

**FISCAL POLICY AND GROWTH IN A SMALL OPEN ECONOMY
WITH ELASTIC LABOR SUPPLY***

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Endogenizing labor supply leads to fundamental changes in the equilibrium structure of the AK growth model of a small open economy. The balanced growth equilibrium is described by two tradeoff loci relating the equilibrium growth rate to the fraction of time devoted to leisure. The implications of endogenous labor supply for fiscal policy are addressed. The effects of distortionary taxes and government expenditure on the equilibrium growth-leisure (employment) tradeoff are analyzed. The responses contrast with both those of a closed economy, having an elastic labor supply, and an open economy with fixed labor supply. Optimal fiscal policy is also characterized.

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1. Introduction

The AK investment-based growth model has been the subject of extensive research during recent years. One feature of this model is that the equilibrium growth rate it generates is responsive to the choice of certain fiscal instruments, particularly the tax rate on capital; see e.g. Jones and Manuelli (1990), Barro (1990), Rebelo (1991), Jones, Manuelli, and Rossi (1993), Ireland (1994), and Turnovsky (1996a). This is in sharp contrast to the neoclassical growth model, in which the equilibrium growth rate is tied to the exogenous population growth rate, and is thus independent of all conventional fiscal instruments. Most of this literature focuses on the closed economy, though several applications to the open economy have also begun to be developed; see Turnovsky (1996b, 1997a), van der Ploeg (1996), Baldwin and Forslid (1999).

One limitation of these new growth models is that with few exceptions they treat the supply of labor as inelastic; that is, labor is either totally fixed, or alternatively, grows at some exogenously determined rate. The few papers to endogenize labor supply deal with closed economies. Benhabib and Perli (1994) and Ladrón-de-Guevara, Ortigueira, and Santos (1997) are concerned with issues of existence of equilibrium, showing how the endogeneity of labor may quite plausibly produce indeterminacy in the Lucas (1988) two-sector model of human capital accumulation. Jones and Manuelli (1995) discuss monetary policy and growth in both one sector and two sector models of capital accumulation and endogenous labor. Turnovsky (1998) analyzes fiscal policy in an economy with endogenous labor, showing how the economy he considers is always on its balanced growth path, one that can be characterized in terms of two tradeoff loci relating the equilibrium balanced growth rate to the fraction of time devoted to leisure. He emphasizes how the endogeneity of labor through the work-leisure choice is crucial in yielding a role to both consumption and wage income taxes in influencing the growth-leisure tradeoff. This is in contrast to the standard AK model where labor is supplied inelastically, in which case these two tax rates are non-distortionary and operate as lump sum taxes.¹

In this paper we introduce an elastic supply of labor into a simple AK investment-based growth model of a small open economy, focusing particularly on the role of fiscal policy in such an

economy. As in a closed economy, endogenizing labor supply leads to important changes in the equilibrium structure of the economy, and has important implications for fiscal policy. But in contrast to the closed economy, the introduction of an elastic labor supply yields a *less*, rather than a more, potent role to distortionary taxes in terms of influencing growth. Not only do the taxes on wage income and consumption continue to act as lump sum taxes, but now in addition, the capital income tax ceases to have any effect on the growth rate of output and capital. All the adjustment to a capital income tax now takes place through the labor-leisure choice. Indeed, with elastically supplied labor, the equilibrium growth rate of output becomes independent of almost all fiscal instruments, including government expenditure, the only instrument to have any influence being the tax on foreign interest income.

The fact that the equilibrium growth rate is invariant with respect to the main fiscal instruments is of interest to the recent debate regarding the empirical relevance of the AK endogenous growth model. Empirical studies by Easterly and Rebelo (1993), Stokey and Rebelo (1995) and Jones (1995), which suggest that the effects of tax rates on long-run growth rates are insignificant, or weak at best, have been taken as evidence against AK models that predict large growth effects from taxation. However, the implications of the present model of a small open economy with endogenous labor supply turn out to be entirely consistent with these empirical findings, which may therefore be viewed as being supportive of this form of AK growth model.

Our analysis of fiscal policy focuses on two general issues. First, we conduct a number of comparative static exercises, analyzing the effects of distortionary tax changes and government expenditure changes (which we take to be on a productive good) on the equilibrium growth-leisure (employment) tradeoff. We find many striking contrasts between the effects of various policy changes as between a closed and open economy. For example, we find that an increase in the tax on capital in the decentralized economy now leads to a decline in leisure (increase in work), whereas in the analogous closed economy just the opposite occurs. Likewise, a lump-sum tax financed increase in government expenditure in the present open economy leads to an increase in leisure (decline in work), again in contrast to the closed economy.

The key mechanism generating these results is the assumption that the small open economy has access to a perfect world capital market that pays a fixed rate of return, r . This rate of return, together with preference parameters, pins down the equilibrium growth rate. The equilibrium equality between the after-tax rates of return on domestic capital and foreign bonds then determines the equilibrium output-capital ratio, which is increasing in labor supply and government expenditure. An increase in the tax on capital reduces the net return on capital and requires an increase in labor supply in order for the equilibrium arbitrage condition to hold. An increase in government expenditure raises the return to capital and requires a decrease in labor supply for this equilibrium condition to be maintained. For convenience, Table 1 summarizes the key qualitative differences of fiscal policy between open and closed economies, for both fixed and elastic labor supplies.

Second, we characterize optimal fiscal policy. If government expenditure is set optimally, then capital income should not be taxed; consumption and leisure should be taxed uniformly. If government expenditure is not at its optimal level, then it introduces a distortion to the domestic capital market and capital income should also be taxed to correct for the distortion so induced. These optimal policies are essentially as in the analogous closed economy. This is hardly surprising since similar distortions are being corrected in the two cases. But, in addition, for the form of government expenditure rule being considered, foreign interest income should remain untaxed.

The paper is structured as follows. Section 2 begins by setting out the equilibrium for a benchmark centrally planned economy, before moving on to consider a decentralized market economy in Section 3. The contrast in the impact of government expenditure policy between these two structures is marked and mirrors that of simpler the AK model. Section 4 analyzes fiscal policy in the decentralized economy, while Section 5 derives the optimal tax structure. Section 6 concludes, while some technical details are provided in the Appendix.

2. The Analytical Framework: Centrally Planned Economy

The small open economy consists of N identical individuals, each of whom has an infinite planning horizon and possesses perfect foresight. Population remains fixed over time. We shall

denote individual quantities by lower case letters, and aggregate quantities by corresponding upper case letters, so that $X = Nx$. We assume that the representative agent is endowed with a unit of time that can be allocated either to leisure, l , or to work, $1-l$, [$0 < l < 1$]. Output of the individual firm, y , is determined by the Cobb-Douglas production function:

$$y = A G (1-l) k^{\alpha} \quad A(G/k) (1-l) k^{\alpha} \quad 0 < \alpha < 1, \quad (1)$$

where k denotes the individual's capital stock, assumed to be infinitely durable, and G denotes the flow of services from government spending on the economy's infrastructure. We assume that these services are not subject to congestion so that G is a pure public good.² The individual firm faces positive, but diminishing, marginal physical products in all factors, non-increasing returns to scale in the private factors, capital and labor, but constant returns to scale in private capital and in government production expenditure.³ We shall assume that government claims a fraction, g , of aggregate output, Y , for expenditure on a productive activity (infrastructure), in accordance with:⁴

$$G = gY \quad 0 < g < 1 \quad (2)$$

Thus combining (1) with (2), and $Y = Ny$, aggregate output in the economy is given by:

$$Y = \left(\frac{g}{N} \right)^{\alpha(1-\alpha)} (1-l)^{\alpha(1-\alpha)} K^{\alpha} \quad (3)$$

where $K = ANk$. Thus aggregate output is proportional to the aggregate capital stock, thereby leading to an equilibrium having ongoing, endogenously determined, growth. The aggregate production function is an AK technology, in which the productivity of the aggregate capital stock depends positively upon the fraction of time devoted to work and the share of productive government expenditure. We shall assume further that labor productivity is diminishing in the aggregate, leading to the additional constraint, $\alpha < 1 - \alpha$.

