The Circuit Theory of Endogenous Money

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Introduction

The theory of the monetary circuit—known as “Circuit Theory”—has made fundamental contributions to the understanding of endogenous money. However, it is also clearly beset by many serious conundrums, not the least of which is an inability to explain how borrowers can manage to service and repay debt, let alone make profits.

In this paper, I argue that these conundrums are chimeras, derived by applying the wrong analytic tools to quite valid economic insights. A deliberately skeletal dynamic model of the monetary circuit shows that firms in a pure credit economy can easily service and repay debt, and make monetary profits—in contrast to assertions to the contrary in many Circuitist papers (Graziani 1989, Bellofiore et al. 2000, Nell 2002).

The model is, I stress, *deliberately* skeletal: causal factors of financial flows that are clearly variables in the real world are treated as constants—with the intention that these will indeed be made variables in a later model. However, just as much is learnt in anatomy by studying skeletons, much can be learnt about the actual monetary systems by studying a stylized system in which the causes of financial instability are absent.

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This is especially necessary for the theory of endogenous money, since its current state implies that "the real world can't possibly work in theory". For example, in making the following statement, Bellofiore et al. 2000 repeat a common claim in Circuitist literature:

> "in the basic circuit approach (describing a closed economy with no government expenditure), firms in the aggregate can only obtain the wage bill they advanced to workers (w_N) and, as a result, it is impossible for all firms to obtain money profits." (Bellofiore et al. 2000: 410)[1]

Yet in the real world, statistical agencies publish national accounts that show corporations making aggregate money profits. The statistical agencies are correct, while Circuit theory—though starting from valid premises—is wrong. We have to strip the financial system back to its "bare bones" to understand why. Firstly, however, I briefly turn to the origins and nature of money to consider a divergence in the analysis of endogenous money that has developed between Circuitists and some Post Keynesians.

Circuitists and Chartalists

Circuitists generally accept the Chartalist or state theory of money position with respect to the origins of money and its modern legal framework (see for example Graziani 2003: 78-80), but build models which at the outset have no government sector—nor any explicit role for the Central Bank (Graziani 2003: 26-32). In this sense, the Circuit approach conflicts with the Chartalist argument that "It is thus impossible to separate the theory of money from the theory of the state" (Wray 2000: 50).

From the Circuitist perspective, a taxing government is a sufficient but not a necessary component of a monetary system. Money, defined as a unit of account whose transfer is accepted as final payment in all commodity and service exchanges, can exist in the absence of a tax-levying state, if Graziani's three conditions for "A real money" are fulfilled:

> since money cannot be a commodity, it can only be a token money;
>
> the use of money must give rise to an immediate and final payment and not to a simple commitment to make a payment in the future; and
>
> the use of money must be so regulated as to give no privilege of seigniorage to any agent. (Graziani 2003: 60)

These conditions lead to the fundamental Circuitist insight that all sales in a monetary economy involve three parties: a seller, a buyer, and a bank which transfers the requisite number of units of account from the buyer's account to the seller's. From the Circuitist point of view, the production and enforcement of a unit of account by a tax-levying state is an embellishment to this fundamental concept of money. The Circuitist starting point of a pure credit economy is thus arguably closer to the essential nature of money, even if so-called "State Money" is the universal norm today, and even if it may be the only viable way to sustainably meet Graziani's third condition in the real world.

However, the failure to date of Circuitists to produce a coherent model of endogenous money could have implied that the Chartalist position was correct, in that a tax-levying state was indeed an essential component of a functional model of money. In fact, as I show below, a functional model of a monetary production economy can be built without either a government sector or a central bank, so long as transfers between private bank accounts are accepted as making final settlement of debts between buyers and sellers.

The basic Circuitist model

Graziani 2003 presents a canonical verbal version of the Circuitist model of a monetary production economy. The model is described as having four agents—"the central bank, commercial banks, firms and wage earners" (26-27) —but despite this, the central bank is given no role in the model itself. The actual model therefore has only three agents.[2]

The model's monetary dynamics commence with "A decision ... by the banks to grant credit to firms, thus enabling them to start a process of production" (27). Graziani argues that the amount of credit demanded by the firms (and supplied by the banks) equals the wage bill for the planned level of production.[3]

Using the borrowed money, capitalists pay workers and put them to work to produce commodities. These are then sold, with consumer goods being sold to workers and investment goods to other capitalists (sales to bankers appear later, after a fashion).

Spending by workers on consumer goods (and also purchases of corporate bonds by workers) return money to the firms, who can then use this money to repay their debt to banks. This repayment of debt destroys money: "To the extend that bank debts are repaid, an equal amount of money is destroyed" (29-30).

The repayment of debt closes the circuit, but this only happens "If wage earners spend their incomes entirely" (including on purchases of corporate bonds). However if they don't, then dilemmas arise:

> If instead wage earners decide to keep a portion of their savings in the firm of liquid balances, firms are unable to repay their bank debt by the same amount. (30)

The next cycle, if it involves an identical scale of production, therefore requires new money, so that the money supply must increase to finance a constant scale of production: the new quantity of money in this second circuit "will be equal to the wage bill plus the new liquid balances set aside by wage earners at the end of the previous cycle" (31).

The above, however, omits the problem of interest on debt! Graziani acknowledges this—in contrast to some Circuitist papers that abstract from the problem, in a manner that is embarrassingly reminiscent of the neoclassical approach to logical conumdrums (Bellofiore et al. 2000: 410—footnotes 8 and 9). It appears that firms are unable to pay interest:

> even in the most favourable case [corresponding to workers spending all their wages], the firms can only repay in money the principal of their debt and are anyhow unable to pay interest. (31)

The solution he proffers, in a monetary model, is a "real" one, that banks are paid in commodities rather than money: "the only thing they can do is to sell part of their product to the banks, which is tantamount to saying that interest can only be paid in kind" (31).

