Basics of Model Predictive Control

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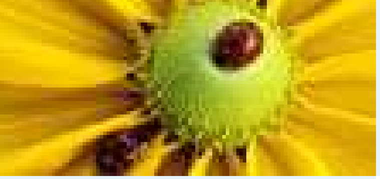


Outline

- Introduction
- Problem Formulation
- Solution to problem
- Examples
- Direction for future work
- Conclusion
- References

Introduction

What is Model Predictive Control?

MPC is a form of control in which the current control action is obtained by solving *on-line*, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state; the optimization yields an optimal control sequence and the first control in this sequence is applied to the plant.

Introduction cont'd

Early Industrial MPC Application

1. Model Predictive Heuristic Control by Richalet et al. 1976 (Adersa)...
2. Dynamic Matrix Control (DMC) by Cutler and Ramaker 1979 (Shell Oil)...
3. Quadratic-Program Dynamic Matrix Control (QDMC) by Cutler et al. 1983 (Shell Oil)...

Introduction cont'd

Academic Research

- Few early theoretical investigations: Klieinmann 1970, Thomas 1975, Chen and Shaw 1982 etc.
- Predictive control theory: Keerthi and Gilbert 1988, Mayne and Michalska 1990 etc.

Introduction cont'd

Industrial Technology

Company	Product name	Description
Aspen Tech	DMC	Dynamic Matrix Control
Adersa	IDCOM	Identification and Command
	HIECON	Hierarchical Constraint Control
	PFC	Predictive Functional Control
Honeywell Profimatics	RMPCT	Robust Model Predictive Control Technology
	PCT	Predictive Control Technology
Setpoint Inc.	SMCA	Setpoint Multivariable Control Architecture
	IDCOM-M	Multivariable
Treiber Controls	OPC	Optimum Predictive Control
Shell Global	SMOC-II	Shell Multivariable Optimizing Control
ABB	3dMPC	
Pavillion Technologies Inc.	PP	Process Perfecter
Simulation Sciences	Connoisseur	Control and Identification Package

Introduction cont'd

Variations of MPC

- Robust MPC - guaranteed feasibility and stability
- Feedback MPC - mitigate shrinkage of feasible region
- Pre-computed MPC - Piecewise-linear solution stored in database or Solve off-line using parametric (linear or quadratic) programming
- Decentralised MPC as used in autonomous air vehicle - Speed up computation.

