MR1832454 (2002k:03072) 03E05 (03E35)
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Consistently there is no non trivial ccc forcing notion with the Sacks or Laver property.
Paul Erdős and his mathematics (Budapest, 1999).
Combinatorica **21** (2001), *no. 2,* 309–319.

From the introduction: "At a recent set theory conference, Boban Veličković asked the following question. Question 1.1: Is there a nontrivial forcing notion with the Sacks property which is also ccc? A 'definable' variant of this question has been answered in [S. Shelah, Israel J. Math. **88** (1994), no. 1-3, 159–174; MR1303493 (96g:03090)]: Every nontrivial Souslin forcing notion which has the Sacks property has an uncountable antichain.

"We show here Theorem 1.2. The following statement is equiconsistent with ZFC: $(*)$ Every nontrivial forcing notion which has the Sacks property has an uncountable antichain.

"Our proof follows the ideas from [op. cit.]. Independently, Veličković has also proved the consistency of $(*)$, following [op. cit.] and some of his works. In fact, he shows that the proper forcing axiom (PFA), and even the open coloring axiom, imply $(*)$.

"Our proof also shows that the following strengthening of $(*)$ is equiconsistent with ZFC: $(**)$ Every nontrivial forcing notion which has the Laver property has an uncountable antichain.

"Note that if cov(meagre) = continuum (which follows e.g. from PFA) then there is a (nonprincipal) Ramsey ultrafilter on ω. The 'Mathias' forcing notion for shooting an infinite subset of ω almost included in every member of this ultrafilter has the Laver property and is ccc, so $(**)$ does not follow from PFA.

"Therefore our result and Veličković's result are incomparable."

References

1. Saharon Shelah: How special are Cohen and random forcings i.e. Boolean algebras of the family of subsets of reals modulo meagre or null, *Israel Journal of Mathematics*, **88** (1994), 159–174. MR1303493 (96g:03090)
2. Jaime Ihoda (Haim Judah) and Saharon Shelah: Souslin forcing, *The Journal of Symbolic Logic*, **53** (1988), 1188–1207. MR0973109 (90h:03035)
3. Saharon Shelah: *Proper and improper forcing*, Perspectives in Mathematical Logic, Springer, 1998. MR1623206 (98m:03002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.