in the period of the shock by less than the increase in the real rate. That is, the model monetary shock generates a "liquidity effect", that is the association of lower output with higher nominal interest rates (this is contrary to many general equilibrium sticky price models which obtain the counterfactual prediction that a monetary tightening actually results in lower nominal interest rates).

Moving to house prices, there are two main counteracting forces driving their dynamics: on the one hand, the rise in real rates and the drop in output push house prices lower; on the other, the possibility of using houses as collateral and factor of production shifts demand away from (non-durable) consumption to (durable) housing, thus raising housing demand relative to consumption. For plausible parameterisations, the first effect dominates the second.

Unsurprisingly, the model also delivers heterogeneity in the response of consumption across agents (see the top row of Figure 4): the fall in house prices associated with a monetary contraction adversely affects borrowers, by affecting their consumption more than creditors'. The consumption Euler equation for the debtors provides the explanation: tight money makes the constraint on borrowing tighter, raising its shadow value. Hence not only is there intertemporal substitution caused by the rise in the real interest rate, but also a further drop in current consumption induced by the rise in λ and λ'', the Lagrange multipliers. Debtors therefore bear most of the brunt of a monetary contraction, while unconstrained households consumption is mildly affected, and rises above the baseline in the transition to the steady state: the real interest rate *falls* in fact below the baseline in the transition (remember that it is lenders' behaviour that prices real bonds in the model). Therefore we have the interesting result that real house prices, whose dynamics are the mirror image of those in real rates, first fall below the baseline thus making the contraction worse and then overshoot above the baseline, anticipating the economy's recovery.[38]

Do credit market frictions still amplify monetary shocks? The bottom row of Figure 3 shows that the initial output response is about 30% greater, and persistence longer, than in the frictionless economy. So amplification continues to hold despite (1) the mild effect of the downturn on the lenders; (2) the accommodative behaviour of the central bank, that now mitigates the fall in output by reducing the nominal rate more quickly after the initial increase.

5.2.2 Responses to an inflation shock

The second row of Figure 3 shows the effects of a positive inflation shock. The rise in inflation generates a positive, but very short-lived (1 period only), positive effect on output due to the wealth redistribution from patient households (lenders) to the entrepreneurs and impatient households (borrowers). Soon these effects die out and output declines, displaying a hump-shaped pattern.

This is in stark contract with the responses that would occur under an inflation shock in an economy where only index bonds are traded and credit market frictions are absent (dashed line). Since the procyclical redistributive effects of inflation are now absent, the fall in output is now immediate and much stronger in magnitude. In other words, the negative correlation between inflation and output that a "supply" shock induces acts as a built-in stabiliser for the economy.

Debt deflation adds therefore a new twist to the theories of "financial accelerator" mentioned in the introduction: while it amplifies demand-type disturbances, it stabilises the shocks that create a trade-off between output and inflation stabilisation for the central bank. I will return to this issue, and to its welfare implications, in Section 6.

5.2.3 Responses to preference and productivity shocks

An exogenous rise in j, the weight on housing services in the households utility function, is a *preference shock* that can proxy for any disturbance pushing house prices up without originating in other sectors of the economy.[39] As such, it can be used to evaluate the impact of an exogenous rise in house prices on the aggregate economy. The top row of Figure 3 tracks the responses to a very persistent increase (quarterly autocorrelation of 0.99) in j. A protracted rise in house prices of 0.4% fuels a short run increase in aggregate output of about 0.07%. Aggregate consumption, not shown in the figure, rises by about 0.03% on impact (compared to a 0.01% in the frictionless economy): in particular, debtors consumption increases (since they can borrow more), while creditors' consumption and housing demand fall.[40] The implied short-run elasticity of aggregate consumption to house prices

[38] Theoretically, it is even conceivable that a monetary contraction will actually *raise* real house prices. In an economy with very high levels of credit and debt, the monetary contraction makes the lenders so better off that their consumption must increase today relative to tomorrow, hence long real rates must actually fall.

[39] Baxter and King (1991) analyse the implications of preference shifts in explaining economic fluctuations at the business cycle frequency, by introducing a parameter into the utility function which varies the individuals marginal rate of substitution (MRS) between consumption and leisure. Shocks to j can have a similar interpretation, since they vary the MRS between consumption and housing.

[40] This point has been emphasised by several authors. See for instance Parker (2000).

(7%) is very close to an estimate reported by Fed's chairman Alan Greenspan,[41] who puts the marginal propensity to consume out of housing wealth in the neighbourhood of 5%, implying a good match of the model with the empirical evidence.[42] Investment increases substantially, thanks to increased entrepreneurial borrowing on the one hand and to the reduction of housing demand in the household sector, which is absorbed by the firms.

The third row of Figure 3 shows the responses to a persistent productivity rise. With markup targeting, output and asset prices increase almost on impact. Labour effort can be shown to fall instead. This result, while surprising from a RBC perspective, is not surprising in the dynamic-neo-keynesian framework. Gali (1999) and Basu, Fernald and Kimball (1999) have observed that in a DNK model a favourable technology shock can lead to a decline in employment. This happens since a favourable supply shock leads to an increase in aggregate demand which is smaller than the increase in productivity, since not all retailers are free to adjust their prices downwards.[43]

5.3 Robustness to parameters selection

Robustness to finance parameters I turn now to the sensitivity of results to different calibrations of the parameters relating to the leverage parameters. I find that high leverage of firms strongly increases the sensitivity of output to monetary disturbances, while high household leverage slightly reduces it.

Consider the parameterisation with a high value for m, the commercial LTV ratio (Figure 4, second row): such a model implies a stronger response of output to a monetary shock. Many authors (e.g. Bernanke, 1992) have emphasised the point that "over-leverage" of households and firms can be a contributor and/or aggravate an economic slowdown. Does high leverage make recessions worse? The answer is yes as far as firms' leverage is concerned: the contraction induces a tight cash-flow squeeze on firms with already high levels of debt and interest burden. The effects can also be quantitatively significant. Moving from $m = 0.6$ (first row, implying a steady state commercial debt-to-output of 25%) to $m = 0.9$ (commercial debt ratio of 48%, second row) increases the output drop by about 50% in relative terms.

Turning to household debt, a high value for m^y (Figure 4, third row) implies a smaller response of output following tight money. In fact, the higher m^y (meaning that constrained households can borrow more against current wage income), the lower the constrained households' willingness to reduce labour supply following a monetary contraction. Consider the optimal response of a constrained household following a rise in the real rate: a higher m^y implies a larger negative wealth effect, which leads to a *higher* level of work effort (which mitigates the drop in output), keeping everything else constant. Analogous considerations apply to the parameterisation with more collateralisable housing wealth. Lastly, when all the m's are set equal to zero (no debt in equilibrium), there is a smaller response of the real variables to tight money (last row).[44]

[41] "Mortgage markets and economic activity", remarks before a conference on Mortgage Markets and Economic Activity sponsored by America's Community Bankers, Washington, D.C., November 2, 1999.

[42] See also the survey by Poterba (2000), who looks at the effects of financial asset prices on consumption. The 5% estimate is after all in the range of values that would be predicted by Friedman's permanent income hypothesis.

[43] Under interest rate targeting, the dynamics of output do not necessarily resemble those of productivity shock in a non-monetary RBC model, and are intrinsically related to the systematic component of monetary policy. For example, if the monetary rule targets output rather than the markup, output and house prices show a hump-shaped response in response to a technology disturbance.