At least bankers get their hands on the physical loot: capitalists, it seems, end up with neither goods nor money.[4] Money profits in the aggregate are zero, and "profits earned by one firm may simply be the mirror image of inefficiencies and consequent losses incurred by other firms" (32).

Starting from precisely the same foundation, I reach contrary conclusions on almost every point above. Though related to the wage bill, the initial amount borrowed is in fact far less than the annual wage bill; money is not destroyed by the repayment of debt (though bank deposits are "destroyed" by loan repayment, and the stock of money available for transactions at any one time is reduced); workers can have positive bank balances without forcing firms to make losses; a constant level of production can be financed with a constant stock of money (consistent with Keynes's vision of a "revolving fund of finance" in Keynes 1937, and concurring with Andresen 2006); firms can easily pay the interest on debt with money, and firms in the aggregate earn money profits.

These contrary—but in essence, very favourable to the Circuitist School—conclusions arise simply from applying the correct form of mathematical analysis to the Circuitist school's brilliant logical insights into the nature of a monetary production economy. The Circuit is fundamentally dynamic, and can therefore only be properly understood using dynamic analysis. Mathematical dynamics are essential here, because the interrelations between entities in a dynamic model are easily mis-specified in verbal analysis: not all of us can be Josef Schumpeter, and track the complicated relations between agents in a monetary system in our heads.[5]

If economists were well-trained in mathematics, I could simply proceed. Unfortunately, the mathematical training of economists is poor because, in economics, even mathematical pedagogy is dominated by neoclassical theory. Since this is fixated on equilibrium, the vast majority even of Post Keynesian economists never learn how to think in terms of "ordinary differential equations" (ODEs), which are the fundamental building blocks of dynamic analysis.

In what follows, I hope to show that learning ODEs is an essential part of the process of escaping from the intellectual handicap of being taught mathematics by neoclassical economists. To make the exposition easier to follow, I will display the model in dynamic balance sheets as well as coupled ODEs, and build the model in a similar sequence to Graziani 2003.

A dynamic model of the Circuit

Graziani's model has three classes of agents—firms, bankers, and workers. Since this is a monetary economy, all three classes have deposit accounts which I indicate as F_D, B_D and W_D respectively. Prior to the making of a loan, all three accounts have zero balances. This "ab initio" situation is shown in Table 1.

Bank Assets & Liabilities				
Time	**Assets**	**Liabilities**		
		Firm Deposit (F_D)	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**
Initial values		**0**	**0**	**0**

Table 1: Initial conditions prior to loan

In step one of the model, banks make loans to the firms. The banks' capacity to loan money arises solely from the social position they occupy, that transfers of their unit of account from one agent's account to another's is accepted as a final payment (Graziani's condition ii above): banks do not need pre-existing reserves or equity (I introduce the topic of reserves later).

Since this is credit money, a debt obligation is created between the firms and banks along with the creation of money. Thus as well as the three deposit accounts, a record of debt is also needed, which I indicate as F_L. This is **not** a bank account as such: it does not contain money, nor can money be paid into it, but it instead records the outstanding obligation of the firms to the banks. It is, however, a record of account, and when the loan is made, an equivalent entry is made in it. Using L to signify the magnitude of the loan, this results in the situation shown in Table 2. This clearly embodies the direct and causal "loans create deposits" perspective of endogenous money, in contrast to the convoluted "deposits enable loans to occur with fractional banking" perspective that dominates neoclassical thinking.

Bank Assets & Liabilities				
Time	Assets	Liabilities		
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)
Start of loan	L	L	0	0

Table 2: Loan issued

Here I divert from the sequence followed by Graziani to consider the topic that Circuistists have to date sidestepped: the monetary payment of interest (Graziani delays considering this to his final step, and argues that interest cannot be paid in money).

At the very minimum, a loan generates an obligation to pay interest to the lender, while a deposit obligates the bank to pay interest to the depositor. I use r_L for the rate of interest on loans and r_D for the rate on deposits, (where $r_L>r_D$). These obligations are shown in Table 3.

Bank Assets & Liabilities				
Time	Assets	Liabilities		
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)
Obligations initiated by loan	$+r_L F_L$	$+r_D F_D$	0	0

Table 3: Loan and deposit obligations

We now move from the instantaneous creation of a stock of initial money L to the flows it initiates. For the loan obligations to be met, flows must occur out of accounts in the system—since there is no other source of money. The firms must therefore pay the loan interest obligation out of their deposit account F_D, while the bank must pay its deposit interest obligation out of its deposit account B_D. The flows occur between these two deposit accounts, and the payment of loan interest is recorded on the asset side of the ledger, resulting in the firms debt stabilising at the level of the initial loan L (I consider repayment of the loan principal later). Since the interest payments flow between the firm and banker deposit accounts, the overall sum of deposit accounts also stabilises at L; but since $r_L > r_D$, the balance shifts from the firms deposit account to the bankers over time. This dynamic is shown in Table 4.

