# New Mathematical Disciplines and Research in the Wake of World War II

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This paper focuses on the significance of the Second World War for the rise and establishment of new disciplines in applied mathematics as well as for the renewed interest and growth in some related subjects in pure mathematics. The mathematical topics involved are mathematical programming, operations research, game theory, the theory of convexity, and the theory of systems of linear inequalities. Connections and interactions between different branches of mathematics on the one hand and between different kinds of driving forces in the development of mathematics on the other hand are discussed. Special emphasis is devoted to the significance of the interplay between practical problem solving and basic research in mathematics proper as a consequence of World War II and the post-war organization of science support in the USA.

## 1 Introduction

The significance of the Second World War for the directions of – and growth in – scientific research in the USA was immense. In the case of mathematics the war gave rise to the emergence and establishment of new disciplines in applied mathematics as well as a renewed interest and growth of research in related subjects in pure mathematics.

The purpose of this paper is to discuss and analyze how mathematical programming, game theory and operations research in applied mathematics,[1] and the theories of convexity and systems of linear inequalities in pure mathematics either came into being or benefited from the scientific mobilization and the subsequent post-war organization of science support in the USA.

It is a story that shows how these particular fields of mathematics evolved through interactions between practical problem solving and basic research brought about by the American post-war military-science symbiosis. It is also a story that illustrates the influence of prominent scientists on the kind of research done and thereby also the kind of knowledge gained.[2]

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[1] Whether operations research is a field of applied mathematics or not will be dicussed in Sect. 5.

[2] I would like to thank Barbara Green and David Volckmann for correcting my English.

## 2 The Struggle to Involve Mathematics in the War Effort

### The scientific mobilization

The American mobilization of science in World War II was spearheaded by Vannevar Bush, who served as President of the Carnegie Institution from 1939, James Conant and Karl T. Compton, President of Harvard and MIT respectively, and Frank Jewett from AT&T's Bell Laboratories. In June 1940 Bush discussed his plan for the organization of scientists for the war effort with President Roosevelt. The result was the establishment of the National Defense Research Committee (NDRC) whose scientific personnel besides Bush, Conant, Jewett, and Compton also included the physicist Richard Tolman from Caltech. The Army and the Navy were represented by Brigadier Admiral George V. Strong and Rear Admiral Harold G. Bowen respectively. The last person in the committee was Coe who was the commissioner of patents.[3]
NDCR operated independently of the military, which meant that it could initiate research projects that had not been requested by the armed forces. In 1941 the Offce of Scientific Research and Development (OSRD) was established with Bush as the leader. OSRD was financed by the Congress and unlike NDRC it had the power not only to initiate research projects but also to actually construct new types of weapons and develop new types of defense systems. Bush wanted to overcome the shortcomings he found in the research as it was conducted within the military which he found ineffective and second rate, ruled by military people with very little appreciation for science and restricted by internal rivalries between the different military departments. His vision was to build a cooperation between the military, industry, and universities that would render scientific research in defense and weapon development more effcient. The way it worked was that the scientists employed by OSRD remained civilians. They were bound to OSRD by contracts and they did their work not in military laboratories but in the universities and in industry.[4] This organizational structure reflects Bush's wish that the science and the scientists should not be controlled by the military. OSRD functioned as a mediating link between the military and the scientists.

### The Applied Mathematics Panel

The people who took the initiative and were responsible for the organization and mobilization of civilian scientists had academic backgrounds in electrical engineering, chemistry, and physics – mathematicians were conspicuous by their absence. There were no mathematicians represented, and there was originally no section for mathematics in either NDRC or OSRD. This does not mean that no

[3] [Zachary, 1997].
[4] [Zachary, 1997].

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mathematician contributed to the war effort; there just was no joint coordination of the services of mathematicians. There were mathematicians working in different military establishments, but it was not until the end of 1942 – well into the war – that an Applied Mathematics Panel (AMP) was created within NDRC. Warren Weaver, the director of the Division of Natural Sciences of the Rockefeller Foundation, was appointed the leader of the panel. Weaver had experience with applied mathematics from his time as professor in the mathematics department at the University of Wisconsin. Before he became the leader of the AMP he had been head of the fire control analysis section of NDRC.[5]

The importance and the success of the AMP has been reported several places, but as pointed out by Larry Owens in his essay *Mathematicians at War: Warren Weaver and the Applied Mathematics Panel, 1942–1945* the panel also had its failures. The story Owens tells shows "the empirical ambitions of those who, like Vannevar Bush, James Conant, and Warren Weaver, took the lead in the mobilization of wartime science."[6] The relatively late involvement of mathematicians should not be taken as a sign that mathematicians weren't interested in offering their services. On the contrary the leaders of the two mathematical societies, AMS and MAA, were very much aware that the expected benefits that would eventually accrue to the different sciences depended on their perceived relative contributions to the war effort. In February 1941 Marston Morse, who was the president of the AMS, presented a paper *Mathematics in the Defense Program* before the National Council of Teachers of Mathematics where he talked about the importance of mathematics in a war situation and how mathematics could be applied. He directed attention towards the first step made by mathematicians to bring mathematics to the service of the country: "About a year ago these societies [AMS and MAA] appointed a committee known as the War Preparedness Committee, to prepare the two societies to be useful to our nation in time of war."[7] The War Preparedness Committee included such able mathematicians as John von Neumann and Norbert Wiener.

Despite attempts to bring the effort of the AMS to the notice of OSRD, mathematics was still not implemented in OSRD, and in March 1942 Morse and Marshall Stone from Harvard University presented Bush, Conant, and Jewett with the memo *Mathematics at War.* As a result a Joint Committee of Mathematics with Marston Morse as chairman was established under the National Academy of Science and the National Research Council. But mathematicians still suffered from lack of recognition, and it got even worse when *Time Magazine* in 1943 – that is after the establishment of the AMP – quoted the science writer George Gray as saying that "the U.S. has been severely handicapped by its shortage of topflight mathematicians."[8] Morse got very angry and wrote back that "The actual fact is that the deficiency lies [...] in the failure of the civilian authorities to use mathe-

[5] [Owens, 1989], [Rees, 1980].

[6] [Owens, 1989, p. 289].

[7] [Morse, 1941, pp. 293–294].

