REVEALING THE NOTION OF STATISTICAL LITERACY WITHIN THE PISA RESULTS

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ABSTRACT

In a globalized world, international evaluations of students´ performance influence political and economical plans. However, the perspectives which support those studies do not consider specific aspects which can provide different interpretations to students´ responses. In this paper we discuss data from the 2003 results for the Programme for International Student Assessment (PISA). We identified that survey items were elaborated only from the mathematical literacy point of view; therefore there is no explicit consideration to statistical literacy as component to be evaluated. In order to discuss this aspect, we present theoretical elements and an empirical analysis based on the PISA 2003 technical report which help to analyse the relationship between mathematical and statistical literacy. In the concluding remarks we emphasise the necessity to reveal statistical literacy within the PISA results.

INTRODUCTION

Over the last decades, statistics has become recognized as distinct discipline. While statistics is a mathematical science, like algebra or geometry, it is no longer seen as a subfield of mathematics (Moore, 2004; Watson, 2006). It applies mathematical tools, however even this is not unique to statistics, e.g. architecture, designing, art. At the same time, there are core ideas, such as variation, data, uncertainty, which are not mathematical in nature. Kline (1985: 501) states that mathematics has to do with certainty, while statistics is a way to handle uncertainty. Whereas the former predicts what must happen in an individual case, the latter can tell us what happens to large groups but does not provide definite predictions about any one given case.

We have to consider that within the field of mathematics education there was – and is – an impressive growth on the conception of mathematics education and even on the notion of mathematics itself. Critical voices on the absolutist view on mathematics have given rise to new conceptions of mathematics and its implications to the field of education. The absolutist view on mathematics is defined by Ernest (1991: 7) as "it consists of certain and unchallengeable truths. According to this view, mathematical knowledge is made up of absolute truths, and represents the unique realm of certain knowledge […]". Beside the absolutist view on mathematics, a number of alternate schools in the philosophy of mathematics arose. Humanistic mathematics, for example, brings in an element of fallibility. Constructivists would stress the fact that mathematics is a product of the human cognition. Intuitionists envisage that mathematics is built up from an empirically neutral mental basis (Van Kerkhove, 2007: 184). Bishop (1988: 18) argues that mathematics can be seen as *essentially a symbolical technology* based on skills or environmental activities of a cultural nature. Another theory on mathematics is rightly labelled *ethnomathematics*, as we see it. Proponents of the naturalistic approach to mathematics claim that mathematical knowledge is rooted in the cultural context of the knower (Pinxten & François, 2007: 214). Mathematics has no longer the statue of an absolute certain discipline. In this sense, it comes closer to the identity of statistics.

The understanding that statistics is not just mathematics has given rise to a new conception of statistical literacy and to a new field of study which is called statistics education. It is closely related to mathematics education, however it is not identical. In this paper we want to emphasise both the similarities and the diversity, which could be seen as paradoxical. Our research is based on the recognition that statistics is not a subfield of mathematics. At the same time we have to recognize that there is a strong relationship between mathematical and statistical literacy. This quasi paradoxical fact generates the question "What can both disciplines learn from each other?" (François & Bracke, 2006). In the following section, we first elaborate on the notion of statistical literacy and its historical growth.

THE NOTION OF STATISTICAL LITERACY

The term statistical literacy can be used to describe the knowledge which people need in order to understand and make decisions based on the analysis of statistics. Haack (1979) states that in order to interpret statistics people need to consider and to scrutinise certain aspects which include the source, the type of data, definition and measurement problems, and certain considerations concerning survey sample. As with most authors who began to develop the concept of statistical literacy, Haack emphasises elements which are basically related to the technical dimension of statistics knowledge. This perspective of statistical literacy seems to be based on accepted academic uses of statistic.

Different authors introduce wider perspectives of statistical literacy related to kinds of statistical skills which are needed by people in everyday life (e.g., Evans, 1992). Wallman (1993: 1) states in her Presidential Address to the American Statistical Association that "statistical literacy is the ability to understand and critically evaluate statistical results that permeate our daily lives–coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions." If we compare this wider perspective on statistical literacy, given by Wallman (1993), with the description of *mathematical literacy* given by the Programme for International Student Assessment (PISA) 2003 establishment, we can see a high correspondence. "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (Organisation for Economic Co-operation and Development, OECD, 2004: 37). In line with this definition, Gal (2002, 2004) emphasises the need for statistical literacy for all citizens who interpret statistics in various everyday situations. For example, Gal suggests that when people read statistics from media they have to make inferences, quite often in the presence of irrelevant or distracting information, and perhaps they also have to apply mathematical operations to data contained in graphs. Figure 1 illustrates Gal's perspective of statistical literacy.

