

Maintenance policy for a two-components system with stochastic dependences and imperfect monitoring

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Abstract

The goal of this work is to optimize a preventive maintenance policy for a multi-component system with imperfect monitoring, i.e the monitoring procedure may exhibit non-detection problems in case of failure.

1 Introduction

The monitoring information (failure time or degradation level) is of great importance for the maintenance decision making. In case of stochastic dependences between components (e.g. the state of each component is correlated to the state of the others (Çinlar, Shaked, and Shanthikumar 1989; Bhattacharya 1996)), it can be shown that the maintenance policies can be significantly improved if the monitoring information is taken into account for the maintenance decision making (Aven and Jensen 1999; Heinrich and Jensen 1996). The model presented in (Heinrich and Jensen 1996) allows to optimize a preventive replacement time only considering partial monitoring: for some components the failure time is totally unknown (whereas it is always perfectly known for the others). In this communication we aim at extending this study to the development of a maintenance model for system with imperfect monitoring: instead of considering the failure time of some components is totally unknown we assume it can be only sometimes unknown because of non-detection events (partial monitoring can be seen as a limit case of imperfect monitoring). The first part of this communication is devoted to the presentation of the multi-component system with stochastic dependences we want to maintain. Numerical experiments show that the performance of a maintenance policy based on the perfect monitoring assumption seriously deteriorates when non-detection events are added. In the second part, we develop the model of a maintenance policy integrating explicitly the imperfect monitoring information: its performance are more robust to non-detection problem.

2 Presentation of the model

We consider a two-component parallel system with exponential lifetimes and stochastic dependences as in (Heinrich and Jensen 1996). When one of the components fails, the extra stress is placed on the surviving one for which the failure rate is switched to a superior one. We note λ_1 and λ_2 the failure rate of component 1 and 2 when both are running, and $\bar{\lambda}_1$ and $\bar{\lambda}_2$ the failure rates when one is failed ($\bar{\lambda}_i > \lambda_i$). External shocks are added with an exponential law (occurrence rate λ_{12}) and destroy the whole system at once. Perfect and instantaneous replacements on the whole system are triggered correctively when it is failed or preventively after a functioning duration of time τ_{prev} . One component is never repaired or renewed alone. The value τ is optimized as a stopping time which minimizes the total expected α -discounted maintenance cost K :

$$K = \mathbb{E}(S_{\tau_1} + e^{(-\alpha\tau_1)}S_{\tau_2} + e^{(-\alpha(\tau_1+\tau_2))}S_{\tau_3} + \dots) = \frac{\mathbb{E}(S_{\tau})}{\mathbb{E}(1 - e^{(-\alpha\tau)})} \quad (1)$$

where S_{τ_i} is the maintenance cost on the i^{th} renewal cycle: $S_t = (c + k * I_{(T \leq t)})e^{(-\alpha t)}$ with I_A the indicator function of the set A , T the lifetime of the whole system, $\tau_i = \min(T, \tau_{\text{prev}})$ the length of the

i^{th} renewal cycle, k the penalty over-cost incurred when the system fails and c , the replacement cost incurred at a renewal. .

The optimal value of τ is found in (Heinrich and Jensen 1996) for different information levels when the failure time of the components is always known for both of them (perfect monitoring), for component 1 only (partial monitoring : component 2 failure time is totally unknown) or for none of them. The failure time for the whole system is always known. Through numerical experiments (Monte-Carlo simulation), we have investigated the robustness of the maintenance policy designed under the perfect monitoring assumption to the occurrence of non-detection on component 2 failure time. When the probability of non-detection increases (i.e. when the monitoring information deteriorates), the performance of the policy drastically collapses. It can become even worst than the performance of the maintenance policy designed under the partial monitoring assumption, i.e when component 2 failure time is totally unknown. Figure 1 sketches an example of the optimal expected discounted cost as a function of the probability p of non-detection.