The representative agent's welfare is given by the intertemporal isoelastic utility function:

$$U = \int_0^{\infty} \frac{1}{1+\rho} ([C/N]^\alpha l^{1-\alpha}) e^{-\rho t} dt \quad (4a)$$

$$\alpha > 0; \quad \rho < 1; \quad 1 > (1+\rho); \quad 1 >$$

where C denotes total aggregate private consumption and the parameter α measures the impact of leisure on the welfare of the private agent.⁵ The parameter ρ is related to the intertemporal elasticity of substitution, s say, by $s = 1/(1 - \alpha)$. The remaining constraints on the coefficients appearing in (4a) are required to ensure that the utility function is concave in the quantities C and l .

The individual agent also accumulates physical capital, with expenditure on a given increase in the capital stock, $i = I/N$, involving adjustment costs (installation costs) which we incorporate in the quadratic (convex) function⁶

$$(i, k) = i + \frac{h}{2} \frac{i^2}{k} = i \left(1 + \frac{h}{2} \frac{i}{k} \right) \quad (5)$$

Aggregating over the N individuals, leads to:

$$(I, K) = I + \frac{h}{2} \frac{I^2}{K} = I \left(1 + \frac{h}{2} \frac{I}{K} \right) \quad (5')$$

This equation is an application of the familiar Hayashi (1982) cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital (rather than its level). The linear homogeneity of this function is necessary if a steady-state equilibrium having ongoing growth is to be sustained.⁷

The aggregate economy accumulates net foreign bonds, B , that pay an exogenously given world interest rate, r , subject to the accumulation equation:

$$\dot{B} = (1 - g)Y + rB - C - I \left(1 + \frac{h}{2} \frac{I}{K} \right) \quad (4b)$$

where we have substituted for the cost function (I, K) . For simplicity we assume that capital does not depreciate, so that the economy also faces the physical capital accumulation constraint:

$$\dot{K} = I \quad (4c)$$

In this section, we consider the equilibrium generated in a centrally planned economy in which the planner chooses K , C , I , and l to maximize the utility of the representative agent, subject to the aggregate resource constraint of the economy, (4b), the capital accumulation equation, (4c), and the aggregate production function, (3). We begin by assuming that the government's share of output used for production, g is fixed arbitrarily; in Section 2.3 below we consider the case where g is set optimally, along with the other decision variables.

In the Appendix we show that the first order optimality conditions can be expressed as:

$$\frac{C}{Y} = \frac{l}{1-l} \frac{(1-g)}{(1-\alpha)} \quad (6a)$$

$$1 + h \frac{I}{K} = q \quad (6b)$$

where q is the shadow price of capital, normalized by the marginal utility of wealth, λ . It thus measures the relative price of capital and is in effect the Tobin q . Equation (6a) restates the equilibrium marginal rate of substitution between consumption and leisure; see (A.1a) - (A.1b). An increase in leisure, l , both raises the marginal utility of consumption and reduces output, leading to an increase in the consumption-output ratio. An increase in the productivity of labor, α , raises the return to labor. This raises the marginal rate of substitution between leisure and consumption, so that given the fraction of time devoted to leisure, it induces an increase in the latter. Finally, an increase in the fraction of output absorbed by the government reduces the amount available for consumption, thus reducing the consumption-output ratio. The latter equates the marginal cost of an additional unit of investment, which is inclusive of the marginal installation cost hI/K , to the relative price of capital. Equation (6b) may be immediately solved to yield the following expression for the rate of capital accumulation:

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{q-1}{h} \quad (7)$$

so that starting from an initial level K_0 , the stock of capital at time t is $K(t) = K_0 e^{\int_0^t (s) ds}$.

Applying the standard optimality conditions with respect to B and K yields the arbitrage relationships

$$\dot{c} = r \quad (8a)$$

$$\frac{(1-g)Y/K}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r \quad (8b)$$

Equation (8a) is the standard Keynes-Ramsey consumption rule, equating the rate of return on consumption to the (given) rate of return on holding a foreign bond.⁸ With \dot{c} and r both being constants, it implies that marginal utility, $u(c)$, grows at a constant rate. Likewise (8b) equates the net rate of return on domestic capital to the rate of return on the traded bond. The former consists of three components. The first is the net output per unit of installed capital, (valued at the price q), while the second is the rate of capital gain. The third element, which is less familiar, reflects the fact that an additional source of benefits of higher capital stock is to reduce the installation costs (which depend upon I/K) associated with new investment. Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must be imposed:

$$\lim_{t \rightarrow \infty} q K e^{-rt} = 0 = \lim_{t \rightarrow \infty} B e^{-rt} \quad (8c)$$

2.1 Macroeconomic Equilibrium

To derive the macroeconomic equilibrium, we begin by taking the time derivatives of: (i) the optimality condition for consumption, (A.1a), (ii) the equilibrium consumption-output ratio, (6a), and, (iii) the production function, (3). This leads to:

$$\left(-1 \right) \frac{\dot{C}}{C} + \frac{\dot{I}}{I} = -r \quad (9a)$$

$$\frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \frac{\dot{I}}{I} + \frac{\dot{I}}{1-I} \quad (9b)$$

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} - \frac{1}{1-g} \frac{\dot{l}}{1-l} = \frac{q-1}{h} - \frac{1}{1-g} \frac{\dot{l}}{1-l} \quad (9c)$$

Combining these equations with (3), (7), and (8b), the macroeconomic equilibrium can be expressed by the pair of differential equations in q and l :

$$\dot{q} = rq - \frac{(q-1)^2}{2h} - (1-g) \left(\frac{g}{1-g} \right)^{\eta(1-g)} (1-l)^{\iota(1-g)} \quad (10a)$$

$$\dot{l} = \frac{1}{F(l)} \left(r - \frac{(1-g)(q-1)}{h} \right) \quad (10b)$$

where $F(l) = \left[1 - \frac{1}{1-g} \right] \frac{1}{l} + 1 - \frac{1}{1-l} > 0$.

The steady state to (10) is obtained by setting $\dot{q} = \dot{l} = 0$ and is therefore characterized by the relative price of capital, q , and the fraction of time devoted to leisure, l , both being constant. Linearizing (10) around its steady-state, we can easily show that the two eigenvalues to the linearized approximation are both positive.⁹ Hence the only bounded equilibrium is one in which both q and l adjust instantaneously to ensure that the economy is always on its balanced growth path (denoted by $\tilde{\cdot}$) namely:¹⁰

$$\tilde{\cdot} = \frac{\tilde{q}-1}{h} = \frac{r-g}{1-g} = s(r-g) \quad (11a)$$

$$\frac{(1-g) \left(\frac{g}{1-g} \right)^{\eta(1-g)} (1-\tilde{l})^{\iota(1-g)}}{\tilde{q}} + \frac{(\tilde{q}-1)^2}{2h\tilde{q}} = r \quad (11b)$$

where $\tilde{\cdot}$ denotes the equilibrium growth rate. The transversality condition, (8c), implies that¹¹

$$\tilde{\cdot} = \frac{\tilde{q}-1}{h} < r, \text{ i.e. } > r - g. \quad (11c)$$

Equation (11a) implies that the equilibrium is one in which domestic output, capital, and consumption all grow at a common rate determined by the difference between the world rate of interest and the domestic rate of time preference, all multiplied by the intertemporal elasticity of

substitution. The form of the expression is analogous to the equilibrium growth rate in the simplest AK model; see Barro (1990). The only difference is that for the small open economy the (fixed) marginal physical product of capital is replaced by the (given) foreign interest rate. Given this growth rate, (11a) determines the equilibrium price of capital, \tilde{q} , which will ensure that domestic capital grows at this equilibrium rate. Having obtained \tilde{q} , (11b) then determines the fraction of time devoted to leisure (employment) such that the marginal physical product of capital ensures that the rate of return on domestic capital equals the (given) world rate of interest. Hence in this small open economy with elastically supplied labor, the growth rate of output and capital is independent of production characteristics such as the productivity parameter, α , and the marginal cost of adjustment, h . Changes in these parameters are reflected in the labor-leisure choice \tilde{l} . In order for the equilibrium to be viable, the implied fraction of time devoted to leisure must satisfy $0 < \tilde{l} < 1$. This will be so if and only if:

$$0 < r + \frac{h}{1-\alpha} (r-\alpha) \frac{r(1-2\alpha)+\alpha}{2(1-\alpha)} < (1-g)(\alpha g)^{\alpha(1-\alpha)} \quad (12)$$

a condition that is plausibly met. For example for the plausible parameter values: $r = 0.06$, $\alpha = 0.04$, $\beta = -1$, $h = 16$, $\gamma = 0.18$, $\delta = 0.08$, $g = 0.08$ inequality (12) reduces to $0 < 0.07 < 0.12$..

