On *1*-soundness and Soundness of Workflow Nets

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Introduction to Workflow Nets

- **Workflow Nets**

  Workflow Nets is a special kind of Petri Nets (proposed by Prof. Aalst) for workflow modeling (control-flow dimension). It specifies the partial ordering of tasks. Tasks are represented by transitions in Petri nets, the ordering between tasks are represented by arcs and places. Workflow nets give a solid theoretical foundation for workflow modeling.

- **Definition (WF-net, by Aalst)**

  A Petri net $PN = (P, T, F)$ is a WF-net iff:

  (1) *PN* has two special places: *i* and *o*. Place *i* is a source place: $\bullet i = \emptyset$. Place *o* is a sink place: $o\bullet = \emptyset$.

  (2) If we add a transition t^* to *PN* so that $\bullet t^* = \{o\}$ and $t^*\bullet = \{i\}$, then the resulting Petri net is strongly connected. (*PN**, the extended net of *PN*)

Introduction to Workflow Nets

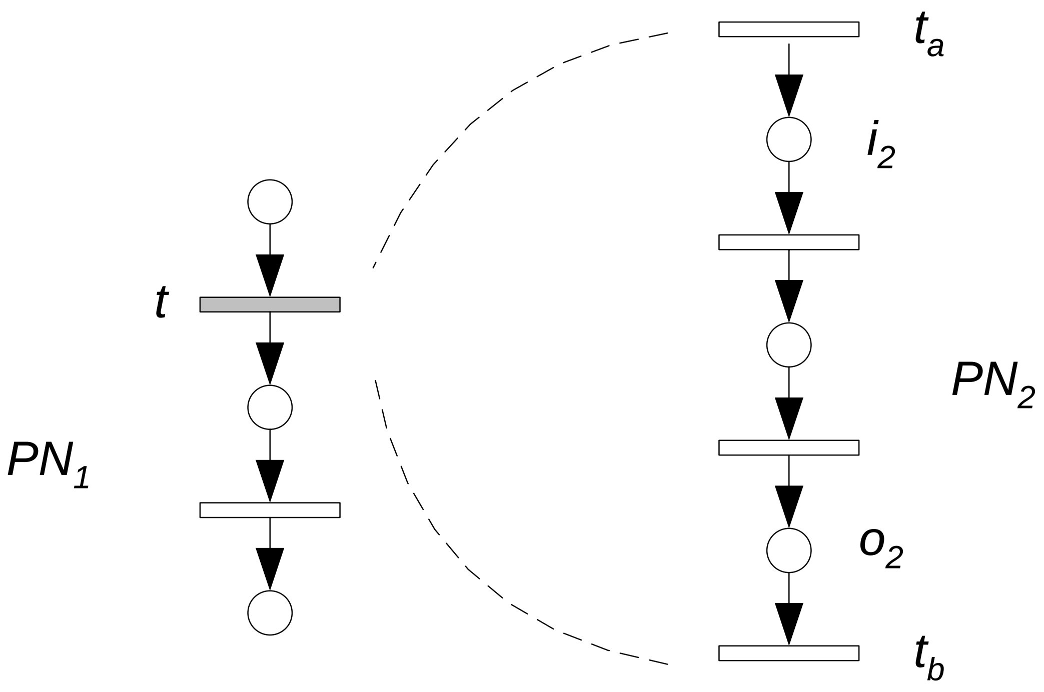
- **Correctness Issues on Workflows**
  No deadlocks
  No dangling tasks
  Termination guaranteed
  …

- **Definition (*1*-soundness, by Aalst).**
  A WF-net *PN* = (*P*, *T*, *F*) is *1*-sound if and only if:
  (1) $\forall M\ ([i] \xrightarrow{*} M) \Rightarrow (M \xrightarrow{*} [o])$
  (2) $\forall M\ ([i] \xrightarrow{*} M \wedge M \geq [o]) \Rightarrow (M = [o])$
  (3) $\forall t \in T\ \exists M, M'\ \ [i] \xrightarrow{*} M \xrightarrow{t} M'$

Introduction to Workflow Nets

- **Composition of Workflow Nets**

  $PN_3 = PN_1 \otimes t \; PN_2$



Introduction to Workflow Nets

- ***1*-soundness is not compositional**

  If we use a *1*-sound WF-net to replace a transition of another *1*-sound one, the result may not be *1*-sound.