Figure 1. A Statistical Literacy Model, Adapted from Gal (2002).

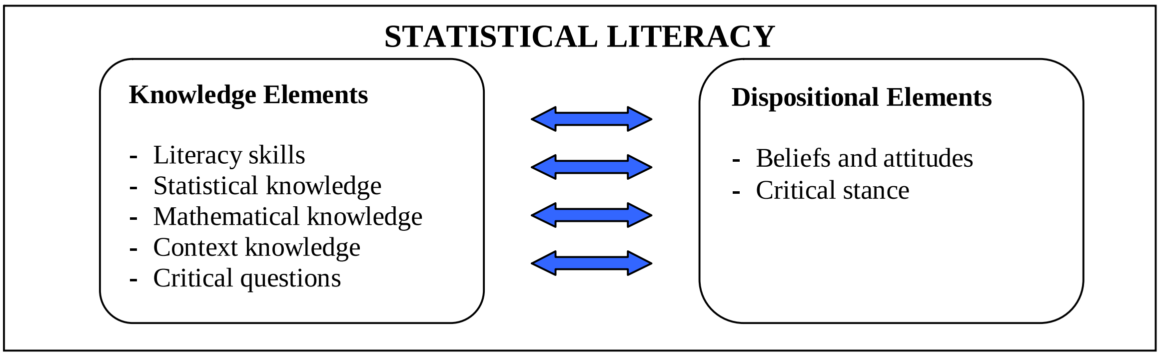

Figure 1 represents two ranges of elements which when combined can enable readers to understand statistical messages. On one side of the diagram there are *knowledge elements* which involve *cognitive* components of the statistical literacy (e.g., rational understanding of the data such as knowing how to decode and make calculations about it). On the other side *dispositional elements* are presented which comprise a range of 'non-cognitive' aspects (e.g., a person who interprets a graph can have knowledge, experiences and beliefs which might differentiate his/her interpretation of the graph). According to Gal, statistical literacy is based on the interaction of the components which comprise each range of elements. Gal's statistical literacy model underlines that the academic or formal schooling background is not the only determinant of use of statistical skills, as it was discussed in other studies (e.g., Monteiro, 2005). To develop statistical literacy, it may be necessary to work with learners in ways that go beyond instructional methods currently in use. To implement all knowledge bases supporting statistical literacy, topics and skills that are normally not stressed at school may have to be addressed (Gal, 2004: 73).

The increasing attention to statistical literacy also raises certain discussion points. For example, Carvalho (2001) emphasises that several authors (e.g., Wallman, 1993) view statistical literacy as a panacea to solve lack of statistical knowledge in several sectors of the society. However, Carvalho highlights the need to discuss issues related to development and transferability of statistical literacy. One important issue related to the role of statistical literacy is associated with the development of active and critical citizens who can read and interpret statistics making connections to different areas and reading the world and its complexity. Therefore, the statistical literacy should enable people to read more than only the data but it should allow people to criticise and propose alternative interpretations to a given set of data. The school systems have a crucial role to develop statistical literacy which enables students to understand why and how statistics is a kind of description of the world (Frankenstein, 1998; Moreira, 2002). Despite the diversity of perspectives and the emergent issues related to statistical literacy, mathematics and statistics educators started to agree that statistical literacy is a specific area beside mathematics literacy.

CORRELATION BETWEEN MATHEMATICAL AND STATISTICAL LITERACY

In this section, we discuss the relationship between mathematical literacy and statistical literacy on empirical data from PISA 2003 survey. Such relationship between these variables is not discussed on standard PISA 2003 reports (e.g., OECD, 2004).

Why PISA 2003 data?

For our purposes, the empirical data from PISA *2003* survey is most interesting because its main focus was on mathematical literacy where the main focus in PISA *2000* and PISA *2006* was respectively on reading and scientific literacy.

Research question

We are interested in the interrelations of the different subscales of mathematical literacy for the PISA 2003 data. In order to investigate the specificity of statistical literacy, we particularly want to answer the question whether the relationship between the *uncertainty* subscale –which is very clearly tied to statistical literacy– and other subscales differs from the other interrelationships (e.g . with reading and science).

