Innovators and Imitators Versus the Bass Model

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Abstract

The Bass model is often thought to represent the diffusion path of a new product when the population is a discrete mixture of innovators who are not influenced by word of mouth and imitators whose adoption decisions are driven solely by word of mouth. Simple analytical expressions and the graphical illustration of a numerical example, however, show that the diffusion path represented by the Bass model is can be quite different from that in a discrete mixture of innovators and imitators. Distinguishing between the Bass model and a mixture model is not just an issue of interpretation, but does matter for descriptive, managerial, and methodological purposes.

For nearly three decades, marketing scientists have used the Bass (1969) model and many variants to understand the diffusion of new products. Over time, the Bass model has not only become "the main impetus underlying diffusion research in marketing" (Mahajan et al. 1993, p. 351), but has also become an important exemplar in marketing science as a discipline. The model has been called "one of the most frequently referred to marketing models (Lilien et al. 1992, p. 471) and possibly "the best-known example of an empirical generalization in marketing" (Uncles et al. 1995, p. G71). The Bass model has two main virtues. First, it fits many data well, though that by itself does not imply one understands what the model actually captures (Gatignon and Robertson 1986; Van den Bulte and Lilien 1997). The second main appeal of the model is its very simple structure that purportedly formalizes the insight from behavioral research that innovation diffusion is a two-step flow process. That is, media influence innovative opinion leaders to adopt, who in turn influence people imitating their behavior to adopt as well (Rogers 1995). The Bass model purportedly formalizes this process into a mathematical model, which—proponents of formal modeling claim—reduces the ambiguity inherent in verbal theories and facilitates the establishment of empirical generalization (Moorthy 1993). Of course, "mathematization" by itself by no means guarantees correct interpretation, though it facilitates critical analysis of assumptions and logic (Sokal and Bricmont 1997).

The objective of this note is to show that the Bass model does not reflect a two-step diffusion process in a population consisting of "innovators" and "imitators," and that this matters. The vivid contrast between the Bass model and a model that truly has innovators and imitators will hopefully end the scholarly confusion. I first present the traditional interpretation of the Bass model as a discrete mixture of two types of consumers, including evidence that some prominent modelers still adhere to this interpretation. Then, I present some characteristics of a diffusion process that truly takes place in a population consisting of a discrete mixture of innovators and imitators. I then contrast the two processes, and conclude with three specific implications.

1. The Bass model and its traditional interpretation

Bass (1969) assumed that innovation acceptance is driven in part by word of mouth. He therefore specified the limiting probability that someone who has not adopted yet does so at time $t+\Delta t$ (with $\Delta t \rightarrow 0$), often referred to as the hazard rate of adoption $h(t)$, as increasing with the proportion of the population that has already adopted at time t:

$$h(t) = p + q\,F(t) \qquad [1]$$

where $h(t)$ is the hazard function, $F(t)$ denotes the cumulative proportion of adopters in the population, and p and q are positive constants.

Bass (1969, p. 216) used the two-step flow hypothesis (Katz and Lazarsfeld 1955) as a behavioral rationale for his model. He posited that p captures the behavior of individuals deciding "to adopt an innovation independently of the decisions of other individuals," and labeled it the coefficient of innovation. Parameter q, on the other hand, was stated to capture the behavior of adopters who "are influenced in the timing of adoption by the pressures of the social system," and was labeled the coefficient of imitation. In other words, the original rationale for the Bass model hinged on the assumption that the total population was a discrete mixture of two groups whose adoption behavior was driven by two different causal processes: innovators with hazard function $h_1(t) = p$ and imitators with hazard function $h_2(t) = qF(t)$.

Today, many discussions still present the Bass model as a discrete mixture of innovators with hazard rate p whose adoptions are not influenced by word of mouth and imitators with hazard rate $qF(t)$ whose adoptions is driven solely by word of mouth (Table 1). A number of papers have appeared who show this to be false. Rather, these papers show,the Bass model assumes a homogeneous population whose adoption behavior is captured by the hazard $p + qF(t)$, but—as Table 1 documents—this rectification has often been neglected. Perhaps this is because none of these papers has appeared in a mainstream marketing outlet, but in conference proceedings (e.g., Bemmaor 1994; Jeuland 1979) or little-read journals such as the now-defunct *Swedish Journal of Economics* and *Mathematical and Computational Modeling* (e.g., Lekvall and Whalbin 1973; Steffens and Murthy 1992; Tanny and Derzko 1988). Whatever its cause, the fact is that many prominent authors still hold and propagate the erroneous belief that the Bass model is a discrete mixture model of innovators and imitators.