- **Definition (*K*-soundness, by Kees van Hee et al.)**

  A WF-net $PN = (P, T, F)$ is k-sound for a natural number k if and only if:

  (1) $\forall M \; ([i^k] \xrightarrow{*} M) \Rightarrow (M \xrightarrow{*} [o^k])$

  (2) $\forall t \in T \;\; \exists M, M' \;\; [i^k] \xrightarrow{*} M \xrightarrow{t} M'$

Introduction to Workflow Nets

- **Definition (*S*oundness, by Kees van Hee et al.)**

  A WF-net *PN* is sound if for all natural number *k*, *PN* is *k*-sound.

- **Soundness is compositional and decidable**

  Kees van Hee et al. proved that soundness is compositional, that is, if we replace a transition in a sound WF-net by another sound one, the result WF-net is also sound. They also proved that soundness is decidable. A decision procedure is proposed. However, it is still to be investigated how to solved the problem of soundness effectively and what complexity the algorithm would have.

- **We find that for some kinds of WF-nets, soundness can be decided effectively**

Basic Properties of Workflow Nets

- **Property (by Aalst).**
  For a WF-nets *PN*, *PN* is *1*-sound iff (*PN**, [*i*]) is live and bounded.

- **Property.**
  If a *1*-sound WF-net *PN* is *k*-sound, then for all natural numbers $p < k$, *PN* is *p*-sound.

- **Property.**
  If a *1*-sound WF-net *PN* is not *k*-sound, then for all natural numbers $p > k$, *PN* is not *p*-sound.

- **Property.**
  For an arbitrary *1*-sound WF-net *PN*, either it is sound or there exists a natural number *k* so that $\forall\ p < k$, *PN* is *p*-sound and $\forall\ q \geq k$, *PN* is not *q*-sound.

Basic Properties of Workflow Nets

- **Property.**

  Let PN_1 be k-sound WF-net, PN_2 be sound WF-net and t be a transition of PN_1, $PN_3 = PN_1 \otimes t\ PN_2$ is also k-sound.

  This property is useful during workflow nets composition when we only want to ensue the *1*-soundness of the resulting WF-net.

Establishing Relationship Between *1*-soundness and Soundness

- **Several specific kinds of WF-nets are examined by Aalst and efficient algorithms are found to decide their 1-soundness**

  Prof. Aalst examined three kinds of WF-nets – free-choice WF-nets, well-handled WF-nets and s-coverable WF-nets. For the former two kinds of WF-nets, the well-formedness of their extended net (*1*-soundness) can be decided in polynomial time. The s-coverable WF-nets is the generalization of the former ones.

- **For the above kinds of WF-nets, can soundness be implied by *1*-soundness?**

Establishing Relationship Between *1*-soundness and Soundness

- **Definition (ST-AC WF-net).**

  A WF-net *PN* is a ST-AC WF-net if *PN** is an asymmetric choice Petri net and every siphon of it contains at least a trap.

- **Properties on ST-AC WF-net**

  For a well-formed ST-AC Petri net, it is live and bounded if and only if every siphon of it is marked (by L. Jiao). Also every minimal siphon of a live and bounded ST-AC net is an S-component of the net (by L. Jiao). For a *1*-sound ST-AC WF-net *PN*, the net system (*PN**, [*i*]) is live and bounded. So the marking [*i*] marks every siphon in the net *PN**. Therefore the marking [i^k] also marks every siphon in *PN** and the net system (*PN**, [i^k]) is live and bounded for any natural number *k*.

Establishing Relationship Between *1*-soundness and Soundness

- **Theorem**

For ST-AC WF-nets, *1*-soundness implies soundness.

(Proof.) Suppose for a *1*-sound ST-AC WF-net *PN*, *PN* is not *k*-sound. The requirement (1) of the *k*-soundness must not hold. So for (*PN*, $[i^k]$), there exists a marking *M* reachable from $[i^k]$ so that $[o^k]$ can not be reached from *M*. In *PN*, let $M \xrightarrow{x} M'$ so that from *M'*, no tokens can be put into place *o*. At *M'* the number of tokens in place *o* must less than *k*. In the system (*PN**, $[i^k]$), the marking *M'* can also be reached from $[i^k]$. Let $M'' = M' - M'|o$, then (*PN**, *M''*) is bounded but not live. But since every minimal siphon in *PN** is an S-component and each contains *k* tokens at $[i^k]$, then at *M''*, each minimal siphon in *PN** must be marked and (*PN**, *M''*) is live. So we get a contradiction.

Establishing Relationship Between *1*-soundness and Soundness

- **Corollary**

  For free-choice and extended free-choice WF-nets, *1*-soundness implies soundness.