With the equilibrium growth rate, \tilde{g} , being determined exogenously by (11a), it immediately follows that an increase in the share of government expenditure claimed by the central planner has no effect either on the growth rate, or on the Tobin q that determines the growth rate of capital. Instead, an increase in g has two offsetting effects on the net social marginal physical product of capital. On the one hand, an increase in government expenditure raises the productivity of capital, while at the same time it absorbs some of the output. The effect on the net productivity of capital depends upon $(\alpha - g)$.¹² In order to maintain the net marginal physical product of capital constant, so that the rate of return to capital remains equal to the (fixed) return on traded bonds, the supply of labor must adjust to offset this effect. That is, the fraction of time devoted to labor must decrease if $\alpha > g$, and increase otherwise. Accordingly, the effect of an increase in g on the time devoted to leisure is given by:

$$\text{sgn} \frac{\tilde{l}}{g} = \text{sgn}(\tilde{l} - g) \quad (13)$$

Substituting (6a) and (7) into (4b), the accumulation of foreign bonds by the economy is

$$\dot{B} = rB + (1-g) \left[1 - \frac{\tilde{l}}{1-\tilde{l}} \frac{\tilde{Y}}{(1-\tilde{l})\tilde{K}} - \frac{\tilde{q}^2 - 1}{2h} \right] K_0 e^{\tilde{r}t} \quad (14)$$

Solving this equation and applying the transversality condition, (8c), implies

$$B_0 + \frac{K_0}{r} \left[(1-g) \left[1 - \frac{\tilde{l}}{1-\tilde{l}} \frac{\tilde{Y}}{(1-\tilde{l})\tilde{K}} - \frac{\tilde{q}^2 - 1}{2h} \right] \right] = 0 \quad (15)$$

This equation is the nation's intertemporal resource constraint. The initial value of its foreign bonds plus the capitalized value of the current account surplus along the balanced growth path must sum to zero. Having determined the equilibrium values of \tilde{l} , \tilde{q} , and \tilde{Y}/\tilde{K} , the intertemporal constraint (15) determines the combination of the initial capital stock, K_0 , and the initial stock of foreign bonds, B_0 , necessary for the equilibrium to be intertemporally viable. If the inherited stocks of these assets violate (15) we assume that the central planner can engage in an initial trade, described by: $dB_0 + \tilde{q}dK_0 = 0$, to bring about the correct ratio. Substituting (15) into (14) we see that the equilibrium stock of traded bonds accumulate at the common equilibrium growth \tilde{r} .

2.2 Comparison with Two Models

Small Open Economy Fixed-Employment AK Model: The above model of the small open economy with elastic labor supply behaves very differently from the basic small open economy AK model with fixed labor supply; see Turnovsky (1996b). Consumption growth in that model remains as given by (11a), and is thus determined by domestic taste parameters and the world interest rate. Investment growth is given by (7), where now \tilde{q} is determined by

$$\frac{(1-g) \left(\frac{g}{\tilde{q}} \right)^{h(1-\alpha)}}{\tilde{q}} + \frac{(\tilde{q} - 1)^2}{2h\tilde{q}} = r$$

In this case, \tilde{q} is determined by the smaller root to this quadratic equation which can be shown to satisfy the transversality condition (11c).¹³ The growth of domestic capital and output now depends upon the domestic production parameters, α and h , and is independent of the taste parameters. In general, consumption and capital (output) grow at different steady rates, with the difference being reconciled by the accumulation of traded bonds. These are subject to transitional dynamics, with the long-run growth rate of traded bonds converging to the larger of these two growth rates. This in turn depends upon the consumer rate of time preference relative to the rates of return on investment.

Equation (11b) determining \tilde{q} implies that in the fixed employment open economy, the common equilibrium rate of growth of domestic output and capital, \tilde{g} , satisfies:

$$\text{sgn} \frac{\tilde{g}}{g} = \text{sgn}(\alpha - g) \quad (16)$$

With labor supplied *inelastically*, the net effect of higher government expenditure is to raise the marginal physical product of capital if and only if $\alpha > g$, thereby leading to an increase in the growth rate of capital and output. By contrast, when labor supply is *elastic*, the equilibrium adjustment to the higher government expenditure is borne entirely by the labor-leisure choice, with the equilibrium growth rate of capital remaining unchanged.

Closed Economy Endogenous Labor Supply Model: The behavior of the small open economy model with elastic labor supply also contrasts markedly with that of the corresponding closed economy model. In that case, Turnovsky (1998), has established the following tradeoff between the effects of government expenditure on growth and leisure:

$$\text{sgn} \frac{\tilde{g}}{g} = \text{sgn}(\alpha - g); \quad \text{sgn} \frac{\tilde{l}}{g} = \text{sgn}(g - \alpha)$$

Assuming, for example, $\alpha > g$, higher government expenditure leads to more growth, as in the fixed employment AK open economy. The higher growth rate raises the wage rate thereby encouraging

agents to substitute work for leisure. This *decline* in leisure in the closed economy is in direct contrast to the *increase* in leisure implied by (13) for the open economy.

2.3 Optimal Government Expenditure

Suppose now that the planner chooses the share of government expenditure, g , optimally in conjunction with C , l , B and K . This leads to the additional optimality condition:

$$\hat{g} = \quad (17)$$

That is, the optimal fraction of output claimed by the government (denoted by \hat{g}) should equal the elasticity of output with respect to the government input. This optimality condition is standard across all models. It obtains both in the fixed-employment and elastic-labor supply closed economy models, as well as in the fixed-employment small open economy model. With elastically supplied labor, (13) and (17) imply that the effect of an increase in government expenditure on leisure depends upon whether government expenditure is above or below its social optimum. Finally, corresponding to the first-best optimal government expenditure policy, (17), the overall first-best optimal consumption-output ratio is

$$\left(\frac{\hat{C}}{\hat{Y}}\right) = \frac{\hat{l}}{1-\hat{l}} -$$

3. Decentralized Economy

We now turn to an individual agent in a decentralized market economy. The agent purchases consumption out of the after-tax income generated by labor, his holdings of domestic capital, and foreign bonds. His objective is to maximize:

$$\int_0^{\infty} \frac{1}{e^{\rho t}} (cl) dt \quad (4a')$$

subject to the individual bond and capital accumulation equations:

$$\dot{b} = (1 - \tau_w)w(1-l) + (1 - \tau_k)r_k k + (1 - \tau_b)rb - (1 + \tau_c)c - i[1 + (h/2)(i/k)] - T/N \quad (18a)$$

$$\dot{k} = i \quad (18b)$$

where: r_k = return to capital, w = real wage rate, τ_w = tax on wage income, τ_k = tax on capital income, τ_b = tax on foreign bond income, τ_c = consumption tax, T/N = agent's share of lump-sum taxes. As before, k refers to the individual agent's holdings of capital, and now c denotes the individual consumption level. We could easily add government bonds, but as in previous models of this type they play no real role; debt is equivalent to lump-sum taxation.