Bank Assets & Liabilities				
Time	**Assets**	**Liabilities**		
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**
Interest flows initiated by loan	$+r_L F_L$ $- r_L F_L = 0$	$+r_D F_D$ $- r_L F_L$	$+r_L F_L$ $- r_D F_D$	**0**

Table 4: Payment of interest

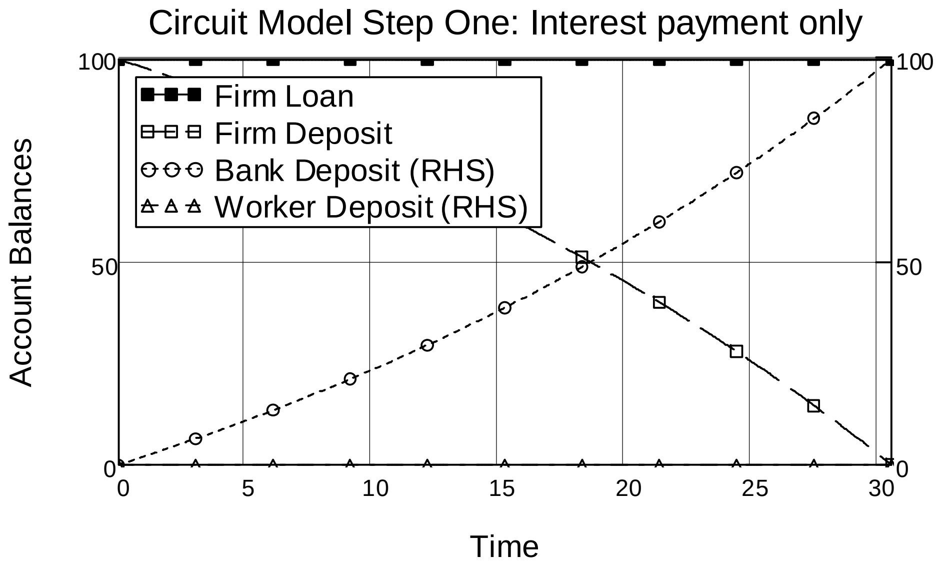
Equation (1.1) states this incomplete system as a set of coupled ODEs. The same balance that is apparent in Table 4 applies here: by inspection it is obvious that the level of debt will remain constant (at the initial value L), as will the sum of deposit accounts, but the money in the firms' account will be transferred to the banks'. At some point, firms' deposit accounts will turn negative—which is of course an unsustainable situation.

$$\begin{aligned} \tfrac{d}{dt}F_L &= 0 \\ \tfrac{d}{dt}F_D &= r_D F_D - r_L F_L \\ \tfrac{d}{dt}B_D &= r_L F_L - r_D F_D \\ \tfrac{d}{dt}W_D &= 0 \end{aligned} \qquad (1.1)$$

Figure 1 shows a simulation of this system in the mathematics program Mathcad. Given the set of example parameter values (L=100, r_L=5%, r_D=3%) while the outstanding loan and the sum of deposit accounts remain at 100 throughout, all the money has been transferred from the firms' deposit account to the bankers' after 30.5 years.

Given	Initial values	Flow dynamics
Firm loan account	$F_L(0) = L$	$\frac{d}{dt}F_L(t) = r_L \cdot F_L(t) - r_L \cdot F_L(t)$
Firm deposit account	$F_D(0) = L$	$\frac{d}{dt}F_D(t) = r_D \cdot F_D(t) - r_L \cdot F_L(t)$
Bank deposit account	$B_D(0) = 0$	$\frac{d}{dt}B_D(t) = r_L \cdot F_L(t) - r_D \cdot F_D(t)$
Worker deposit account	$W_D(0) = 0$	$\frac{d}{dt}W_D(t) = 0$

$$\begin{pmatrix} F_L \\ F_D \\ B_D \\ W_D \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} F_L \\ F_D \\ B_D \\ W_D \end{pmatrix}, t, Y\right]$$



$F_D(Y) = 0$ $\quad B_D(Y) = 100$ $\quad W_D(Y) = 0$ $\quad F_D(Y) + B_D(Y) + W_D(Y) = 100$

Figure 1: Simulation of interest payment only model in Mathcad

This outcome possibly explains why Circuitists have been loathe, to date, to acknowledge the need to pay interest in their models of the monetary circuit: the situation seems hopeless for firms. However, this is only because firms have not yet done anything with the borrowed money.

In fact, it has been borrowed to finance production, which involves both buying inputs from other firms, and paying wages to workers. This in turn is done in order to evoke a stream of purchases from other firms, workers and bankers from which the firms hope to make a net profit.

The issue of production, and the transactions enabling it and emanating from it, is another area of great confusion in Circuitist writings.[6] The key confusion is one of stocks and flows, starting from believing that the size of the initial loan (the stock L) is equal to the wage payments needed to hire the workforce (a flow). Instead, the wage bill is related, not to the initial loan, but to the rate of outflow of money from firms' deposit accounts that is used to pay wages. Calling this rate of outflow w, an amount $w.F_D$ is transferred per unit of time (per year in this model) from firms to workers as wages.

Bank Assets & Liabilities				
Time	Assets	Liabilities		
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)
Interest flows initiated by loan	$+r_L F_L$ $-r_L F_L=0$	$+r_D F_D$ $-r_L F_L$	$+r_L F_L$ $-r_D F_D$	0
Wage flow to initiate production		$-w. F_D$		$+w. F_D$

Table 5: Spending to finance production

The relationship between money and wages is thus not "the credit initially granted [L, a stock] is totally turned into wages [$w.F_D$, a flow]" (Graziani 2003: 29). Instead, in this skeletal model, wages equal a constant times the balance in the firms' deposit account.[7] Given the relationship between the initial loan and the balance in the firms' account, the annual wages paid can be substantially greater than the initial loan.

With workers now having positive bank balances, they too are receipients of interest income. Though in the real world workers normally get lower deposit rates than firms, for simplicity I will use the same rate of interest r_D here. A flow of $r_D.W_D$ is therefore deducted from the bankers' account and deposited into the workers' account.

