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On the resonance and influence of the tides in Ungava Bay and Hudson Strait

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[1] The tides of Leaf Basin in Ungava Bay may be the world's highest. An analysis of the frequency dependence of the response to outside forcing, a normal mode analysis, and a study of the damped oscillation of an initial disturbance, suggest that the Ungava Bay/Hudson Strait region has a natural period of about 12.7 hours and so is close to resonance with the tidal forcing. The implications for regional and global tides in the present, past, and future are explored. **Citation:** Arbic, B. K., P. St-Laurent, G. Sutherland, and C. Garrett (2007), On the resonance and influence of the tides in Ungava Bay and Hudson Strait, *Geophys. Res. Lett.*, *34*, L17606, doi:10.1029/2007GL030845.

1. Introduction

[2] The tidal range in Leaf Basin, Ungava Bay, in northern Quebec (Figure 1) can reach 16.8 ± 0.2 m (from *O'Reilly et al.* [2005] based on 311 days of observations, from 28 June 2001 to 5 May 2002 at 58°44.0′N, 69°50.8′W, by D. DeWolfe). Within error bars, this equals the range of 17.0 ± 0.2 m in the upper Bay of Fundy [*O'Reilly et al.*, 2005] so that Ungava Bay may be regarded either as runner-up or winner of the title for the world's highest tides. Moreover, the region comprising Ungava Bay, Hudson Strait, Hudson Bay and the northern Labrador Sea dissipates more tidal energy than any other region of the world's ocean [*Egbert and Ray*, 2001], thus influencing the global tidal response. Also, the tides in this region are very sensitive to changes in sea level, as shown by three recent papers on paleotides [*Arbic et al.*, 2004; *Egbert et al.*, 2004; *Uehara et al.*, 2006], with potential implications for both ice-sheet dynamics [*Arbic et al.*, 2004] and ocean mixing [*Egbert et al.*, 2004] during ice ages. Our goal is to understand the regional tides of the present-day.

[3] The Bay of Fundy/Gulf of Maine system is thought to have a resonant period of about 13.3 hours, close to the 12.4 hour period of the M_2 tide and thus partly accounting for the large tides there [*Garrett*, 1972; *Ku et al.*, 1985]. Ungava Bay has also been suspected of being close to resonance [*Drinkwater*, 1986], but there has been no quantitative analysis of this or its implications. In this paper we use three approaches to quantify the resonant period of Ungava Bay: (1) a response analysis in which the observed frequency dependence of the response of the Ungava/Hudson system to outside tidal forcing is fitted to a simple model, (2) a numerical normal mode analysis, and (3) a numerical initial value problem in which, starting from high (or low) tide inside the bay, the sea level is allowed to oscillate freely but with damping by internal friction and by radiation into an otherwise quiescent ocean.

2. Response Analysis

[4] The dominant tide in the Ungava/Hudson system, as in the Bay of Fundy, is M_2 (the principal lunar semi-diurnal constituent with frequency ω_M = 1.932 cycles per day (cpd)). Contributions to the tidal range from the diurnal constituents are small, but there are significant contributions from S_2 (the principal solar semi-diurnal constituent with frequency ω_S = 2.000 cpd) and N_2 (the larger lunar elliptic constituent with frequency ω_N = 1.896 cpd). Tidal constants are based on record lengths of 311, 237, 365, and 364 days, respectively, for Leaf Basin, the south and north sides of Hudson Strait, and Churchill on the western shores of Hudson Bay (Figure 1).

[5] As a first step in a response analysis, we take the forcing to be represented by the tides observed at five locations (Figure 1) close to each other on the Labrador continental shelf [*Smithson*, 1992], with record lengths varying from 343 to 390 days. At each station we have the amplitude H of each tidal constituent and its phase lag G (in degrees with respect to Greenwich transit of the astronomical forcing). For pairs of constituents, we examine changes in the amplitude ratios and phase differences from outside to inside the Ungava/Hudson system (Table 1). We see that M_2 is amplified more than S_2 from outside to inside, and more than N_2 as the ratio of N_2 to S_2 amplitudes changes little. This enhanced amplification of M_2 suggests that there is a resonance at a frequency close to M_2, an interpretation supported by the change in phase lag from outside to inside being more for S_2 than for M_2, and more for M_2 than for N_2, as one expects for a simple harmonic oscillator near resonance.