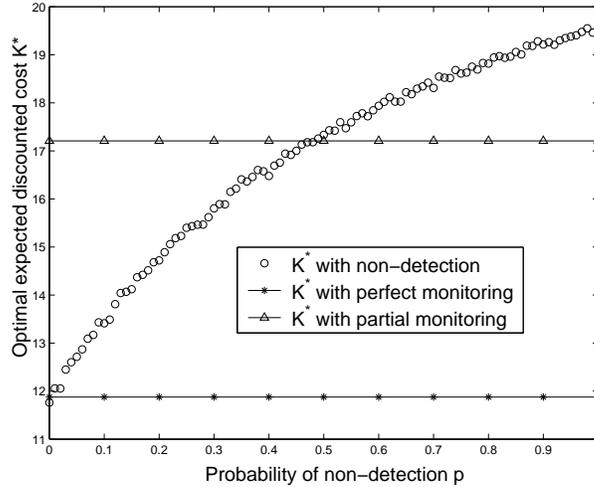


Figure 1: Optimal expected discounted cost K^* as a function of the probability of non-detection p with $\lambda_1 = 1, \lambda_2 = 3, \bar{\lambda}_1 = 1.5, \bar{\lambda}_2 = 3.5, \lambda_{12} = 0.5, \alpha = 0.08, c = 0.1, k = 1$.

3 Maintenance model with imperfect monitoring

The deterioration of the performance of the previous policy in case of imperfect monitoring motivates the development of a maintenance policy which explicitly takes into account non-detection events. The new preventive maintenance, optimized also as a stopping time, exploits correctly the available information even there is non-detection.

3.1 Imperfect monitoring structure

We assume component 2 is imperfectly monitored because of non-detection, i.e its failure time is not detected with a probability p ($p = 1$ corresponds to partial monitoring). Component 1 is perfectly monitored and the failure time of the whole system is always known. Such a structure has been chosen in order to compare the performance of the maintenance policy to the one of (Heinrich and Jensen 1996) with partial monitoring. But the following calculations can be developed for non-detection on component 1 only or on both of them. The variables we consider are only those given by the monitoring: For component 1 and for the system we have the the real lifetimes X and Z and for component 2, we have

an observed lifetime Y^o different from the real lifetime Y . For any event A :

$$\begin{aligned} P(Y^o = Y) &= 1 - p & P(A, Y^o > t) &= P(A, Y > t)(1 - p) + P(A)p \\ P(Y^o = \infty) &= p & P(A, Y^o \leq t) &= P(A, Y \leq t)(1 - p) \end{aligned} \quad (2)$$

3.2 Global structure of the optimization process

In the case of perfect monitoring and if S_t has a smooth semi-martingale representation (Aven and Jensen 1999) (which is true on mild condition on the lifetime of the system T). Then we have:

$$S_t = c + \int_0^t I_{T>s} \alpha e^{(-\alpha s)} r_s ds + M_t \quad (3)$$

where $r_s = \frac{1}{\alpha}(-\alpha c + \lambda_s k)$ with λ_s the failure rate of the system and M_t is a uniformly integrable martingale such that $M_0 = 0$. Then upper and lower bounds b_u and b_l can be found for the minimum cost K^* . If r_t has a non-decreasing or bath-tub-shaped paths and $r_0 \leq bl$, then the optimization problem is equivalent to solve the following system (Heinrich and Jensen 1996):

$$\begin{aligned} \tau_K &= \min \left[\inf(t \in \mathbb{R}_+ : r_t \geq K), T \right] \\ K^* &= \inf \left(K \in \mathbb{R} : K \mathbb{E}(1 - e^{-(\alpha \tau_K)}) - \mathbb{E}(S_{\tau_K}) \geq 0 \right) \end{aligned} \quad (4)$$

In the case of imperfect monitoring we can have the same formulation of the optimization problem. Instead of using the exact failure rate λ_t obtained with perfect monitoring, we use the conditional failure rate λ_t^o obtained with imperfect monitoring. Here imperfect monitoring corresponds to non-detection on component 2 failure time. Three cases are considered to calculate λ_t^o which depend on the conditions given by the observable variables X , Y^o , and Z :

$$\begin{aligned} \lambda_t^1 &= P(\text{System failure on } [t, t+h] \mid \text{No system failure on } [0, t] \text{ and } X > t, Y^o > t, Z > t) \\ \lambda_t^2 &= P(\text{System failure on } [t, t+h] \mid \text{No system failure on } [0, t] \text{ and } X \leq t, Y^o > t, Z > t) \\ \lambda_t^3 &= P(\text{System failure on } [t, t+h] \mid \text{No system failure on } [0, t] \text{ and } X > t, Y^o \leq t, Z > t) \end{aligned}$$