  (An extended free-choice net is also an asymmetric choice net. For a *1*-sound extended free-choice net *PN*, (*PN**, [*i*]) is live and bounded, so every siphon of *PN** must contain a trap (Commoner's Theorem). So a *1*-sound extended free-choice WF-net is also a *1*-sound ST-AC WF-net)

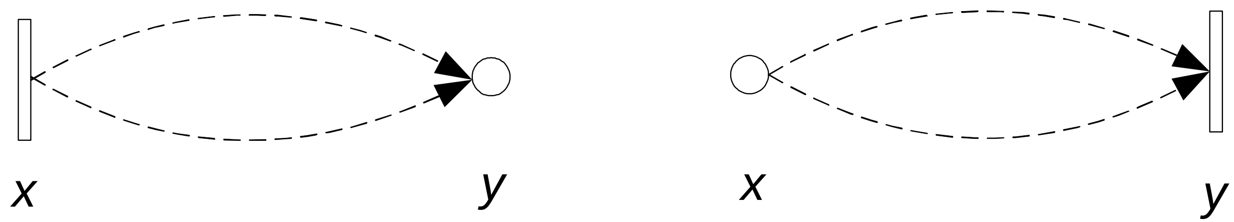
- **Corollary**

  For free-choice and extended free-choice WF-nets, their soundness can be decided in polynomial time.

Establishing Relationship Between *1*-soundness and Soundness

- **Definition (Well-handledness, WH WF-nets, by Aalst)**

  A Petri net *PN* is well-handled if for any pair of nodes *x* and *y* such that one of the nodes is a place and the other a transition and for any pair of elementary paths C_a and C_b leading from *x* to *y*, if C_a and C_b have only nodes *x* and *y* in common, C_a and C_b must be identical. A WF-net *PN* is a well-handled WF-net if *PN** is well-handled.



Establishing Relationship Between *1*-soundness and Soundness

- **Definition (Conflict free, ENSeC net, ENSeC WF-net)**

  Let *PN* be a Petri net and $C = <n_1, \ldots, n_k>$ be a path in *PN*, *C* is conflict-free iff for any transition n_i of the path, $j \neq i -1 \Rightarrow n_j \notin \bullet n_i$. Let *PN* be a Petri net, *PN* is an Extended Non-Self Controlling (ENSeC) net iff for every pair of transition t_1 and t_2 such that $\bullet t_1 \cap \bullet t_2 \neq \emptyset$, there does not exist a conflict-free path leading from t_1 to t_2. A WF-net *PN* is an ENSeC WF-net if *PN** is an ENSeC net.

- **Properties on ENSeC WF-net**

  For ENSeC Petri net system (*PN*, *M*), if it is live and bounded then *PN* is S-coverable. If (*PN*, *M*) is bounded, it is live if and only if every minimal siphon is a marked state-machine at *M*. For a *1*-sound ENSeC WF-net *PN*, (*PN**, [*i*]) is live and bounded, so (*PN**, $[i^k]$) is live and bounded for any natural number *k*.

Establishing Relationship Between *1*-soundness and Soundness

- **Theorem**

  For ENSeC WF-nets, *1*-soundness implies Soundness

  (Proof.) Let *PN* be a *1*-sound ENSeC WF-net, suppose *PN* is not *k*-sound. For *PN*, we can find a marking *M'* reachable from $[i^k]$ so that from *M'*, no more tokens can be put into place *o*. In the system (*PN**, $[i^k]$), let *M''* = *M'*-*M'*|*o*, then (*PN**, *M''*) is not live. But since every minimal siphon is an state-machine at $[i^k]$, at *M''* they must also be marked, so (*PN**, *M''*) is also live.

- **Corollary**

  For well-handled WF-nets, *1*-soundness implies soundness.

  (A well-handled WF-net is also an ENSeC WF-net, by Prof. Aalst)

- **Corollary**

  For well-handled WF-nets, their soundness can be decided in polynomial time.

