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REVISING THE LOGIC OF LOGICAL REVISION

(Received in revised form 19 April 1999)

1. INTRODUCTION

Michael Dummett's realism debate is a semantic dispute about the kind of truth conditions had by a given class of sentences. According to his semantic realist, the truth conditions are potentially verification-transcendent in that they may obtain (or not) despite the fact that we may be forever unable to recognize whether they obtain. According to Dummett's semantic anti-realist, the truth conditions are of a different sort. Essentially, for the anti-realist, that the truth conditions obtain (whenever they do) is a matter that is always recognizable by us in principle. On this view, truth cannot outrun all possible human knowledge. Unsurprisingly, the outcome of the debate is sometimes said to hinge on whether all truths are knowable.¹ More carefully the point of contention is the following *knowability principle*:

(*KP*) All understood truths of the given class are knowable by us in principle.

Dummett's defense of anti-realism – i.e., some version of *KP* – relies crucially upon a certain semantic molecular thesis and upon Wittgensteinian considerations about the publicity of meaning. But his controversial defense need not be discuss here, since Dummett's argument for *KP* is not the centerpiece of the present essay. My primary concern is with the *consequences* of adopting a Dummettian anti-realism, in particular the consequences that its adoption

* A draft of this material was presented to the Eastern Division of the APA. I am especially grateful to Neil Tennant, Stewart Shapiro, Crispin Wright and Jon Cogburn for their comments, suggestions and insights. For expressing worries that led to revisions herein, I am also grateful to Risto Hilpinen, Robert Kraut, and William Taschek.



has for logical reform. To this extent my inquiry bears on any theory according to which truth is taken to be constrained by possible knowledge.²

Dummett's anti-realism is most notoriously associated with the call to revise classical logic. Yet, it remains unclear how anti-realism is supposed to motivate the obligation to revise. Does the obligation arise in light of a anti-realist *refutation* of a classical principle? Or does it arise because once we embrace anti-realism some classical principles turn out merely to lack a certain kind of justification or privileged epistemic status that logicians standardly claim for them?

The primary aim here is to determine whether and how Dummettian anti-realism yields a commitment to revise classical logic. It seems that important attempts to make the revisionist's point fail. These are the attempts made by Michael Dummett and Crispin Wright. The negative thesis is that, given the resources provided by either Dummett or Wright, choice of logic is not a realism-relevant feature – i.e., logical revision is not a consideration that is enjoined by one's stance on the possibility of verification-transcendent truth. In fact, *it is not clear* that either Dummett or Wright provides a consistent set of anti-realist commitments from which to argue. The positive thesis is that anti-realism *does* entail revision. The task is to get the logical structure of the argument straight.

I shall suppose, as Dummett does, that any discourse in dispute consists entirely of meaningful and understood indicatives that are neither vague nor ambiguous. Partly for this reason, partly for the sake of simplicity, and partly to track Dummett's original interests, I focus my discussion on mathematical discourse.³

2. RESTRICTION ON A MEANINGFUL SEMANTIC DEBATE

Dummett mentions the following by way of preserving a meaningful semantic debate:

...the dispute can arise only for classes of statements for which it is admitted on both sides that there may not exist evidence either for or against a *given statement*.⁴

Dummett's claim is that both the realist and the anti-realist are committed to the undecidability of some sentences of the disputed class.

The reason, according to Dummett, is that *only with the possibility of undecidable sentences can the philosophical difference be discerned between the realist and anti-realist*. The realist will be the one asserting that the undecidable sentence has truth conditions that determinately obtain or fail to obtain despite evidential deficiencies. The anti-realist will demur on this score.

Given a class of sentences each of which is decidable, in contrast, the knowability principle is neutral on the metaphysical nature of the truth conditions for those sentences. In such cases robust truth conditions just are (or at least, are co-present with) verification conditions. And so, to characterize a difference that makes a meaningful difference, the adoption of an undecidability thesis (or some relevant degree of epistemic modesty) is required by those participating in the debate.

Formal characterizations of Dummett's undecidability thesis (*Und*) and the knowability principle (*KP*) will be helpful.

Let ' φ ' and ' ψ ' be place-holders for mathematical sentences.