[Table 1 about here]

Of course, one might weasel-word oneself out of the problem by noting that, even though the Bass hazard rate $p + qF(t)$ applies to everyone, there will be some people adopting at time 0 when $F(t) = 0$ and everyone has hazard rate p. Hence, one might *a posteriori* call the people adopting at the very beginning of the process, when social influence is nil, "innovators" and label all later adopters as "imitators" (e.g., Hunt 1983, p. 201-202; Mahajan et al. 1990, p. 41). Such a re-definition of the terms innovator and imitator based on adoption time rather than causal drivers of adoption behavior does not conflict with the Bass specification, but—unfortunately—this is not the interpretation found throughout the literature. A Table 1 shows, Horsky and Simon (1983) or Day (1986) define innovators based on causal behavior rather than adoption time. The review paper by Mahajan, Muller and Bass (1993, p. 351), explicitly notes that "the Bass model conceptually assumes that 'innovators' or buyers who adopt exclusively because of mass-media communication or external influence are present at any stage of the diffusion process." Lilien, Kotler ad Moorthy (1992, p. 469) similarly imply that innovators adopt throughout the diffusion process, rather than only at the very beginning of the diffusion process.

Another ad hoc re-interpretation to align the innovator-imitator dichotomy with a mathematically correct interpretation of the Bass model is to posit that each individual is a mixture of an innovator and an imitator (Table 2), such that $p + qF(t)$ describes the behavior of each member of an homogenous population. Strangely enough, this "within-individual dichotomy" scenario, when mentioned, is often presented as a matter of mere interpretation rather than mathematical necessity.

[Table 2 about here]

2. Diffusion Hazards in a Discrete Mixture of Innovators and Imitators

So, if a product diffusing through a population consisting of innovators and imitators can not be represented by the Bass hazard function, how does its diffusion process really look like? Actually, modeling hazard rates in a population consisting of two types of actors has a long tradition (e.g., Cox 1959). Let the population be a discrete mixture of I different types, and let the proportion of actors of type i ($i = 1, \ldots, I$) in the population at the outset of the process ($t = 0$) be $\theta_i > 0$ ($\Sigma_i\ \theta_i = 1$). The cumulative distribution function for the time to adopt in this mixed population is then given by (Bain and Engelhardt 1991):

$$F_m(t) \ = \Sigma_i\ \theta_i\ F_i(t)\ . \qquad [2]$$

Similarly, the density function is also a weighted average given by

$$f_m(t) \ = \Sigma_i\ \theta_i\ f_i(t)\ . \qquad [3]$$

In contrast, the population hazard function is not a weighted average of the hazards of the mixture components, but is given by:

$$h_m(t) = [\, \Sigma_i\, \theta_i f_i(t)\,] \,/\, [\Sigma_i\, \theta_i\, (1\text{-}F_i(t))\,]$$

$$= [\, \Sigma_i\, \theta_i\, h_i(t)\, (1\text{-}F_i(t))\,] \,/\, [\Sigma_i\, \theta_i\, (1\text{-}F_i(t))\,]\,. \qquad [4]$$

Finally, the probability that an adoption at time *t* is by someone of type *k* given that it occurs at time *t*, say $\phi_k(t)$, is (Cox 1959):

$$\phi_k(t) = \theta_k f_k(\mathrm{t}) \,/\, [\, \Sigma_i\, \theta_i f_i(t)\,]\,. \qquad [5]$$

Applying these general results to our case of interest featuring two types of actors ($I = 2$), innovators with proportion θ and adoption hazard $h_1(t) = p$ and imitators with proportion $(1\text{-}\ \theta)$ and adoption hazard $h_2(t) = qF_m(t)$, or $qF(t)$ for short, leads to the following conclusions:

1. The Bass model with population hazard function $h(t) = p + qF(t)$ does not describe the diffusion process in a population consisting of two types of adopters with hazard functions $h_1(t) = p$ and $h_2(t) = qF(t)$, respectively (nor does it describe a process with $h_1(t) = p \,/\, \theta_1$ and $h_2(t) = (q \,/\, \theta_2)F(t)$). This follows from Eq. (4): $h_m(t) \neq \Sigma_i\, h_i(t)$.
2. The intercept of the density function and hazard function is lower for the mixture model than for the Bass model (i.e., p). This follows from Eqs. (2-4): $f_m(0) = h_m(0) = \theta p < p$.