2.1. Simple Model

[6] We assume that the frequency response of a system that is largely in one mode near resonance can be represented as [*Garrett*, 1972; *Sutherland et al.*, 2005]

$$\eta(\mathbf{x}, t; \omega) = \mathcal{R}_e\left[A(\omega)S(\mathbf{x})R(\omega)e^{-i\omega t}\right], \tag{1}$$

where A(ω) is the (complex) forcing H exp (iG) for the tide outside the system. The mode has a complex spatial dependence $S(\mathbf{x})$, which cancels out when considering tidal

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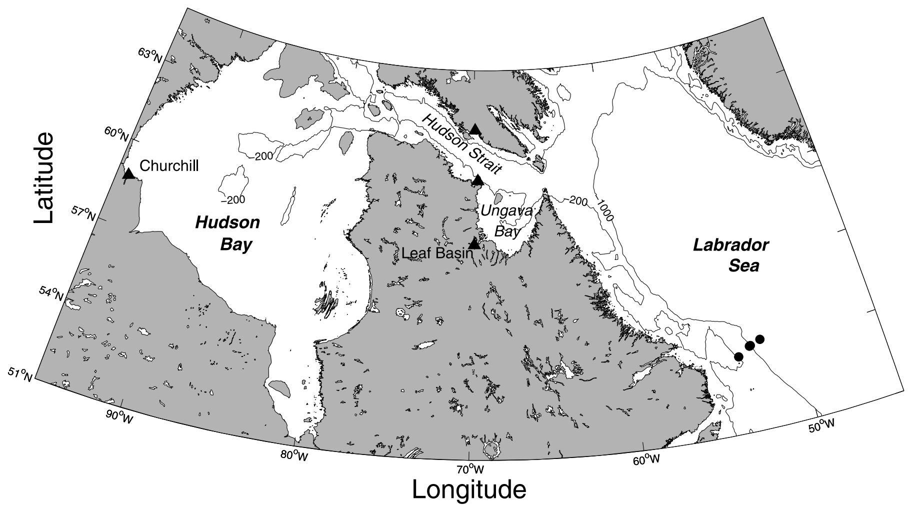

Figure 1. Map of the area. The bathymetric contours for 200 m and 1000 m are shown. Stations inside the Ungava/Hudson system are shown by triangles. The circles denote the five Labrador stations (some locations are close together).

ratios from outside to a single station inside, and a natural frequency ω_0. The response function R(ω) is given by

$$R(\omega) = \left[\frac{\omega_0 - \omega}{\omega_0} - \frac{1}{2} i (cQ)^{-1}\right]^{-1}. \qquad (2)$$

Damping is represented by the quality factor cQ where $c = 1$ for M_2 but is a number to be determined, between 2/3 and 1, for N_2 and S_2. This allows for a mix of damping by radiation into the exterior ocean, which is linear and would have $c = 1$, and damping by the standard quadratic friction inside the bay which would damp small constituents (N_2, S_2) more than the dominant constituent M_2, by 50% if the amplitude ratio is large [*Jeffreys*, 1970; *Inoue and Garrett*, 2007].

[7] We thus have four inputs, the changes in amplitude ratio and phase difference for two pairs of the three constituents (we take S_2, M_2 and S_2, N_2), from which to determine the three unknowns ω_0, Q, c. A least squares fit for the changes from the Labrador shelf to Leaf Basin gives ω_0 = 1.94 cpd (period 12.4 hours), Q = 9.4, and c = 0.79, suggesting a resonant frequency very close to M_2, though most of the amplification of M_2 compared with N_2 and S_2 (Figure 2) comes from the reduced frictional damping. The value of c implies a mix of frictional and radiative damping. Using the average of the two Hudson Strait stations to represent the Ungava/Hudson system would lead to 1.92 cpd, 7.0, and 0.95 for ω_0, Q, c respectively, and using Churchill would lead to 1.93 cpd, 11.8, and 0.90 for ω_0, Q, c respectively. Thus the response analysis indicates a resonant period very close to that of the M_2 tide, but with uncertain values of Q and c.