[44] Interestingly, when loan to values ratios become very close to 1, lenders are made so better off from a rise in the nominal interest rate that their consumption must initially rise in the transition to the new steady state, hence the real

Two points deserve emphasis: first, this quantitative experiment clearly underestimates the potential magnitude of the credit channel if the central bank responds to output gap, as assumed in the benchmark parameterisation. With pure inflation targeting ($r_x = 0$), I find that the relative fall in output is about 3 times as big in the high m case. Increasing the persistence of the monetary shock or moving m closer to 1 raises the relative multiplier even more. This could explain why even small differences in the debt burden to output ratio can generate large differences in output sensitivity to monetary policy. For instance, they could shed light on why the Great Depression was so much worse than the 1921-22 recession, given that the private debt-output ratio was 156% at the onset of the first and 120% at the beginning of the second.[45] Secondly, the model successfully captures the distinction that Cole and Ohanian (2000) draw between *debt burden* (firm) effect and *wealth transfer* (household) effect of an increase in the real value of debt following tight money. While they argue that there is no presumption that these two offsetting effects can reduce aggregate activity, the simulations seem to highlight that the first effect is very likely to dominate the second.

Robustness to technology and preference parameters. The specification with α close to 1 boils down to a model with entrepreneurs and unconstrained households only, thus shutting off the effect that house price fluctuations can have on household borrowing and consumption decisions. The responses of output and house prices in this model are slightly bigger than in the benchmark model.

The model with high η's – implying a lower elasticity of labour supply – reduces asset price and output responses to monetary shocks. While the model is robust to changes in η'', the labour supply elasticity of impatient households, it requires a relatively high labour supply elasticity of lenders for the real rate to rise following a monetary contraction. If lenders did not vary their labour supply response following a monetary shock, they could be actually made better off by a rise in the nominal interest rate, so that their consumption would rise. For this to happen, the real interest rate should decline after tight money. While theoretically interesting, this result is hardly found in the data: this is why I have to disregard choices of a high η' as plausible for a calibration. After all, it is often recognised that the microeconomic evidence on variations in hours worked is at odds with the elasticities built in RBC models.[46]

The model with a high elasticity ν of output to real estate yields more amplification and persistence of the monetary shock. In principle, this could imply that, by increasing ν, one could generate, through borrowing constraints, an arbitrarily high degree of amplification. However, a high value of ν yields a poor match of the first moment properties of the data with their empirical counterparts. As seen above, the higher ν, the higher the value of commercial real estate and commercial mortgage debt over GDP. Finally, the calibration with low capital adjustment costs implies, perhaps unsurprisingly, a bigger response of GDP.

6 Systematic monetary policy and policy frontiers

Inflation and output volatility, asset price targeting and monetary policy rules. I consider now how different monetary policy rules affect output and inflation variability in the economy under exam when subject to technology, monetary, preference and inflation shocks. If the central bank controls the short-term interest rate, inflation and technology shocks (which generate a negative

rate has implausibly to fall in the wake of a monetary contraction. However the amplification result still holds.

[45] These data were taken from Cole and Ohanian (2000).

[46] King and Rebelo (2000) and Rotemberg and Woodford (1997) make similar points.

correlation between output and inflation) will force the monetary authority to face a trade-off between the variability of output and that of inflation. Analogously, if the relevant welfare measure is the markup instead of output, only inflation shocks will generate a markup-inflation variability trade-off.

My purpose is to assess how house price targeting affects the volatility of markup (or output) and inflation. Similar questions have been recently raised by a number of authors, including Bernanke and Gertler (BG, 1999, 2001), Cecchetti, Genberg, Lipsky, and Wadhwani (CGLW, 2000) and Gilchrist and Leahy (GL, 2001): these studies look at whether central banks should respond to stock prices (Tobin's q) using the model framework proposed by Bernanke, Gertler and Gilchrist (2000) − which I briefly review in the discussion of the literature below −. CGLW and BG enrich such an environment considering the possibility of fundamental and non-fundamental (bubbles) disturbances that drive asset prices. BG and GL find a very weak case for responding to stock prices. CGLW, instead, find support for a policy rule that targets stock prices.[47] Yet, none of the above studies calculates a complete efficient frontier for different specifications of the policy rule: the differences among these findings might reflect, in part, the lack of an explicit welfare analysis.[48]

I calibrate the variance of each disturbance (see Table 5.1) so that the second moments of the theoretical time series under the benchmark Taylor rule are roughly similar to their empirical counterparts. The combination of borrowing constraints and adjustment costs for capital guarantees that the variances of consumption and investment relative to GDP are plausible. As mentioned before, a relatively high variance for the preference shock[49] suffices to generate a volatility of house prices about twice as big as the output volatility, consistently with the data.

Next, I compute the inflation-output volatility frontiers for alternative parameterisations of the interest rate rule, subject to an upper bound on interest rate volatility, in the spirit of Levin, Wieland and Williams (1999). The class of rules I consider is:

$$\widehat{R}_t = r_r \widehat{R}_{t-1} + (1 - r_r) \left[r_q \widehat{q}_t + (1 + r_\pi) \widehat{\pi}_t - r_x \widehat{X}_t \right]$$

To begin, I set $r_q = 0$ as in the traditional formulation and calculate the efficient frontier, often known as the Taylor curve (Taylor, 1979). I then evaluate how the Taylor curve is shifted when house prices are included in the feedback rule, that is r_q is allowed to differ from zero.

How does house prices targeting affect the Taylor curve? It is instructive to see how targeting house prices affects the Taylor curve. For the time being, r_r, the inertial coefficient, is fixed to 0.8. Figure 5 displays how we move in the inflation-markup standard deviation space as we change the systematic response to house prices, r_q, in the feedback rule. There are $3 \times 3 = 9$ segments in the Figure. Each of them is generated by fixing a different value for r_x and r_π (0.2, 0.5 or 0.8, so that the median values correspond to the Taylor recommendation) and varying r_q from -0.1 to 0.2. In most of the segments, a greater value for r_q (above zero) brings about a movement south-east along each the lines. In other words, a greater weight placed on house prices reduces markup and increases inflation volatility. Only when the weight placed on inflation is low (so that inflation volatility is high,

[47] What seems driving the difference between these conflicting results are the different assumptions about the type of shocks hitting the economy and the nature of the bubble shock. See Bernanke and Gertler (2001) for a discussion.

[48] Filardo (2000, 2001) calculates efficiency frontiers in a much simpler reduced-form macroeconomic model with asset prices concluding that a monetary policy generally benefits from responding to asset prices.

[49] The variance of preference shock is unimportant for the results, although it needs not to be empirically implausible. Cochrane (1994) suggests that "consumption shocks", for instance, might accounts for the bulk of economic fluctuations more than technology or monetary disturbances.

as is the case of the two lines towards the bottom-right corner of Figure 5, obtained for $r_\pi = 0.2$), a greater weight on asset prices leads to unambiguous welfare losses.

To sum up, Figure 5 hints that asset price targeting could be potentially welfare enhancing when the central bank places a strong preference on output gap targeting, provided that it is already sufficiently aggressive against inflation. The result is probably not entirely surprising, since asset price stabilisation in this economy implies lower variability in the decisions of production and consumption of the agents who can engage less in consumption smoothing, namely entrepreneurs and constrained households. However, Figure 5 does not tell us whether targeting house prices moves us inside or outside the constrained optimal frontier which is achievable when r_q is set to zero.