Bank Assets & Liabilities				
Time	**Assets**	**Liabilities**		
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**
Interest flows initiated by loan	$+r_L F_L$ $-r_L F_L=0$	$+r_D F_D$ $-r_L F_L$	$+r_L F_L$ $-r_D F_D$	0
Wage flow to initiate production		$-w. F_D$		$+w. F_D$
Interest income flows from wages			$-r_D. W_D$	$+r_D. W_D$

Table 6: Incomes from production

To complete the model, we have to include the flow of transactions from workers and bankers to capitalists that purchase the goods flowing (implicitly in this model) in the opposite direction. Here I use ω for the rate at which spending flows from workers' deposit accounts to firms', and

β for the corresponding rate of spending by banks. The amounts $\omega.W_D$ and $\beta.B_D$ are therefore deducted from workers and banks accounts respectively and credited to the firms' account.

The basic model is finally complete, and as shown by the Social Accounting Matrix column, all transactions are properly accounted for. The components of the basic coupled ODE model can now be read down the columns of the final 4 rows of Table 7.

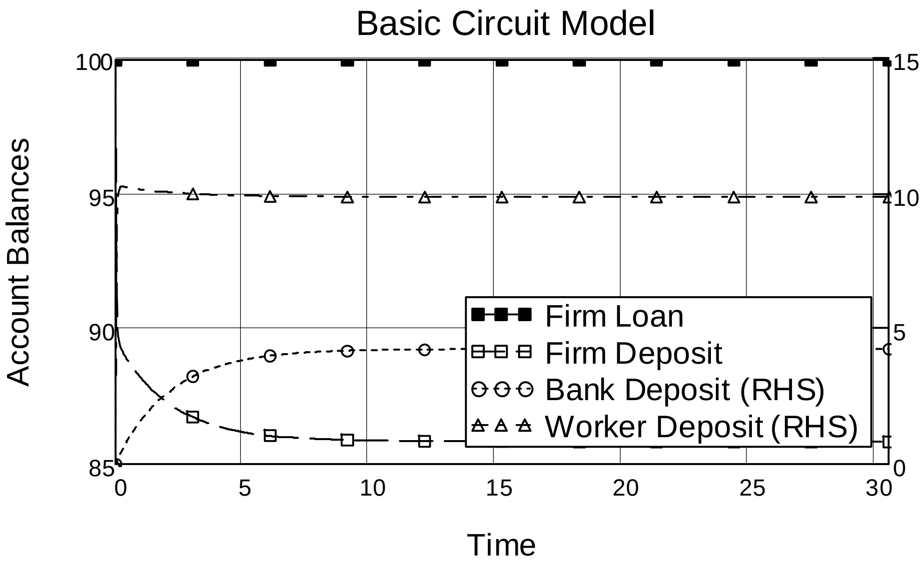
Bank Assets & Liabilities					
Time	**Assets**	**Liabilities**			**SAM**
	Firm Loan (F_L)	**Firm Deposit (F_D)**	**Banker Deposit (B_D)**	**Worker Deposit (W_D)**	**Sum**
Interest flows initiated by loan	**0**	$+r_D.F_D$ $- r_L.F_L$	$+r_L.F_L$ $- r_D.F_D$	**0**	**0**
Wage flow to initiate production		$-w. F_D$		$+w. F_D$	**0**
Interest income flows from wages			$- r_D. W_D$	$+r_D. W_D$	**0**
Flows from sale		$+\omega.W_D + \beta.B_D$	$-\beta.B_D$	$-\omega.W_D$	**0**

Table 7: Transactions complete the basic model

In coupled ODE form, the model is as shown in Equation (1.2).

$$
\begin{aligned}
\tfrac{d}{dt}F_L &= 0 \\
\tfrac{d}{dt}F_D &= \left(r_D F_D - r_L F_L\right) - w \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) \\
\tfrac{d}{dt}B_D &= \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
\tfrac{d}{dt}W_D &= w \cdot F_D + r_D \cdot W_D - \omega \cdot W_D
\end{aligned}
\tag{1.2}
$$

The model can now be simulated (see Figure 2; the additional parameter values used here are w=3, ω=26 and β=0.5), and since it is a linear model, its equilibrium can also be derived symbolically (see equation (1.3))



$F_D(Y) = 85.83$ $B_D(Y) = 4.255$ $W_D(Y) = 9.915$ $F_D(Y) + B_D(Y) + W_D(Y) = 100$

Figure 2: Basic Circuit model

As is now obvious, the basic Circuitist model with a single injection of endogenous money is consistent with sustained economic activity over time—contra Graziani 2003, an increasing supply is not needed to sustain constant economic activity (this confirms the result in Andresen 2006). However the amounts shown here are transaction account balances: we do not yet know whether these are compatible with sustained incomes over time.

$$\begin{bmatrix} F_{L_E} \\ F_{D_E} \\ B_{D_E} \\ W_{D_E} \end{bmatrix} = \begin{bmatrix} L \\ \frac{L\cdot(\omega - r_D)\cdot(\beta - r_L)}{(w+\omega - r_D)\cdot(\beta - r_D)} \\ \frac{L\cdot(r_L - r_D)}{\beta - r_D} \\ \frac{L\cdot w\cdot(\beta - r_L)}{(w+\omega - r_D)\cdot(\beta - r_D)} \end{bmatrix} = \begin{bmatrix} 100 \\ 85.83 \\ 4.255 \\ 9.915 \end{bmatrix} \tag{1.3}$$

Income dynamics

Fortunately, two income flows are easily associated with particular transactions in equation (1.2): wages and interest income. Annual wages are equal to $w \cdot F_D$ and bank interest income is $r_L F_L$ (which equal 257.489 and 5 per annum respectively in this simulation). Wages and interest income are thus positive and sustained in this model; what about profits?