Obtaining further analytic results is difficult because the imitators' adoption hazard is a function not of the proportion of imitators but of the proportion of the entire population (both innovators and imitators) that have adopted. Hence, the imitators' diffusion process is not a logistic process and closed form results are not available. Still, as illustrated below, numerical analysis suggests that the mixture model has a number of additional features not shared by the Bass model when q is sizably larger than p, as found in most applications of the Bass model.

3. The hazard does not monotonically increase with $F_m(t)$: it first increases, then decreases.
4. The hazard reaches a lower asymptote equal to p as $t \rightarrow \infty$.

5. The last adopters are more likely to be "innovators" rather than "imitators."

These results, especially the last one, may seem counter-intuitive at first, but are easy to explain. Once $F_m(t)$ is sufficiently large such that $qF_m(t) > p$, imitators that have not adopted yet do so with a higher likelihood than innovators who have not adopted yet. As a result, the numerator in Eq. (4) becomes dominated by the term $qF_m(t)$, and $h_m(t)$ increases with $F_m(t)$, especially if imitators make up a large proportion of the total population (i.e., θ is small). However, imitators' being more likely to adopt than innovators also implies that the product reaches saturation sooner among imitators. As the number of imitators who have not adopted yet depletes rapidly, imitators begin to account for a smaller proportion of all adoptions still occurring. Manipulating Eq. (5), one can see that once the ratio $f_2(t) / f_1(t)$ falls below $\theta/(1-\theta)$, adoptions are more likely to come from innovators rather than imitators. As $t \to \infty$ and $f_2(t)$ becomes negligibly small compared to $f_1(t)$, the probability that adoptions are made by innovators rather than imitators increases to 1. Hence, the last adopters are likely to be innovators rather than imitators, and the asymptote of the mixture hazard equals p. I emphasize that the last three results require q to be sufficiently larger than p. Very different patterns occur when p is about equal or larger than q.

3. Numerical Example

It is informative to plot and compare the hazard, density, and distribution function of the (homogeneous) Bass model with those of a mixture model consisting of both innovators and imitators. Figure 1 plots these functions using $p = .03$ and $q = .40$, values close to the average found in empirical applications of the Bass model. The differences between the mixture model and the Bass model are striking. The mixture hazard function has a very different shape than the

Bass hazard, and is always lower. As a result, the diffusion is much faster in a homogeneous population than in a mixture of innovators and imitators, for given values of *p* and *q*.

[Figure 1 about here]

Figure 2 plots the density functions for both models, but also varies the proportion of innovators in the mixture. As $\theta \rightarrow 0$, the mixture model increasingly resembles a purely imitative logistic model. Such a logistic model is similar in shape to the Bass model when *q*/*p* is high. However, the fewer innovators there are, the longer it takes before the imitation snowball effect gains critical mass among the large group of imitators. The net effect is that the diffusion process in the two models becomes increasingly similar as $\theta \rightarrow 0$, apart from a rightward shift of the mixture compared to the homogenous population. In other words, facing a mixed population with very few innovators rather than a homogenous population behaving according to the Bass model, does not much affect the diffusion path <u>once it takes off</u>, but results in a delay of the product's take-off time. Thus, for values of *p* and *q* found in typical marketing applications, the Bass model generates a diffusion path that is very different from a discrete mixture of innovators and imitators, and does so regardless of the value of θ. Managers believing that they face markets exhibiting a chasm characterized by pure innovators and pure imitators should therefore refrain from using parameters obtained with the Bass model to forecast the penetration path of their product or to normatively optimize their marketing resource allocation.