Let $P\varphi$ mean ' φ is mathematically verified', i.e. 'There is a proof of φ that has been correctly recognized as such by now, or at least, we have at present a proof of the fact that we can construct a proof of φ .'

Let $\diamond P\varphi$ mean ' φ is mathematically verifiable', i.e. ' φ is a consequence of our current mathematical knowledge, or of what is mathematically known at some future time.'⁵

Here is a first stab at the logical form of the undecidability requirement, suggested by the Dummett quote above:

$$(Und) \quad (\exists\varphi)(\neg\diamond P\varphi \wedge \neg\diamond P\neg\varphi).$$

In this context a mathematical indicative is *undecidable* just in case it is independent of all possible mathematical knowledge. The undecidability requirement tells us that there exists a sentence that cannot in principle be proved or disproved.

The knowability principle, which says that all truths are knowable, may be expressed as follows:

$$(KP) \quad (\forall\varphi)(\varphi \rightarrow \diamond P\varphi).$$

3. DUMMETT'S ARGUMENT FOR LOGICAL REVISION

In light of the above considerations, Dummett's argument for revision may be reconstructed as follows.⁶ Let us suppose that some indicative of the given class is undecidable. By accepting the law of excluded middle, one accepts the truth or falsity of every sentence, so the undecidable sentence is either true or false. First, suppose it is false. Then it follows from the knowability principle that we could prove it false. But, we cannot prove it false, since it is undecidable. Second, suppose the undecidable sentence is true. Then it follows from the knowability principle that we could recognize it as true. Again, this is in contradiction with its undecidability! But then we have absurdity resting on excluded middle, the knowability principle, and the undecidability thesis. Something must go. Recall that for Dummett, rejecting the undecidability thesis renders the debate meaningless. So the anti-realist must reject the generalized form of the law of excluded middle.⁷

$$\frac{\overline{LEM}^{(1)} \quad KP \quad Und}{\vdots} \frac{\perp}{\neg LEM}^{(1)}$$

Despite appearances, this logical strategy ends in disaster for the revisionist. As it turns out, an *intuitionistically* acceptable reductio exists resting merely upon *KP* and *Und*.⁸ Importantly, the contradiction resting on *KP* and *Und* is intuitionistically acceptable. No exclusively classical principles are employed.⁹

Notice that it does not help the anti-realist to weaken the statement of undecidability to $\nabla(\exists\varphi)(\neg\Diamond P\varphi \wedge \neg\Diamond P\neg\varphi)$, where ∇ is read as the epistemic operator 'for all we know.' Call this alternative reading *potential undecidability*, or ∇Und . We now know, via the above proof, that it intuitionistically follows from *KP* that there does not exist an undecidable sentence. And we *know* that there is no such sentence, if we know that *KP* is true. This is simply an instance of the epistemic principle that says that if we currently know φ and we currently know that ψ is a consequence of φ , then we

currently know ψ . Dummett of course will suggest that we know KP from Wittgensteinian considerations about the publicity of meaning. Granting this, it follows that we now know that there does not exist an undecidable sentence. But then it is not the case that, for all we know, there is such an undecidable sentence. So, weakening the undecidability requirement to ' $\nabla(\exists\varphi)(\neg\Diamond P\varphi \wedge \neg\Diamond P\neg\varphi)$ ' does not rescue the revisionist.

Alternatively, the intended interpretation of the potential undecidability may be this: $(\exists\varphi) \nabla (\neg\Diamond P\varphi \wedge \neg\Diamond P\neg\varphi)$. But, a similar logical objection is forthcoming. I leave that reductio to the reader.