Introduction cont'd

Basic structure of MPC

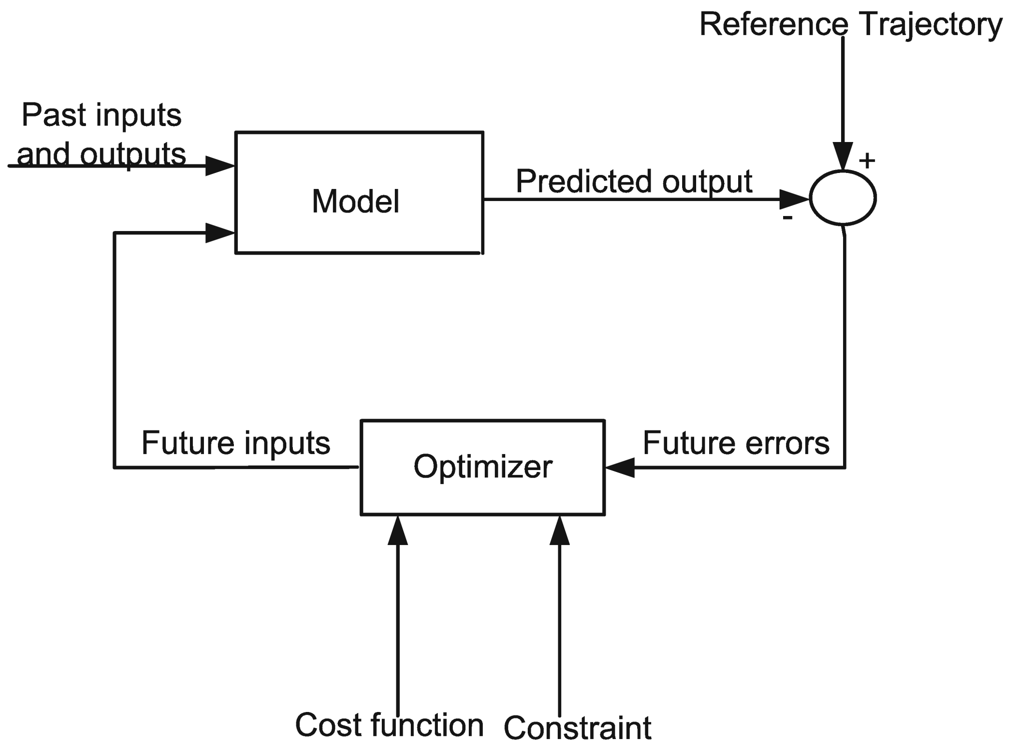


Figure 1: Basic Structure of MPC

Introduction cont'd

Components of MPC

- Prediction model
- Objective function
- Obtaining the control law

Introduction cont'd

What makes MPC successful in industry

1. It handles multivariable control problems naturally
2. It can take account of actuator limitations
3. It allows operation closer to constraints, hence increased profit
4. It has plenty of time for on-line computations
5. It can handle non-minimal phase and unstable processes
6. It is an easy to tune method and
7. It handles structural changes.

Introduction cont'd

Characteristics of MPC

- Moving horizon implementation
- Performance oriented time domain formulation
- Incorporation of constraints and
- Explicit system model used to predict future plant dynamics.

Introduction cont'd

Types of MPC

- Linear MPC

1. Uses linear model:

$$\dot{x} = Ax + Bu$$

2. Quadratic cost function:

$$F = x^T Q x + u^T R u$$

3. Linear constraints:

$$Hx + Gu < 0$$

4. Quadratic program

- Nonlinear MPC

1. Uses nonlinear model:

$$\dot{x} = f(x, u)$$

2. Cost function can be nonquadratic: $F(x,u)$
3. Nonlinear constraints: $h(x,u) < 0$
4. Nonlinear program

Introduction cont'd

Applications of MPC

- Distillation column
- Hydrocracker
- Pulp and paper plant
- Servo mechanism
- Robot arm ...

Problem Formulation

- Model of Plant in State Space

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= C_y x(k) \\
z(k) &= C_z x(k)
\end{aligned} \tag{1}
$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^l, y \in \mathbb{R}^{m_y}, z \in \mathbb{R}^{m_z}$

- A Basic Formulation of Cost function

$$
V(k) = \sum_{i=H_w}^{H_p} \|\hat{z}(k+i\backslash k) - r(k+i\backslash k)\|^2_{Q(i)} + \sum_{i=0}^{H_u-1} \|\Delta\hat{u}(k+i\backslash k)\|^2_{R(i)} \tag{2}
$$

Problem Formulation cont'd

$$
\begin{bmatrix} \hat{x}(k+1\backslash k) \\ \vdots \\ \hat{x}(k+H_u\backslash k) \\ \hat{x}(k+H_u+1\backslash k) \\ \vdots \\ \hat{x}(k+H_p\backslash k) \end{bmatrix} = \underbrace{\begin{bmatrix} A \\ \vdots \\ A^{H_u} \\ A^{H_u+1} \\ \vdots \\ A^{H_p} \end{bmatrix} x(k) + \begin{bmatrix} B \\ \vdots \\ \sum_{i=0}^{H_u-1} A^i B \\ \sum_{i=0}^{H_u} A^i B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i B \end{bmatrix} u(k-1)}_{\text{past}}
$$

$$
+ \underbrace{\begin{bmatrix} B & \cdots & 0 \\ AB+B & \cdots & 0 \\ \vdots & \ddots & \cdots \\ \sum_{i=0}^{H_u-1} A^i B & \cdots & B \\ \sum_{i=0}^{H_u} A^i B & \cdots & AB+B \\ \cdots & \cdots & \cdots \\ \sum_{i=0}^{H_p-1} A^i B & \cdots & \sum_{i=0}^{H_p-H_u} A^i B \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k\backslash k) \\ \vdots \\ \Delta\hat{u}(k+H_u-1\backslash k) \end{bmatrix}}_{\text{future}} \qquad (3)
$$