Establishing Relationship Between *1*-soundness and Soundness

- **The s-coverable WF-nets are the generalization of the free-choice and well-handled WF-nets, does their *1*-soundness implies soundness?**

  We only have the partial results on the SMA (state-machine-allocatable) WF-nets, a subset of s-coverable WF-nets

- **For SMA WF-nets, their *1*-soundness implies Soundness**

- **For SMA WF-nets, their soundness can be decided in polynomial time**

Establishing Relationship Between *1*-soundness and Soundness

- **Does *1*-soundness imply soundness for s-coverable WF-nets?**

- **Does *1*-soundness imply soundness for asymmetric-choice WF-nets?**

  Liveness monotonicity does not hold for asymmetric-choice net since there may be siphons that do not contain any trap in a live asymmetric-choice net. However, we believe that restricted liveness monotonicity (Let *PN* be an AC-net, (*PN*, $[i^k]$) is live if (*PN*, $[i]$) is live) does hold for asymmetric-choice net. Such a property may be necessary in the prove if *1*-soundness does imply soundness for AC WF-nets.

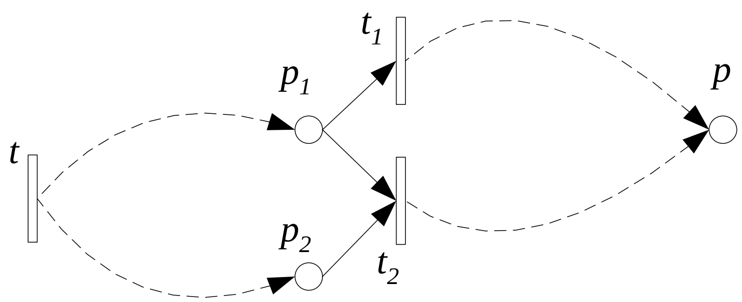
Well-handled with Regular Iteration Nets

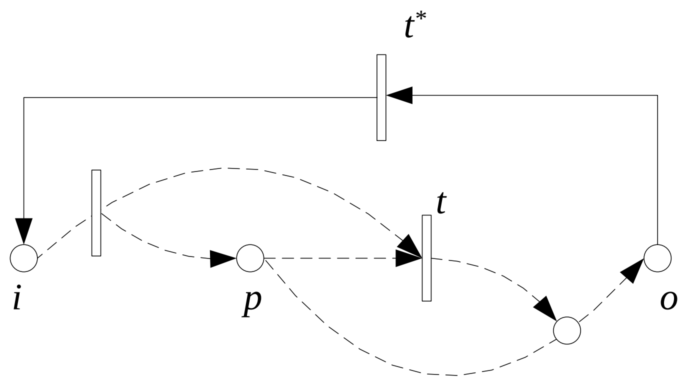
- **Definition (Well-handled and Acyclic Workflow Nets)**

  A WF-net *PN* is WA WF-net if *PN* is well-handled and acyclic

- **Property**

  For a WA WF-net *PN*, *PN* is a free-choice WF-net and also a well-handled WF-net





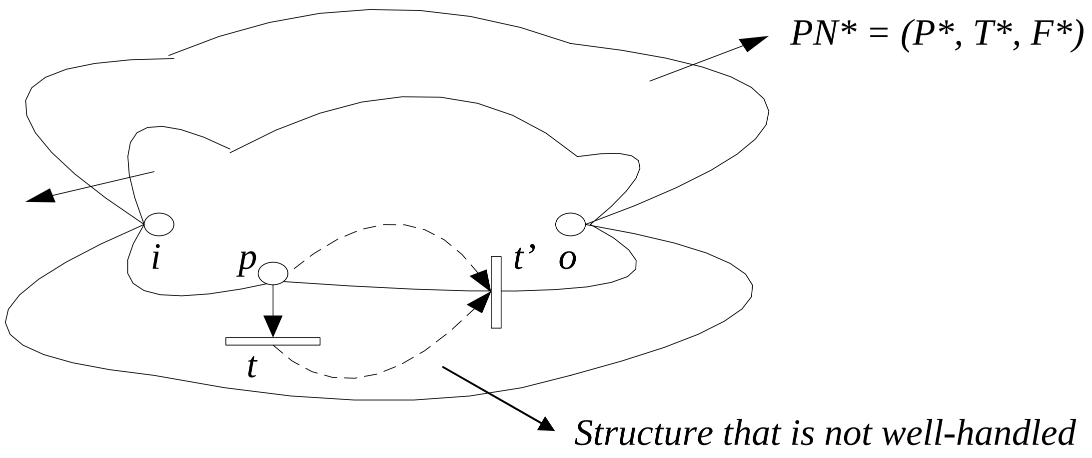
Well-handled with Regular Iteration Nets

- **Theorem**

  For a WA WF-net *PN*, *PN* is sound

  (Proof.) Since *PN** is well-handled, no circuit of *PN** has PT- or TP-handle. So (*PN**, [*i*]) is bounded and covered by s-component (by J. Esparza). *PN** is free-choice, we proved that every minimal siphon of *PN** must be a trap. Thus, (*PN**, [*i*]) is live.