Efficient frontiers for different specifications of the Taylor rule. The next step is to compute constrained efficient frontiers[50] for alternative values of r_q. Optimal control theory suggests that a rule that includes also a feedback from house prices to interest rates should be desirable. The questions are: what are the optimal values for r_q? to what extent targeting house prices can generate lower inflation and output gap volatility? Figure 6 plots four Taylor curves in the inflation-markup (output gap) standard deviation space obtained for four different values of r_q, namely $-.1, 0, .1$ and $.2$. The frontier with $r_q = 0$ is the one that performs better. Making the grid narrower confirms this finding: the optimal value of r_q is zero up to the second decimal.[51] In other words, if the central bank wants to stabilise output gap fluctuations, it can achieve a superior outcome by targeting the output gap directly rather than house prices.[52]

Such a lesson appears consistent with Bernanke and Gertler (2001) finding that, by responding to asset prices, a central bank could worsen economic outcomes. In their paper, the signal-to-noise ratio of asset prices is too high for asset prices to yield on their own information for the central bank.[53] Here, asset prices matter in that they transmit and amplify a range of disturbances to the real sector. However, their effects are all captured by a bigger response of output (or markup) to given disturbances. To the extent that the monetary policy authority wants to minimise variations in output fluctuations, a more appropriate policy is to target output directly, rather than choosing routes that are more complex.

Matters are different in a pure inflation targeting framework: if the central bank cannot respond to fluctuations in the output gap (for instance, because such fluctuations cannot be observed), so that $r_x = 0$, targeting house prices allows the monetary policy authority to achieve a more favourable trade-off. Figure 7 illustrates how a positive weight on house prices "stretches" the Taylor curve allowing a lower markup volatility as the weight on asset prices is increased.

[50] This frontier derived by varying r_r, r_π and r_x only is only constrained optimal, in the sense that it is the best achievable frontier in the family of frontiers derived by responding only to inflation and markup and past interest rates.

[51] Other results are that, unsurprisingly, a higher inflation (output gap) coefficient generates a lower volatility of inflation (output gap), while the optimal inertial coefficient r_r is in a neighbourhood of 0.8. These results are in line with the findings by Levin, Wieland and Williams (1999) in their analysis of optimal policy rules across a range of different structural macro models of the US economy.

[52] Matters are only slightly different when the same results are looked at in the inflation-output space. For high levels of inflation variance (when r_π is low), a small positive reaction to house prices appears now desirable.

The monetary policy loss of responding to house prices appears non-negligible. Keeping markup standard deviation constant, the increase in inflation standard deviation from responding to asset prices with $r_q = 0.1$ is about 8%.

[53] This point has also been made by Vickers (2000) and Gilchrist and Leahy (2001), among the others.

Do debt indexation or equity financing improve welfare? In principle, debt indexation or equity could offer a simple policy device to reduce the costs of inflation and to alleviate the trade-off.[54] Section 3.8 has shown how the model can deliver a smaller response of output to monetary shocks when debt contracts can be indexed to the price level or borrowers can raise equity.

Real contracting reduces the response of output to monetary and credit disturbances. Yet, it does not dampen output fluctuations in response to supply disturbances. Consider in fact a positive inflation shock: this disturbance causes a more negative response of output in the real bond than in the nominal bond model, since, while demand declines in both cases, entrepreneurs do not get the full benefit of lower real repayments (thanks to the inflation effect) to the creditors as before. In the real bond model, this is due to the fact that repayments are higher because the realised real interest rate is higher than the ex ante interest rate.

In the equity model, the answer is less clear-cut, since repayments are tied to real asset prices and to asset holdings, whose response depend on the parameters of the Taylor rule; when monetary policy is very aggressive against inflation (r_π high, so that inflation volatility is low), the nominal bond model delivers a better outcome than the equity model; the reverse is true when monetary policy is very aggressive against output/markup (r_x high, so that output volatility is low). As a consequence, the Taylor curves for the nominal bond model and the equity model cross each other.

Thus, from a welfare point of view, a shift to real indexation is a better option for the economy only for the type of disturbances (demand ones) that monetary policy can better deal with. Altogether, when both demand and supply shocks are present, the Taylor curves in an economy with real bond issuance lie above those for an economy with nominal contracting only, as shown in Figure 8. The differences between the equity model and the benchmark model are less marked: equity (nominal debt) slightly improves welfare only when the central bank aims at keeping output gap (inflation) volatility low.

7 Concluding remarks

Summary of the findings. This paper has modelled the asset market in a spirit close to Kiyotaki and Moore (1997) and monetary policy according to the dynamic-neo-keynesian approach in a framework featuring heterogeneous agents, debt, asset prices and nominal rigidities. The strength of the model is that it formalises in a simple and tractable way aspects of traditional intuition about debt, the asset market and the economy dating back at least to Fisher (1933) and summarised in the quote from Tobin at the beginning of the paper. It has also the virtue of adding heterogeneity in preferences, found in numerous micro studies, in the context of a general equilibrium model of consumer and firm behaviour in presence of credit constraints.

The paper shows how the impact on inflation and asset prices of monetary policy redistributes wealth among agents and acts as a powerful amplification mechanism. However, the amplification mechanism is not constant across shocks: debt deflation stabilises supply shocks, thus making the economy less volatile when the central bank controls the interest rate. The paper also explains how equity financing and debt indexation can reduce the response of the economy to demand disturbances. In addition, it shows how higher firm leverage increases output sensitivity to monetary shocks, while household leverage reduces it. The effects of most of the shocks are transmitted through changes in agents' financial positions and fluctuations in asset prices: however, even if asset prices changes affect

[54] For a broad discussion of the issues regarding debt indexation, see Fischer (1975) and the references therein.

the business cycle, the monetary policy authority may not want to respond to such changes, so long as asset prices themselves are not the ultimate goal of monetary policy.

Related papers. There is an enormous literature on the links between financial factors, monetary policy and the macroeconomy, alongside a number of recent clear and extensive surveys (see for instance Section 6 in Bernanke, Gertler and Gilchrist, 2000 and Trautwein, 2000). Here I pick a number of papers that are more directly related to mine.

The closest antecedents of this work are Kiyotaki and Moore (KM, 1997), Carlstrom and Fuerst (CF, 1997) and Bernanke, Gertler and Gilchrist (BGG, 2000). KM construct a dynamic economy in which durable assets serve as factor of production and collateral for loans. They show how the interaction between asset prices, credit limits and investment can magnify the effect of temporary productivity shocks on aggregate output. CF introduce in an otherwise standard RBC model agency costs in the creation of new capital, which sells at a premium over the production cost because of the possibility of bankruptcy. They show that, unlike the traditional RBC model, agency costs can generate a hump-shaped response of output to productivity disturbances. BGG embed both these considerations in the dynamic neo-keynesian tradition, by incorporating credit market frictions in an otherwise conventional monetary business cycle model, where monetary shocks can affect both the price level and the shadow price of *variable* capital goods. In turn, capital goods prices affect the cost of external financing for the firm. They show that such a model can generate a large degree of amplification compared to a model in which financial frictions are absent.[55]

Putting aside the technical details, my model adds several dimensions to these stories. First, and unlike BGG, rather than focusing on the capital's shadow price (u_t in my model notation), I add a fixed asset, which can be thought of as real estate, in the model. The asset is available in fixed supply, so that I can directly focus on the effect of changes in its price on collateral limits and aggregate economic activity. Such price changes, which affect and are affected by agents' financial positions, have a more natural interpretation than the capital's shadow value, which reflects the firm's inside valuation of future investment projects. Next, I endogenise and introduce frictions on the household side and calibrate the model to obtain plausible and clear-cut steady state values for the stock of household and firm asset holdings and debt in the economy. In particular, I relax the strong assumption (shared by nearly all models that feature a financial accelerator) that the only indebted agents are those who own the economy capital stock.[56] In this way, I am in a better position to analyse the implications of, say, different levels of leverage or exogenous asset price changes for the business cycle.[57]

In addition, unlike BGG, I sidestep the derivation of the intraperiod optimal financial contract and consider debt obligations as set in nominal terms. Although I make no progress in justifying why this is case, this allows me to uncover a new propagation mechanism, going through the debt-

[55] However, one of the key elements to generate this result is their maintained assumption that agency costs apply to the entire (old and new) capital stock of the economy (which is owned by the entrepreneurs), so that the implied economy "commercial debt" is probably implausibly high.

