Detectability of Discrete Event Systems [1]

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Abstract

In this paper, we investigate the detectability problem in discrete event systems. We assume that we do not know initially which state the system is in. The problem is to determine the current and subsequent states of the system based on a sequence of observation. The observation includes partial event observation and/or partial state observation, which leads to four possible cases. We further define four types of detectabilities: strong detectability, (weak) detectability, strong periodic detectability, and (weak) periodic detectability. We derive necessary and sufficient conditions for these detectabilities. These conditions can be checked by constructing an observer, which models the estimation of states under different observations. The theory developed in this paper can be used in feedback control and diagnosis. If the system is detectable, then the observer can be used as a diagnoser to diagnose the failure states of the system.

Key Words: Discrete event systems, state estimation, observability, detectability.

1. Introduction

Discrete event systems have been studied for more than twenty years. Various problems have been investigated, especially those problems related to supervisory control [2, 3, 4, 7, 11], such as controllability, observability, co-observability, and normality. However, there is one problem that has not been fully investigated. This is the problem of how to estimate or determine

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the current and subsequent states of the system based on observations. The partial reason that this problem has not been fully investigated is that what is important in supervisory control is the information on sequences or traces of events. That is why observability of discrete event systems is defined on traces rather than on states [7]. We say that a language representing the desired behavior of a discrete event system is observable if for any two traces in the language that look the same to a controller (that is, they have the same projection), the control action following these two traces is consistent (that is, it cannot be the case that an event is desirable after one trace but not desirable after another). Obviously, the above definition of observability is unrelated to the estimation of states. Similarly, co-observability and normality (a stronger version of observability) are also unrelated to the estimation of states.

However, there are some applications of discrete event systems where the state estimation problem is important. We realize this especially in medical applications that we are investigating [8]. In medical applications, it is important to know the state of a system (representing, for example, the disease stage of a patient). State estimation is also important in diagnosis. One approach to the diagnosis problem is to model failures as unobservable events and diagnosability requires that one can determine the occurrence of a failure event after finite number of event observations [12, 13]. However, we can also approach the diagnosability problem by defining some failure states [6, 14]. Then the diagnosability problem becomes a problem of estimating states. Yet another application of the state estimation problem is in remote and distributed systems, where it is desirable for a central station to be able to determine the state of a remote system under limited communications. For all these applications, in this paper, we will investigate the state estimation problem.

State estimation is first studied using automata in an abstract sense in [15] and the idea is followed by many others, including us. In that sense, our paper can be viewed as an extension of the work presented in [15]. However, our paper is more specific and investigates state estimation in a general discrete event system framework. State estimation problem is also studied [10] and [9]. Our work is different from [10] and [9] because we investigate the state estimation problem from all possible angles which includes the results of [10] and [9] as special cases. In particular, we define two types of detectabilities: detectability where we know the current and subsequent states of the system after some finite number of observations and periodic detectability where we know the state of the system periodically. One reason for defining periodical detectability is that when not all events are observable, knowing the current state does not imply knowing the subsequent states. We consider both strong detectability, where state can be determined for all possible trajectories of the system, and (weak) detectability, where state can be determined for some trajectories of the system. One reason for defining (weak) detectability is that if the system is weakly detectable, then we may be able to control it within certain trajectories so that it is strongly detectable.

In general, there are two types of outputs: event output and state output. For event output, we assume, as in supervisory control, that some events are observable and some are not. For state observation, we assume that there is a many-to-one output mapping from the state set to a state output set. Therefore, by observing state output, we know which subset of states the system is in, but we do not know exactly which state it is in because the output mapping is not one-to-one (otherwise the state estimation problem is trivial). Thus there are four possible cases of event and state observations: (1) All events are observable and no state is observable. (2) All events are observable and some states are observable. (3) Some events are observable and no state is observable. (4) Some events are observable and some states are observable. The detectability when all events are observable and partial states are observable (Case 2) is partly investigated in [10] and the periodic detectability when some events are observable and no state is observable (Case 3) were partly investigated in [9], which they call observability. We investigate both detectability and periodic detectability for all the above four cases. To the best of our knowledge, our results have not been obtained before. The most related recent works are reported in [1, 5].