A siphon PH that is not a trap. PH = (H, •H, •F) in which •F* = F*∩ ((H × •H) ∪(•H ×H))*



Well-handled with Regular Iteration Nets

- **Definition (Well-handled with Regular Iteration Nets)**

  (1) A WA WF-net is a WRI WF-net

  (2) Let PN_1 and PN_2 be two WRI WF-nets, $PN_3 = PN_1 \otimes t\ PN_2$ is a WRI WF-net.

  (3) Let PN_1 and PN_2 be two WRI WF-nets, $PN_3 = PN_1 \otimes t\ PN_2*$ is a WRI WF-net

  (4) WRI WF-nets could only be obtained by (1), (2) and (3)

- **Theorem**

  WRI WF-nets are sound workflow nets.

  (Proof. Let PN_1 and PN_2 to two *1*-sound and safe WF-nets, $PN_3 = PN_1 \otimes t\ PN_2$ or $PN_3 = PN_1 \otimes t\ PN_2*$, it's easy to see that PN_3 is also *1*-sound and safe. Moreover, WRI WF-nets are free-choice WF-nets, so their *1*-soundness implies soundness)

Well-handled with Regular Iteration Nets

- **WRI WF-nets support hierarchical modeling of workflows naturally**

  (1) First, the sketch of a workflow process is modeled by a WA WF-net, those iterations and subnets to be refined are represented by special transitions.

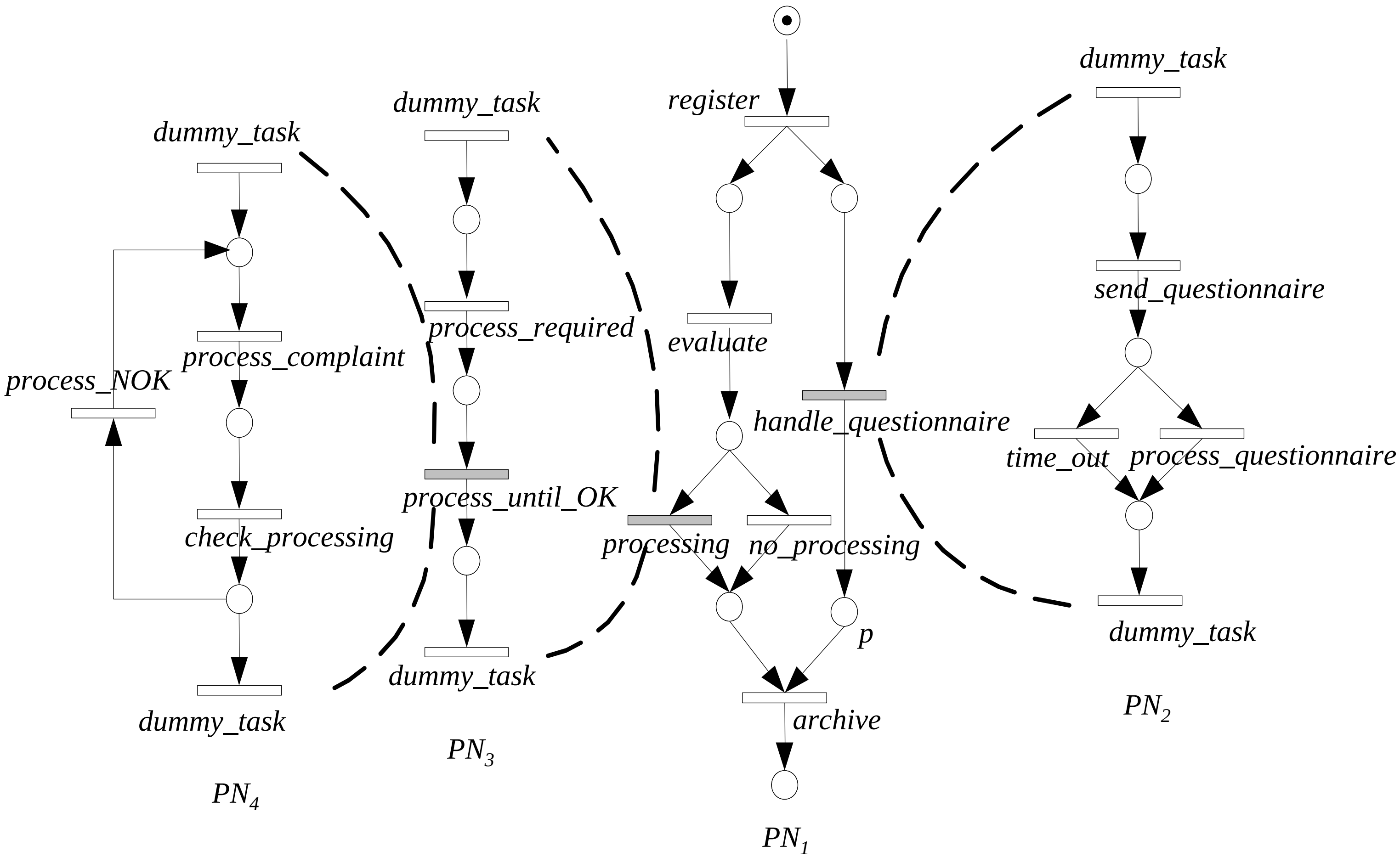
  (2) Those special transitions are replaced by WA WF-nets or by WA WF-nets' extended nets in which special transitions may also exist to represent the subnets or iterations to be modeled next.