[56] In this dimension, my approach is closer to Diaz-Giménez et al. (1992), who develop an OLG computable general equilibrium model of the banking sector, where banks lend to businesses and young households who purchase houses, and old households save for retirement. However, the model answers a set of questions (such as the welfare costs of alternative government policies) different from those analysed in the paper.

[57] Another paper worth mentioning is Aoki, Proudman and Vlieghe (2001). Using the BGG framework, they analyse the links between capital prices and household consumption under the assumption that capital producers transfer lump-sum dividends to the households. They however fall short of providing a fully consistent analysis of the interaction between house prices on the one hand and investment, consumption and labour supply decisions on the other.

deflation: such a mechanism *amplifies* the effect of shocks that induce a positive correlation between inflation and output (such as monetary shocks), but at the same time serves as a *stabiliser* for those that generate a negative correlation between the two variables (such as inflation shocks). This is, to my knowledge, a new result in the financial accelerator literature. Interestingly, it mirrors a well-known result of the wage indexing literature going back as far as Gray (1976), who showed how wage indexation was predominantly appropriate (from the viewpoint of output stability) in an economy subject predominantly to nominal shocks, but could have been destabilising in an economy subject to real shocks.

Directions for future research. Let me turn now to directions for future research. One limitation of the model is that the timing of the model impulse responses, with the peaks in output and asset price responses occurring in the period of the shock, is at odds with the hump-shaped responses found in the VAR estimates. It would be interesting to explore which sorts of frictions in the factor and goods markets as well as agents' decision lags are needed to get across this problem, and how the policy implications of the last section are robust to these alterations.

In addition, the assumption that the supply of real estate is fixed in the aggregate while entirely elastic within sectors is probably overly restrictive, especially since structures investment is a sizeable and volatile component of aggregate GDP in most economies. I have studied versions of the model with adjustment costs in the housing market. However, if the overall supply of real estate is fixed, adjustment costs solve one problem only by creating another: while quantities respond less, prices respond much more, and often with implausible magnitudes.

Furthermore, in the set-up of this paper, constrained (unconstrained) agents are constrained (unconstrained) forever. A chief issue would be to extend the model so that agents end up facing binding constraints in certain states of the world but not in others; such an avenue of research could also look into the issue of the asymmetry of business cycles, although, by taking linearity away from the model, it would also make matters more complicated.

Finally, it could be of interest to construct a welfare theoretic justification (à la Rotemberg and Woodford, 1997) for the policy aims in an economy with credit constraints. The maintained assumption here is that output (or markup) variability and inflation variability are the only two "bads" to care about: however, to the extent that asset prices play a crucial role in the agents' decision-making process, they could also matter per se in the economy, thus representing independent goals of monetary policy.

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A Different assumptions about the debt contracts

This appendix sketches how different assumptions about indexation of debt contracts affect the relevant equations in the basic model of the paper.

A.1 Benchmark model (nominal bond)

The entrepreneur borrows B_t each period, while repaying $R_{t-1}\frac{E_{t-1}(P_t)}{P_{t-1}}B_{t-1} = R_{t-1}B_{t-1}$, where E_{t-1} denotes the expectation operator conditional on information available at time $t-1$. Expressed in nominal units, the entrepreneur flow of funds and borrowing constraint are respectively:

$$\begin{aligned} P_t^w Y_t + B_t &= P_t c_t + Q_t\left(h_t - h_{t-1}\right) + R_{t-1}B_{t-1} + W_t L_t \\ B_t &= mE_t\left(Q_{t+1}h_t/R_t\right) \end{aligned}$$

in real terms:

$$\begin{aligned} \frac{Y_t}{X_t} + b_t &= c_t + q_t\left(h_t - h_{t-1}\right) + \frac{R_{t-1}}{\Pi_t}b_{t-1} + w_t L_t \\ b_t &= mq_{t+1}\Pi_{t+1}/R_t \end{aligned}$$

A.2 Real bond

If debt is indexed to the price level, the entrepreneur borrows B_t but pays back the *realised* real value of his debt, equal to R_{t-1} times $\frac{P_t}{P_{t-1}}B_{t-1}$. Flow of funds is

$$P_t^w Y_t + B_t = P_t c_t + Q_t\left(h_t - h_{t-1}\right) + R_{t-1}\frac{P_t}{P_{t-1}}B_{t-1} + W_t L_t$$

divide by price level to obtain the new flow of funds, where inflation has disappeared:

$$\frac{Y_t}{X_t} + b_t = c_t + q_t\left(h_t - h_{t-1}\right) + R_{t-1}b_{t-1} + w_t L_t \tag{A.1}$$

The entrepreneurial new Euler equation is

$$c_t^{-\rho} = E_t\left(\gamma R_t c_{t+1}^{-\rho}\right) + \lambda_t R_t$$

whereas Euler equation for the unconstrained households is

$$\left(c_t'\right)^{-\rho'} = E_t\left[\beta R_t\left(c_{t+1}'\right)^{-\rho'}\right] \tag{A.2}$$

so that now unconstrained households behaviour prices *nominal* bonds. A.1 and A.2 replace respectively 3.5 and 3.1.

A.3 No asset price channel

To consider what happens when borrowing limits are present but borrowing is no longer tied to the value of the asset, consider an entrepreneur who can borrow an amount $B_t = P_t z/R$, fixed in nominal terms. In real terms, his budget and borrowing constraints are

$$\begin{aligned} \frac{Y_t}{X_t} + b_t &= c_t + q_t\left(h_t - h_{t-1}\right) + \frac{R_{t-1}}{\Pi_t}b_{t-1} + w_t L_t \\ Rb_t &= z \end{aligned}$$

the Entrepreneur first order conditions for consumption and housing choice become respectively:

$$\begin{aligned} \frac{1}{c_t^{\rho}} &= E_t\left(\frac{\gamma R_t}{\Pi_{t+1}c_{t+1}^{\rho}}\right) + \lambda_t R \\ \frac{1}{c_t^{\rho}}q_t &= E_t\left[\frac{\gamma}{c_{t+1}^{\rho}}\left(\nu\frac{Y_{t+1}}{X_{t+1}h_t} + q_{t+1}\right)\right] \end{aligned}$$