To reveal profits, we need to consider what the term w represents. As well as being equivalent to wages, it also represents that part of the net surplus from production that accrues to workers. The net surplus—in monetary terms—itself depends on how rapidly money invested in production returns to firms. In Marx's terms, it represents the time lag between extending M and receiving M+ (assuming, as I do in this skeletal model, that the process occurs smoothly). This could be a period of, say, 4 months between financing production and receiving the complete proceeds of sale of output—again something that would be a variable in a more complex model. There are thus two components to w: the share of the net surplus (in Sraffa's sense of the surplus, in which wages and profits are entirely paid out of the net surplus from the input-output process) from production going to workers, and the rate of turnover from M to M+, given by technical conditions of production and the time taken for the sale of physical commodities. I use s for the share of surplus accruing to the owners of firms (so that the share going to workers is thus 1-s), and P for the lag between M and M+.[8] We therefore have the relation given by equation (1.4):

$$w = s \cdot (1 - P) \tag{1.4}$$

With w set to 3 in the simulation above, a hypothetical value of s of 0.4 (which corresponds to a "rate of surplus value" in Marx's terms of 67%) yields a value for P of 5 (which means that the lag between spending M and making M+ is 1/5th of a year or 2.4 months). The monetary value of net output per annum is thus $P.F_D$ (which equals 429.15 in equilibrium, given the parameter values in the model) which is split between workers and the owners of firms in the ratio (1-s):s. In this debt-finance only model, the owners of firms then have to pay interest on their outstanding debt to banks. Using Π, W and I to signify profits, wages and interest income respectively, the income flows of the model in equilibrium are:

$$\begin{bmatrix} \Pi_E \\ I_E \\ W_E \end{bmatrix} = \begin{bmatrix} s \cdot P \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ r_D \cdot L \\ (1-s) \cdot P \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \end{bmatrix} = \begin{bmatrix} 166.66 \\ 5 \\ 257.49 \end{bmatrix} \tag{1.5}$$

Firms thus do make net profits, which, though related to the size of the initial loan, can be substantially larger than this amount (and profits are substantially larger than the servicing cost of debt). Economic activity also continues indefinitely at an equilibrium level with a single injection of endogenous money: additional money is not needed to sustain economic activity at a constant level. This contradicts Graziani's assertion that additional money would be needed if workers retained positive bank balances (Graziani 2003: 31), but confirms Keynes's intuition that a “revolving fund of a more or less constant amount” can finance sustained economic activity (Keynes 1937b: 248).

The size of the initial loan L can also be related to the equilibrium value of wages generated by the loan:

$$L = W_E \cdot \frac{\left((1-s)\cdot P + \omega - r_D\right)\cdot\left(\beta - r_D\right)}{(1-s)\cdot P \cdot \left(\omega - r_D\right)\cdot\left(\beta - r_L\right)} = 100 \qquad (1.6)$$

Two more issues remain to be considered: the impact of debt repayment, and the modelling of growth.

Debt repayment and bank reserves

According to Graziani—and almost all theorists in endogenous money—the repayment of debt destroys the money that was created with it (Graziani 2003: 29-30). I consider this by adding an additional term R_P to represent the repayment of debt. If we relate this to the level of outstanding debt[9], then the amount $R_P.F_L$ is deducted from the firms’ only source of money, F_D. Yet to where does it go?

Here Graziani’s third condition for the existence of money comes into play: “the use of money must be so regulated as to give no privilege of seigniorage to any agent” (Graziani 2003: 60). This repayment therefore cannot be made to the existing bankers’ deposit account B_D, since banks use this account to finance spending on commodities. It must therefore go to a separate, capital account: the banks’ reserve account, which I call B_R.

Reserves, once created by the repayment of loans, will be relent. This amount will be deducted from the banks’ reserve account and deposited in the firms’ deposit account—and a matching entry will be made in the firms loan record of account. The complete relations are shown in Table 8.

Bank Assets & Liabilities					
Time	Assets	Liabilities			SAM
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)	Income
Repayment of debt	$-R_L.F_L$	$-R_L.F_L$			$-R_L.F_L$
Relending of reserves	$+L_R.B_R$	$+L_R.B_R$			$+L_R.B_R$

Bank Reserves		
Time	Reserve Account	Capital
Repayment of debt	$R_L.F_L$	$+R_L.F_L$
Relending of reserves	$-L_R.B_R$	$-L_R.B_R$

Table 8: Repayment and relending

The repayment of loans therefore does not "destroy" money, but transfers it out of income accounts—where it can be used for expenditure—to a reserve account. Once there, it is an unemcumbered asset of the banks which can then be re-lent—though not spent directly on commodities or services. This adds an important additional insight to the concept of endogenous money: not only do "loans create deposits", but "the repayment of loans creates reserves".

This results in the model shown in equation (1.7):

$$
\begin{aligned}
\tfrac{d}{dt}F_L &= +L_R \cdot B_R - R_L \cdot F_L \\
\tfrac{d}{dt}F_D &= \left(r_D F_D - r_L F_L\right) - (1-s)\cdot P \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + \left(L_R \cdot B_R - R_L \cdot F_L\right) \\
\tfrac{d}{dt}B_D &= \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
\tfrac{d}{dt}W_D &= (1-s)\cdot P \cdot F_D + r_D \cdot W_D - \omega \cdot W_D \\
\tfrac{d}{dt}B_R &= +R_L \cdot F_L - L_R \cdot B_R
\end{aligned}
\qquad (1.7)
$$

The simulation results for this model are shown in Figure (with a shorter time span to show the initial dynamics). The new parameters R_L and L_R were given the values of 2 and 3 respectively.