[8] *Time Magazine*, November 29, 1943, quoted in [Owens, 1989, p. 293].

matics at an early time, in adequate numbers and in the proper way."[9] When mathematics through AMP finally was implemented in NDRC the leaders of NDRC asked Weaver to be the chief of the panel without first discussing it with either the AMS or the Joint Committee. Karl T. Compton wrote to Weaver in October 1942 asking him to take on the appointment as chairman of AMP and gave the following assesment of the Joint Committee:

> [...] thus far the mathematicians as a group have not been brought into the war picture very effectively. For example; the committee headed by Marston Morse has not been effective; I think this is because it is too "pure."[10]

The fact is, according to Owens, that "the leaders of OSRD [...] seem to have expected little from the committee [the Joint Committee chaired by Morse] and doubted that a comfortable working relationship with the AMS was generally possible."[11] The reason why the leaders of OSRD apparently did not expect very much of the Joint Committee headed by Morse seems to be buried in the dichotomy between pure and applied mathematics. Ward Davidson from NDRC reported after a first meeting with the Joint Committee that he could not "bridge the wide gap between the view of an engineer and those of a 'pure' mathematician. My imagination just didn't go far enough to understand how problems that seem to me to be rather practical could be handled effectively in the quite rare atmosphere of abstract mathematical thinking."[12]

Whether these problems of overcoming the breach between applied and pure mathematics were self-fulfilling or not, it seems that Weaver had some problems handling the more eccentric ones of the mathematicians. In Owens' essay a discussion between Weaver and Stone about the priorities between suitable personality and good mathematics illustrates some of the administrative problems Weaver faced as leader of the AMP. There is the story about how he "bent over backwards" to apply Wiener's expertise, the problems he had with John von Neumann, and with Jerzy Neyman, whose contract in the end was terminated by the AMP. There were more than hurt feelings at stake here on the part of the AMS. The expected flow of money to mathematics after the war was dependent on how well mathematics was represented in the war effort. There was also a concern that applied mathematics would benefit at the expense of pure mathematics.[13] Before the war only a very few mathematicians in the USA working in academia were interested in applied mathematics and the few who were, were not considered to be doing first class mathematics. The state of affairs before the war can be summarized by the words of Professor Prager, who gave the following description in 1972:

[9] *Time Magazine*, December 20, 1943, quoted in [Owens, 1989, p. 293]

[10] Quoted in [Williams, 2001, p. 168].

[11] [Owens, 1989, pp. 294–295].

[12] Letter from Davidson to Conant, quoted from [Owens, 1989, p. 295]

[13] See [Owens, 1989, pp. 291–292].

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> [...] their number [professional mathematicians interested in applications] was extremely small. Moreover, with a few notable exceptions, they were not held in high esteem by their colleagues in pure mathematics, because of a widespread belief that you turned to applied mathematics if you found the going too hard in pure mathematics.[14]

Despite the problems and disagreements the AMP did succeed in making mathematics play an important role in the war. One of its more celebrated accomplishments was its program for educating mathematicians to serve in operations research groups. This will be discussed below in Sect. 3.

## A programming planning problem

Linear programming was a new branch of applied mathematics that – in the USA – came into being as a direct consequence of mathematicians' war work. It was not done under contract with the AMP but by some of the mathematicians employed directly by the armed forces.[15] The source was a concrete practical problem within the US Air Forces,[16] a logistic problem that eventually led to the mathematical theory of linear programming, and from there to mathematical programming.[17] The person normally associated with the origin of linear programming in the USA is George B. Dantzig. Dantzig was one of the mathematicians hired directly by the armed forces for the war effort. In 1941 he egan working at the Combat Analysis Branch of the United States Air Force Headquarters Statistical Control under the leadership of Tex Thornstons. During the war Dantzig worked on what was called "programming planning" methods to calculate Air Force programs. An Air Force program was a kind of activity plan. In 1951 Dantzig gave the following explanation of the nature and purpose of such a program:

> The levels of various activities such as training, maintenance, supply, and combat had to be adjusted in such a manner as not to exceed the availability of various equipment items. Indeed, activities should be so carefully phased that the necessary amounts of these various equipment items were available when they were supposed to be available, so that the activity could take place.[18]

Dantzig's job during the war was to teach Air Force staff how to compute these programs.

Programming planning was a practical problem in the Air Forces. After the war it entered – with the help of the military – into the universities where it became

[14] Quoted in [Rees, 1980, p. 607].

[15] For a history of the Russian contribution see [Brentjes, 1976b], [Charnes and Copper, 1961].

[16] The U.S. Air Force was not established until after the war; Air Forces refers to the aviation branches of the Army, Navy, and Marine Corps. See [Shell, forthcomming].

[17] The history of the development of linear programming has been reported several places. See e.g. [Dantzig, 1982, 1991], [Lenstra et al., 1991], [Grattan-Guiness, 1970, 1994]

[18] [Dantzig, 1951, p. 18].

subject to mathematical theorizing. It turned into the subject of applied mathematics called linear programming and expanded from there to the broader field of mathematical programming. This will be discussed in Sect. 4

## 3 The Struggle to Include Operations Research in OSRD

Operations research (OR) and its significance during the war are often mentioned as one of the great successes of the AMP. OR was "invented" in Britain in connection with responses to the growing German air force. In 1936 the British Army and Air Force established a joint research center – the Bawdsey Research Station – for the development of radar as a tool for air defense.[19] In July 1938 the British completed a test which revealed that while radar was technically an effcient tool in air defense, there was nevertheless an important operational problem:

> the Superintendent of Bawdsey Research Station, A. P. Rowe, announced that although the exercise had again demonstrated the technical feasibility of the radar system for detecting aircraft, its operational achievements fell far short of requirements. He, therefore, proposed that research into the operational – as opposed to the technical – aspects of the system should begin immediately.[20]

– and this launched one of the first operations research groups in England.