The production function remains as specified in (3), while the government continues to tie its expenditure levels to aggregate output as in (2). The equilibrium wage rate and return to capital satisfy the marginal product conditions:¹⁴

$$w = \frac{y}{(1-l)} = \frac{y}{(1-l)} \quad (19a)$$

$$r_k = \frac{y}{k} = (1 - \tau_k) \frac{y}{k} \quad (19b)$$

Carrying out the optimization for the consumer and aggregating over the N identical representative agents leads to the corresponding optimality conditions (see Appendix):

$$\frac{C}{Y} = \frac{1 - \tau_w}{1 + \tau_c} - \frac{l}{1-l} \quad (6a')$$

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{q-1}{h} \quad (7')$$

together with the dynamic efficiency conditions:

$$(1 - \tau_b)r = \dots \quad (8a')$$

$$\frac{(1 - \tau_k)(1 - \tau_b)Y/K}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r(1 - \tau_b) \quad (8b')$$

The parallels between (6a'), (7'), (8a') and (8b') in the decentralized economy, and (6a), (7), (8a), and (8b) in the centrally planned economy are clear. The main difference is that in the decentralized economy the agent takes the size of government as given independent of his decisions, and responds to tax incentives. The fact that a higher tax on consumption reduces the consumption-income ratio is straightforward and familiar from models having inelastic labor supply. But in addition, a higher tax on labor income, for given leisure, l , also reduces the consumption-output ratio. This is because for given output a higher wage tax reduces the income available for private consumption. Equations (8a') and (8b') equate the net after-tax rate of return on bonds to the net-after-tax private rate of return to capital to the rate of return on consumption and to the net after tax return on capital.

In the absence of debt, tax revenues and government expenditures must satisfy the balanced budget condition:

$$\tau_w Nw(1-l) + \tau_k r_k K + \tau_b rB + \tau_c C + T = gY \quad (20)$$

Summing (18a) over the N individuals and combining with (20) leads to the aggregate resource constraint (4b).¹⁵

3.1 Macroeconomic Equilibrium

Following the procedure as set out in Section 2, the macroeconomic equilibrium in the decentralized economy may again be expressed as a pair of first order differential equations in q and l . Again the two eigenvalues to the linearized system are positive and both q and l adjust instantaneously to ensure that the system always lies on its balanced growth path, now expressed by:

$$\dot{q} = \frac{\tilde{q} - 1}{h} = \frac{r(1 - \tau_b) - \tilde{q}}{1 - \tau_b} \quad (11a')$$

$$\frac{(1 - \tau_k)(1 - \tau_c) \left(\frac{g}{\tilde{q}} \right)^{\alpha(1 - \tau_c)} (1 - \tilde{l})^{1 - \tau_c}}{\tilde{q}} + \frac{(\tilde{q} - 1)^2}{2h\tilde{q}} = r(1 - \tau_b) \quad (11b')$$

where the transversality condition now implies:

$$\tilde{r} = \frac{\tilde{q} - 1}{h} < r(1 - \tau_b)$$

As before, these three equations jointly determine the steady-state equilibrium growth rate, the Tobin q , and leisure. Eliminating \tilde{q} from (11a'), the equilibrium can be conveniently summarized by a pair of relationships between \tilde{r} and l , illustrated in Fig 1.¹⁶ The first of these, described by the horizontal locus RR expresses the equilibrium growth rate in terms of its determinants, namely the taste and interest rate parameters. The second, illustrated by PP, describes the tradeoff between the equilibrium growth rate and the fraction of time devoted to leisure that ensures the equality between the after-tax rate of return to capital and the after-tax rate of return to bonds. This curve is nonlinear (quadratic) as indicated in the figure. A feasible equilibrium, where $0 < l < 1$, will exist if and only if:¹⁷

$$0 < r(1 - \tau_b) + \frac{h}{1 - \tau_b} (r(1 - \tau_b) - \tilde{r}) \frac{r(1 - \tau_b)(1 - 2\tau_k) + \tilde{r}}{2(1 - \tau_k)} < (1 - \tau_k)(1 - \tau_b) (g)^{\frac{1}{1 - \tau_k}} \quad (12')$$

The viability of the equilibrium requires that the nation's intertemporal budget constraint (15) be met and this requires that the initial ratio of traded bonds, B_0 , to capital, K_0 , be set appropriately. In the decentralized economy we assume that this occurs through initial lump-sum taxation (if necessary), of the form $dT_0 + dB_0 + \tilde{q}dK_0 = 0$, whereby the private agent is forced to readjust his portfolio to attain the intertemporally viable ratio consistent with (15). Using the equilibrium conditions (19), the government's balanced budget (20) can be expressed as:

$$T = gY - \tau_w Y - \tau_k(1 - \tau_b)K - \tau_b rB - \tau_c C \quad (21)$$

thus determining the required level of lump sum taxes at each point in time. Along the balanced growth path, this is of the form:

$$T(t) = (aK_0 + bB_0)e^{-\tilde{r}t} \quad (22)$$

where a, b are constants, easily derived from the balanced growth equilibrium.

4. Fiscal Shocks in the Decentralized Economy

This section uses the model to analyze the effects of various fiscal shocks.

Tax on Capital Income: This is illustrated in Fig 2. A higher tax on capital causes the PP locus to rotate through the point T from PP to P'P'. The equilibrium thus moves from A to B. With the equilibrium growth rate, and the equilibrium price of capital, \tilde{q} , fixed, the higher tax on capital reduces the after-tax return to capital. In order to maintain equilibrium among rates of return, the productivity of capital must be increased. This is achieved by an increase in the fraction of time devoted to labor, that is, by a decline in leisure.

This response contrasts with the fixed-employment open economy AK model, where the higher tax on capital reduces the equilibrium growth rate of output. It also contrasts with the analogous closed economy with endogenous labor, in which the higher tax on capital also reduces the growth rate, but is accompanied by a reduction in the time devoted to labor, i.e. an increase in the time allocated to leisure. Hence we conclude that the effect of a higher capital income tax on employment (leisure) depends critically upon whether the economy is open or closed.

Increase in Government Expenditure: This is illustrated in Fig 3 and operates in precisely the opposite way to a higher capital income tax. The move is represented by the increase in leisure from A to B, with the growth rate remaining unchanged. The contrast in this response with both the fixed-employment small open economy and the endogenous labor closed economy continue to apply.

Tax on Interest Income: This is also illustrated in Fig. 3 and leads to a downward shift in the RR curve together with a leftward rotation in the PP curve through T. These two effects compound and the combined response is summarized by the movement from A to D. First, the higher tax on foreign interest reduces its net return and the equilibrium growth rate directly. If the after-tax rates of return on domestic capital and foreign bonds are initially in equilibrium, domestic capital now yields a higher net rate of return. To restore equilibrium the productivity of domestic capital must be reduced and this is achieved by a switch from work to leisure. This initial effect is reflected by the

movement along AC in the figure. In addition, however, the lower growth rate means a reduction in the Tobin q . This further increases the rate of return on domestic capital relative to the net return on foreign bonds. To restore equilibrium the productivity of domestic capital must be further reduced, and this is accomplished by a further increase in leisure. This part of the adjustment is represented by the horizontal movement CD.

Taxes on Wages and Consumption: These have no effect on either the growth rate or on employment. Their only effect is on the consumption-output ratio, which is reduced. In this respect, the two taxes act very much like lump-sum taxes, as in the fixed employment AK model of the open economy. But these effects contrast with the closed economy elastic labor supply model, when both the wage tax and the consumption tax operate precisely as does the capital income tax, namely they reduce growth and increase leisure. The difference arises because of the fact that in the open economy: (i) the equilibrium growth rate, and (ii) the equilibrium relationship between the rate of return to capital and bonds are both independent of these two tax rates.

The fact that the growth rate is independent of most income taxes (except τ_b) offers an interesting perspective to the following issue. As we observed at the outset, one of implications of the basic endogenous growth model, a feature that distinguishes it from the traditional neoclassical model, is that its equilibrium growth rate varies inversely with distortionary income taxes. The fact that empirical evidence by Easterly and Rebelo (1993), Stokey and Rebelo (1995) and Jones (1995) does not support this implication, has been used as evidence against these endogenous growth models. Our results suggest some caution might be required in reaching this conclusion. For small economies facing a perfect world capital market, equilibrium growth rates are in fact independent of most tax rates. Instead, such economies respond to changes in tax rates through variations in their equilibrium labor-leisure choice.