Since the anti-realist must deny even a relatively weak formulation of the undecidability thesis, the meaningfulness of the debate is threatened. That is to say that Dummett does not satisfy his own necessary condition on a meaningful semantic debate. But there are alternative formulations of the debate. I want to turn now to Crispin Wright's formulation of the revisionist's argument.¹⁰

4. WRIGHT'S ARGUMENT FOR LOGICAL REVISION

In Chapter two of *Truth and Objectivity*¹¹ Wright begins his characterization of the argument for revision with a valid proof of a decidability thesis resting on excluded middle and the knowability principle. I represent these assumptions using schematic letters, as Wright does. Wright calls the knowability principle 'the epistemic constraint on truth.' The constraint is this:

$$(EC) \quad \varphi \rightarrow \Diamond P\varphi$$

which says ' φ , only if it is possible in principle for us to prove that φ .' The decidability thesis may be represented in the following way:

$$(Dec) \quad \Diamond P\varphi \vee \Diamond P\neg\varphi$$

which says either φ or its negation is provable. Here then is the overall structure of Wright's proof:

$$\begin{array}{c} \text{Instance of LEM:} \\ \underbrace{\Diamond P\varphi \vee \neg\Diamond P\varphi} \quad \underbrace{(EC) \quad \varphi \rightarrow \Diamond P\varphi} \\ \vdots \\ (Dec) \quad \Diamond P\varphi \vee \Diamond P\neg\varphi \end{array}$$

We may grant, on behalf of the anti-realist, that we currently *know* that the epistemic constraint is true. Again, my interest is more with the consequences than with the motivations for anti-realism. If it is true that we currently know that *the law of excluded middle* obtains, then (in acknowledging the above proof) it follows that we currently know that every mathematical indicative or its negation is in principle provable by us. That is, we know that the above decidability thesis obtains.¹²

So what does this result have to do with logical revision? Well, Wright tells us that the above conclusion – viz., that *Dec* is currently known – is

in contradiction with the a priori unwarrantability of the claim that the scales of in principle available evidence must tilt, sooner or later.¹³

That is, having knowledge of the truth of the decidability thesis is inconsistent with some other debate-neutral commitment (which Wright expresses as the ‘a priori unwarrantability’ of the decidability thesis). The task then is to get clear on what exactly this debate-neutral commitment comes to. What is this form of epistemic modesty that is endorsed by both the realist and the anti-realist but is inconsistent with decidability? The text underdetermines the answer to this question.

I think Wright is saying that the decidability thesis does not really have the epistemic status that the above result attributes to it. This of course would cast doubt on the epistemic status of the law of excluded middle for the anti-realist. So, the ultimate strategy is not that the decidability thesis is provably false, and so the law of excluded middle is false. Rather, the strategy is to say that since *for all we really know the decidability of the discourse could be false*, the exclusively classical principle is not currently known to be true.

The intuition behind Wright’s debate-neutral commitment, at first glance, appear to be that *undecidability* is epistemically possible. This commitment is very intuitive. Most of us endorse the humble admission that as far as we can immediately determine ‘the scales of in principle evidence [may never] tilt one way or the other, between each [mathematical] statement and its negation.’ In other words, upon perusal of our current set of things known, we find nothing that explicitly contradicts the negation of the decidability thesis. But this very fact contradicts the above result, which says that we do cur-

rently know that decidability obtains. The reasoning is, simply, that it is absurd to think that some claim is known (by us right now), if as far as we can at present tell the negation of the claim is consistent with what is known (by us right now).

So we have a contradiction resting on three assumptions: (i) it is known that the law of excluded middle obtains, (ii) it is known that the epistemic constraint obtains, and (iii) for all we know, decidability is false:

$$\begin{array}{c} \underbrace{K(LEM) \quad K(EC)} \\ \vdots \\ \hline \frac{K(Dec) \quad \nabla \neg(Dec)}{\perp} \end{array}$$

where $K(x)$ means ‘ x is currently known by us’ and $\nabla(x)$ says ‘as far as we can at present tell, x is consistent with what is at present known’—i.e., ‘ x does not explicitly contradict any y such that Ky ’ (whence, ‘ $\nabla \neg Dec$ ’ entails ‘ $\neg K(Dec)$ and $\neg K \neg \neg Dec$ ’).

Assumption (iii) is supposed to be debate-neutral. So, it follows for the anti-realist that the law of excluded middle is not really a known law of logic.