[Figure 2 about here]

4. Discussion

Figures 1 and 2 leave little doubt that the Bass model does not represent the diffusion of an innovation in a population consisting of innovators with adoption hazard *p* and imitators with adoption hazard *qF(t)*. The graphs clearly show that the homogeneity vs. dichotomy assumption is not an issue of mere interpretation, but that it generates quite different diffusion paths. Given the very good data fitting performance of the Bass model, this casts doubt that the two-step flow hypothesis applies to data series to which the Bass model has been fit successfully.

A second finding of this paper is to refute the claim by Mahajan et al. (1990) and Lilien et al. (1992, p. 469) that "as the process continues, the relative number of innovators diminishes monotonically with time. Imitators … increase relative to the number of innovators as the process continues." Either the Bass model is true, and there are no innovators and imitators, or these is indeed such a dichotomy, in which case the proportion of innovators does not decrease monotonically over time. This has obvious implications for optimal advertising policies: in a market consisting of pure innovators and imitators, there is less reason to decrease advertising over time than in a homogenous population, since the word-of-mouth snowball effect does not affect innovators, but only imitators.

Finally, this paper's findings also invalidate the procedure by Mahajan, Muller and Srivastava (1990) to use the Bass model to separate adoptions due to external influence from those due to internal influence. Separating adoptions based on an internal vs. external causal process requires the existence of two different pools of potential adopters, and is hence in conflict with the Bass model's assumptions. The only thing that the Bass model allows Mahajan and his associates to do (and what they actually do) is to compute an index of the relative importance of these two influences operating on each member of a homogenous population of adopters. Specifically, they construct a ratio variable with a rather complex expression which can be simplified to $p / [p + qF(t)]$. This alternative notation makes clear that what they construct is not the ratio of adoptions due to innovative tendencies at time t to the total adoptions due to both innovative and imitative tendencies at time t (Mahajan et al.'s interpretation), but the ratio of the importance of innovative tendencies to the entire set of influences driving people's decision to adopt at time *t*, when people are homogenous and behave according to the adoption hazard function $h(t) = p + qF(t)$. In other words, Mahajan et al.'s ratio does not capture the relative size of two different types of adoptions (innovative vs. imitative, or internally vs. externally induced), but the relative size of the innovative and imitative tendencies at work in the process of adoption in a homogeneous population. As these three implications show, getting the interpretation of the Bass model right matters for advancing the field of diffusion research in its theory, policy guidance, and methodology.

References

Bain, Lee J. and Max Engelhardt (1991*), Statistical Analysis of Reliability and Life-Testing Models, 2nd. ed*. New York: Marcel Dekker.

Bass, Frank M. (1969), "A New Product Growth Model for Consumer Durables," *Management Science*, 15 (January), 215-227.

Bemmaor, Albert C. (1994), "Modeling the Diffusion of New Durable Goods: Word-of-Mouth Effect versus Consumer Heterogeneity," in *Research Traditions in Marketing*, Gilles Laurent, Gary L. Lilien, and Bernard Pras, eds., Boston, MA: Kluwer Academic, 201-223.

Cox, D.R. (1959), "The Analysis of Exponentially Distributed Life-Times with Two Types of Failure," *Journal of the Royal Statistical Society, Ser. B*, 21 (2), 411-421.

Day, George S. (1986), *Analysis for Strategic Market Decisions*. St. Paul, MN: West.

Gatignon, Hubert and Thomas S. Robertson (1986), "Integration of Consumer Diffusion Theory and Diffusion Models: New Research Directions," in *Innovation Diffusion Models of New Product Acceptance*, Vijay Mahajan and Yoram Wind, eds., Cambridge, MA: Ballinger, 37-59.

Horsky, Dan and Leonard S. Simon (1982), "Advertising and the Diffusion of New Products," *Marketing Science*, 2 (Winter), 1-17.

Hunt, Shelby D. (1983), *Marketing Theory: The Philosophy of Marketing Science*. Homewood, IL: Irwin.

Jeuland, Abel P. (1979), "Epidemiological Modeling of Diffusion of Innovation: Evaluation and Future Directions of Research," in *Proceedings of the 1979 Educators' Conference*, Neil Beckwith et al., eds. Chicago: American Marketing Association, 274-278.

Katz, Elihu and Paul F. Lazarsfeld (1955), *Personal Influence: The Part Played by People in the Flow of Mass Communications*. New York: The Free Press.