One of the characteristics of the operations research groups was their mixed composition of expertise. They consisted of representatives from various scientific fields like mathematics, chemistry, biology, and physics. Also different kinds of engineers joined these groups. The job of OR groups during the war was not to invent new kind of weapons but to analyze what went on in the field and suggest ways to optimize the use of existing military equipment. The first groups in England focused primarily on the use of radar equipment in air defense and anti-submarine warfare. By the summer of 1941 their accomplishments were generally valued and accepted by the Royal Air Force (RAF) who by then set up operations research groups to be spread out over the various units of the RAF. Eventually these groups were also implemented in the British army and navy and their work expanded to include strategic and logistic planning as well.[21]

In the course of the war, operations research was imported into the USA and – eventually – incorporated in OSRD in October 1943 through the formation of Offce of Field Service. Even today members of American operations research groups tend to emphasize Vannevar Bush and OSRD as central agents in bringing about operations research in the USA. Contrary to this flattering picture painted by some of these members themselves, Eric P. Rau suggests in his paper *The Adoption*

[19] See [Larnder, 1979, pp. 4–5] and [Rosenhead, 1989, p. 89].

[20] [Lardner, 1979, p. 8].

[21] [Fortun and Schweber, 1993, p. 602], [Rau, 2000, p. 59].

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*of Operations Research in the United States During World War II* that the implementation of operations research groups in the USA military actually happened not because of Bush and OSRD but rather in spite of him and his organization.[22] From the beginning of 1941 Bush already knew for certain about the significance of the English operations research groups. NDRC had by then established an offce in London with people who had direct contact to the English operations research groups from which they reported back to Bush.[23] Two of NDRC's own people, the Princeton physicist H. P. Robertson and J. E. Burchard, an architect from MIT, brought back to the USA information about the data on blast damage collected by an English OR group. Robertson strongly urged that similar kinds of work should be started in the USA.[24] Bush however did nothing to introduce OR groups into the American military services. Rau convincingly demonstrates the resistance put forward by Bush against the introduction of OR groups in the OSRD. According to Rau the nature of OR did not fit into Bush's very carefully constructed system for scientific mobilization.

The conflict was due to "incompatible strategies for organizing research and development for the war effort."[25] Bush had very carefully structured the OSRD in such a way that scientists were shielded from governmental influence. Scientists were not under military command; they were under the leadership of civilians independent of the government. The reason for this was of course to protect the civilian research institutions against future governmental interference. The consequence, in effect, was the creation of a gap between the scientists who developed the new types of weapons and the military people who were going to use these inventions. In Britain OR originated as a conscious attempt to bridge this gap, but the incorporation of OR in OSRD would necessarily break down the boundary between users and developers, which would shake the very foundation of Bush's organization.

Within the first year after the USA entered the war, the military services had already begun to establish their own OR groups and OSRD began to receive requests for scientific personnel to participate in OR groups. Bush held the opinion that it was up to the armed forces themselves to administer and recruit people for the OR groups they wanted.[26] As it turned out the recruitment was a problem, and Bush's own leaders within the various sections under OSRD began to oppose Bush on the OR-issue. For example the physicist John T. Tate who chaired the NDRC section for sub-surface warfare supported Captain Baker's formation of the Antisubmarine Warfare Operations Research Group (ASWORG) through a contract with Columbia University in New York. ASWORG became one of the best known OR groups. It was headed by Philip Morse, a physicist from MIT, who became very instrumental in the establishing of OR as an academic discipline

[22] See [Rau, 2000].
[23] [Rau, 2000, p. 62], [McCloskey, 1987, p. 911], [Fortun and Schweber, 1993, p. 603].
[24] [Rau, 2000, p. 62], [Fortun and Schweber, 1993, p. 603].
[25] [Rau, 2000, p. 57].
[26] [Rau, 2000, p. 70].

in the universities in the USA in the immediate post-war period.[27] Also Warren Weaver disagreed with Bush on the question of OR, and along with other NDRC sections the AMP began to develop their own programs for educating scientists to become future OR personnel. In the end Bush relented and in October 1943 the Offce of Field Service (OFS) was created as a section under OSRD. In the USA operations research seems to have been closely connected to mathematics during the war. Half of the people in Morse's ASWORG group were mathematicians and the AMP's program for educating OR personnel is counted as one of the Applied Mathematics Panel's great successes.[28] According to the mathematician Barkley Rosser OR was regarded as mathematics in the military:

> The Air Force Generals and Navy Admirals thought it [operations research] was wonderful stuff. You could not have convinced one of them that it was not mathematics.[29]

But it was not only military people who linked OR with mathematics. As we shall see in Sect. 5, Philip Morse also thought of it as a new branch of applied mathematics.

## 4 The Post-War Organization of Science

The end of the war also meant the end of OSRD. There was a common concern that the vitality and flourishing of wartime research would dissolve in the post-war period. The scientists would go back to the kind of work they did before the war with the consequence that the research cooperation within the military-university-industry complex, which had proved itself so productive during the war, would simply disappear. Not surprisingly there was a shared belief that the USA had to be strong scientifically in order to be strong militarily.[30] The secretary of the Navy, James V. Forrestal, brought up the question of how to continue the cooperation between the universities and the military in peacetime in his annual report to President Truman in 1945:

> In peace, even more than in war, scientists owe to their nation an obligation to contribute to its security by carrying on research in military fields. The problem which began to emerge during the 1944 fiscal year is how to establish channels through which scientists can discharge this obligation in peace as successfully as they have during the war [...] The Navy believes that solution to this problem is the establishment by law of an independent agency devoted to longterm, basic military research, securing its own founds from Congress and responsive to, but not dominated by, the Army and Navy [...] The Navy so firmly believes in the importance of this solution to the future welfare of the

[27] On the formation of ASWORG see [McCloskey, 1987], [Miser, 1986], [Morse, 1986].

[28] [Rees, 1980, p. 617].

[29] [Rosser, 1982, p. 510].

[30] [Rees, 1977a, p. 104].

> country that advocacy of it will become settled Navy policy [...] The Navy feels so deeply about the importance of the solution of this problem, that it requests your intervention, guidance and support on this problem, which transcends the responsibility and authority of any single department.[31]

Also Bush prepared a strategy for the organization of post-war research and education. His proposal *Science: The Endless Frontier* was delivered to President Truman in 1945. Basically it contains a plan for government support of science organized in the spirit of OSRD – through a National Science Foundation – based on contracts to secure the independence of the universities and the industry.[32] A main point in Bush's report was to stress the significance of basic – not necessarily military – research:

> basic research leads to new knowledge. It provides scientific capital. It creates the fund from which the practical applications of knowledge must be drawn. [...] today it is truer than ever that basic research is the pacemaker of technological progress. [...] A nation which depends upon others for its new basic scientific knowledge will be slow in its industrial progress and weak in its competition in world trade, regardless of its mechanical skill.[33]

The National Science Foundation was not established until 1950 and in the meantime the military services initiated different channels for supporting scientific research. There were two primary places where the new mathematical techniques that emerged during the war became the subject of military funded basic research, Project RAND and the Offce of Naval Research (ONR).