5. Optimal Fiscal Policy

Comparing the equilibria of the decentralized and centrally planned economies enables us to characterize the first best optimal tax policy. First, setting:

$$\hat{b} = 0 \quad (23a)$$

$$1 - \hat{\tau}_k = \frac{1 - g}{1 - \tau_k} \quad \text{i.e.} \quad \hat{\tau}_k = \frac{g - \tau_k}{1 - \tau_k} \quad (23b)$$

the pair of equations (11a',b') coincide with (11a,b), thus ensuring that the steady-state growth rate and leisure time in the decentralized economy will replicate those of the centrally planned economy. These optimal tax rates are identical to those obtained for the fixed employment AK model. They correct for the distortions (if any) induced by government expenditure in financial markets; see Turnovsky (1996a).

From (17) and (23b) it is seen that the tax on capital income depends upon the deviation in the aggregate share of government expenditure from its optimum. The intuition for this result is as in Turnovsky (1996a), involving the deviation between the social and private returns to capital accumulation. The social return to accumulating a marginal unit of capital is:

$$r_s = (1 - g) \frac{Y}{K} \quad (24a)$$

This consists of the gross marginal product of capital, less the induced claim by government. This measure takes account of the fact that as capital increases and output expands, the size of the government also expands in accordance with (2). The private return to capital in the decentralized economy is the after-tax rate of return

$$r_p = (1 - \tau_k) \frac{y}{k} = (1 - \tau_k)(1 - \tau_k) \frac{y}{k} = (1 - \tau_k)(1 - \tau_k) \frac{Y}{K} \quad (24b)$$

This expression assumes that the private agent operating in a decentralized economy treats the impact of his own rate of capital accumulation on aggregate government expenditure as negligible.

The optimal tax on capital is then determined so as to equate the private and social rates of return; i.e. set $r_p = r_s$.

If aggregate government expenditure exceeds the optimum the cost of the resources utilized by the government exceed the benefits and it has a net adverse effect on the return to capital. The private agent fails to recognize this and overinvests relative to the optimum. This is corrected by imposing a positive tax on private capital income. If government expenditure is below its optimum the benefits exceed the costs and capital needs to be subsidized. If it is at its optimum, the costs just match the benefits, there are no spillovers from government expenditure to the capital market, and capital income should remain untaxed; i.e. $\hat{\tau}_k = 0$.

The optimal tax on foreign interest income, $\hat{\tau}_b$, is zero, because government expenditure is not tied to interest income. If it were, similar distortions to those involving capital would arise, and when g is away from its optimum, foreign interest income would need to be taxed as well.

Comparing (6a) and (6a'), and having replicated l through the appropriate tax rates on capital and interest, the consumption-output ratios in the two economies will be replicated if and only if:

$$\frac{1 - \hat{\tau}_w}{1 + \hat{\tau}_c} = \frac{1 - g}{1 - l} \quad (25)$$

The relationship between the optimal tax on wage income and on consumption reflects the effects of externalities on the marginal rate of substitution between consumption and leisure. The marginal rates of substitution (MRS) in the centrally planned and in the decentralized economies (subscripted by c and d), obtained by dividing (A.1b) by (A.1a) and (A.4b) by (A.4a), are:¹⁸

$$MRS_c = \frac{1 - g}{1 - l} \frac{Y}{1 - l}; \quad MRS_d = \frac{1 - \hat{\tau}_w}{1 + \hat{\tau}_c} \frac{Y}{1 - l}$$

and the latter will mimic the former if and only if (25) holds.

When government expenditure is set optimally, [i.e. $g = g^*$], (25) reduces to

$$\hat{\tau}_w = -\hat{\tau}_c \quad (25')$$

That is, the tax on labor income must be equal and opposite to that on consumption. Interpreting the tax on wage income as a negative tax on leisure, (25') says that in the absence of any externality, the optimal tax structure requires that the two utility enhancing goods, consumption and leisure, be taxed uniformly. This result can be viewed as an intertemporal application of the Ramsey principle of optimal taxation; see Deaton (1981), Lucas and Stokey (1983). If the utility function is multiplicatively separable in c and l , as we are assuming here, then the uniform taxation of leisure and consumption is optimal.

The first best optimal integrated fiscal policy is characterized by (17), (23), and (25'). Substituting these conditions into (11a) and (11b), the corresponding first-best equilibrium growth rate, \hat{r} , price of capital, \tilde{q} , and associated time devoted to leisure, \hat{l} , are given by

$$\hat{r} = \frac{r - \delta}{1 - \delta}; \quad \tilde{q} = 1 + h \frac{r - \delta}{1 - \delta};$$

$$(1 - \delta) \left(\frac{r - \delta}{1 - \delta} \right)^{\alpha(1 - \delta)} (1 - \hat{l})^{1 - \alpha(1 - \delta)} = r + 1 + h \frac{r - \delta}{1 - \delta} - \frac{h}{2} \left(\frac{r - \delta}{1 - \delta} \right)^2 \quad (26)$$

The optimal tax rates and expenditure shares must also be consistent with the government budget constraint (21). Setting $\hat{b}_b = \hat{b}_k = 0$, $\hat{b}_c = -\hat{b}_w$ and evaluating (19a), (6a) at the optimum, yields: $N\hat{w} = \hat{Y}/(1 - \hat{l})$; $\hat{C} = (\hat{l}/(1 - \hat{l}))(\alpha/Y)\hat{Y}$ so that (21) may be expressed as:

$$\hat{b}_w \left(1 - \frac{\hat{l}}{1 - \hat{l}} \right) \frac{1}{\alpha} \hat{Y} + T = \hat{Y} \quad (21')$$

Any combination of \hat{b}_w and T consistent with this equation will satisfy the government budget constraint. This can be feasibly achieved without lump-sum taxation, ($T = 0$), if and only if $(1/\alpha)(\hat{l}/(1 - \hat{l})) > 1$. A sufficient condition for this to be met that any plausible economy will satisfy is that the optimal consumption-output ratio, (\hat{C}/\hat{Y}) , exceeds the optimal government production expenditure-output ratio, $\hat{g} = \frac{1}{\alpha}$. In this case, (21') will be met by subsidizing the labor-leisure choice and applying an exactly offsetting tax to consumption:¹⁹

$$\hat{w} = -\hat{c} = \frac{1}{\left[1 - (1/\lambda)(\hat{l}/(1-\hat{l}))\right]} < 0 \quad (27)$$

Finally, to sustain the first-best optimum, the government will also need to levy an initial one-time lump-sum tax so as ensure that the initial configuration of assets, consistent with the nation's intertemporal government budget constraint is attained.

6. Conclusions

Most endogenous growth models assume that labor is supplied inelastically. This is true for the bulk of the literature that deals with closed economies; it is even more true for the sparser literature focusing on the open economy. This assumption is not only unrealistic, but it has strong, yet misleading, policy implications. In this paper we have introduced an elastic labor supply, determined by the labor-leisure tradeoff, into an AK growth model of a small open economy. We show that the macroeconomic equilibrium is such that the economy lies continuously on its balanced growth path, which under plausible conditions always exists. This equilibrium is determined as follows. Domestic consumption, capital and output all grow at a common rate determined by taste parameters, together with the after-tax rate of return on foreign bonds. Given this growth rate and the associated price of capital, the fraction of time devoted to leisure (work) is then determined so as to ensure that the return to capital equals the return to foreign bonds.

Our main focus has been on the implications of the elastic labor supply for fiscal policy. We find that the long-run growth rate is independent of almost all domestic fiscal instruments, except the tax on foreign bonds. The essential exogeneity of this growth rate is thus consistent with the empirical evidence on this issue by a number of authors. It does, however, contrast sharply with the fixed-employment small open economy AK model in which consumption grows as in this model, but output and capital grow at an independent rate that depends not only upon technological parameters, but also upon both the tax on capital and productive government expenditure.