Problem Formulation cont'd

The predictions of z is

$$\begin{bmatrix} \hat{z}(k+1\backslash k) \\ \vdots \\ \hat{z}(k+H_p\backslash k) \end{bmatrix} = \begin{bmatrix} C_z & 0 & \dots & 0 \\ 0 & C_z & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_z \end{bmatrix} \begin{bmatrix} \hat{x}(k+1\backslash k) \\ \vdots \\ \hat{x}(k+H_p\backslash k) \end{bmatrix} \tag{4}$$

Solution to problem

Unconstrained problem with measured state :The cost function to be minimized is

$$V(k) = \sum_{i=H_w}^{H_p} \|\hat{z}(k+i\backslash k) - r(k+i\backslash k)\|^2_{Q(i)} + \sum_{i=0}^{H_u-1} \|\Delta\hat{u}(k+i\backslash k)\|^2_{R(i)} \quad (5)$$

$$\Rightarrow \|\mathcal{Z}(k) - \mathcal{T}(k)\|^2_{\mathcal{Q}} + \|\Delta\mathcal{U}(k)\|^2_{\mathcal{R}}$$

$$\mathcal{Z}(k) = \begin{bmatrix} \hat{z}(k+H_w\backslash k) \\ \vdots \\ \hat{z}(k+H_p\backslash k) \end{bmatrix} \mathcal{T}(k) = \begin{bmatrix} \hat{r}(k+H_w\backslash k) \\ \vdots \\ \hat{r}(k+H_p\backslash k) \end{bmatrix} \Delta\mathcal{U}(k) = \begin{bmatrix} \Delta\hat{u}(k+i\backslash k) \\ \vdots \\ \Delta\hat{u}(k+H_u-1\backslash k) \end{bmatrix}$$

Solution to problem cont'd

$$\mathcal{Z}(k) = \Psi x(k) + \Upsilon u(k-1) + \Theta\Delta\mathcal{U}(k)$$
$$\xi(k) = \mathcal{T}(k) - \Psi x(k) - \Upsilon u(k-1)$$

$$\begin{aligned}
V(k) &= \|\Theta\Delta\mathcal{U}(k) - \xi(k)\|^2_{\mathcal{Q}} + \|\Delta\mathcal{U}(k)\|^2_{\mathcal{R}} \\
&= [\Delta\mathcal{U}(k)^T\Theta^T - \xi(k)^T]\mathcal{Q}[\Delta\mathcal{U}(k)\Theta - \xi(k)] + \Delta\mathcal{U}(k)^T\mathcal{R}\Delta\mathcal{U}(k) \\
&= \xi(k)^T\mathcal{Q}\xi(k) - 2\Delta\mathcal{U}(k)^T\Theta^T\mathcal{Q}\xi(k) + \Delta\mathcal{U}(k)^T[\Theta^T\mathcal{Q}\Theta + \mathcal{R}]\Delta\mathcal{U}(k)
\end{aligned}$$

$$\Delta\mathcal{U}(k)_{opt} = 0.5\mathcal{H}^{-1}\mathcal{G} \qquad (6)$$

Solution to problem cont'd

Constrained problem with QP formulation : The system is subject to constraints of the form:

$$E \begin{bmatrix} \Delta\mathcal{U}(k) \\ 1 \end{bmatrix} \leq 0 \tag{7}$$

$$F \begin{bmatrix} \mathcal{U}(k) \\ 1 \end{bmatrix} \leq 0 \tag{8}$$

$$G \begin{bmatrix} \mathcal{Z}(k) \\ 1 \end{bmatrix} \leq 0 \tag{9}$$

The constraints are converted to a single linear inequality of the form:

$$\begin{bmatrix} \mathcal{F} \\ \Gamma\Theta \\ W \end{bmatrix} \Delta\mathcal{U}(k) \leq 0 \begin{bmatrix} -\mathcal{F}_1 u(k-1) - f \\ -\Gamma[\Psi x(k) + \Upsilon u(k-1)] - g \\ -w \end{bmatrix} \tag{10}$$