My objection to this characterization of the argument for revision, which I am here attributing to Wright, is that the anti-realist cannot endorse the epistemic possibility that decidability is false. He cannot, because his epistemic constraint on truth is inconsistent with that possibility. More importantly, *EC* is *intuitionistically* inconsistent with the negation of the decidability thesis, $\neg(\Diamond P\varphi \vee \Diamond P\neg\varphi)$.¹⁴ Granting, on behalf of the anti-realist, that *EC* is currently known, it then follows that it is not epistemically possible that $\neg(\Diamond P\varphi \vee \Diamond P\neg\varphi)$ – i.e., we cannot possibly know that Wrightian decidability is false. And so, we have failed to locate the debate-neutral commitment that is needed to ground the debate. A clearer account of revisionism remains wanting.

5. LOGICAL REVISION REVISITED

My present goal is to offer such an account – to show that the logical constraints on a meaningful revisionism debate can be met. It will suffice to provide formal characterizations of an exclusively classical principle of logic, the anti-realist's thesis, and some degree of epistemic modesty, such that

- (i) jointly these three commitments entail absurdity through reasoning that both parties accept,
- (ii) no proper subset of these three commitments entails a contradiction through reasoning that both parties accept, and
- (iii) the epistemic modesty or 'undecidability' in question is of a form that both the classicist and the anti-realist could and do readily adopt.

Conditions (i) through (iii) are ineliminable adequacy criteria on any characterization of a coherent and interesting argument for revision. My efforts above aimed to show that it is all but clear whether either Dummett or Wright has met condition (ii) and condition (iii).

Nevertheless, I believe these three criteria can be satisfied. One needs to begin with the following jointly inconsistent commitments to characterize the debate: with respect to the given class of mathematical indicatives φ ,

- (1) we know that for all φ , φ or $\neg\varphi$,
- (2) we know that if φ is true, φ is provable, and
- (3) it is not currently known that every indicative is such that it or its negation can be proven by us in principle.

Commitment (3) says that $(\forall\varphi)(\Diamond P\varphi \vee \Diamond P\neg\varphi)$ has not yet been explicitly verified, and is meant to capture the anti-realist's very weak obligation to demur at the decidability of the discourse. Commitment (3) may be what Wright had in mind when he declared the 'unwarrantability of the claim that the scales of in principle available evidence must tilt, sooner or later.' Perhaps this formulation of the commitment serves as a logical clarification of Wright's logically ambiguous declaration.

Formally, I express the three commitments as shown:

Exclusively Classical Thesis	$K(\forall\varphi)(\varphi \vee \neg\varphi)$
Dummettian Anti-Realism	$K(\forall\varphi)(\varphi \rightarrow \Diamond P\varphi)$
Epistemic Modesty	$\neg K(\forall\varphi)(\Diamond P\varphi \vee \Diamond P\neg\varphi)$.

It can easily be proven that the exclusively classical thesis and anti-realism jointly entail a new expression of decidability, $K(\forall\varphi)(\Diamond P\varphi \vee \Diamond P\neg\varphi)$. But this formulation of decidability – $K(\forall\varphi)(\Diamond P\varphi \vee \Diamond P\neg\varphi)$ – and of modesty – $\neg K(\forall\varphi)(\Diamond P\varphi \vee \Diamond P\neg\varphi)$ – contradict each other. The anti-realist, at this point, denies the exclusively classical thesis.

$K\forall\varphi(\varphi \vee \neg\varphi)$	Anti-Realism: $K\forall\varphi(\varphi \rightarrow \Diamond P\varphi)$
\vdots	Epistemic Modesty: $\neg K\forall\varphi(\Diamond P\varphi \vee \Diamond P\neg\varphi)$
$K\forall\varphi(\Diamond P\varphi \vee \Diamond P\neg\varphi)$	\perp
$\neg K\forall\varphi(\varphi \vee \neg\varphi)$ Revisionism	$\neg K\forall\varphi(\varphi \vee \neg\varphi)$

Importantly, both the anti-realist and the classicist would endorse this new form of modesty. They would endorse it, just because it is so modest. The principle simply amounts to the humble recognition that we have not yet confirmed that each understood mathematical claim or its negation is humanly provable in the long run.