The other interesting contrast is with the closed economy. In the basic fixed employment AK model due to Barro (1990), for example, taxes on wages and consumption have no effect; they operate like lump-sum taxes. Endogenizing labor supply in that model ensures that these tax rates have real distortionary effects, operating in a qualitatively similar way as the capital income tax. Introducing elastic labor supply in the small open economy has quite the opposite effect. Not only do the tax on wages and consumption remain nondistortionary, but now the tax on capital income has no effect on the growth rate. It does affect leisure, although now in precisely the opposite way to how it does in the closed economy. The bottom line is that how fiscal policy impacts on the equilibrium growth rate of an endogenous growth model depends upon the nature of the economy.

Finally, the optimal tax structure in the decentralized economy has been characterized. In general, the optimal distortionary tax rates will depend upon the chosen levels of government expenditure relative to its optimum. These rates are set in response to externalities that this expenditure generates in: (i) financial markets, and (ii) the consumption-leisure choice. If government expenditure is chosen optimally the first externality vanishes. In that case, the optimal tax rate on capital income is zero and leisure and consumption should be taxed uniformly, in accordance with established principles of public finance. Under the assumption we have made that the level of government expenditure is set independently of foreign interest income, the latter should remain untaxed. However, if government expenditure were tied to domestic GNP, rather than GDP, this would cease to be the case, and foreign interest income would have to be taxed in much the same way as would domestic capital income.

Table 1

Summary of Qualitative Effects of Fiscal Shocks in a Decentralized Economy

Increase in Productive Government Expenditure, g

	Closed Economy		Open Economy		
	\tilde{c}	\tilde{l}	\tilde{c}	\tilde{k}	\tilde{l}
fixed labor	+	0	0	+	0
endogenous labor	+	-	0	0	+

Increase in Capital Income Tax, τ_k

	Closed Economy		Open Economy		
	\tilde{c}	\tilde{l}	\tilde{c}	\tilde{k}	\tilde{l}
fixed labor	-	0	0	-	0
endogenous labor	-	+	0	0	-

Increase in Labor Income Tax, τ_w , or Consumption Tax, τ_c

	Closed Economy		Open Economy		
	\tilde{c}	\tilde{l}	\tilde{c}	\tilde{k}	\tilde{l}
fixed labor	0	0	0	0	0
endogenous labor	-	+	0	0	0

NB: \tilde{c} , \tilde{k} , denote the equilibrium growth rates of consumption and capital (output) respectively. These are distinct for the open economy with fixed labor supply; they are identical in all other cases.

APPENDIX

This Appendix provides the optimality conditions that form the basis of the equilibrium. The centrally planned and decentralized market economies are discussed in turn.

A.1 Centrally Planned Economy

The central planner's optimization problem is to choose C, l, Y , and K to maximize:

$$\int_0^{\infty} e^{-\rho t} [N^{\alpha} C^{1-\alpha} l^{\alpha} (1-g)^{\alpha} (1-l)^{1-\alpha} K^{\alpha} + rB - C - I] e^{-\rho t} dt + \frac{h}{2} \frac{I}{K} - \dot{B} + q e^{-\rho t} [I - \dot{K}]$$

The optimality conditions consist of the following:

$$N^{\alpha} C^{-\alpha} l^{\alpha} = \lambda \tag{A.1a}$$

$$N^{\alpha} C^{1-\alpha} l^{-\alpha} = \frac{1-g}{1-l} \left(\frac{1-g}{1-l} \right)^{\alpha(1-\alpha)} (1-l)^{(1-\alpha)-1} K^{\alpha} = \frac{1-g}{1-l} \frac{Y}{1-l} \tag{A.1b}$$

$$(1 + h(I/K)) = q \tag{A.1c}$$

$$r = -\dot{\lambda} + \lambda \tag{A.1d}$$

$$(1-g)Y/K + (h/2)(I/K)^2 = -\dot{q} + q \tag{A.1e}$$

Equations (A.1a) - (A.1c) imply

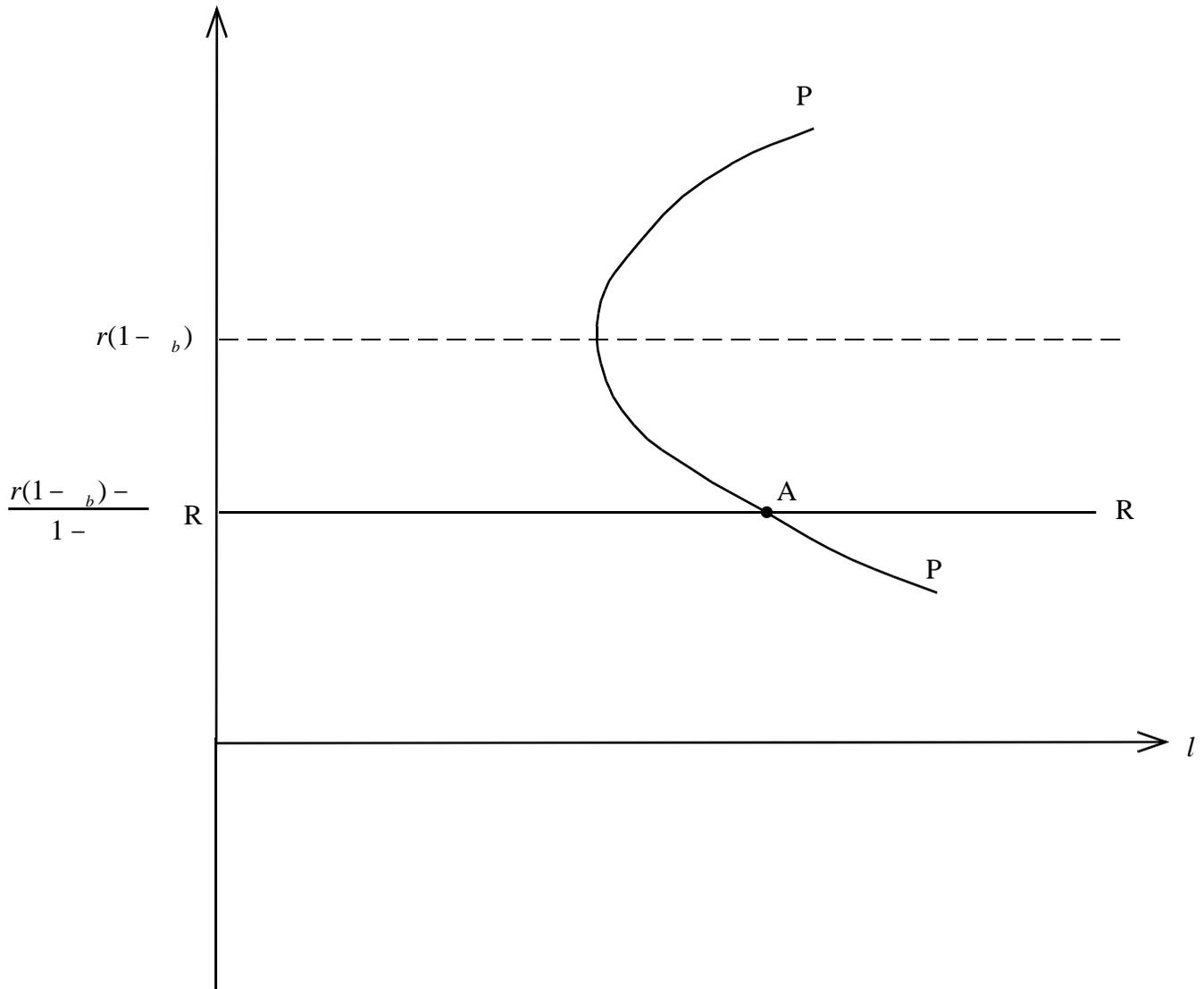
$$\frac{C}{Y} = \frac{l}{1-l} \frac{(1-g)}{(1-l)} \tag{A.2a}$$

$$1 + h(I/K) = q, \text{ where } q = q/l \tag{A.2b}$$

Equations (A.1d) and (A.1e) imply:

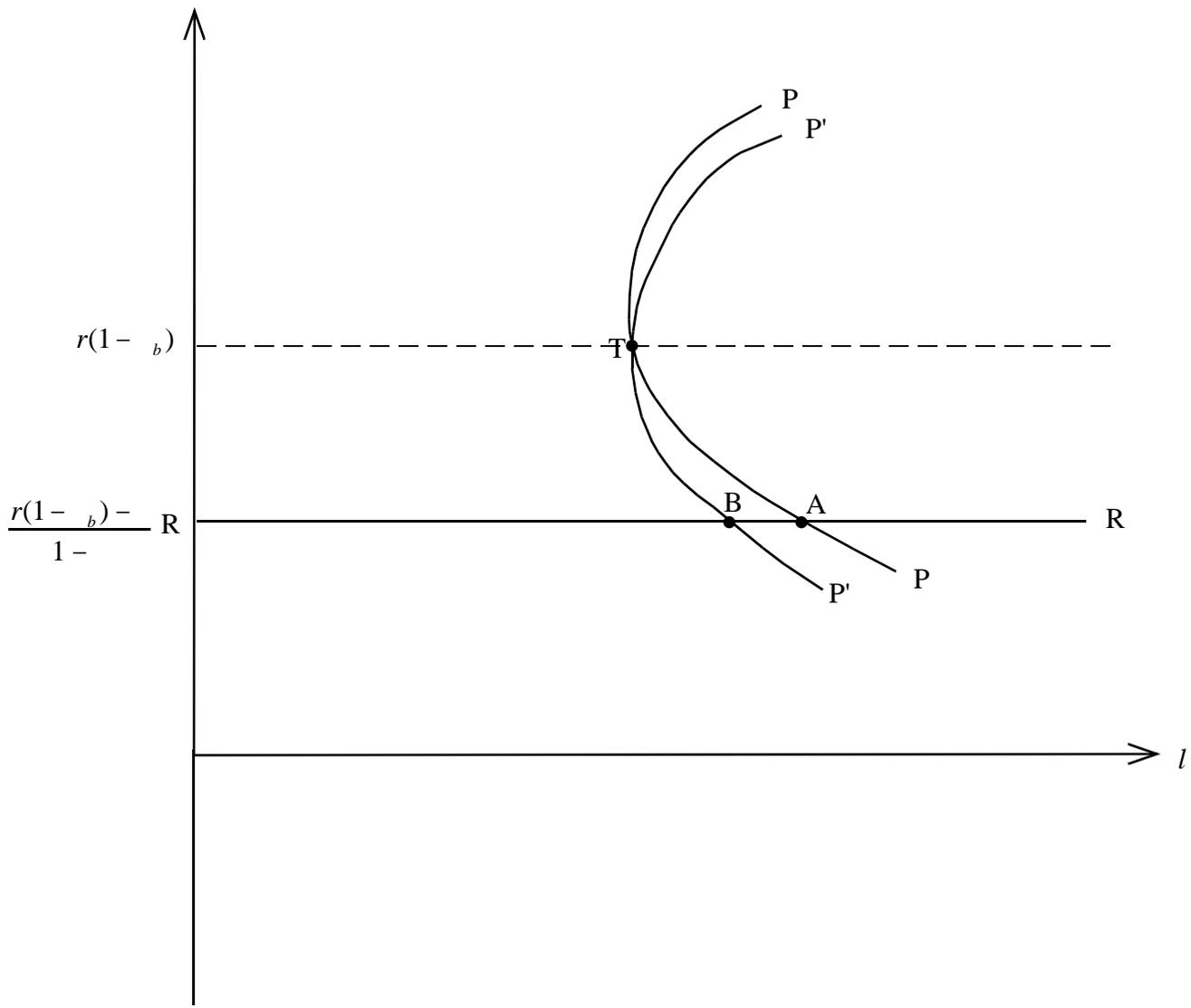
$$\dot{I} = -r \tag{A.3a}$$

$$\frac{(1-g)(Y/K)}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r \tag{A.3b}$$

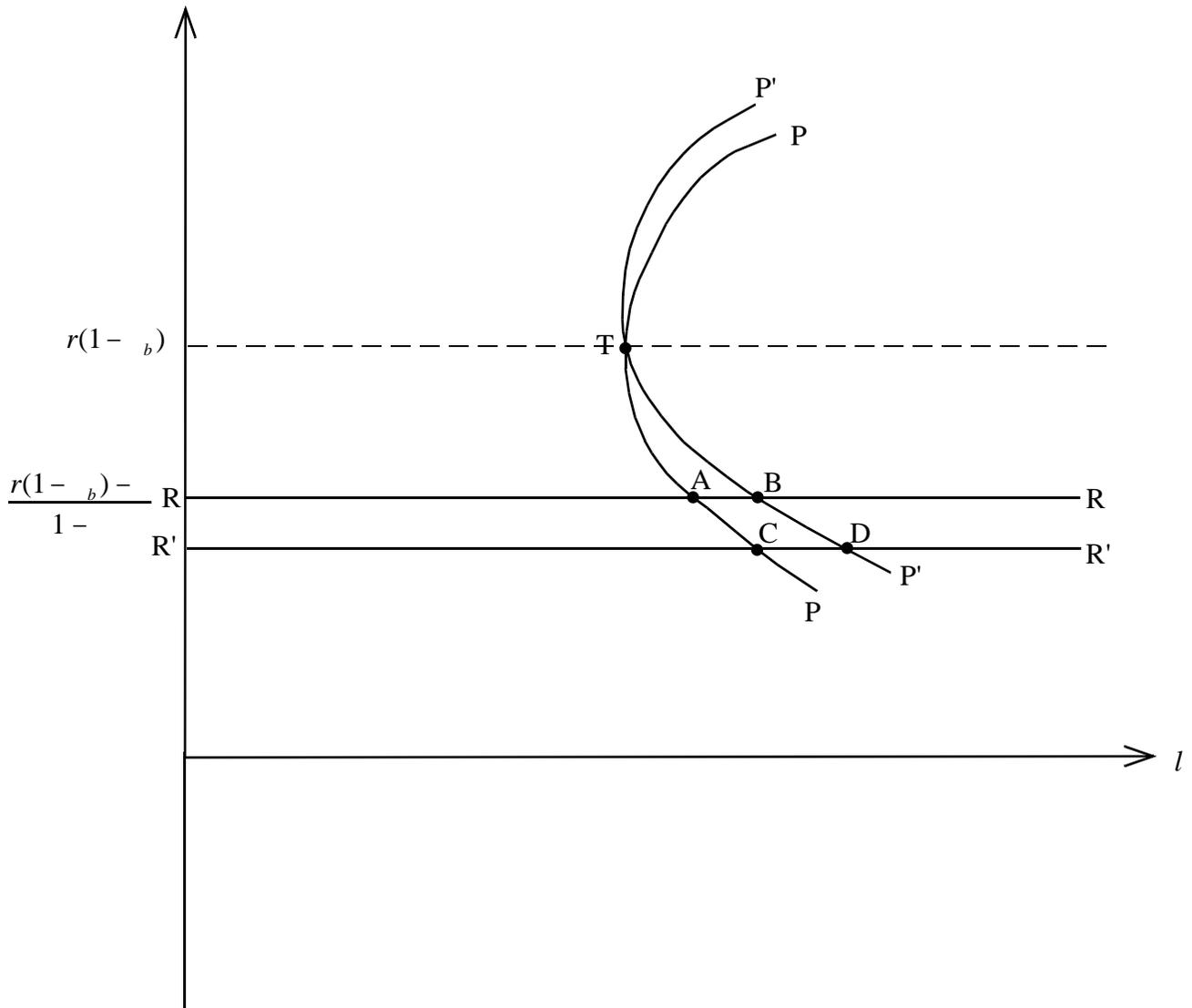


Equilibrium Growth and Leisure

$r \sim 1$



Increase in Tax on Capital
Fig 2



Increase in Government Expenditure and Tax on Foreign Interest
 Fig 3

A.2 Decentralized Economy

The representative consumer's optimization problem is to choose c, l , and k to maximize:

$$\begin{aligned} (1/\beta) \int_0^T c e^{-\beta t} + e^{-\beta T} (1 - \beta w)(1-l) + (1 - \beta k)r_k k + (1 - \beta b)rb - (1 + \beta c)c - i + \frac{h}{2} \frac{i}{k} - \frac{T}{N} - \dot{b} \\ + q e^{-\beta t} [i - \dot{k}] \end{aligned}$$