Solution to problem cont'd

The constrained optimization problem to be solved is

$$\min_{\Delta\mathcal{U}(k)} \Delta\mathcal{U}(k)^T \mathcal{H} \Delta\mathcal{U}(k) - \mathcal{G}^T \Delta\mathcal{U}(k) \tag{11}$$

subject to the inequality

$$\begin{bmatrix} \mathcal{F} \\ \Gamma\Theta \\ W \end{bmatrix} \Delta\mathcal{U}(k) \leq 0 \begin{bmatrix} -\mathcal{F}_1 u(k-1) - f \\ -\Gamma[\Psi x(k) + \Upsilon u(k-1)] - g \\ -w \end{bmatrix} \tag{12}$$

Example

A randomly generated system with state space model parameters:

$$A = \begin{bmatrix} -0.1267 & -0.3357 & 0.0958 & -0.1723 \\ -0.4877 & 0.3487 & 0.0511 & 0.6393 \\ 0.0367 & 0.3482 & -0.0547 & -0.0399 \\ 0.0842 & -0.0110 & 0.2125 & 0.0334 \end{bmatrix}, \; B = \begin{bmatrix} 0.9501 \\ 0.2311 \\ 0.6068 \\ 0.4860 \end{bmatrix},$$

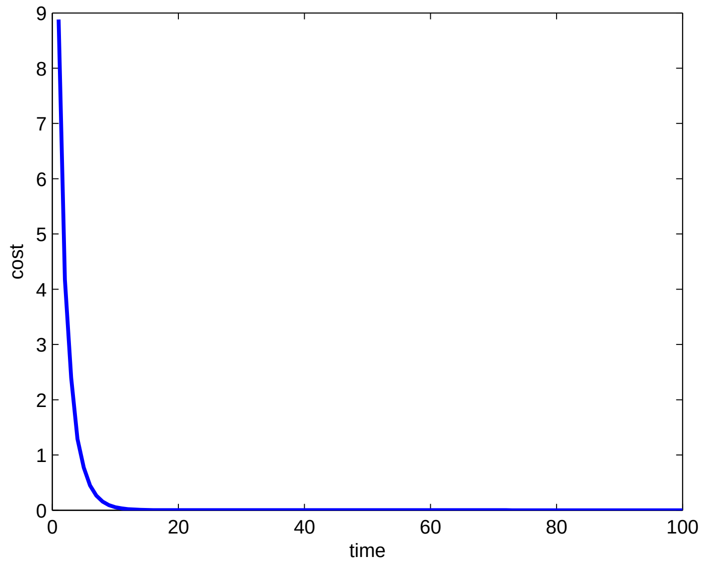
$$Cy = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C_z = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$H_p = 5; H_u = 3; H_w = 1; Q > 0; R > 0$ and using the following constraints