How is it that this new formulation of epistemic modesty succeeds where our original formulation of Wrightian modesty fails? Notice that it is the extra expressive power of quantified propositional logic that blocks the anti-realistically unwelcome contradiction between the modesty principle and anti-realism. The Wrightian understanding failed to bring these further resources to bear, and as a result leaves itself vulnerable to the aforementioned inconsistency. We now see that the proper interpretation of the revisionism debate must utilize, not only standard modal, but also quantified propositional logic.¹⁵

It appears then that these two principles (anti-realism and modesty) are intuitionistically consistent with one another. Though there is a classical proof of absurdity resting on the new formulation of modesty and anti-realism, the result is not intuitionistically

acceptable.¹⁶ The reason is that the classical proof of absurdity requires that we infer the existence of an undecidable sentence from the claim that not all sentences are decidable. And simply, the negation of a universal $\neg(\forall\varphi)\Psi(\varphi)$ does not intuitionistically entail an existentially quantified negation $(\exists\varphi)\neg\Psi(\varphi)$.¹⁷ So the reductio analogous to the one wielded against Wright is blocked in this new context. Thus, we may finally have an intuitive expression of epistemic modesty (or undecidability) that the anti-realist can live with.

6. SOME PROBLEMS, OLD AND NEW

6.1. *The Fitch Paradox of Knowability*

The *Fitch Paradox of Knowability* has threatened the knowability principle for decades. What the paradox suggests is that the knowability principle is inconsistent with a very weak commitment to our own non-omniscience – viz., that there is a sentence φ such that φ but it has not yet been proven that φ .¹⁸ Nevertheless, there are several solutions to this paradox in the literature, none of which are incompatible with the results herein.¹⁹

6.2. *The Problem of Shared Content*

A deeper problem remains for any characterization of the revisionism debate. It is the *Problem of Shared Content*. In the context of this chapter, the problem rears its head in the following way. The theses of anti-realism and epistemic modesty classically, but not intuitionistically, entail absurdity. To the extent that these two principles incur different logical commitments for the classical realist than they do for the intuitionistic anti-realist, the realist and anti-realist mean different things by ‘anti-realism’ and ‘epistemic modesty.’ The participants in the debate are talking past one another.²⁰

Recall that the debate is to be constrained by the aforementioned adequacy criteria (last section), if it is to make sense at all. A disagreement is not genuine if it does not occur against a backdrop of agreement. In particular, the Dummettian participants (or any opposing interlocutors who wish to disagree) must agree, for the sake of meaningful disagreement, to share standards of rationality.

Our adequacy conditions (i) and (ii) remind us of this important fact. And any differences in interpretation (of the logical constants) that do not violate these conditions are not differences that make a relevant difference, since they do not affect the outcome of the debate. Importantly, the problem of shared content, as formulated above, focuses on reasoning that is not acceptable to *both* parties. So, it focuses on reasoning that is not acceptable to *either* party in the context of the Dummettian disagreement. A classicist sincerely and meaningfully disagreeing with the anti-realist about anti-realism cannot invoke logical norms that the anti-realist finds unfavorable. But, to the extent that the participants agree to have their dispute against a backdrop of shared logical commitments, they can ward off criticisms about shared content that are grounded in differences of logical commitment. Any interesting solution to the problem of shared content (i.e., any solution that preserves a point of contention while retaining interest for both the classical realist and the revisionary anti-realist) will have to begin there.

It may be suggested that my solution begs the question in favor of the anti-realist, since I motivate intuitionistic restrictions on a coherent revisionism debate. But these restrictions do not necessarily beg the question. They would, only if in accepting them we would, in affect, decide the debate in favor of the revisionist, and only if there were no *independent* reasons for accepting them.

But there appear to be good independent reasons for motivating the aforementioned restriction. On my view, for instance, intuitionistic restrictions on the revisionary debate are necessary if the disputants are to communicate at all. To the extent that both parties debate only using patterns of inference that both parties accept, their preferred semantics will not mark a difference (say, in interpretation of the logical constants) that makes a relevant difference. It will not make a relevant semantic difference, since in this context their inferential behavior will not reflect one semantics at the cost of another. And most importantly, such inferential restrictions will not load the dice in favor of the revisionist, since in this context the realist will still have the option of embracing the law of excluded middle and rejecting anti-realism in the face of the basic (semantically neutral) aporia.²¹