Table 1. Phase Differences in Degrees and Amplitude Ratios for the Labrador Stations (Taking the Average) and Stations Within the Ungava/Hudson System

	$G(S_2) - G(N_2)$	$G(S_2) - G(M_2)$	$H(N_2)/H(S_2)$	$H(M_2)/H(S_2)$
Labrador	49.6	33.2	0.53	2.42
Leaf Basin	92.4	64.9	0.59	3.11
North Hudson Strait	88.1	54.6	0.59	2.91
South Hudson Strait	88.4	56.0	0.60	2.91
Churchill	108.6	69.4	0.66	3.4

2.2. Correction for Back Effect

[8] We have so far assumed that the sites chosen for the forcing are unaffected by the response inside the bay. The Ungava/Hudson system is so large, however, that its back effect on the open ocean may be significant. In other words, what we have taken as "input" may be partly "output". To investigate this, we have used a forward global tidal model [*Arbic*, 2005], with a 0.5° resolution, in one run with Hudson Strait open and another with it closed off. We find large

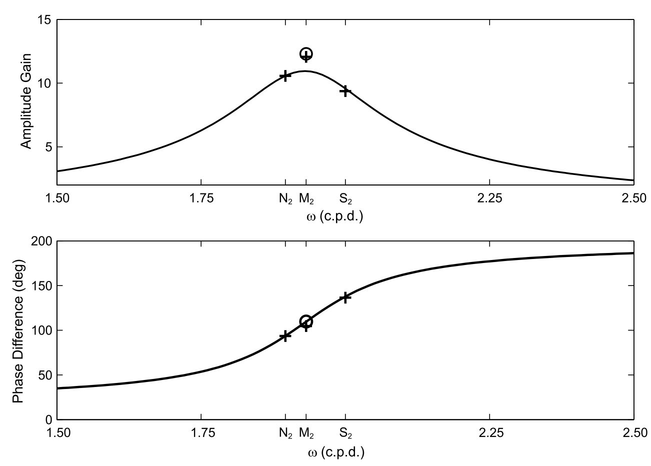

Figure 2. The amplitude and phase of the response function $R(\omega)$ for the best fits of ω_0, Q and c using the offshore pelagic forcing. The + signs show the values for N_2, M_2 and S_2. The circles denote the model gain for M_2 with c = 0.79; it is displaced from the solid line which describes the response for the other constituents which have c = 1.