Methodology

We are using PISA 2003 data which already exists. What is being undertaken is a secondary analysis of the results from that data/study. Data are available at the OESO databank (OECD, 2005). PISA result scores use ‘Raw Test scores’ based on the techniques of modern item response modelling. We will investigate the research question with an analysis by two variables:

1) at the level of the country;
2) at the level of the individual (pupil level).

The analysis/testing will be done by *Pearson product-moment correlation coefficients.* PISA 2003 results do not have an explicit consideration to statistical literacy as a specific component to be evaluated. Statistical literacy knowledge is only approached from the point of view of mathematical literacy. Within the PISA survey, four areas of mathematics are defined, each of them corresponding to a subfield of mathematics:

1) space and shape, related to spatial and geometric phenomena and relationships,
2) change and relationship scale, related to mathematical manifestations of change, functional relationship and dependency among variables,
3) quantity scale, related to numeric phenomena and quantitative relationship and patterns, and
4) uncertainty scale, related to probabilistic and statistical phenomena and relationships (OECD, 2004).

The first area often draws on the curriculum of geometry, the second relates most closely to algebra, the third is most associated with arithmetic and the fourth area phenomena are the subject of statistics and probability. Hence, the uncertainty scale is very clearly tied to statistical literacy. In the following part we will refer to this scale as "*statistical literacy*" (we include quotation marks to indicate that this is not the original name of the subscale). We will investigate the research question with an analysis by two variables:

1) at the level of the mean performance of all participating country;
2) at the level of the mean performance of all individual participating students.

At the first level (countries), we analyse the interrelations of the four subscales of mathematical literacy: *space and shape*, *change and relationship*, *quantity*, and "*statistical literacy*". Next, we relate each scale to the general mathematics score: *mathematical literacy¹*, which refers to the original PISA 2003 mathematical literacy scale. Finally, we relate each scale to a weighted mathematics score. To exclude the correspondent subscale part from the general mathematics score, we created a variable *mathematical literacy²* that calculates –for each country– the weighted sum of three subscales (excluding the corresponding subscale).

Analysing the data associated with the student, we concentrated on the correlations between the four mathematical scales, based on the data available in the *Technical Report* (OECD, 2005).

For both variables (countries and students), we also related the mathematical literacy scales to the *reading* and *science* scale.

Data Analysis

The data which we analysed are available at the OESO databank (OECD, 2005). PISA result scores use 'Raw Test scores' based on the techniques of modern item response modelling. This makes it possible to construct a scale of mathematical performance, to associate each assessment item with a point score on this scale according to its estimated difficulty and to assign each student a point score on the same scale representing his or her estimated ability.

To facilitate the interpretation of the scores assigned to students, the scale was constructed to have an average score among OECD countries of 500 points, with about two-thirds of students across OECD countries scoring between 400 and 600 points (OECD, 2004: 45). To illustrate these data, in Table 1 we present the ranking for the first ten countries (De Meyer, Pauly, & Van de Poele, 2004; OECD, 2004).

Table 1. Ranking of countries by mean performance on the mathematics subscales (OECD, 2004)

Space and Shape		Change and Relationship		Quantity		"Statistical literacy"	
Country	*Mean*	*Country*	*mean*	*Country*	*Mean*	*Country*	*Mean*
Hong Kong	558	Netherlands	551	Finland	549	Hong Kong	558
Japan	553	Korea	548	Hong Kong	545	Netherlands	549
Korea	552	Finland	543	Korea	537	Finland	545
Switzerland	540	Hong Kong	540	Liechtenstein	534	Canada	542
Finland	539	Liechtenstein	540	Macao-China	533	Korea	538
Liechtenstein	538	Canada	537	Switzerland	533	New Zealand	532
Belgium	530	Japan	536	Belgium	530	Macao-China	532
Macao-China	528	Belgium	535	Netherlands	528	Australia	531
Czech Republic	527	New Zealand	526	Canada	528	Japan	528
Netherlands	526	Australia	525	Czech Republic	528	Iceland	528

In Table 2 we present the correlations between the mean performance on the four scales for all participating country (n=40).

Table 2. Correlations between the mean performance on the four scales for all participating countries (n=40).