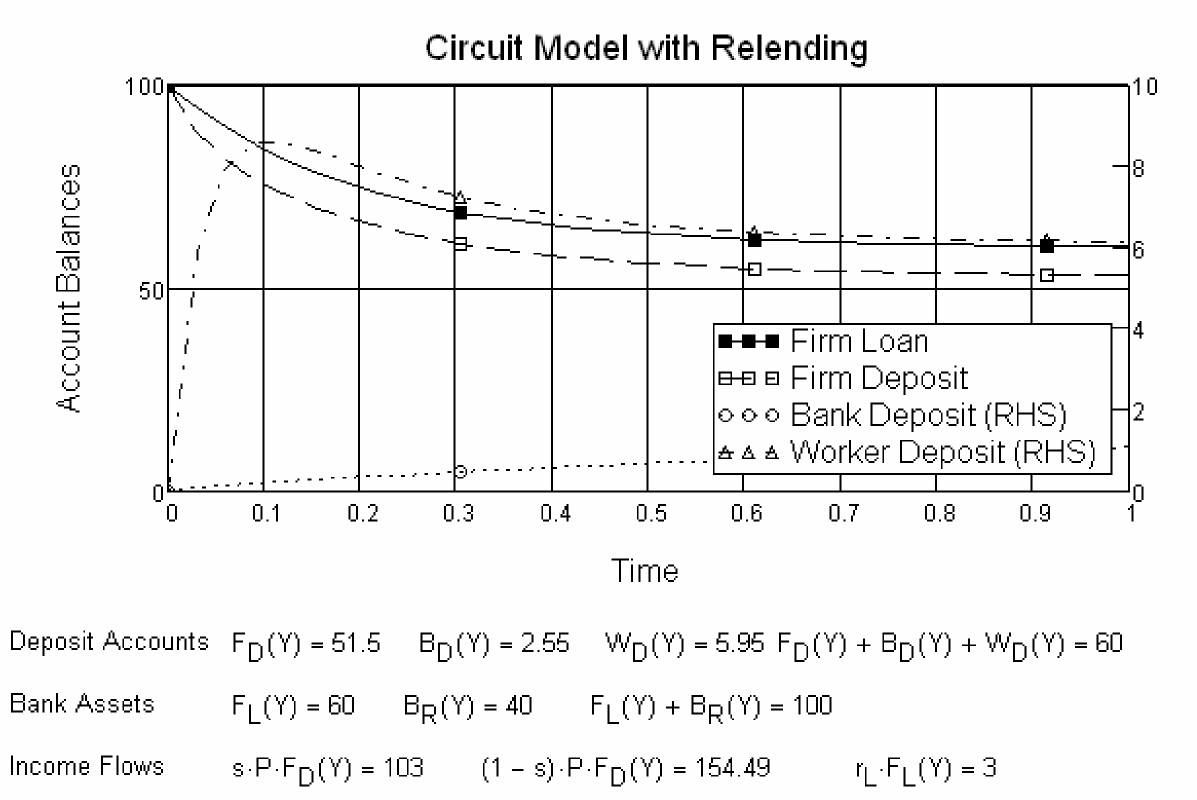


Figure 3: Model with repayment and relending

The equilibrium values are shown in Equation (1.8):

$$\begin{bmatrix} F_{L_E} \\ F_{D_E} \\ B_{D_E} \\ W_{D_E} \\ B_{R_E} \end{bmatrix} = \frac{1}{L_R + R_L} \cdot \begin{bmatrix} L_R \cdot L \\ L_R \cdot \dfrac{L \cdot (\omega - r_D) \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ L_R \cdot \dfrac{L \cdot (r_L - r_D)}{\beta - r_D} \\ L_R \cdot \dfrac{L \cdot (1-s) \cdot P \cdot (\beta - r_L)}{((1-s) \cdot P + \omega - r_D) \cdot (\beta - r_D)} \\ R_L \cdot L \end{bmatrix} = \begin{bmatrix} 60 \\ 51.5 \\ 2.55 \\ 5.95 \\ 40 \end{bmatrix} \tag{1.8}$$

It is obvious that money is not destroyed, but turned into reserves that are then available for relending. However there is a reduction in money in circulation at any one time, equivalent to the proportion of debt that has been repaid. Given the parameters used in this simulation, the amount of circulating money is reduced from 100 to 60 units.

It is thus not money that is "destroyed" by the repayment of debt, but deposits in income accounts. This in turn reduces the amount available for the financing of production, reducing all incomes—including that of banks. The equilibrium levels of income are now:

$$\begin{bmatrix} \Pi_E \\ I_E \\ W_E \end{bmatrix} = \begin{bmatrix} 103 \\ 3 \\ 159.49 \end{bmatrix} \tag{1.9}$$

Growth

At this stage, the model accords with Keynes's verbal analysis of the "revolving fund of finance":

> If investment is proceeding at a steady rate, the finance (or the commitments to finance) required can be supplied from a revolving fund of a more or less constant

> amount, one entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment. (Keynes 1937b: 248)

The final topic is how to model endogenous money in a growing economy, when "decisions to invest are (e.g.) increasing" and "the extra finance involved will constitute an additional demand for money." (Keynes 1937b: 248).