Equations (L4) and (L6) are now replaced by:

$$\widehat{q}_t = \gamma\widehat{q}_{t+1} + (1-\gamma)E_t\left(\widehat{Y}_{t+1} - \widehat{h}_t - \widehat{X}_{t+1}\right) + \rho\widehat{c}_t - \rho E_t\widehat{c}_{t+1} \tag{L.4'}$$

$$\widehat{b}_t = 0 \tag{L.6'}$$

A.4 Equity participation

Under (partial) equity financing, the value of real estate "external finance" is still $mE_t\left(Q_{t+1}h_t\right)/R_t$. The entrepreneur signs a loans contract for $(1-n)$ share of this amount, borrowing an amount $B_t = (1-n)\,mE_t\left(Q_{t+1}h_t\right)/R_t$ and repaying back $(1-n)\,mR_{t-1}B_{t-1}$ from the previous period. He issues equity for the remaining part, $nmE_t\left(Q_{t+1}h_t\right)/R_t$, paying back nmQ_th_{t-1} from the previous period. In real terms, his flow of funds and borrowing constraint become:

$$
\begin{aligned}
\frac{Y_t}{X_t} + b_t + nmq_{t+1}\frac{\Pi_{t+1}}{R_t}h_t &= c_t + q_t\left(h_t - h_{t-1}\right) + nmq_th_{t-1} + \frac{R_{t-1}}{\Pi_t}b_{t-1} + w_tL_t \\
R_tb_t &= (1-n)\,mq_{t+1}h_t\Pi_{t+1}
\end{aligned}
$$

After tedious algebra, one can show that the only equation to change in the model is the log-linearised flow of funds:

$$
\frac{c}{Y}\widehat{c}_t = \frac{b}{Y}\widehat{b}_t + \frac{(1-n)\,Rb}{Y}\left(\widehat{\pi}_t - \widehat{R}_{t-1} - \widehat{b}_{t-1}\right) + \frac{nRb}{Y}\left(\widehat{q}_t + \widehat{h}_{t-1}\right) + \frac{\nu}{X}\left(\widehat{Y}_t - \widehat{X}_t\right) - \frac{qh}{Y}\left(\widehat{h}_t - \widehat{h}_{t-1}\right) \tag{L.3"}
$$

the model without equity obtains when $n = 0$.

Figure 1: SVAR effects of monetary shocks

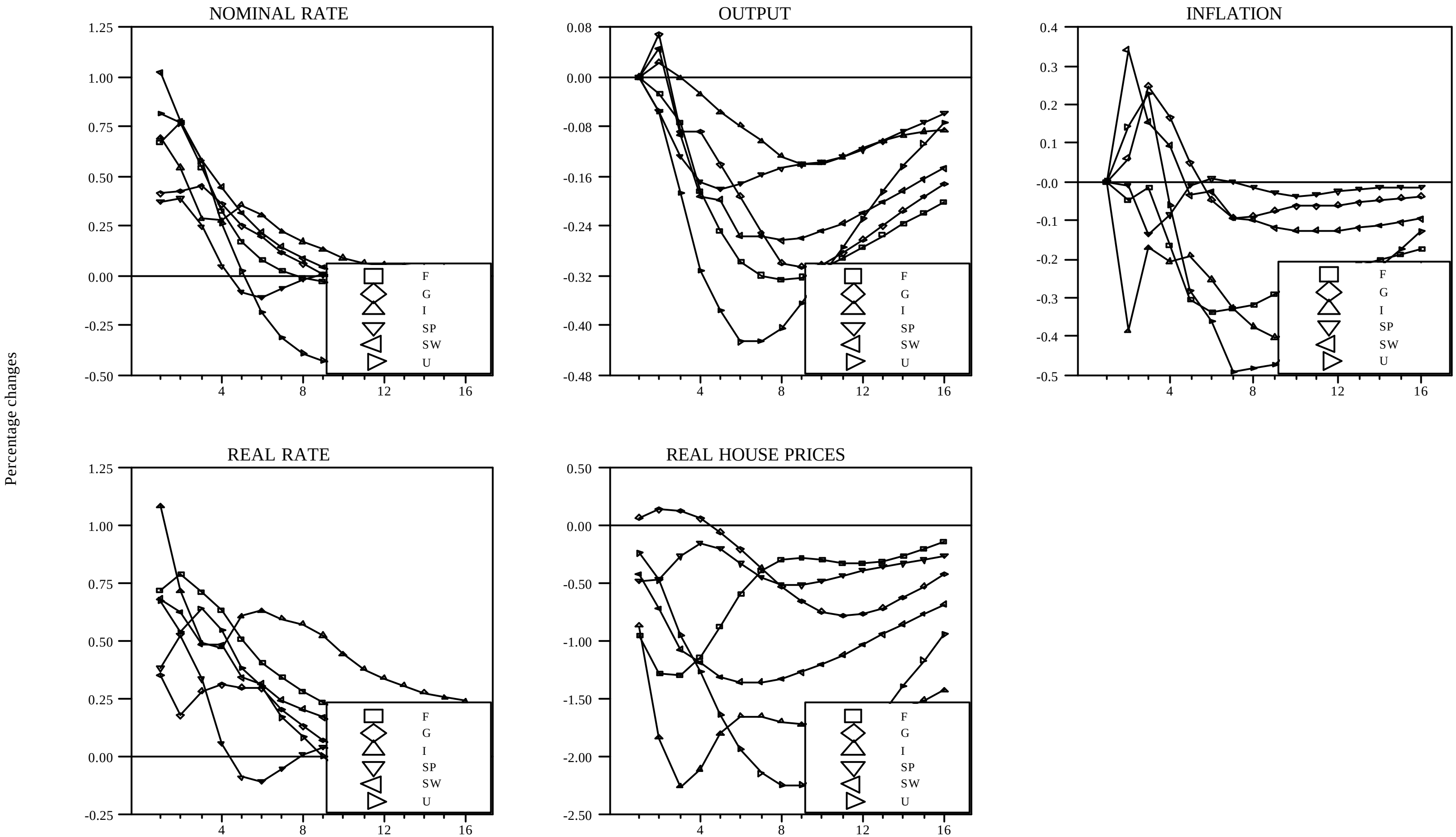

Figure 2: Effects of a monetary shock in the basic two agents' model
Comparison between the benchmark (top row) against alternative models