## RAND and game theory

Game theory was not "invented" during the war, and it doesn't seem to have played any significant role in the mathematics used and developed for the war effort. However, it benefitted tremendously from the importance attached to mathematics and operations research after the war.[34]

In the 1920s von Neumann developed a theory for two-person zero-sum games. His theory was published in 1928 in the paper *Zur Theorie der Gesellschaftsspiele* and – as the title indicates – the object under consideration at that point was parlor games. On the first page of the paper von Neumann pointed towards the possibility of a much wider scope for game theory because – as he phrased it – under the right circumstances "any event [...] may be regarded as a game of strategy if one looks at the effect it has on the participants."[35] In a footnote he characterized the game

[31] Quoted in [Rees, 1977a, p. 104].

[32] [Dupree, 1986, p. 213].

[33] Quoted in [Schweber, 1988, p. 14].

[34] According to the historian Robert J. Leonard ASWORG should have used game-theoretic analysis in two applications; see [Leonard, 1992, p. 65].

problem as "the principal problem of classical economics: how is the absolutely selfish "homo economicus" going to act under given external circumstances?"[36] The theory was built up on the concept of strategy. Von Neumann discussed at length what should be understood by a "best" or an "optimal" strategy. The main result in the paper is the so-called minimax theorem, which proves the existence of optimal strategies to every finite zero-sum two-person game. For such games a value $V$ can be assigned, which is the average gain that one player can expect to win from the other player, regardless of what strategy the opponent chooses.[37]

The really significant development in game theory took place during the war when von Neumann and another refugee, the Austrian economist Oskar Morgenstern, co-authored the book *Theory of Games and Economic Behavior.*[38] The book was written with economists in mind. Morgenstern, and von Neumann explicitly stated so in the opening phrase of the book and in Morgenstern's diary one can follow his attempts to bring the theory to the attention of economists. Comments in Morgenstern's diary show that the economists were at best just not interested in game theory even though they expressed a hostile attitude in the first decade after the book was printed.[39] Even though the book was written and parts of the theory were developed in the mist of the war, it seems that they did not think of war applications in the process. There are no references to possible war applications in Morgenstern's diaries. Yet game theory became the main subject of mathematical research at the RAND Corporation, the nest of military support of science. Why was that?

According to the historian Philip Mirowski, the disregard shown by economists brought von Neumann to search for another "home" for game theory. Given the time, the place, and the concept of optimal strategies for winning a game, which fitted perfectly into the war context, and given von Neumann's multiple connections, reputation, and influence within the military-science complex during the war, the military context was an obvious choice.[40] Project RAND in Santa Monica, California became the most important home for game theory. This project originated in March 1946 by the initiative of Army Air Force Chief of Staff Henry H. "Hap" Arnold and Donald Douglas, the president of Douglas Aircraft. In the beginning the project functioned as a subsidiary of Douglas Aircraft but in 1948 Project RAND became a free-standing nonprofit corporation, a so-called "think-tank".[41]

In the first decade after the war RAND was the center for mathematical research in game theory. The first mathematicians working there were recruited

[35] [von Neumann, 1928, (1959, p. 13)].

[36] [von Neumann, 1928, (1959, p. 13, footnote no. 2)].

[37] For a historical account of von Neumann's conception of the minimax theorem see [Kjeldsen, 2001]. For the history of game theory see also [Weintraub, 1992].

[38] [von Neumann and Morgenstern, 1944].

[39] [Mirowski, 1991, p. 239].

[40] [Mirowski, 1991].

[41] See [Hourshell, 1997, pp. 241–242], [Smith, 1969].

mainly from the Applied Mathematics Panel. It counted people like John Williams, Morris Girschich, Olaf Helmer, and Ed Paxton among its theorists. John von Neumann himself was a consultant for RAND as well as Weaver and Wilks, who had been the leader of AMP's Statistical Research Group at Princeton. Wilks also referred his student Ted Harris to RAND. This group at RAND was the first established group of game theorists and they all either came from the war work or had connections to mathematicians who had been involved with OSRD. The group at RAND held lengthy summer sessions in game theory and collaborated with another military financed project – the logistic project – in Princeton which included people like Albert Tucker, John Nash, Martin Shubik, and Harold Kuhn.[42] This project will be discussed below.

Besides the promise that game theory could provide optimal strategies for military problems, there were other connections as well. It turned out that a two-person zero-sum game is mathematically equivalent to the mathematical model of the programming planning problem or – as it soon became known as – a linear programming problem. This connection had a lot of impact on further mathematical research both applied and pure, and again – as will be elaborated in the next section – von Neumann was instrumental in the series of events that started that line of development.

## The mathematics program of the Offce of Naval Research

In 1946 the Navy established the Offce of Naval Research (ONR) which in the first four years of its existence became the main sponsor of government supported research in the USA. ONR continued the research practice of OSRD by supporting research done at the universities through contracts with individual scientists.[43]

Mina Rees, who had served as technical aid to Warren Weaver at the AMP, was asked by the ONR to take on the position as head of the mathematics department of ONR and as such she became a very influential figure in the post-war mathematical community in the USA.[44] Mina Rees has reported her recollections regarding her time at ONR in several places, and it seems that she at the outset was skeptical about the idea of military funding of mathematical research in peacetime. She simply didn't think mathematicians would let the military finance their peacetime research. Nevertheless she accepted the position at ONR because – according to her recollections – she found it extremely important for the further development of mathematics in the USA to be actively engaged in the ONR program. Judging from her own writings she was very concerned about whether the Navy would support basic research in mathematics proper without any relevance to the Navy. She didn't want the ONR program to fragment the community of mathematicians who

[42] See [Mirrowski, 1991] and [Leonard, 1992].

[43] For historical accounts on ONR see [Sapolsky, 1979], [Schweber, 1988], [Old, 1961].