The optimality conditions consists of the following:

$$c^{-1} l = (1 + \beta c) \quad (\text{A.4a})$$

$$c l^{-1} = (1 - \beta w)w \quad (\text{A.4b})$$

$$[1 + h(i/k)] = q \quad (\text{A.4c})$$

$$(1 - \beta k)r_k = -\dot{k} \quad (\text{A.4d})$$

$$\frac{(1 - \beta k)(1 - \beta)(y/k)}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r(1 - \beta b) \quad (\text{A.4e})$$

Wages and the return to capital are set in accordance with:

$$w = y/(1-l) = G k^{1-\alpha} (1-l)^{-1} = y/(1-l) \quad (\text{A.5a})$$

$$r_k = y/k = (1 - \alpha)G k^{-\alpha} (1-l) = (1 - \alpha)y/k \quad (\text{A.5b})$$

As in the centralized economy, we derive:

$$\frac{c}{y} = \frac{C}{Y} = \frac{1 - \beta w}{1 + \beta c} = \frac{l}{1-l} \quad (\text{A.6a})$$

$$i/k = I/K = ((q-1)/h) \quad (\text{A.6b})$$

Combining (A.4c), and (A.4e) implies the following equality between the after-tax rates of return:

$$\frac{(1 - \beta k)(1 - \beta)(y/k)}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r(1 - \beta b) = -\dot{k} \quad (\text{A.7})$$

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¹Brief treatments of endogenous labor supply in a closed economy are also provided by Rebelo (1991) and Stokey and Rebelo (1995).

²Barro and Sala-i-Martin (1992) and Turnovsky (1996a) analyze the case of a public good subject to congestion. Government expenditure is introduced as a flow. One can argue that productive government expenditure is more appropriately introduced as a stock, in which case the equilibrium is characterized by transitional dynamics; see Futagami, Murata, and Shibata (1993). Turnovsky (1997b) pursues this for a small open economy in which production is represented by an AK technology with inelastic labor.

³It is well known that in order to assure ongoing growth the production function must be linearly homogeneous in the factors that are being accumulated. This requires that, in equilibrium, government expenditure be tied to the capital stock and that the production function have constant returns to scale in capital and government expenditure.

⁴This paper is part of a growing literature that introduces government expenditure as a productive input; see Barro (1990), Barro and Sala-i-Martin (1992), Futagami, Murata, and Shibata (1993), and Turnovsky (1997b). Much of the literature analyzing the effect of government assumes that it is utility-enhancing or wasteful and examples of this in a small open economy (with fixed labor supply) are provided by Turnovsky (1997a, Chapter 5). In this case it is straightforward to show that the impact of taxes on growth and employment are identical to those derived below. With the productivity of capital being independent of government expenditure in either of these cases, government now has no effect on equilibrium employment.

⁵In general the introduction of leisure will be consistent with a balanced growth equilibrium as long as the utility function is of one of the two following forms [see Ladrón-de-Guevara, et al (1997)]:

$$U(C,l) = (\gamma)C^{-\gamma} f(l) \text{ for } \gamma < 1, \quad \gamma = 0 \text{ or } U(C,l) = \log C + f(l), \text{ for } \gamma = 0$$

The constant utility formulation being adopted here is of this class.

⁶Most dynamic models of small open economies facing perfect international capital markets assume adjustment costs on investment, in order to obtain non-degenerate dynamics; see Turnovsky (1997a).

In the case of the AK model with fixed labor assumed by van der Ploeg (1996) and Turnovsky (1996), this is necessary to permit the productivity of domestic capital to deviate from the return on foreign bonds. We maintain this assumption here, to preserve comparability with these earlier models, although the endogeneity of the labor supply also introduces flexibility to the domestic productivity of capital. However, the introduction of adjustment costs adds little complication. They can be eliminated by assuming $h = 0$, in which case, $q = 1$. All qualitative results remain unchanged.

⁷Costs of adjustment are often formulated in terms of the level of investment. This formulation violates the homogeneity required to maintain a steady-state growth path. For more discussion of different formulations of the adjustment cost function, see Hayashi (1982), who also provides some empirical evidence supporting the homogeneity assumption.

⁸Implicitly, in most of our discussion we assume that $B > 0$ so that the representative agent is a net lender abroad. However, our analysis applies equally to the case where $B < 0$, so that the agent is a net borrower.

⁹Linearizing (10) one can easily establish that the determinant and the trace to this linearized system, Δ , Tr satisfy: $\Delta = (1-g)(1-\alpha)(Y/K)((1-\alpha)/h)1/F(l) > 0$; $Tr = r - ((q-1)/h) > 0$. This implies that the two eigenvalues are real and positive.

¹⁰ This local instability of the dynamic path depends in part upon our assumptions of a Cobb-Douglas production function and constant elasticity utility function, and justifies our focus on that equilibrium in the present analysis. For more general production functions one cannot dismiss the

possibility that the dynamics has a stable eigenvalue, giving rise to potential problems of indeterminate equilibria. In a model with both physical and nonhuman capital Benhabib and Perli (1994), Ladrón-de-Guevara, Ortigueira, and Santos (1997) show how the steady-state equilibrium may become indeterminate. Other authors have emphasized the existence of externalities as sources of indeterminacies of equilibrium; see Benhabib and Farmer (1994).

¹¹This is met under plausible conditions. It is certainly met if $\sigma < 0$, i.e. if the intertemporal elasticity of substitution is less than unity, a condition that virtually all empirical studies confirm. For example, Hall (1988) estimates the intertemporal elasticity of substitution in consumption to be around 0.1. These results obtained for the United States are confirmed in a more recent study by Patterson and Pesaran (1992) who also obtain slightly higher estimates (0.4) for the United Kingdom.

¹²The effect of g on the net productivity of capital is given by $(1-g)(g)^{\sigma(1-\sigma)} / g$.

¹³It is easy to demonstrate from this equation that the presence of convex costs of adjustment may preclude the existence of a balanced growth equilibrium; see Turnovsky (1996b) for further discussion.

¹⁴Aggregating the individual production functions (1), aggregate output is determined by

$$Y = Ny = AG [N(1-l)]^{\alpha} K^{1-\alpha} = AG L^{\alpha} K^{1-\alpha}$$

where $L = N(1-l)$ = aggregate supply of labor. The equilibrium real wage and rate of return on capital are determined by:

$$w = Y/L = Y/L = y/(1-l); \quad r = Y/K = (1-\alpha)Y/K = (1-\alpha)y/k.$$

¹⁵Note that equation (18a) ignores the residual income, attributable to the fixed factor, land say, associated with the non-increasing returns to scale to private capital and labor. Strictly speaking, this needs to be taken into account in deriving the aggregate resource constraint (4b) in the decentralized economy.

¹⁶A similar graphical representation can also be given in the centrally planned economy.

¹⁷Assuming the parameter values applied to equation 12, and in addition the tax rates $\tau_k = \tau_b = 0.28$, (12') reduces to $0 < 0.04 < 0.09$.

¹⁸In comparing the equation pairs (A.1a), (A.1b) and (A.4a), (A.4b), we must correct for the fact that the optimality condition (A.1a) is with respect to aggregate consumption, while (A.4a) is with respect to individual consumption.

¹⁹In the implausible case where $\tau > (\hat{C}/Y)$, (27) implies that labor income should be taxed and consumption subsidized at more than 100%. Obviously this is infeasible and lumpsum taxation would be required to sustain the optimum.