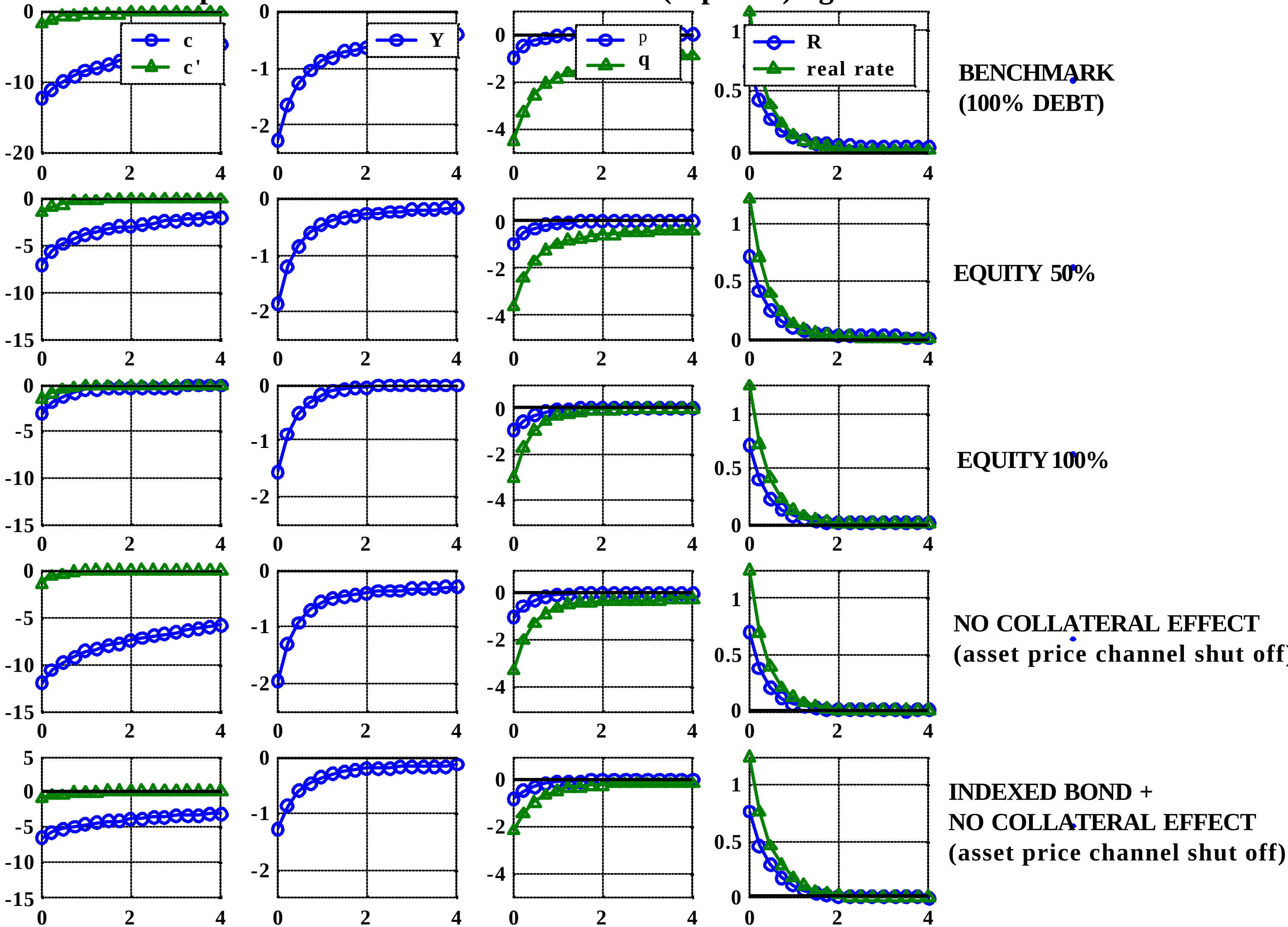

Figure 3: responses to all shocks, benchmark (solid) vs frictionless model (dashed)