[44] For a historical account on Rees and her role in the history of the U.S. government's role in funding research in the mathematical sciences see [Shell, forthcoming]. Also [Williams, 2001] has a chapter on Mina Rees.

feared that applied mathematics would benefit from the wartime at the expense of pure mathematics.[45]

In a note in the Bulletin of the AMS from January 1948 Mina Rees announced "the philosophy which has determined the mathematical research projects which ONR is sponsoring":

> The Offce of Naval Research is committed primarily to the support of fundamental research in the sciences, as contrasted with development, or with applications of known scientific results. [...] It is natural, however, that the most obvious types of mathematical research which would seem to warrant Navy support would be research in applied directions.[46]

This "philosophy" is reflected in the budget where 4/5 of the annual expenditure went to research in applied mathematics and computing devices. But in the note Mina Rees emphasized that basic research in mathematics proper was deemed important by the Navy, and actually more than 1/3 of the contracts dealt with projects with theoretical objectives.

## Scientific computation of optimum programs

In the meantime the Air Force had re-employed George Dantzig to continue the work on calculating Air Force programs. The emergence of the computer had a profound influence on this work because it made it possible to build some kind of objective into the programs, such as the possibility of choosing the "best" program among feasible ones with respect to some objective, for example, the least expensive program.[47] The Air Force then established project SCOOP, which stood for Scientific Computation of Optimum Programs. The main persons in the group were Dantzig and Marshall Wood – an expert on programming procedures. Later John Norton and Murray Geisler joined the project. In 1949 Marshall Wood gave the following description of the project:

> Early in 1947 the Air Comptroller's Offce undertook a concerted attack on this problem, establishing the Planning Research Division. [...] The work of this Division, now designated as PROJECT SCOOP [...], was directed to four main problem areas:
>
> a) The systematic and comprehensive identification and quantitative evaluation of interrelationships among Air Force activities, objectives, and limitations, usually expressed in the form of planning factors;
> b) The development of a system of equations, expressing these interrelationships explicitly in mathematical form;
> c) The development of mathematical computing techniques for the solution of these systems of equations, so as to construct a program which will accom-

[45] See [Albers and Alexanderson, 1985], [Rees, 1977a, 1977b].

[46] [Rees, 1948, p. 1].

[47] [Dantzig, 1991].

plish our objectives to the maximum extent possible within the external limitations of funds, industrial capacity, etc.;

d) The development and construction of high speed electronic computing machines adequate to perform in a few days the computations required for the equations for a complete Air Force program.[48]

Through the military-academic complex this problem got introduced into the world of university mathematics and it appears that the person who pulled the strings was – again – John von Neumann who served both as a consultant for the various military establishments and was on the advisory board for ONR.

## An ONR project in linear programming and game theory

The Air Force programming group had built a mathematical model for the programming problem which resulted in the mathematical problem of optimizing a linear function subject to linear inequality constraints.[49] Dantzig was encouraged to seek the advice of von Neumann on the problem of finding an algorithm that could solve such a programming problem, and in the fall of 1947 Dantzig introduced von Neumann to the Air Force programming problem.[50] In the book *Theory of Games and Economic Behavior* von Neumann had based the theory of two-person zero-sum games on the theory of convexity and linear inequalities, and – according to Dantzig – von Neumann immediately suggested that a two person zero sum game is equivalent to a programming problem.[51]

A month later von Neumann circulated the note *Discussion of a Maximum Problem* in which he transformed a problem of maximizing a linear function constrained by linear inequalities into the problem of finding a solution to a system of linear inequalities.[52] This shows that von Neumann was interested in the mathematics underneath the programming problem. A couple of months later – that is in the spring of 1948 – Dantzig revisited von Neumann at the Institute for Advanced Study to discuss the possibility of setting up a university-based project to study further the linear programming problem, game theory, and the underlying mathematical structure.[53]

Mina Rees has described the occurrence of this university-based project as follows:

> [...] when, in the late 1940's the staff of our offce became aware that some mathematical results obtained by George Dantzig, who was then working for the Air Force, could be used by the Navy to reduce the burdensome costs

[48] Quoted in [Brentjes, Ph.D. thesis, p. 177].

[49] [Dantzig, 1949]

[50] [Dantzig, 1982, 1988].

[51] [Dantzig, 1982, p. 459].

[52] [von Neumann, 1947]

[53] [Albers and Alexanderson, 1985, pp. 342–343].

> of their logistics operations, the possibilities were pointed out to the Deputy Chief of Naval Operations for Logistics. His enthusiasm for the possibilities presented by these results was so great that he called together all those senior offcers who had anything to do with logistics, as well as their civilian counterparts, to hear what we always referred to as a "presentation". The outcome of this meeting was the establishment in the Offce of Naval Research of a separate Logistics Branch with a separate research program. This has proved to be a most successful activity of the Mathematics Division of ONR, both in its usefulness to the Navy, and in its impact on industry and the universities.[54]

The project began in the summer of 1948 as a trial project with Albert W. Tucker, a mathematician from Princeton, as the principal investigator. Together with two students, Harold W. Kuhn and David Gale, Tucker read von Neumann and Morgenstern's book on game theory and studied von Neumann's note on a maximum problem.[55] Their work resulted in the paper *Linear Programming and the Theory of Games* which they presented at the first conference on linear programming held in Chicago in the summer of 1949.[56]

The project continued with support from ONR until 1972[57] and it gave rise to a substantial amount of research both in what traditionally will be counted as applied as well as pure mathematics. This project is a prime example of the effect of the military-university cooperation formed by the post-war organization of science support. By initiating research on the programming problem and game theory by university mathematicians usually engaged in pure mathematical research, the field expanded, and basic research in mathematics proper – pure and applied – was the result.

In their first work on linear programming and game theory, Kuhn, Tucker, and Gale proved the main theoretical theorem in linear programming, the so-called duality theorem. This result and its connection to the important minimax theorem in two-person zero-sum games were interesting from a mathematical point of view and changed the scientific status of linear programming.[58] Its connection to game theory, to the theory of systems of linear inequalities, and to convex analysis broadened the field and made linear programming an interesting potential mathematical research area. As I have argued elsewhere, I find that this change in scientific status was very important for the further development of the theory of mathematical programming.[59] Until Tucker and his group got involved, the driving force behind the development had been from a practical point of view, the solving of the Air Force programming problem. Tucker, Kuhn, and Gale's work on the other hand was done within a university context of theoretical mathemati-

[54] [Rees, 1977a, p. 111].