Lekvall, Per and Clas Wahlbin (1973), "A Study of Some Assumptions Underlying Innovation Diffusion Functions," *Swedish Journal of Economics*, 75 (December), 362-377.

Lilien, Gary L., Philip Kotler and K. Sridhar Moorthy (1992), *Marketing Models*. Englewood Cliffs, NJ: Prentice-Hall.

------------ and Arvind Rangaswamy (1998), *Marketing Engineering: Computer-Assisted Marketing Analysis and Planning*. Reading, MA: Addison-Wesley.

Mahajan, Vijay, Eitan Muller and Frank M. Bass (1993), "New-Product Diffusion Models," in *Marketing*, J. Eliashberg and G.L. Lilien, eds. Amsterdam: North-Holland, 349-408.

------------, ------------, and Rajendra K. Srivastava (1990), "Determination of Adopter Categories by Using Innovation Diffusion Models," *Journal of Marketing Research*, 27 (February), 37-50.

Moorthy, K. Sridhar (1993), "Theoretical Modeling in Marketing," *Journal of Marketing*, 57 (April), 92-106.

Rogers, Everett M. (1995), *Diffusion of Innovations*, *4th ed*. New York: The Free Press.

Sokal, Alan D. and Jean Bricmont (1997), *Impostures intellectuelles*. Paris: O. Jacob.

Steffens, P.R. and D.N.P. Murthy (1992), "A Mathematical Model for New Product Diffusion: The Influence of Innovators and Imitators," *Mathematical and Computational Modeling*, 16 (4), 11-26.

Tanny, S.M. and N.A. Derzko (1988), "Innovators and Imitators in Innovation Diffusion Modeling," *Journal of Forecasting*, 7 (4), 225-234.

Uncles, Mark, Andrew Ehrenberg and Kathy Hammond (1995), "Patterns of Buyer Behavior: Regularities, Models, and Extensions," *Marketing Science*, 15(3, pt. 2), pp. G71-G78.

Van den Bulte, Christophe and Gary L. Lilien (1997), "Bias and Systematic Change in the Parameter Estimates of Macro-level Diffusion Models," *Marketing Science*, 16 (4), 338-353.

Table 1. The innovator-imitator interpretation of the Bass model is still dominant

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“ [Diffusion] research implies the existence of two groups of new product buyers: those who adopt the product independently of others—the innovators—and those who are influenced by others—the imitators. These premises will also serve as the basis of our model…. [A hazard rate model of the form h(t) = $p + b$ ln(advertising(t)) + $qF(t)$] represents the *average* probability of purchase across the population of nonadopters, which is composed at any point in time of both innovators and imitators.”

Horsky and Simon (1983, pp. 2-3).

“The basic notion is that in every time period, there will be both innovators and imitators buying the new product. Innovators are not influenced by promotional activity. As the diffusion process continues, the relative number of innovators diminishes. This tendency is offset by the imitators who are influenced by the number of previous buyers and thus become increasingly important as time passes.”

Day (1986, p. 87)

“[T]he Bass model assumes that potential adopters of an innovation are influenced by two means of communication—mass-media communication and word-of-mouth communication. In its development, it further assumes that the adopters of an innovation comprise of two groups. The first group is influenced only by the mass-media communication (external influence) and the second group is influenced only by the word-of-mouth communication (internal influence). Bass termed the first group ‘innovators’ and the second group ‘imitators’.”

Mahajan et al. (1993, p. 351)

“The first term, $p(m-N(t))$ … represents adoptions due to buyers who are not influenced in the timing of their adoption by the number of people who have already bought the product … The second term $(q/m)N(t)(m-N(t))$ … represents adoptions due to buyers who are influenced by the number of previous buyers.”

Mahajan et al. (1993, pp. 353-354)

“Key elements in the Frank Bass (1969) forecasting model are (1) adopters due to mass media messages (p), (2) adopters due to interpersonal communication channels (q), and (3) an index of market potential (m) for the new product.”

Rogers (1995, p. 81)

“[$qF(t)$] … represents the extent of favorable interactions between the innovators and the other adopters of the product (imitators).”