  (3) We continue the above modeling process until there is no more iterations and subnets to be refined in our workflow model.

  (4) By the definition of WRI WF-net, the workflow model we get is a WRI WF-net and its soundness is ensured. At each step, the verification task is rather simple.

Well-handled with Regular Iteration Nets

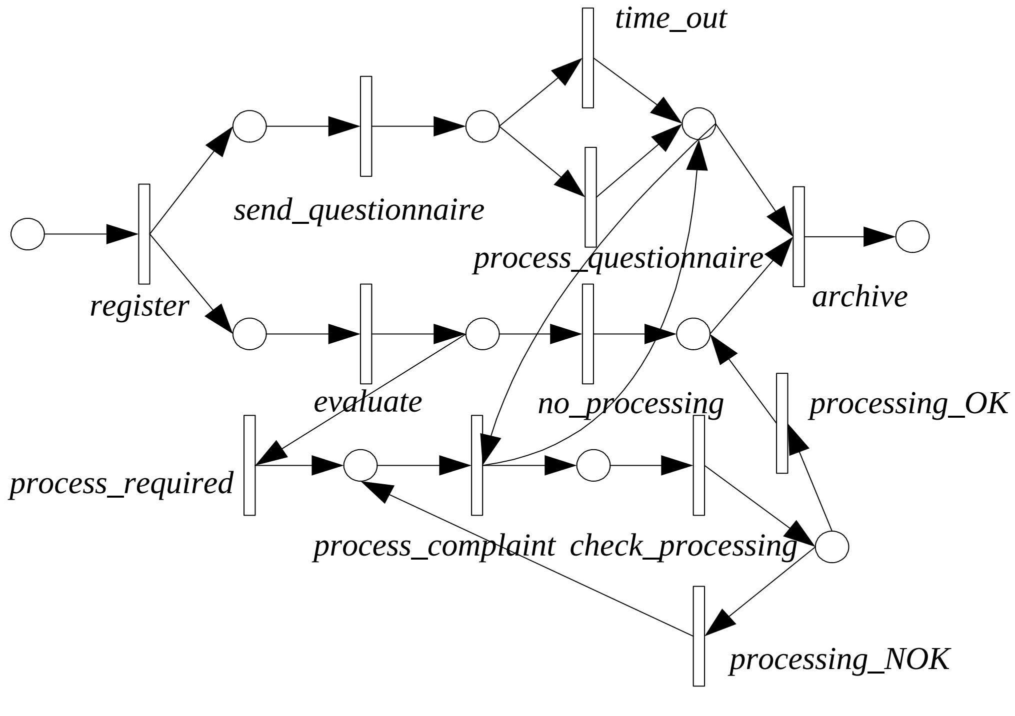
- **An example using WRI WF-nets modeling workflows**



Well-handled with Regular Iteration Nets

- **WRI WF-nets do not fit for modeling all workflow models.**

  When complex synchronizations exist in workflow models, it may be hard to use WRI WF-nets to model them.



Conclusion

- **In this paper we**

  (1) Examined the relationship between *1*-soundness, *k*-soundness and soundness of workflow nets

  (2) Proved that for several kinds of WF-nets, their soundness can be decided effectively

  (3) Proposed a specific workflow model – WRI WF-nets which are inherently sound. Gave a way to use WRI WF-nets modeling workflows hierarchically.

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Thank you !