$$-10 \leq u(k) \leq 10$$
$$-2 \leq \Delta u(k) \leq 2$$
$$-3 \leq z(k) \leq 5$$

with initial state $x_0 = [0.5 \;\; -0.5 \;\; 0.2 \;\; -0.2]^T$

Example cont'd



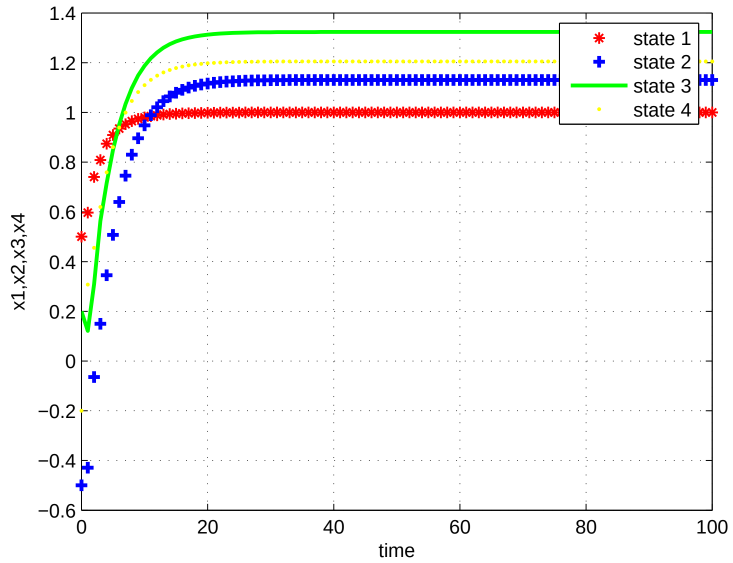


Figure 2: Cost function for unconstrained case

Figure 3: States for unconstrained case

Example cont'd

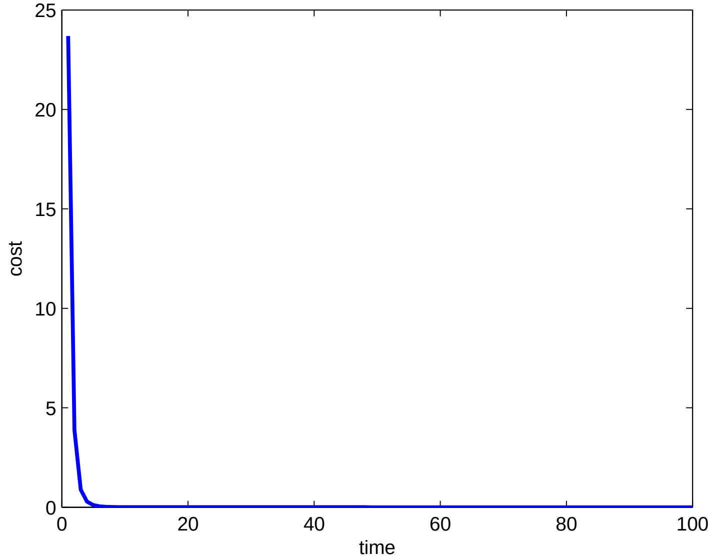


Figure 4: Cost function for constrained case

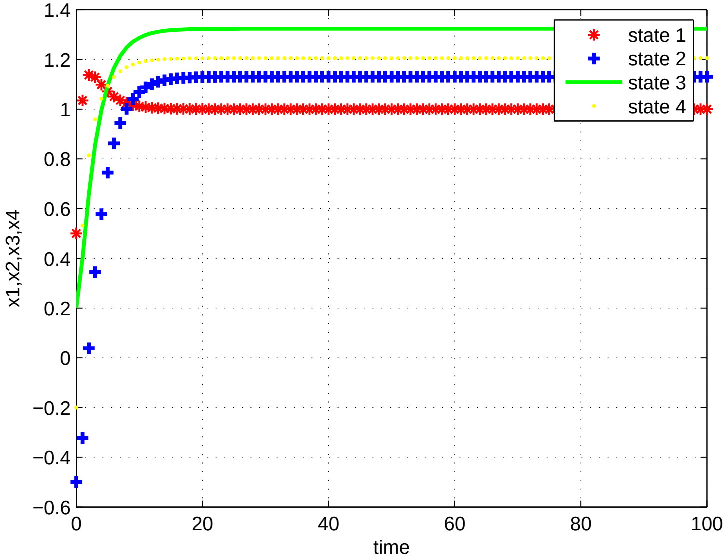


Figure 5: States for constrained case

Direction for future work

- Consider normbounded uncertain system of the form:

$$x(k+1) = Ax(k) + Bu(k) \tag{13}$$
$$y(k) = Cx(k),$$
$$A = A_0 + \Delta \text{ and } B = B_0 + \Delta \tag{14}$$

- It is not possible to minimize the cost and as such we will consider an upper bound on the cost
- Use LMIs which are of the form

$$F(x) = F_0 + \sum_{i=1}^{l} x_i F_i > 0, \tag{15}$$

to minimize this upper bound

Conclusion

- The importance of MPC has been shown
- We have mentioned some practical application of MPC
- It can handle constraints
- We have shown the components of MPC
  - Model
  - Objective function
  - Obtaining the control law
- We have outlined a future direction for research concerning Robust Model Predictive Control

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Thank You