Subscale	Space and Shape	Change and Relationship	Quantity	"Statistical Literacy"
Space and Shape	1,00			
Change and Relationship	0,98	1,00		
Quantity	0,98	0,98	1,00	
"Statistical Literacy"	0,94	0,97	0,95	1,00
Mathematical Literacy[1]	0,99	0,99	0,99	0,98
Mathematical Literacy[2]	0,98	0,99	0,98	0,96
Reading	0,90	0,94	0,92	0,96
Science	0,96	0,97	0,95	0,94

Note: *Mathematical Literacy*[1] refers to the original PISA 2003 mathematical literacy scale. *Mathematical Literacy*[2] refers to the weighted sum of the three subscales, each time excluding the corresponding subscale in the column.

The extremely high correlations in Table 2 are remarkable. All subscales are very highly correlated. These values suggest that there is no specificity for the "statistical literacy" subscale. Although the correlation between the subscale *"statistical literacy"* and general *mathematical literacy*[2] (excluding the corresponding variable) is the lowest of all mathematical subscales, it is still extremely high (r = 0.96). The PISA 2003 *reading* and *science* scales show –as expected– the lowest correlations with all Mathematics subscales. Still, the correlations are very high.

In Table 3 we present the correlations between the mean performance on the four scales for all participating students (n = 276 165). For this analysis, we use the available data from the Technical Report (OECD, 2005: 190).

Table 3. Correlations between the mean performance on the four scales for all participating students (n = 276 165) (OECD, 2005: 190).

Variable	Space and Shape	Change and Relationship	Quantity	"Statistical Literacy"
Space and Shape	1,00			
Change and Relationship	0,90	1,00		
Quantity	0,90	0,93	1,00	
"Statistical Literacy"	0,89	0,92	0,90	1,00
Reading	0,68	0,74	0,73	0,74
Science	0,74	0,77	0,76	0,78

As was the case at the country level, at the student level all subscales are very highly correlated. The magnitude of the correlation is similar (around 0.90) for all subscales, again indicating no specific behaviour for the "statistical literacy" subscale.

Correlations between the mathematical literacy subscales and the PISA 2003 *reading* and *science* scales are lower in magnitude but also rather high for all subscales (ranging between 0.68 and 0.78).

CONCLUSIONS AND FURTHER RESEARCH

Statistical literacy knowledge is often seen only from the point of view of mathematical literacy. One of the best examples is the PISA research where items related to probabilistic and statistical phenomena and relationships appear under the cover of a mathematical 'uncertainty' scale. Looking at the differences and at the similarities between mathematical and statistical literacy at the theoretical level, there is an increasing tendency to agree that statistical literacy is a specific area which is delineating beside mathematics literacy. On the other hand, looking at the PISA 2003 data, we recognize an extremely high relationship between statistical and mathematical literacy for both variables (country and student). These data show that countries and students that have a high (or low) score on (subscales of) mathematical literacy in general also have a high (or low) score on statistical literacy.

A first hypothesis for this high relation is that there is one factor we could call *general literacy* behind the data. Educators in general have to take up the challenge to prepare students to be literate consumers, statistically and mathematically. Mathematical and statistical literacy are highly related. Mathematics teachers and statistics teachers are challenged in the same way. In this sense we want to stay that a reciprocal influence between mathematics and statistics educators should be fruitful.

A second hypothesis for the high relation between statistical and mathematical literacy within PISA 2003 data is the emphasis on *application* within the PISA survey. In future research, comparing statistical and mathematical literacy in the PISA survey with The Trends in International Mathematics and Science Study (*TIMSS*), where *calculation* rather than *application* is more central, could give us further interesting insights.

While in literature statistics has become recognized as distinct discipline, it has not in the international comparative PISA research which assesses how far 15-year-olds in schools –near the end of compulsory education– have acquired some of the knowledge and skills that are essential for full participation in society. However, looking at the different sections of mathematical literacy, the section in PISA on *uncertainty* is very clearly tied to statistical literacy. Why shouldn't theoretical developments of statistical literacy –as a distinct discipline– become attuned to contemporary international research on pupils' skills? Why shouldn't we reveal the notion of statistical literacy within the PISA results?

Further research is recommended at the level of the students' scores. Since about 90 per cent of performance variation occurs within countries, country averages give only part of the picture (OECD, 2004: 60). Furthermore, an analysis at lower level is needed. Subscales other than *Uncertainty* also include aspects related to statistics. For instance, all items that require combinatorics are included in the *Quantity* subscale and some (graphs of) relationships in the *Change and Relationship* subscale have a statistical nature. Therefore we could say that PISA results should reveal the notion of statistical literacy within the interpretation of results and in PISA reports.

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