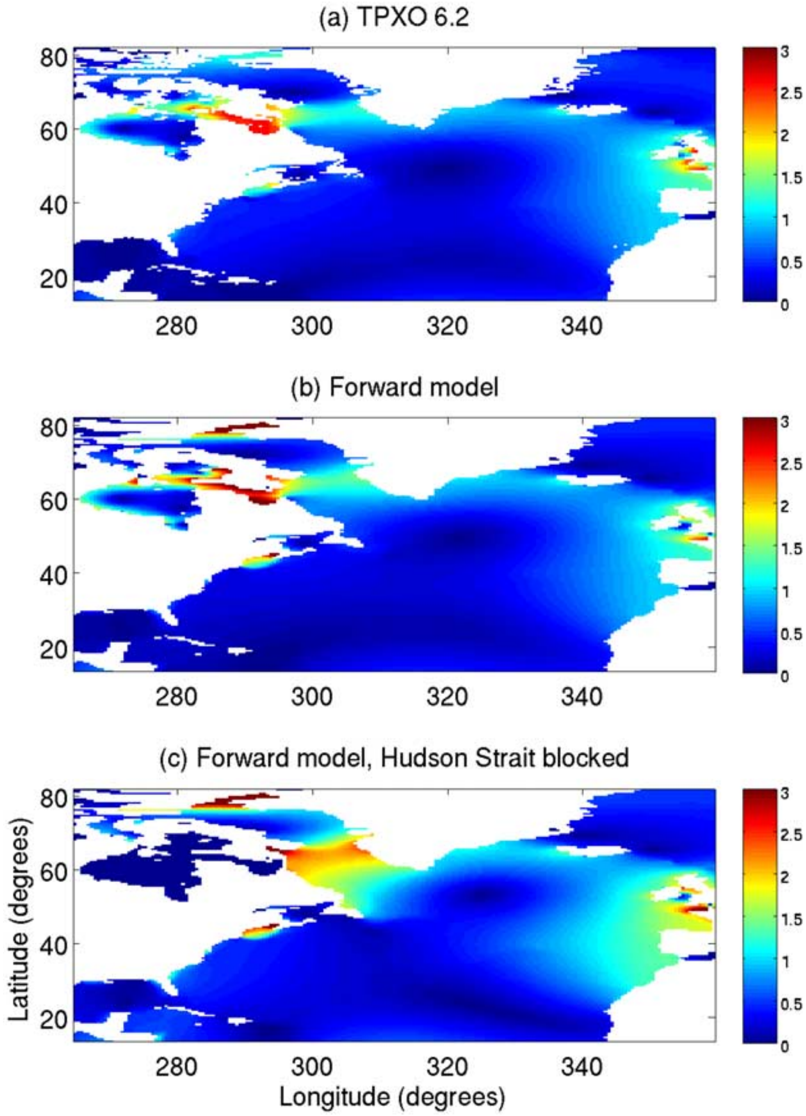

Figure 3. The M_2 tidal elevation in metres in the North Atlantic, (a) based on a tide model constrained by satellite altimetry [*Egbert et al.*, 1994], (b) from a forward global numerical model for the present geography, and (c) from this model with the entrance to Hudson Strait blocked.

changes in the Labrador Sea (Figure 3); the increase is consistent with the earlier finding [*Arbic et al.*, 2004] that such increases occur in response to a large decrease in global mean sea level, which effectively shuts off the Ungava/Hudson system. Substantially increased tides are seen even as far away as the British Isles, further reinforcing the point that the Ungava/Hudson system exerts a strong control on oceanic tides. (Note that the forward modeled tide in Nares Strait (west of Greenland) is much too large, as shown in Figure 3. The reason for this is currently under investigation but is not a matter of concern here as one simulation with Nares Strait carved out all the way to the Arctic, and another with the Strait completely filled in, show that the Labrador Sea tide is little affected by the state of Nares Strait.)

[9] We can allow for these changes by writing the observed tide outside the system as

$$A'(\omega) = A(\omega) + BA(\omega)R(\omega) \tag{3}$$

where $A(\omega)$ is the true forcing, $R(\omega)$ is the frequency response from (2), and B is a (complex) linear coefficient defined as

$$B = \left(\frac{A'_{\text{mod}}}{A_{\text{mod}}} - 1\right)R^{-1}(\omega) \tag{4}$$

where A'_{mod} and A_{mod} are the computed (complex) amplitudes at the outside station for the model runs with Hudson Strait open and blocked respectively. The factor $A'_{\text{mod}}/A_{\text{mod}} - 1$ is calculated from the model at the pelagic station at 54.17°N, 52.13°W. We now repeat our earlier analysis, but replacing AR in (1) with $A'R(1 + BR)^{-1}$. The results tend to give a resonant period slightly less than 12 hours, but this and the values of Q and c are sensitive to small changes in B and whether it is assumed constant (as one might expect to a good approximation for a radiating Kelvin wave) or whether the factor $A'_{\text{mod}}/A_{\text{mod}} - 1$ is determined separately from the model for each frequency.