6.3. *The Problem of Gödelian Optimism*

Another problem remains for the above account of the Dummettian disagreement. There is logical space for the epistemically optimistic classicist to advocate *KP* while embracing the full range of classical principles. This position goes at least as far back as Descartes, in his *Discourse on Method*, where he says,

there is nothing so far removed from us as to be beyond our reach, or so hidden that we cannot discover it²²

Or more recently, as Stewart Shapiro has emphasized,²³ the position is adopted by Hilbert:

However unapproachable these problems may seem to us and however helpless we stand before them, we have, nevertheless, the firm conviction that the solution must follow by . . . logical processes . . . This conviction of the solvability of every mathematical problem is a powerful incentive to the worker. We hear the perpetual call: There is a problem. Seek its solution. You can find it . . . for in mathematics there is no *ignorabimus*.²⁴

And the position is adopted by Gödel:

[T]hose parts of mathematics which have been systematically and completely developed . . . show an amazing degree of beauty and perfection. In those fields, by entirely unsuspected laws and procedures . . . means are provided . . . for solving all relevant problems This fact seems to justify what may be called 'rationalistic optimism'.²⁵

Gödelian optimism, as Shapiro calls it, is marked by the joint acceptance of the knowability principle and classical logic. My worry is that the occupation of this position by eminent philosophers is itself evidence that I have gone afoul in my characterization of the Dummettian terrain. After all, if my modesty principle is as modest as I claim, then anybody would accept it. But if modesty is inconsistent with the joint acceptance of the knowability principle and classical logic (i.e., with Gödelian optimism), then the Gödelian optimist would not accept it.

So it seems that either my modesty requirement is not readily embraced by all, or it is so embraced and Gödelian optimism needs to be re-evaluated for coherence. If modesty is not readily embraced, then it is not clear that both the realist and the anti-realist endorse it. But then, the meaningfulness of the semantic debate is threatened;

adequacy condition (iii) on a meaningful semantic debate will not have been satisfied. On the other horn of the dilemma, if the modesty in question is readily adopted, then it is readily adopted even by the Gödelian optimist. But the argument for revision provided above apparently shows that a commitment to Gödelian optimism plus modesty is logically inconsistent. But then positions occupied by Descartes, Hilbert and Gödel are *a priori* untenable.

The solution is to accept the second horn of the dilemma and make one clarification. Modesty is perfectly modest, and so is probably embraced even by the optimist. The optimist will admit that we do not currently *know* that all meaningful mathematical claims are decidable by us one way or the other. But this is not necessarily inconsistent with his position. In the face of the above argument for revision the Gödelian optimist can deny that the knowability principle has the epistemic status that the anti-realist supposes it to have. The optimist says that the knowability principle is not known but, rather, is avowed for pragmatic reasons, or just plain gets us up in the morning to work on our favorite mathematical problems. The suggestion is that *KP* serves, for the optimist, as an expression of optimism rather than as a statement of fact. Or at the very least, the optimist does not claim for *KP* the special epistemic status that the anti-realist claims for it. The optimistic classicist is not committed to the strong modalized form of the knowability principle, $\Box\forall\varphi(\varphi \rightarrow \Diamond P\varphi)$, required in the argument for revision (where box is read, “It is known *a priori* that”, or “it is known that”). Classical optimism, then, does not contradict epistemic modesty.

So my claim is that epistemic modesty is modest enough to warrant its endorsement by the relevant parties, and it is strong enough to play the logical role that the anti-realist intends for it. On this understanding, the above account of the revisionism debate evades the earlier criticisms that I used against plausible readings of Dummett and Wright on this issue. A commitment to Dummettian anti-realism, then, apparently does oblige us to reform our logic. This becomes clear once the logic of the debate is made explicit – that is, once we formulate the Dummettian disagreement within quantified propositional logic, while at the same time satisfying the adequacy criteria on a meaningful and interesting debate. So, if we have succeeded in doing this, then we now know that it follows

logically from semantic anti-realism that some logical principles – standardly taken to be known – are in fact unknown. Or conversely, we find that an unyielding commitment to the full range of classical principles has the metaphysical cost of semantic realism.