[55] Personal interview with H. W. Kuhn, Princeton, April 23, 1998.

[56] [Gale et al., 1951].

[57] Personal interview with H. W. Kuhn, Princeton, April 23, 1998.

[58] For an analysis of how this changed the scientific status of linear programming see [Kjeldsen, 1999, 2000b].

[59] See [Kjeldsen, 1999, 2000b].

cal research. Kuhn and Tucker got stuck on the project and they began to work on an extension of the duality theorem to the more general nonlinear case. They published their joint work in 1950 in the paper *Nonlinear Programming* – a classic in mathematical programming – that launched the theory of nonlinear programming.[60] Kuhn and Tucker did not prove a duality result for nonlinear programming but they did prove the so-called Kuhn-Tucker theorem about necessary conditions for a point $x_0$ to be an optimal solution to a nonlinear programming problem.[61] The different circumstances under which linear and nonlinear programming originated is a remarkable consequence of the military-university cooperation. In contradiction to what was the case with linear programming, there was no direct external, applicational, motivation for the development of nonlinear programming. The subsequent practical usefulness of nonlinear programming notwithstanding, it originated as an interest in pursuing a purely mathematical issue of generalization and understanding.

The work on game theory within Tucker's group can be followed in a series of papers published in several volumes of the *Annals of Mathematics Studies* under the title *Contributions to the Theory of Games*.[62] The cooperation between RAND and the logistic project under ONR is reflected in these volumes, since approximately half of the papers in each volume were done under contract with the ONR while the other half were written by RAND people.

## Research in pure mathematics

Subjects of more pure character were also pursued within the military context. Theodore Motzkin's thesis *Beiträge zur Theorie der linearen Ungleichungen*[63] was translated into English both at RAND by D. R. Fulkerson and in the logistics project by S. Bargmann.[64] In 1956 a series of eighteen papers exploring "various aspects of one mathematical theme, the theory of linear inequalities" was published, and again part of the research was supported by ONR and part of it was done at RAND. As is explicitly stated in the introduction the research was motivated by the developments of game theory and linear programming.[65]

The theory of convexity is another field of theoretical mathematics that gained renewed interest as a consequence of the Logistic Research Project financed by ONR. Von Neumann had based the theory of two-person zero-sum games in

[60] [Kuhn and Tucker, 1950a].

[61] See [Kjeldsen, 2000a] for the argumentation for the claim that the work Kuhn and Tucker published in their *Nonlinear Programming* paper was motivated by a wish to extend the duality result for linear programming.

[62] See [Kuhn and Tucker, 1950b], [Kuhn and Tucker, 1953], [Dresher and Tucker, 1957], and [Luce and Tucker, 1959].

[63] [Motzkin, 1936].

[64] See the introduction to Motzkin's collected works in [Cantor et al., 1983], and [Billera and Lucas, 1978, p. 5].

[65] [Kuhn and Tucker, 1956, p. v].

*Theory of Games and Economic Behavior* on the geometrical theory of convexity. The theory of convex functions also seemed to be a promising tool in the newly emerged field of nonlinear programming. As mentioned above Kuhn and Tucker did not succeed in expanding their duality theorem for linear programming to the general nonlinear case but they were able to prove that if the involved functions are concave (and differentiable) there will be complete equivalence between the nonlinear programming problem and the saddle value problem for the corresponding Lagrangian function, suggesting the possibility of a duality result for concave functions.[66]

One of the experts on the theory of convexity at that time was Werner Fenchel from the University of Copenhagen in Denmark. He happened to be visiting the USA in the academic year of 1950/51 with extended visits first in California at the University of Southern California and Stanford. He ended his US visit in Princeton, first as a member of the Institute for Advanced Study and then as a visiting professor at Princeton University.[67] Tucker invited Fenchel to give a series of lectures on convex sets and functions within the logistic project.[68] The lecture notes from Fenchel's course became a source of inspiration and influenced widely the further research in the theory of convexity. Significant for the theory of mathematical programming was the fact that Fenchel was able to derive a duality result for "a generalized programming problem" as he phrased it.[69] The background for this result, which is often termed Fenchel duality, was a small paper published in 1949 where Fenchel introduced the concept of conjugate convex functions.[70] Fenchel originally introduced this concept with the purpose of examining the mathematical structure underlying various inequalities – like the Hölder inequality – that appears in analysis. In Princeton Fenchel was introduced to the new field of mathematical programming, and by applying his concept of conjugate convex functions on this type of questions Fenchel derived the first duality result in nonlinear programming.[71] Fenchel did not explore this any further, but his lectures and the notes had quite an influence on the following development of the theory of convexity in the USA and in the developing of convex programming. R. T. Rockafellar in particular was very much inspired by Fenchel's Princeton lectures. He used the Fenchel duality to build a duality theory for convex programming based on Fenchel's concept of conjugate convex functions.[72]

66 [Kuhn and Tucker, 1950a], [Kjeldsen, 2000a].

67 [Fuglede, 1989, p. 167].

68 [Fenchel, 1953, Acknowledgement], [Fenchel, Bidrag til de konvekse funktioners teori, BOX 2, Folder: Manuskripter om konvekse mængder og funktioner. Non-dated, but from the years 1953/54.], [Letter from Tucker to Fenchel, June 11, 1951, BOX 1.].

69 [Fenchel, 1953, p. 105].

70 [Fenchel, 1949].

71 [Fenchel, 1953, pp. 105–106].

72 See [Rockafellar, 1970].

# 5 The Interactions between Operations Research, Mathematical Programming, and Game Theory in the Post-War Period

The discussion in Sect. 3 focused on the success of and importance ascribed to operations research during the war. After the war both the military and the academics who had been involved with operations research during the war were very anxious to secure the field in the post-war period and transform it into an academic discipline. Both game theory and mathematical programming were almost immediately considered as essential items in the toolkit of operations research and, as such, benefited from the importance attached to operations research as a direct consequence of the war work.