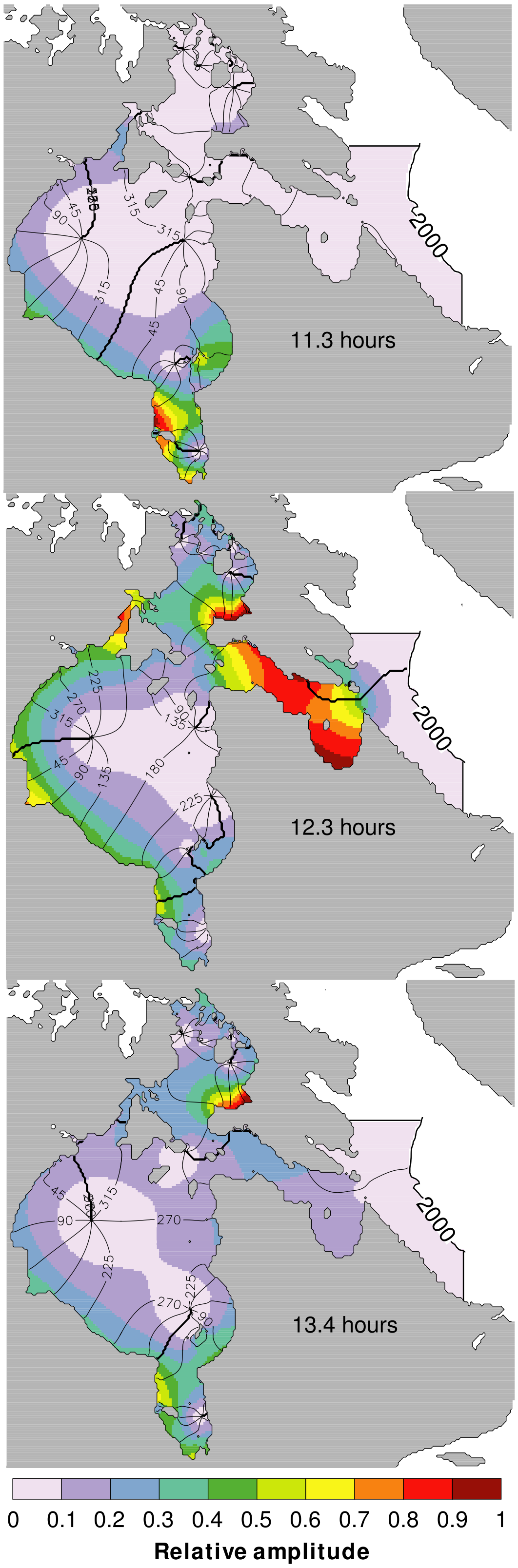

Figure 4. The phase (in degrees) and amplitude of near-semidiurnal period normal modes of the Ungava/Hudson system, with a boundary condition of zero elevation at the offshore 2,000 m contour and lateral boundaries crossing the continental shelf. The periods are as shown. The amplitude in each case is scaled to have a maximum of 1.

[10] The conclusion from the admittance analysis is thus that the resonant period is close to the 12.4 hours of the M_2 tide, but that an accurate determination of this period, and of Q and c, is made difficult by the complicated response of the North Atlantic to the back effect of the Ungava/Hudson system.

3. Normal Modes

[11] Our next approach is to use the shallow water equations of motion with a boundary condition of zero elevation at an open boundary in the Labrador Sea to calculate the normal modes of the Ungava/Hudson system which have periods close to semidiurnal [*Platzman*, 1975; *Gavino*, 1984]. As shown in Figure 4, there is a mode which is concentrated in Ungava Bay and Hudson Strait and has a period of 12.3 hours. (A computed mode with zero elevation at the western end of Hudson Strait, as well as on the Labrador Shelf, is very similar in appearance but with a slightly shorter period of 12.0 hours.) The other modes with periods near semidiurnal are much less concentrated in Ungava Bay and Hudson Strait. We therefore assume that the 12.3 hour mode is the one being excited by the tides, but note that the computed period does not include an "end correction". This is difficult to estimate for a complicated geometry but is always a small increase if the mode is radiating into a large ocean. Thus the numerical normal mode calculation indicates a resonant period close to, but slightly more than, the 12.4 hours of the M_2 tide. The model does allow for internal friction, but inaccurately as this is linearized. Moreover, it does not allow for radiative damping. Thus we cannot estimate Q or c from this approach.