Accounting for growth integrates Basil Moore's "Horizontalism" into the Circuitist framework. As Moore argues, firms negotiate "lines of credit" with banks that enable them to expand the available money, subject to the same sum being added to their outstanding debt. New money is thus created by an addition of an identical sum to to the firms' deposit and loan accounts Using F_I (for "Firms' Investment") to signify the rate, and relating this to the level of firms' deposit accounts,[10] this introduces a new term $F_I.K_D$ into the columns for F_L and F_D in the final table. I have included the creation and simultaneous transfer of this new money in the banks' reserve account simply to indicate that the endogenous creation of money by firms depends upon the legal right they have negotiated with banks to expand their borrowings.[11]

Bank Assets & Liabilities					
Time	Assets	Liabilities			SAM
	Firm Loan (F_L)	Firm Deposit (F_D)	Banker Deposit (B_D)	Worker Deposit (W_D)	Income
Investment by firms	$+I_F.F_L$	$+I_F.F_L$			$+I_F.F_L$

Bank Reserves		
Time	Reserves	Capital
Investment by firms	$+I_F.F_L - I_F.F_L$	0

Table 9: Endogenous creation of new money

There is no offsetting transfer between income and capital accounts in this case, so that the term $F_I.K_D$ causes a net increase in the money stock: it is an endogenous source of growth. As a result, rather than having a zero sum, the complete SAM has a positive sum equal to the amount of new money $I_F.F_L$ being created each year. The overall model, as shown in Equation (1.10), is therefore "dissipative"—in the language of modern dynamic analysis—rather than "conservative", which has important implications for the feasible behaviour of the complete model that will be built on this skeleton.

$$
\begin{aligned}
\tfrac{d}{dt}F_L &= +L_R \cdot B_R - R_L \cdot F_L + F_I \cdot F_D \\
\tfrac{d}{dt}F_D &= \left(r_D F_D - r_L F_L\right) - (1-s)\cdot P \cdot F_D + \left(\omega \cdot W_D + \beta \cdot B_D\right) + \left(L_R \cdot B_R - R_L \cdot F_L\right) + F_I \cdot F_D \\
\tfrac{d}{dt}B_D &= \left(r_L F_L - r_D F_D\right) - r_D \cdot W_D - \beta \cdot B_D \\
\tfrac{d}{dt}W_D &= (1-s)\cdot P \cdot F_D + r_D \cdot W_D - \omega \cdot W_D \\
\tfrac{d}{dt}B_R &= +R_L \cdot F_L - L_R \cdot B_R
\end{aligned}
\tag{1.10}
$$

Though the amount of money and debt in this final model grow exponentially over time, the same relations hold between debt and income deposits, while the overall money stock includes both the sum of deposit accounts and the amount in banks' reserves. At the end of the simulation period (30 years), the endogenous money stock has grown from 100 to 379.13, 228.78 of which is in circulation between firm, bank and worker income accounts, and 150.35 of which is in the banks' reserve account.

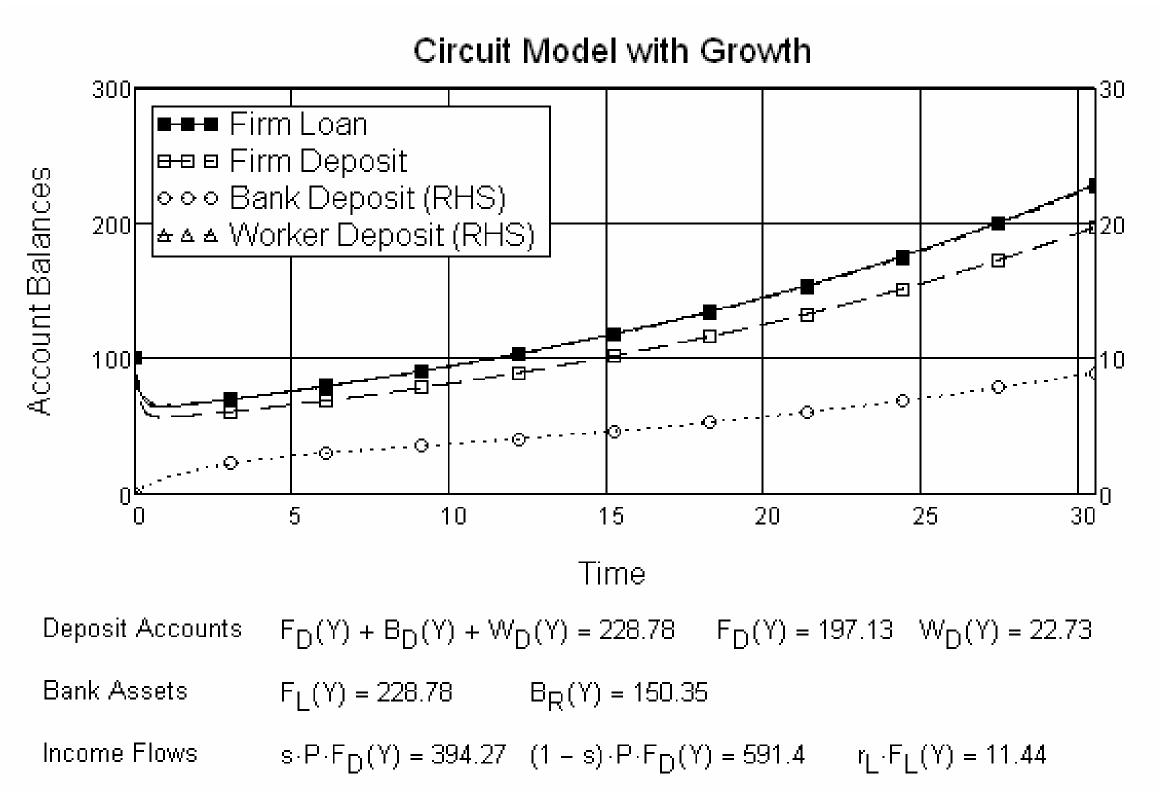


Figure 4: Model with growth

From parameters to behaviours

Like a biological skeleton, this model is designed to have muscles attached, in that its fixed parameters can be replaced by nonlinear behavioral relations that mimic those of real economies. Two that deserve special mention are R_L and F_I,, representing respectively the rate of relending by banks and the rate of new money creation driven by firms.