Lilien and Rangaswamy (1998, p. 198)

Table 2. Some authors make present the homogeneity vs. dichotomy assumption in the Bass model as a matter of interpretation rather than mathematical consistency

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[A hazard rate model of the form h(t) = $p + b$ ln(advertising(t)) + $qF(t)$] represents the *average* probability of purchase across the population of nonadopters, which is composed at any point in time of both innovators and imitators. The specification of the average probability does not preclude the possibility that if no such totally distinct segments exist, each individual is a mixture of an innovator and an imitator and accepts information from all sources."

Horsky and Simon (1983, p. 3)

"One traditional interpretation of this model has both innovators and imitators buying the product. The innovators are not influenced in their purchase timing by the number of persons who have already bought … Imitators are influenced by the number of previous buyers … Another, more recent interpretation has 'internal' and 'external' influences going on simultaneously, even within the same individual."

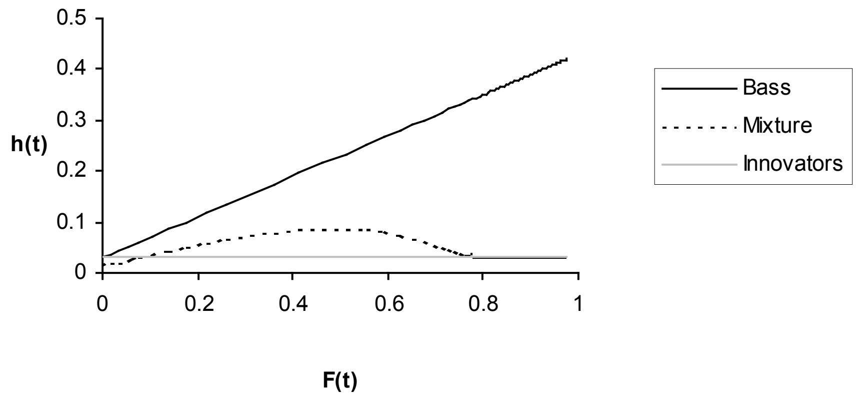
Lilien et al. (1992, p. 469)

"[A] question can be raised as to whether the Bass model really captures the communication structure between two assumed groups of adopters of 'innovators and 'imitators'."

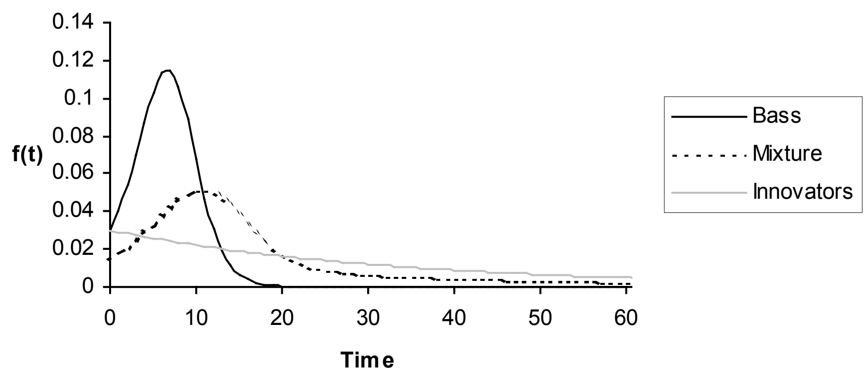
Mahajan et al. (1993, p. 355)

Figure 1. Comparing the Bass and Mixture Models (p = .03, q = .40, θ = .50)