Other cases are not considered since they imply a failure before t . The conditional probabilities are calculated with Bayes formula and Equations 2. We obtain:

$$\begin{aligned} \lambda_t^1 &= \lambda_{12} + \frac{p\lambda_2 \bar{\lambda}_1 [e^{(\lambda_1 + \lambda_2 - \bar{\lambda}_1)t} - 1]}{\lambda_1 + \lambda_2 - \bar{\lambda}_1 + p\lambda_2 [e^{(\lambda_1 + \lambda_2 - \bar{\lambda}_1)t} - 1]} = g(t) \\ \lambda_t^2 &= \lambda_{12} + \bar{\lambda}_2 \\ \lambda_t^3 &= \lambda_{12} + \bar{\lambda}_1 \\ \lambda_t^o &= \lambda_t^1 I_{(X>t, Y^o>t, Z>t)} + \lambda_t^2 I_{(X<t, Y^o>t, Z>t)} + \lambda_t^3 I_{(X>t, Y^o<t, Z>t)} \end{aligned} \quad (6)$$

According to the value of λ_t^o , the function $r(t)$ and the stopping time τ_K can be of three different forms:

$$\begin{aligned} \text{If } K &\leq (\lambda_{12} + \bar{\lambda}_1) \frac{k}{\alpha} - c & : \tau_K &= \min \left[X, Y^o, \inf(t \in \mathbb{R}_+ : g(t) \frac{k}{\alpha} - c > K), Z, T \right] \\ \text{If } (\lambda_{12} + \bar{\lambda}_1) \frac{k}{\alpha} - c &< K \leq (\lambda_{12} + \bar{\lambda}_2) \frac{k}{\alpha} - c & : \tau_K &= \min[X, Z, T] \\ \text{If } K &> (\lambda_{12} + \bar{\lambda}_1) \frac{k}{\alpha} - c & : \tau_K &= \min[Z, T] \end{aligned} \quad (7)$$

As the average cost K is calculated by Monte-Carlo simulations, the three above conditions on K can not be explained only as conditions on the cost parameters and on the failure rates. The average cost is calculated in each case and the minimum one which verifies the associated condition is the optimal one. The optimization process in the first case when $K \leq (\lambda_{12} + \bar{\lambda}_1) \frac{k}{\alpha} - c$ gives an optimal couple (K^*, τ^*) where the stopping time has the following form:

$$\tau^* = \min[X, Y^o, b^*, Z, T] \quad \text{with} \quad b^* = \inf(t \in \mathbb{R}_+ : g(t) \frac{k}{\alpha} - c > K^*) \quad (8)$$

In this later case the optimal value τ^* depends directly on the probability of non detection p since $g(t)$ is a function of p . The imperfect monitoring influences the optimal preventive maintenance and imperfect information is explicitly taken into account.

3.3 Numerical results

On Figure 2 we can see the expected cost of the policy optimized with imperfect monitoring assumption. The parameters are the same as on Figure 1 and correspond to the case $K \leq (\lambda_{12} + \bar{\lambda}_1) \frac{k}{\alpha} - c$, i.e when imperfect monitoring influences directly the optimal preventive maintenance. Contrary to the policy of (Heinrich and Jensen 1996), this one is logically always better as the one based on partial monitoring assumption.

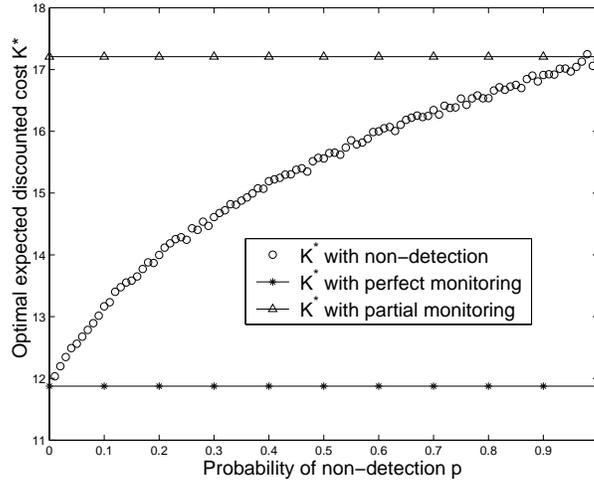


Figure 2: Optimal expected discounted cost K as a function of the probability of non-detection p with $\lambda_1 = 1, \lambda_2 = 3, \bar{\lambda}_1 = 1.5, \bar{\lambda}_2 = 3.5, \lambda_{12} = 0.5, \alpha = 0.08, c = 0.1, k = 1$.

4 Conclusion

A model is proposed to integrate non-detection into the maintenance optimization process. The policy obtained preforms satisfactorily in comparison with policies based on the perfect or partial monitoring assumption and further works aim to develop an analytical expression for the associated total expected discounted cost K .

References

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