## The establishment of operations research as an academic discipline

One of the exercises was to "move" operations research into peaceful applications. In 1947 Charles Kittel published a paper about "The Nature and Development of Operations Research" in *Science* where he gave the following definition of operations research:

> Operations Research is a scientific method for providing executive departments with a *quantitative basis for decisions*. Its object is, by the analysis of past operations, to find means of improving the execution of future operations.[73]

and expressed his hope for its future:

> It is hoped that the publication of this paper will serve to stimulate the establishment of operations research groups in the United States for the advancement of peaceful objectives. This powerful new tool should find a place in government and industry. [74]

As we saw in Sect. 3, operations research was popular within the military and the Office of Naval Research supported its move into the universities through the logistic project. According to Fred Rigby, who was ONR's leader of the logistic project, ONR exerted an immense influence on operations research and its development:

> We did indeed influence the introduction of operations research into business schools. The subdiscipline called management science is our invention, in quite a real sense. That is, we and our contract researchers recognized its potentials, planned its early growth, and, as it turned out, set the dominant pattern in which it has developed. [75]

[73] [Kittel, 1947, p. 150].
[74] [Kittel, 1947, p. 153].
[75] Quoted in [Rees, 1977a, p. 111].

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Also the National Research Council contributed to the growth of operations research by forming a committee under its section of applied mathematics to "further its [operations research] development and applications outside the armed forces."[76] This was done in practice by the granting of Ph.D. scholarships, financial support to conferences etc.

Philip Morse from MIT was a key person in the shaping of operations research as an academic discipline. Already from 1948 he had two courses running at MIT and he also introduced MIT's summer sessions in operations research, two-week courses for civil servants and people working in industry. From 1952 Johns Hopkins University had a program in operations research and from 1954 it was possible to earn a Ph.D in the field.[77]

## Different opinions on the role of mathematics in operations research

Mathematical programming and game theory benefitted from Morse's strong influence on the early establishment of operations research because Morse continued to emphasize the importance of mathematical research for operations research. Not everyone agreed with him on the importance of mathematics. In the journals of the Operations Research Societies of America (ORSA) and Britain respectively as well as in the proceedings from the international conferences there was a continuing debate about what operations research actually was. Morse tried at some point to stop the arguing by simply defining operations research to be what operations research people do:

> We should no longer have trouble explaining the scope and methods of operations research to the layman. We already can say: operations research is the activity carried on by members of the Operational Research Society; its methods are those reported in our journal.[78]

Morse was a persistent advocate for the inclusion of game theory and linear programming in operations research. In 1953 he wrote about *Trends in Operations Research* in ORSA's journal where he pointed towards game theory as a very important tool that should be further developed. Morse wrote that:

> Linear programming is rapidly becoming an important theoretical tool in economics; it deserves equal or greater exploitation in operations research.[79]

And two years later, in 1955, he maintained that "Just as with any other field of science, we are finding that we need our own kind of mathematics"[80] in operations research. How serious this issue was for Morse can be seen from his appeal to a growing generation of operations researchers:

[76] [Fortun and Schweber, 1993, p. 611].
[77] [Morse, 1956], [Roy, 1956].
[78] [Morse, 1953, p. 159].
[79] [Morse, 1953, p. 169].
[80] [Morse, 1955, p. 383].

> But linear programming is only one part of a larger theory of optimal programming, which covers such subjects as dynamic programming, some aspects of search theory and, probably, of game theory. It is hard to visualize, right now, all the mathematical aspects of this broader subject, because they haven't been investigated as yet in any detail. [...]
>
> It is not hard to foresee the considerable usefulness of this general theory in solving many operations problems, particularly those concerned with planning. But a great deal of basic research will be needed before the theory will be able to answer our practical needs. Some of the fundamental mathematics has not yet been developed and a great number of the algorithms for solving specific problems have not yet been worked out. Much of this basic work can probably best be done as a long-term study, not subject to the short-term deadlines and crises which occur in the study of immediate, practical problems. It needs a good many man-years of concentrated work. Operations research needs this sort of research. No branch of science can continue to grow unless its underlying theory is continuously being expanded.[81]

Morse regarded mathematics as not only the theoretical basis of operations research, but actually more than that. In 1948 he characterized operations research as a "new field of applied mathematics ..."[82]

A look at the papers published in *Operations Research* during the first two decades of its existence reveals that mathematical programming, especially linear programming, played an important part in operations research. This is confirmed in the proceedings from the first international meetings and from the content of the early textbooks where mathematical programming is presented.[83]

Not everyone shared the opinion of Morse regarding the role of mathematics in operations research. Many expressed their discontent with the importance attached to mathematics. Already in 1953 Norman Hitchman warned against this tendency in ORSA's journal:

> One main caution seems to stand out glaringly. It concerns the emphasis which we place on certain specialized fields as to their value in the operations research team. It is not an uncommon observation of today to note the very great emphasis given to mathematical and physical sciences. Our new society, ORSA, for example, is playing a preeminent part in creating the impression that mathematics and physics are almost synonymous with operations research itself.[84]

In 1956 W. N. Jessop warned against "the placing of emphasis on mathematical methods and on highly abstract treatments of general situations" also in ORSA's journal. When it comes to linear programming Jessop also held the opinion that

[81] [Morse, 1955, p. 383].

[82] [Morse, 1948, p. 621].

[83] See [Banbury and Mailand, 1961], [Dunod, 1963], [Hertz and Melese, 1966], [Churchman et al., 1957], [Sasiene et al., 1959], and [Hillier and Lieberman, 1974].

[84] [Hitchman, 1953, p. 242].

there was too much focus on the developing of theory “a subject [linear programming] so delightful to the pure mathematician that many papers appear to have had their origin in sheer exuberance unsullied by any thought of a factual situation.”[85]

Despite the quarrels linear programming became – and still is – a widely used tool in operations research. The simplex algorithm developed by Dantzig for solving linear programs was and is used in many sorts of branches of industry.