NOTES

¹ See, for instance, Crispin Wright, *Truth and Objectivity* (Harvard, 1992), pp. 4–5, or Neil Tennant, *The Taming of The True* (Oxford University Press, 1997), p. 204.

² I have in mind, for instance, one incarnation of Putnam’s “internal realist” who claims that truth is that which would be justified under epistemically ideal circumstances, and the Peircean pragmatist who believes that truth is “the opinion which is fated to be ultimately agreed upon by all who investigate.” See Putnam’s *Reason, Truth and History* (Cambridge, 1981), and Peirce’s *Reasoning and the Logic of Things* (Cambridge, 1992).

³ Innovative attempts to extend the discussion to empirical discourse are found in Crispin Wright, op. cit. (1992), Chapter 2, and in Neil Tennant, op. cit. (1997), Chapter 12.

⁴ “Realism” in Dummett’s *Truth and Other Enigmas* (Harvard, 1978), p. 155. (my emphasis); first presented to the Philosophical Society in Oxford in 1963.

⁵ I leave the notions of proof and consequence unarticulated for the time being. One might run the revisionist’s argument with either classical or intuitionistic notions. For more specific varieties of the knowability principle see Tennant, *The Taming of the True* (Oxford University Press, 1997), Chapter 7.

⁶ An expression of the following argument may be found in “The Philosophical Basis of Intuitionistic Logic,” essay 14 in *Truth and Other Enigmas* (Harvard University Press, 1978); first presented to the Logic Colloquium in Bristol, 1973.

⁷ The underbrace and ellipsis indicate where I have compressed a proof for economy.

⁸ Tennant discusses this result in op. cit. (1997), p. 181. Dummett also acknowledges the inconsistency in his later writings. See p. 17 of *Elements of Intuitionism* (Oxford, 1977). I thank an anonymous referee for calling this latter reference to my attention.

⁹ Here is the unabbreviated result:

Assn.#	Line	Justification on which line rests
1	(1) $(\forall\varphi)(\varphi \rightarrow \Diamond P\varphi)$	Assn. <i>KP</i>
2	(2) $(\exists\varphi)(\neg\Diamond P\varphi \wedge \neg\Diamond P\neg\varphi)$	Assn. <i>Und</i>
3	(3) $\neg\Diamond P\psi \wedge \neg\Diamond P\neg\psi$	Assn. for $\exists E$ (parametric instance of 2)
3	(4) $\neg\Diamond P\psi$	$\wedge E$ 3
3	(5) $\neg\Diamond P\neg\psi$	$\wedge E$ 3

1	(6)	$\psi \rightarrow \diamond P\psi$	$\forall E$ 1
1, 3	(7)	$\neg\psi$	MT 4, 6
1	(8)	$\neg\psi \rightarrow \diamond P\neg\psi$	$\forall E$ 1
1, 3	(9)	$\neg\neg\psi$	MT 5, 8
1, 3	(10)	CONTRADICTION	$\neg E$ 7, 9
1, 2	(11)	CONTRADICTION	$\exists E$ 2, 3–10

¹⁰ For the sake of simplicity and to bypass a number of red herrings, I take a few liberties in reconstructing the argument from Chapter 2 of Wright's *Truth and Objectivity*. I diverge from the text in that I focus only upon the debate over the class of meaningful, unambiguous, non-vague, mathematical indicatives. Also, for those of us harboring a Quinean discomfort with talk of the *a priori*, I make no use of the concept of *a priori* knowledge. And for those of us suspicious of an anti-realist who readily avails himself of Tarski's T-schema and the "commutation of truth and negation", I make no important use of the truth-predicate. But my criticisms do not depend on taking these liberties.

¹¹ Wright, *Truth and Objectivity* (Harvard, 1992) p. 43.

¹² This is another instance of the non-contentious epistemic principle that allows us to claim knowledge of the known consequences of known premises.

¹³ Wright, *op. cit.* (1992), p. 42.