1a. Hazard functions



1b. Density functions



1c. Cumulative distribution functions

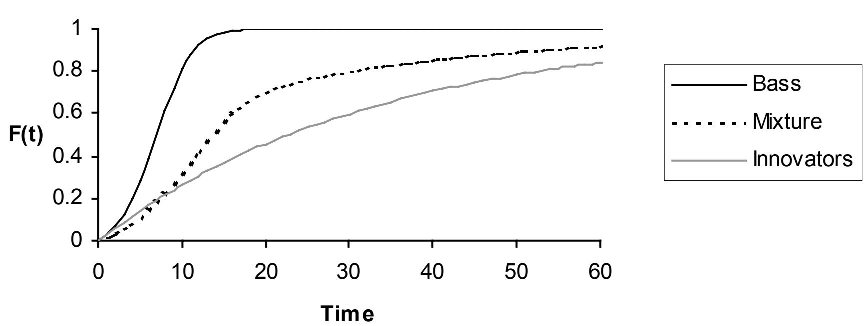
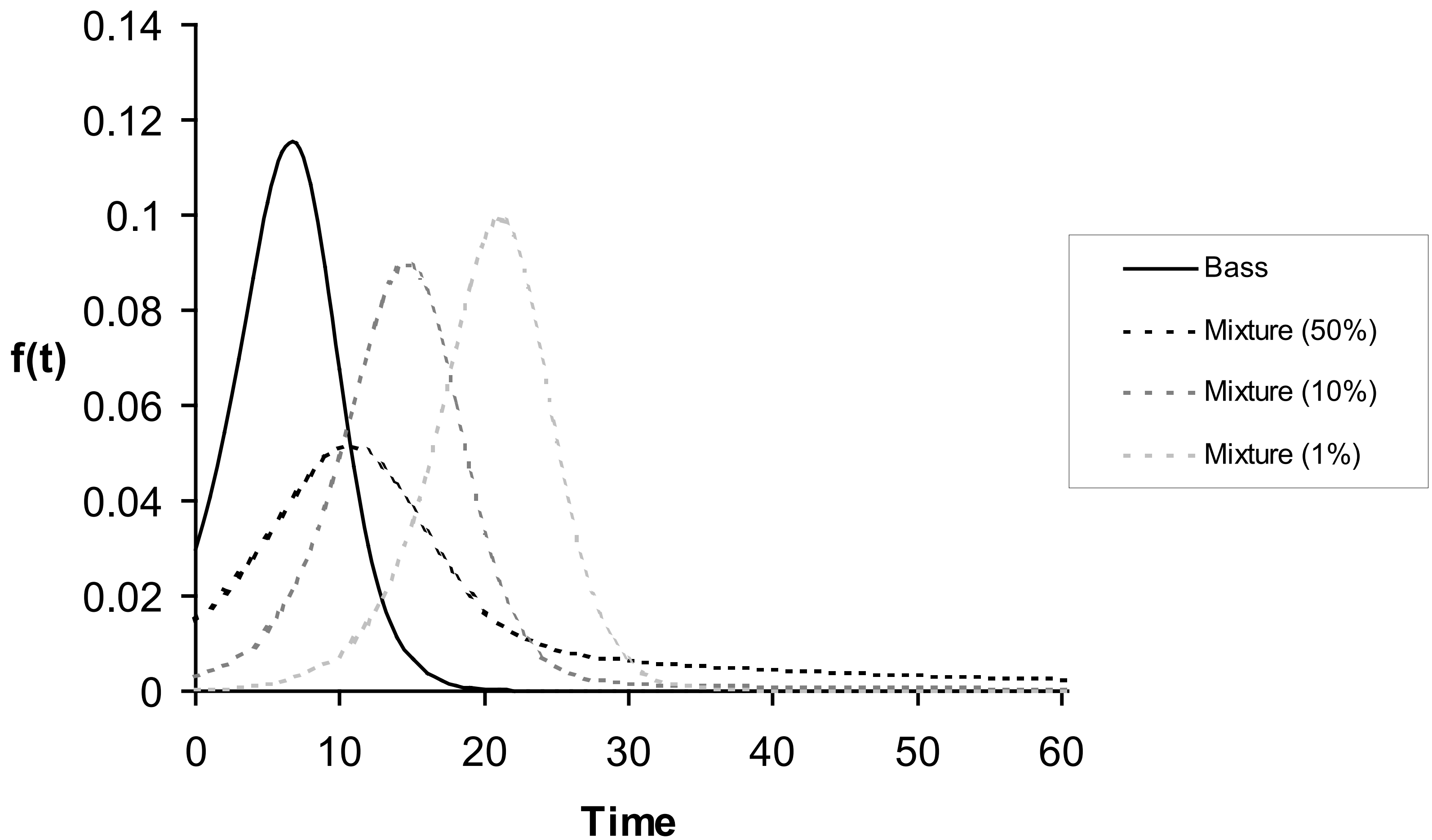


Figure 2. How the proportion of innovators (θ x 100%) affects the diffusion path in a mixed population



Note: p = .03, q = .40