## The usefulness of game theory in operations research

Game theory was also imbedded in operations research, and again John von Neumann seems to have played a crucial role. During the war von Neumann functioned – and was often used – as a consultant for Morse’s operations research group (ASWORG) and according to Fortun and Schweber, he introduced game theory into operations research:

> By the end of the war the new game theoretic methods that had been developed by von Neumann and Morgenstern were added to the toolkit and mathematical techniques that operations research scientists deployed. These proved very valuable, and game theoretic approaches took on great importance after the war.[86]

Morse considered game theory an important tool in operations research, just as he advocated mathematical programming. The first courses in operations research at MIT also included game theory,[87] and that was also the case in operations research education at Johns Hopkins.[88] An analysis of the content of the early textbooks as well as a comparison of bibliographies of game theory and operations research reveals – as phrased by R. Rider – that operations research and game theory “shared much territory”.[89] There were, however, criticisms of the value of game theory. In her paper *Operations Research and Game Theory: Early Connections* Rider questions the usefulness of game theory in practical problem solving in operations research. Churchman, Ackoff, and Arnoff wrote in their textbook from 1957 in a conclusion to their chapter on game theory that:

> Very little has been accomplished by way of applying the theory.[90]

The same conclusion was reached more than twenty years later by George R. Lindsey in his talk *Looking Back over the Development and Progress of Operational Research* held at the Eighth International Conference in Operations Research

[85] Quoted from [Rider, 1992, p. 231 and 234].
[86] [Fortun and Schweber, 1993, p. 604].
[87] [Rider, 1992].
[88] [Roy, 1956].
[89] [Rider, 1992, p. 227].
[90] [Churchman et al, 1957, p. 557].

where he gave the following evaluation of the significance of game theory in operations research:

> Direct application of the theory of games to the solution of real problems has been rare, and its chief uses have been to offer some insight and understanding into the problems of competition (without actually solving them), and to provide mathematicians with new fields to conquer. Many important real problems involve more than two opponents, are not zero-sum, and exceed the bounds of the most developed versions of game theory.[91]

## 6 Conclusion

Mathematical programming and operations research have their origin directly in scientific work for the war effort. Their continued growth and final establishment as scientific disciplines in American universities in the post-war period developed as a consequence of the military financing of science after the war. Much of this work was done in projects directly financed and established by military means like the Project RAND and the ONR's logistic project. One of the major journals – the *Naval Research Logistics Quarterly* – was also financed by the military.

The construction of the post-war organization of science with the military-university cooperation inherited from the war allowed mathematics of a more pure character to benefit from the military science support and to experience a growth in research. The theory of convexity and the theory of linear inequalities are two examples of that.

Mina Rees, the head of the mathematics department of the ONR, originally did not think that mathematicians would let the military pay for their peacetime research, but she was indeed very successful in the way she constructed and managed the mathematical department of ONR. When asked, Kuhn expressed the opinion that the military had no direct influence on the mathematical work he did within the logistic project. The military did not demand him to work on the solution of concrete problems. The work he did was not classified but published in journals, proceedings, and books. The financial support from ONR was spent on summer salaries, conferences, guests, and traveling expenses. Kuhn characterizes the support from ONR as just an improvement of the working conditions at the universities. In the day to day work it made no difference where the money was coming from. He was free to investigate the aspects of mathematical programming and game theory that he found theoretically interesting, regardless of potential applications.[92] Actually in the first four years of its existence the ONR supported promising scientific research projects without regard to direct applicational value or possible military applications. Instead the support from ONR was justi-

[91] [Lindsey, 1979].

[92] Personal interview with Kuhn, Princeton, April 23, 1998.

fied within the scientific discipline itself and often there were no explicit strings attached to the support.[93] That changed gradually after 1950.

But the logistic project at Princeton was not – at the outset – initiated by Tucker but by the Armed Forces. Even though the mathematicians in the group did not work on explicitly formulated problems given by the military, the theoretical mathematical research questions they pursued within the project had to belong to or be justified by issues in the field of mathematical programming and game theory. By giving money to promote research in certain areas, the military had a huge influence on the growth and direction of mathematical research after the war.

One may also wonder whether the selection of theorems the mathematicians chose to prove was influenced by the military context. According to Philip Mirrowski that was the case with game theory. In his paper *When Games Grow Deadly Serious* he argues that the military connections went all the way into the core of game theory and shaped its content and he claims that "a social-constructivist history of game theory is not only possible; it is the only plausible candidate."[94] Reading Mirrowski one gets the impression that since the economists were not interested in game theory, von Neumann turned to the military context and actively placed game theory at the doorstep of RAND.[95] The military connection created an artificial focus on – among other things – the minimax solution and made it the ultimate solution concept in game theory,[96] which makes sense in a military context of "capitalists versus communists, USA versus USSR."[97] Mirrowski concludes that the military influence hindered the theory in its "proper" development, that it postponed, for example, the realization that the minimax solution was not *the* solution concept but only one of many possible solution concepts. John Nash, who worked as a consultant at RAND in the early 1950's, published his Equilibrium Point Theorem (which is the central solution concept in game theory) in 1950, that is quite early in the development of game theory, indicating that the restraint was not too damaging. The military context might have influenced the content of game theory, but there is another level, I think, namely the question of what can be proven – given the axioms and the logical rules – within such a mathematical system, and that is indifferent to the context.[98] Eventually game theory moved into economics where it is very much alive today.

From the discussion in Sect. 5 it follows that game theory was not able to meet its promises as a useful tool in operations research. Also the mathematical programming people felt a need to have their own home. In 1970 they began publishing their own journal *Mathematical Programming* and two years later they organized the Mathematical Programming Society. During the 1970s mathematical programming became established with a professional society, journal, prizes, and a growing number of monographs and textbooks.

93 [Sapolsky, 1979, p. 386], [Schweber, 1988, p. 8].
94 [Mirrowski, 1991, p. 233].
95 [Mirrowski, 1991, p. 240–242].
96 [Dresher, 1961, p. 79], [Mirrowski, 1991, p. 244].
97 [Mirrowski, 1991, p. 246].
98 The context might influence the choice of the axioms and maybe also the logical rules.

This story also demonstrates the influence of individual scientists like John von Neumann and Philip Morse. von Neumann was involved with military work from 1937 when he became a consultant to the Ballistics Research Laboratory of the Army Ordinance Department.[99] From then on his connections with the military expanded rapidly, and he died as one of the most powerful scientists in the USA.[100]

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[100] [Heims, 1980, p. 275]. Besides being involved with mathematics during the war he was also involved with the Manhattan Project.

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