¹⁴ Here is the result:

Assn.#	Line		Justification on which line rests
1	(1)	$\neg(\diamond P\varphi \vee \diamond P\neg\varphi)$	Main Assn. for reductio ($\neg Dec$)
2	(2)	$\varphi \rightarrow \diamond P\varphi$	Instance of <i>EC</i>
3	(3)	$\neg\varphi \rightarrow \diamond P\neg\varphi$	Instance of <i>EC</i>
4	(4)	φ	Subordinate Assn. for reductio
2, 4	(5)	$\diamond P\varphi$	MP 2, 4
2, 4	(6)	$\diamond P\varphi \vee \diamond P\neg\varphi$	$\vee I$ 5
1, 2, 4	(7)	CONTRADICTION	$\neg E$ 1, 6
1, 2	(8)	$\neg\varphi$	Subordinate $\neg I$ 4–7
1, 2, 3	(9)	$\diamond P\neg\varphi$	MP 3, 8
1, 2, 3	(10)	$\diamond P\varphi \vee \diamond P\neg\varphi$	$\vee I$ 9
1, 2, 3	(11)	CONTRADICTION	$\neg E$ 1, 10

¹⁵ Risto Hilpinen has objected (as commentator to an earlier presentation of this material to the Eastern Division meeting of the APA, 1997) that even these resources are not sufficient to characterize the debate. His view is that a satisfactory resolution to the Fitch Paradox of Knowability "requires conceptual resources beyond the standard modal logic and quantification theory." Nevertheless, there are several solutions to the paradox, not all of which require the employment of further conceptual resources. See, for example, Tennant's solution in *op. cit.* (1997), Chapter 8.6.

¹⁶ The said classical reasoning goes like this. Modesty tells us that we do not currently know that decidability obtains. So, for all we know decidability is false;

it is epistemically possible that $\neg(\forall\varphi)(\Diamond P\varphi \vee \Diamond P\neg\varphi)$. But then it follows classically that it is possible that $(\exists\varphi)\neg(\Diamond P\varphi \vee \Diamond P\neg\varphi)$. And, as we saw earlier, the existential formulation of undecidability is inconsistent with anti-realism.

¹⁷ The point is that a reduction of some universally quantified statement $(\forall\varphi)\Psi(\varphi)$ to absurdity does not entail, for the intuitionist, that we have constructed some instance and effectively shown that it fails to satisfy the relevant open sentence $\Psi(\)$. Blocking this move from $\neg(\forall\varphi)\Psi(\varphi)$ to $(\exists\varphi)\neg\Psi(\varphi)$ allows the anti-realist to assert the modesty principle without fear of the unwelcome contradiction.

¹⁸ See Frederic Fitch, "A Logical Analysis of Some Value Concepts," *The Journal of Symbolic Logic* 28 (1963), p. 138.

¹⁹ Some such treatments of the paradox are found in Timothy Williamson, "Intuitionism Disproved?", *Analysis* 42 (1982), pp. 203–207, in Wright *Realism, Meaning and Truth* (Blackwell, 1986), pp. 426–427, in Tennant, op. cit. (1997), esp. Chapter 8.6, and in Sten Lindström, "Situations, Truth and Knowability: a Situation-Theoretic Analysis of a Paradox by Fitch," in E. Ejerhed and S. Lindström (eds.), *Logic, Action and Cognition: Essays in Philosophical Logic* (Kluwer Academic Publishers, 1997), pp. 183–210.

²⁰ This formulation of the problem was put to me by Crispin Wright during 1997 Graduate Symposium, held in his honor at The Ohio State University.

²¹ Pierluigi Miraglia has convinced me that the problem of shared content has other manifestations that perhaps are not soluble by the above means. We attempt to resolve this semantic conundrum more systematically in our "Semantic Anti-Realism and the Problem of Shared Content" (unpublished typescript).

²² From Descartes' "Discourse on Method" in *The Rationalists: Descartes, Spinoza, Leibniz* (Doubleday, 1974), p. 52.

²³ Shapiro, "Anti-Realism and Modality" in *Philosophy of Mathematics: Proceedings of the 15th International Wittgenstein-Symposium*, ed. Johannes Czermak (Verlag Holder-Pichler-Tempsky, Vienna, 1993), esp. pp. 284–285.

²⁴ Hilbert's "Mathematical Problems" lecture, 1900.

²⁵ See H. Wang's *From Mathematics to Philosophy* (Routledge and Kegan Paul, London, 1974), pp. 324–325.

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