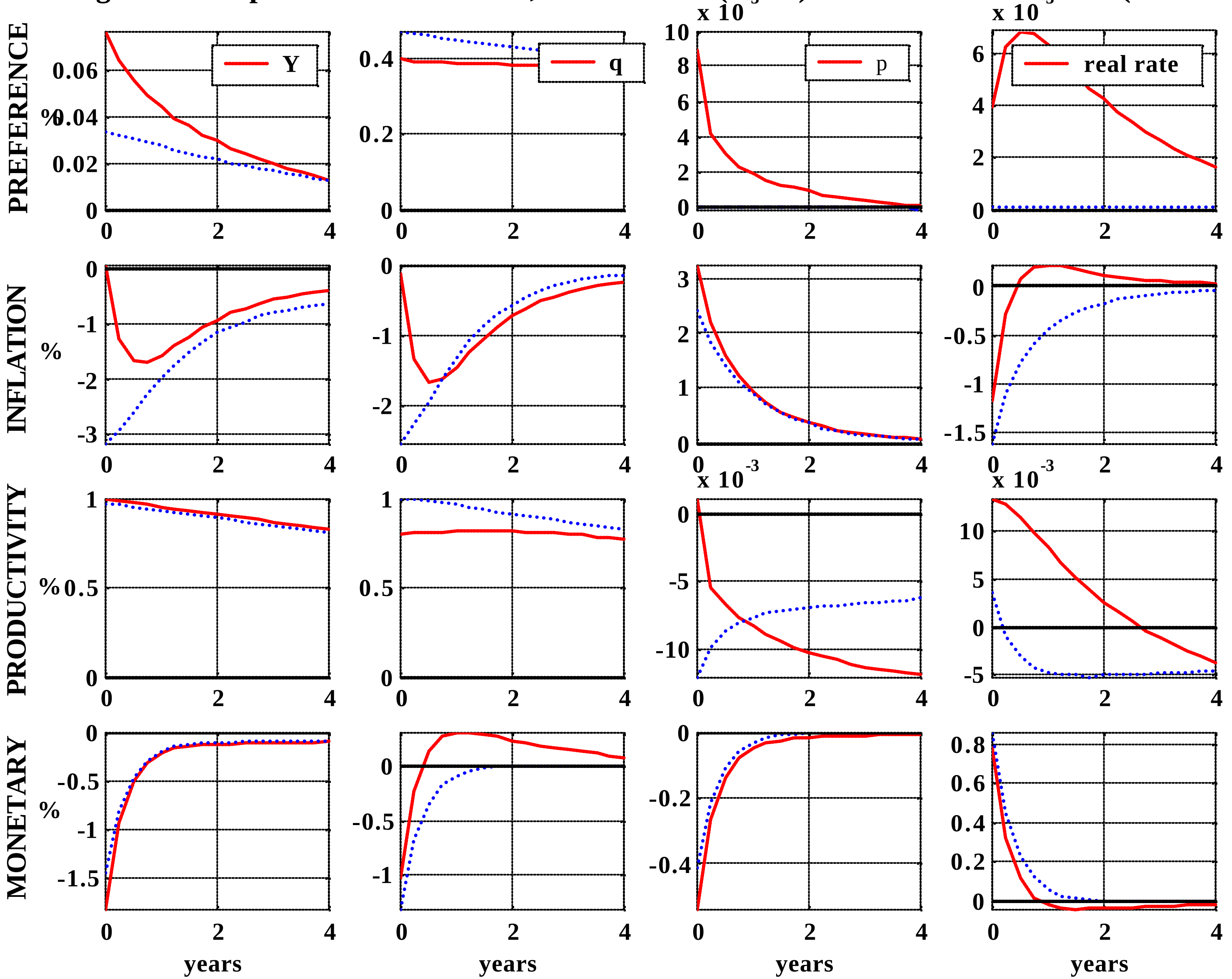

Figure 4: sensitivity analysis, monetary shock, wrt leverage

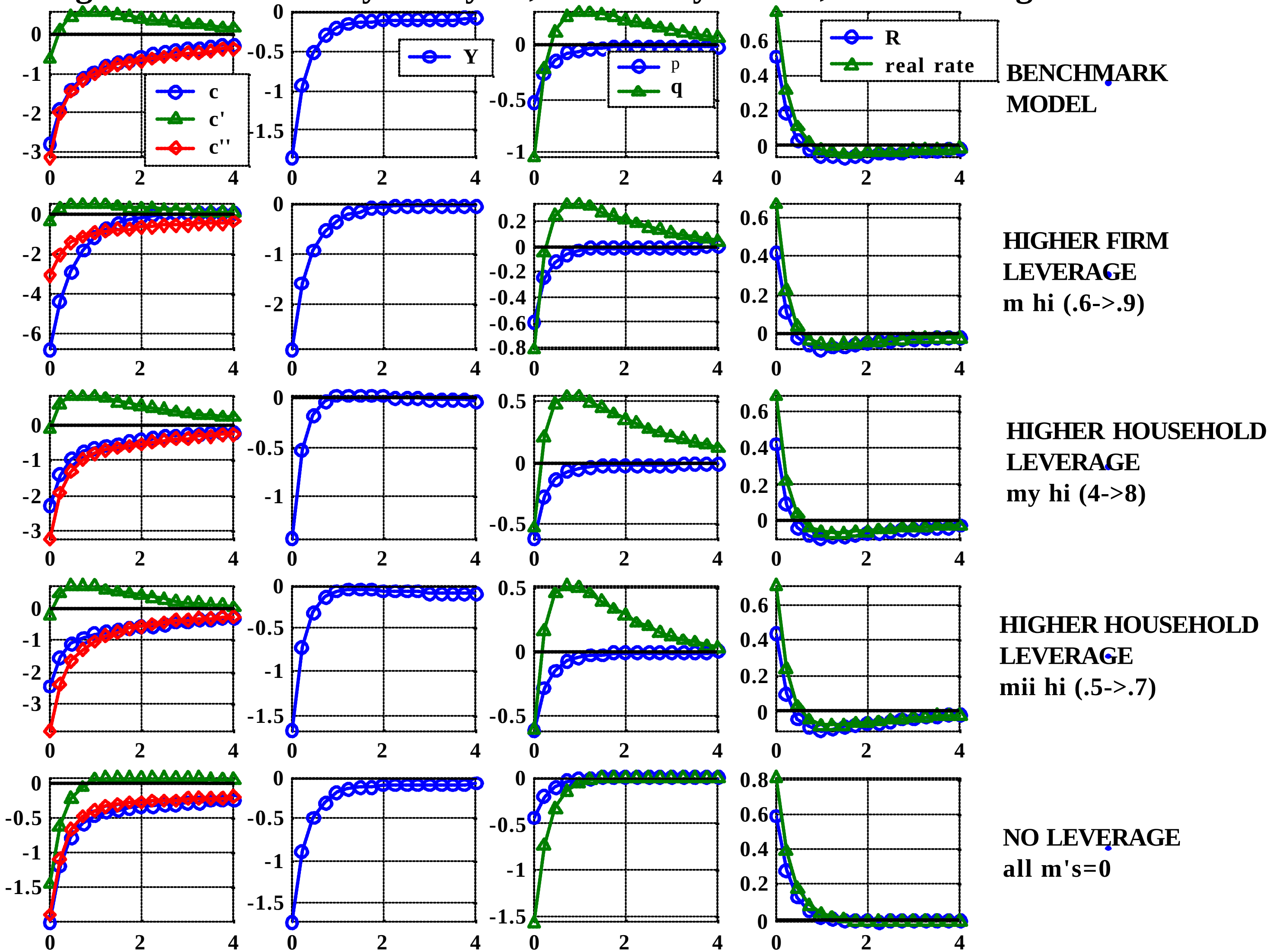

Figure 5: Effect of r_q changes on the frontier [each line, fix r_p & r_x]

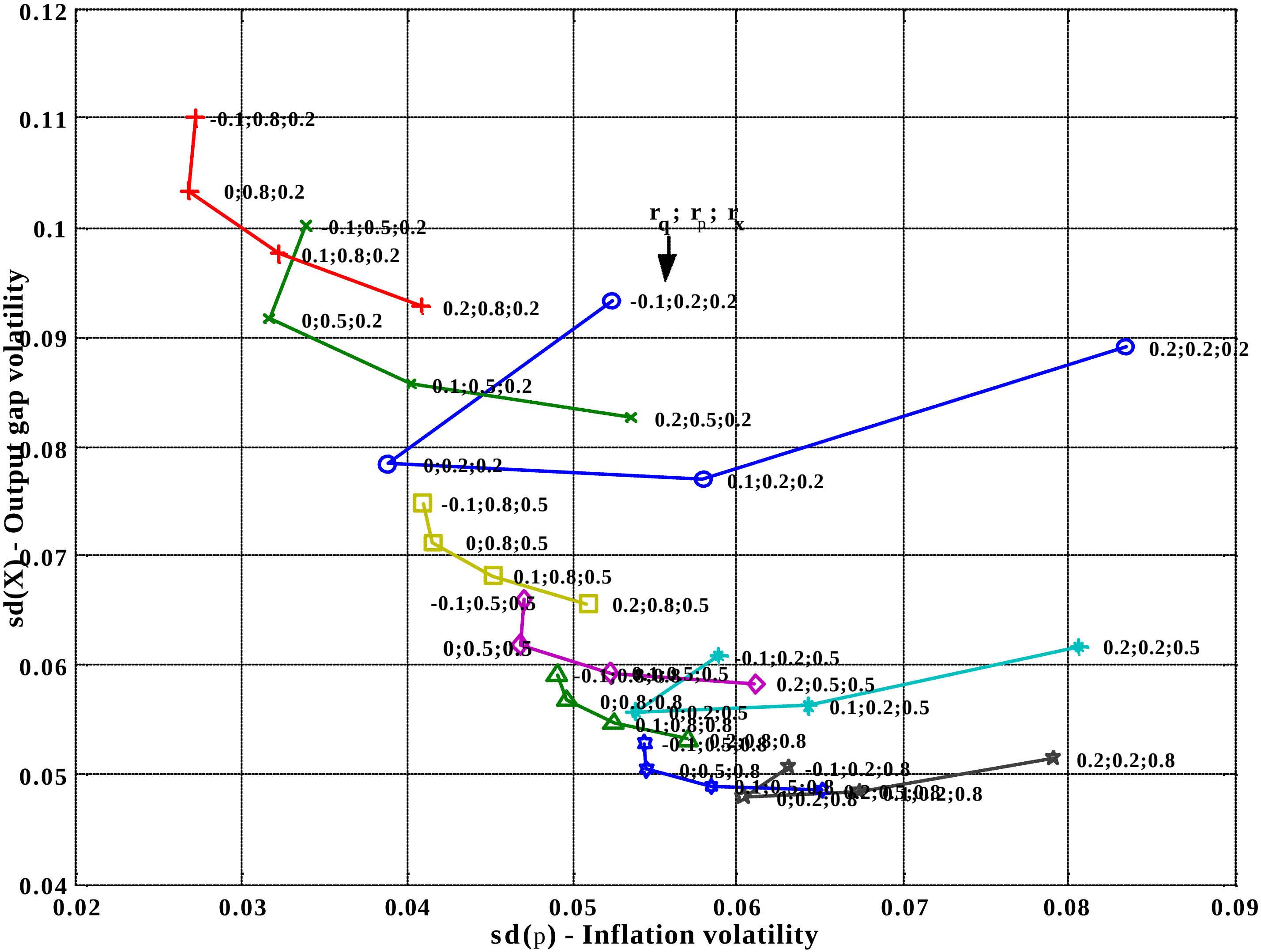

Figure 6: policy frontiers, different values of r_q [sd(R) indicated]