4. Initial Value Problem

[12] Perhaps the most reliable way to establish the natural period of oscillation and its damping rate and damping mechanism is to initialize the global model with a disturbance inside the system close to that of the normal mode calculated above, and then let the disturbance freely oscillate and decay. Fitting a function $A_0 \exp(-rt) \cos \omega_0 t$ to the first day of output, where A_0 is the initial amplitude and t is time, to the model sea level in Leaf Basin gives a period $T = 2\pi/\omega_0$ of 12.7 hours for oscillations started at either low or high tide (see auxiliary material for details[1]). The damping rates r correspond to Q values of 4.3 and 4.9 respectively, and should apply to the observed tide, which has quadratic internal friction with drag coefficient 0.0025, as the initial amplitude is chosen to correspond to that of the observed tide. Our best estimates for the period and Q are therefore about 12.7 hours and 4.6.

[13] The damping is approximately 2/3 from internal friction and 1/3 from radiation. Thus of the total energy loss rate of 410 GW obtained from the initial energy content of 13.7×10^{15} J and a decay rate corresponding to a Q of 4.6, approximately 270 GW are lost to friction. This compares well with the 261 TW estimated from the TPXO model in *Egbert and Ray* [2001], albeit for a region that also

[1]Auxiliary materials are available in the HTML. doi:10.1029/2007GL030845.

included the northern Labrador Sea, though this is not likely to contribute much to the total.

5. Discussion

[14] Although uncertainty remains, it is evident that one reason for the extreme tides of Ungava Bay is that the Ungava/Hudson system is close to resonance, with the key mode mainly confined to Ungava Bay and Hudson Strait. If we take our best estimate of the resonant period to be 12.7 hours, then $1 - \omega/\omega_0$ in (2) is −0.02, considerably less in magnitude than the value of 0.11 for $\frac{1}{2}Q^{-1}$ if $Q = 4.6$. Thus the tidal amplitude is limited more by friction than by being off-resonance.

[15] Because the natural period of 12.7 hours is longer than the tidal period, some further amplification of the tides could occur if mean sea level were to rise, reducing the natural period and bringing the system even closer to resonance. Approximating the system by a bay with the mean depth of 154 m, it would take a sea level rise of 7 m to reduce the period from 12.7 hours to the M_2 period of 12.4 hours, but, this would only increase the tidal range by 2%, using (1) and (2), if we assume the same value for Q. A reduction of friction due to deeper water could, of course, increase Q, leading to a larger increase in the tides. Further modeling is required to evaluate this. If the natural period were to decrease to less than 12.1 hours, pushing the system farther away from resonance than at present it could lead to a decrease in tidal range. We also note that the change of 7 m required for a significant change in the resonant period is small compared with changes since the last ice age, so that tidal resonance is a comparatively recent phenomenon. This is consistent with the results on paleotides cited earlier.

[16] A large-scale tidal power project in Leaf Basin or other locations near the head of Ungava Bay could, by effectively shortening the system, also reduce its natural period and lead to an increase in the overall tidal range in the system, but this is likely to be very small, particularly as the natural frictional dissipation of about 270 GW is so large compared with the few GW that might be extracted.

[17] The behavior of the tides in the Ungava/Hudson system and their effect on global tides is made more interesting by the finding, from a frequency sweep with the global forward tide model, that the North Atlantic also has a natural resonant period near 13 hours (as suggested earlier by admittance analysis for Bermuda [*Garrett and Greenberg*, 1977; *Heath*, 1981; *Platzman*, 1991]). Interpretation and understanding of global and regional tides may be enhanced by further investigation in terms of coupled oscillators, with near-resonant oscillators in extensive shallow seas, such as the Ungava/Hudson system, having a significant influence on the near-resonant response of deep ocean basins.

[18] **Acknowledgments.** We thank Ron Solvason of the Canadian Hydrographic Service for providing tidal constants for the Canadian stations. B. K. A. thanks the National Science Foundation (OCE-0623159) and the Naval Research Laboratory (N000173-06-2-C003) for support. Computations for sections 2.2 and 4 were performed on the supercomputer cluster at NOAA/GFDL. P. S. thanks Canada's Natural Sciences and Engineering Research Council (NSERC). G. S. and C. G. thank NSERC and the Office of Naval Research.

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