The latter provides the "Horizontalist" aspect of this skeletal model, and in a general model would be a nonlinear function of firms' expectations of profits (see Keen 1995, 2000). The former reflects the Structuralist emphasis on the active role of banks in the credit system. In a financial crisis, this would tend towards zero, while during a period of euphoric expectations the rate of relending would accelerate.

This illustrates another advantage of dynamic modelling over the conventional diagrammatic and static methods that Post Keynesian and Circuitist economists have in the past applied. Diagrammatic methods are necessarily "two dimensional", while static methods make it difficult, if not impossible, to examine causal relations—even when they are correctly specified, which is rarely if ever the case. On the other hand, this properly specified dynamic model enables the integration of the Horizontalist and Structuralist approaches (which could be further embellished by making the spread between r_L and r_D a variable).

Conclusion

Though my conculsions contradict accepted Circuitist dogma, they are beneficial to Circuitist thinking in general. The underlying Circuitist intuition into the nature of money is right, and a coherent monetary model of production can be built from that initial logical foundation. The results are consistent with common sense: firms can and do make monetary profits, and they can service debt—at least hypothetically.

In the real world, there is clearly a tendency for firms to accumulate excessive debt that Minsky's Financial Instability Hypothesis captures, and this skeletal model deliberately omits. However the model has been designed to allow the incorporation of Minsky's insights, once many other factors that are treated here implicitly are added—including production, markup pricing, capital asset production and pricing, expectations formation.

The model can also easily be extended to incorporate a government and central bank, where the government's taxes and expenditures and the central bank's "lender of last resort" functions generate fiat money, in addition to the credit money created by the banks and firms. It is thus not in conflict with Chartalism, but sees the state generation of money as secondary to credit money.

There is still much to be learnt from studying the skeleton itself, however, and a more complete but still skeletal model with production, prices and stocks. Issues such as the relationship of the money stock to output—the “velocity of money”—can be derived from it, and it can also aid in deducing probable behavioural functions (for example, the fact that debt repayment actually reduces bank income makes it likely that banks are less interested in having old loans repaid than in extending new ones). Thanks to Graziani’s fundamental insights into the nature of endogenous money, and armed with the basic tools of dynamic analysis, we may finally be able to lay bare the monetary anatomy of capitalism.

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[1] Similar conclusions are reached in numerous other Circuitist papers from Graziani 1989 on. Rochon puts the problem well: The existence of monetary profits at the macroeconomic level has always been a conundrum for theoreticians of the monetary circuit… not only are firms unable to create profits, they also cannot raise sufficient funds to cover the payment of interest. In other words, how can *M* become *M*`? (Rochon 2005: 125).

[2] The Central Bank properly enters the Circuitist model when the banking sector is expanded, so that a seller can deposit the proceeds of a sale in a different bank to that of the buyer. This necessitates a clearing house between banks, which is the primary role of a Central Bank in the Circuitist model. In this paper, for the sake of simplicity I omit inter-bank dynamics.

[3] He also asserts that "there is a correspondence between the wage bill paid and the cost of produced commodities", and that "the bank debt of the firms in a single instant of time ... is ... equal to the money value of ... semi-finished products plus inventories" (28). I will consider this proposition in a subsequent paper.

[4] Bellofiore et al. concur on the inability of firms in the aggregate to make money profits, but propose that they make physical profits—again, a "real" outcome in a supposedly "monetary" model.

[5] I regard Schumpeter's Theory of Economic Development as the outstanding—and possibly only—instance of an economist flawlessly developing a dynamic, and indeed evolutionary, model verbally. Schumpeter is of course a major inspiration for the Circuitist school, along with Keynes and Marx.

[6] For example, Graziani 2003 implies that firms pay wages by transferring the whole of the sum of borrowed money from their account to the workers account ("Thus, albeit in an indirect way, the credit initially granted is turned

totally into wages"; Graziani 2003: 28), and the time dynamics of wage payment are poorly specified: it could be that the amount L is transferred instantly from F_D to W_D; or it could be that a flow of payments are made to workers over time. In general, the concepts of stocks and flows are confused in Circuitist literature: an instantaneous transfer of the borrowed money from firms to workers is a stock transfer. However wages—and all other incomes—are in fact flows, as is expenditure by firms to finance production.

[7] Later I apply Graziani's position that "the demand for bank credit coming from producers depends only on the wage rate and on the number of workers that firms intend to hire" (29) to calculate the size of the initial loan L as a function of the equilibrium wage bill

[8] Again, in a more complete model, each of these stages of the process would have their own equation with its own dynamics; here, for reasons of simplicity and exposition, they are all collapsed into the values of s and P.

[9]It could equally be related to the level of F_D.

[10] It could as easily be related to the level of outstanding loans, and would doubtless have a more complex causal link in a full dynamic model.

[11] In a full model, this could be given a rationing ceiling; however I believe that a better way to indicate banks' "structuralist" control over lending is to replace R_L with a variable dependent upon financial conditions.