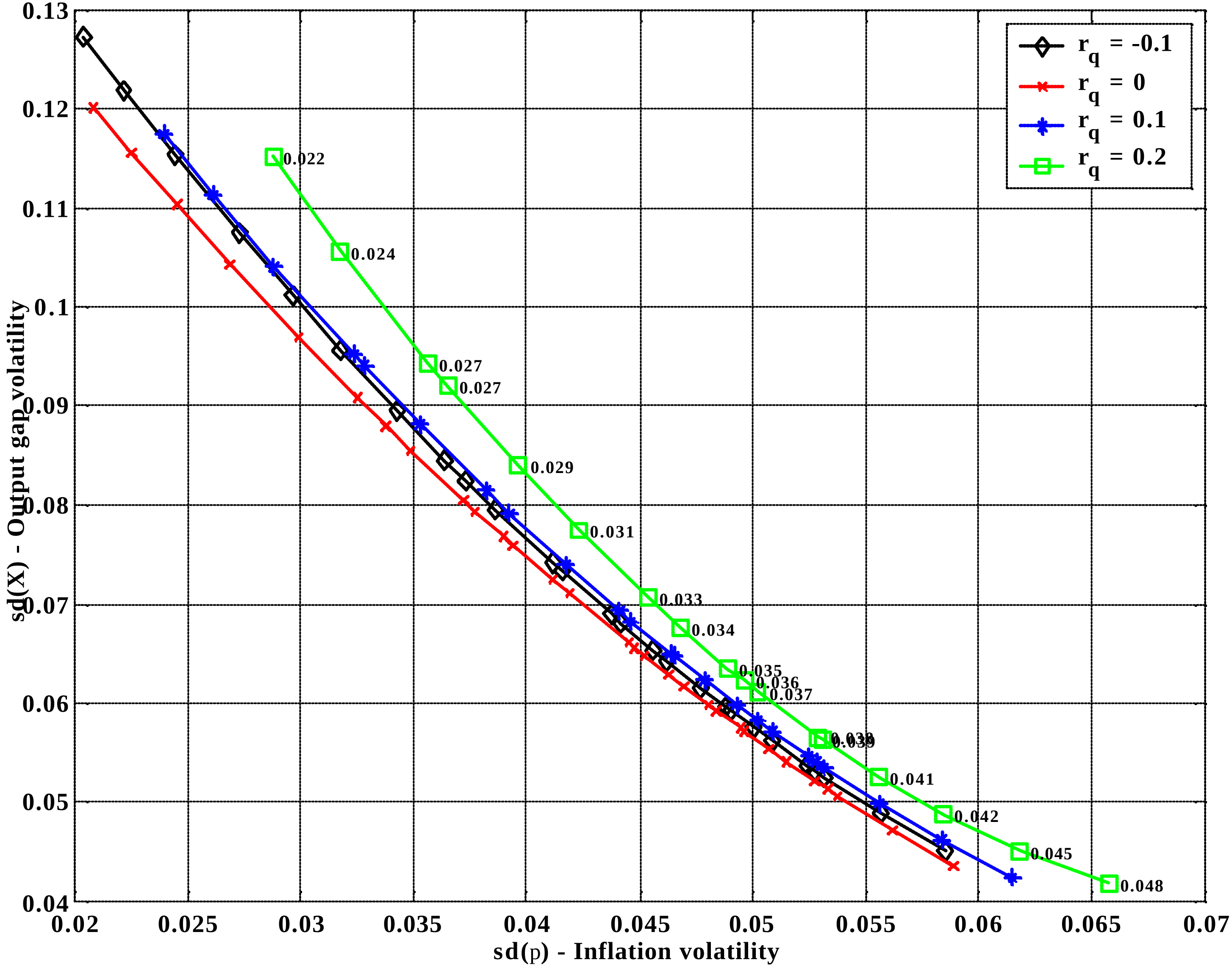

Figure 7: frontier, different values of r_q, central bank does not target output gap

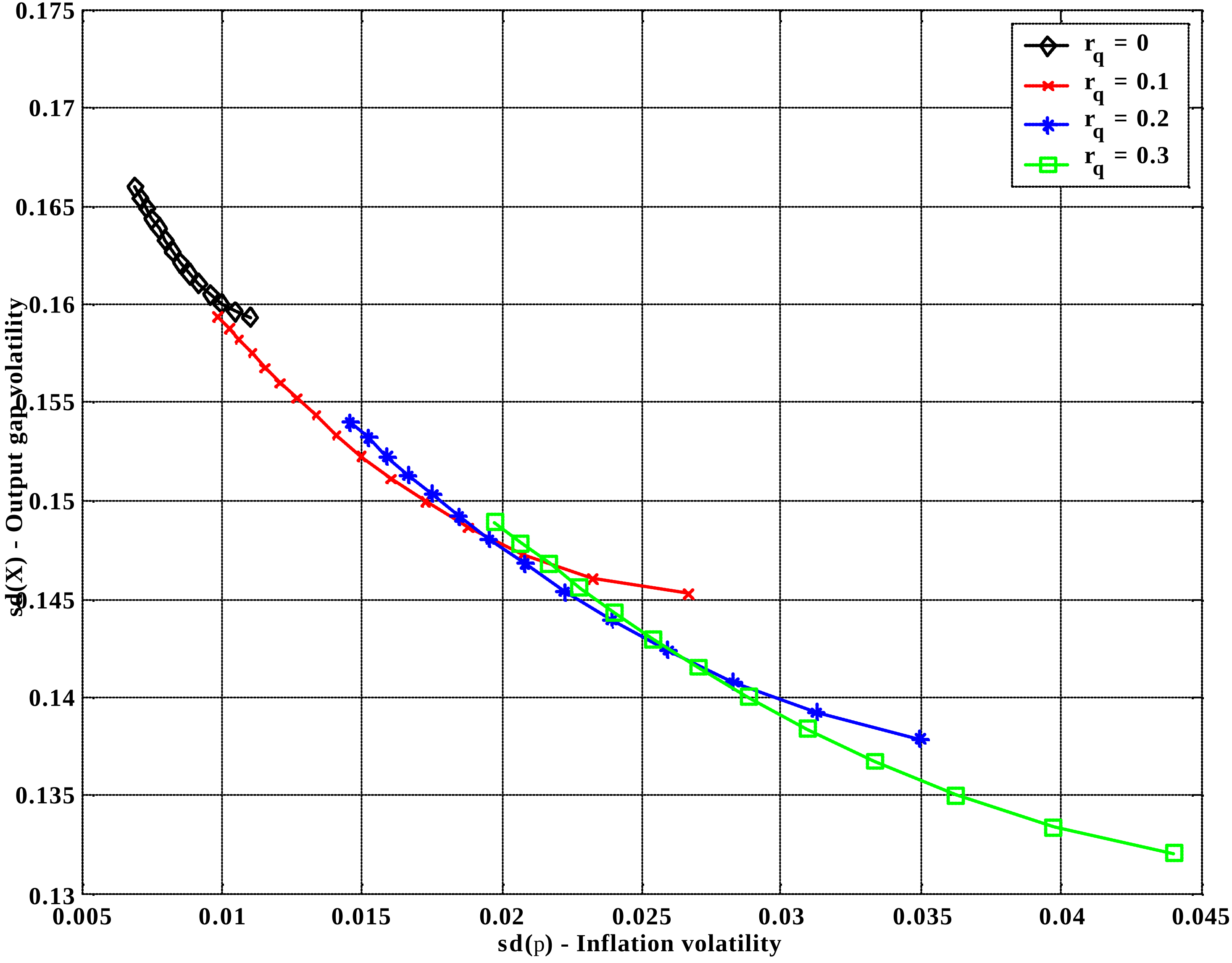

Figure 8: frontier, different assumptions about the debt contract