The paper is organized as follows. In Section 2, we present the model of discrete event systems and the output mechanisms. In Section 3, we study detectability for complete event observation and no state observation. In Section 4, we study detectability for complete event observation and some state observation. In Section 5, we study detectability for partial event observation and no state observation. In Section 6, we study detectability for partial event observation and some state observation. Some remarks and conclusions are given in Section 7.

2. Discrete Event Systems

Discrete event systems are used to model systems with discrete states and events. States represent conditions and status of a system. For example, the states of a machine may consist of idle, working, and down; the status of a patient may include excellent, fair, and poor. Events represent changes in the system, action taken by external agents, and other activities of significance. For example, turning on or off a machine is an event; so is machine breaking down or being repaired. Similarly, improvement or deterioration of patient's conditions is an event; so is administrating a drug or treating a patient. To model a discrete event system, we often use an automaton (also called state machine or generator) [3]:

$$G = (Q, \Sigma, d),$$

where Q is the set of discrete states, Σ the set of events, and $d : Q \times \Sigma \to Q$ the transition function describing what event can occur at one state and the resulting new state. An equivalent way to define the transition function is to specify the set of all possible transitions: $\{(q, s, q') : d(q, s) = q'\}$. With a slight abuse of notation, we will also use d to denote the set of all possible transitions and write $(q, s, q') \in d$ if $d(q, s) = q'$ is defined.

In many applications of discrete event systems, it is desirable to know the current state of the system. If we do not know the current state of G, we need to estimate it. The estimation is based on observation of some events and/or some states. The event observation is described by the projection $P : \Sigma^* \to \Sigma_o^*$, where Σ_o is the set of observable events. The state observation is described by the output map $h : Q \to Y$, where Y is a (finite) output set. The discrete event system with the event and state observation is described by

$$(G, P, h, \Sigma_o, Y)$$

where $G = (Q, \Sigma, d)$. The question is whether we can estimate the current and subsequent states based on the event and state observations.

To avoid unnecessarily complicated technicalities in our ensuing development, we will assume that $G = (Q, \Sigma, d)$ is deadlock free, that is, for any state of the system, at least one event is defined at that state: $(\forall q \in Q)(\exists s \in \Sigma)\, d(q, s)$ is defined. This assumption is also made in [9] and can be relaxed at the expense of more complicated notations and proofs. We also make another assumption that no infinite strings exist whose events are all unobservable. In other words, no loops in G contain only unobservable events.

3. Complete Event Observation and No State Observation

We first consider the state estimation problem with complete event observation and no state observation, that is, we consider the case $\Sigma_o = \Sigma$ and $Y = \text{emptyset}$. Since all events are observable and G is deterministic, if we can determine the current state of G, then we can determine the state of G afterwards. In other words, the reason for not knowing the state of G is due to the uncertainty in the initial state. The state estimation problem in this case can be stated as follows.

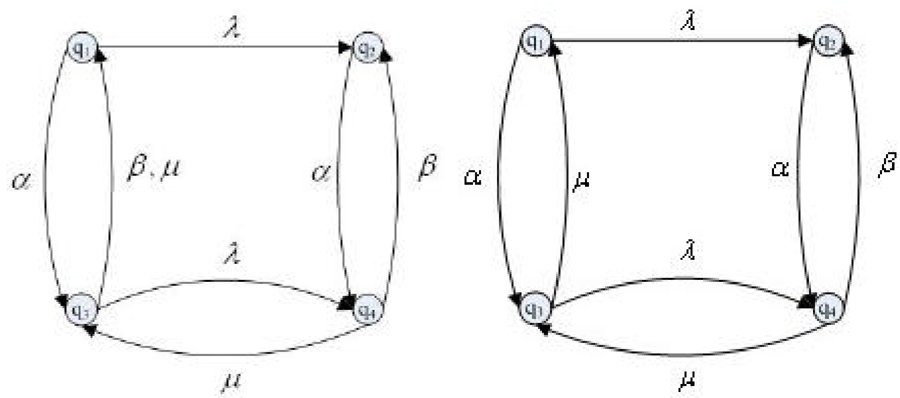
State Estimation Problem 1

Given a discrete event system

$$(G, P, h, \Sigma_o, Y)$$

we do not know the initial state of $G = (Q, \Sigma, d)$. We have complete event observation and no state observation, that is, $\Sigma_o = \Sigma$ and $Y = \text{emptyset}$. Can we determine the current state of the system after a finite number of event observations?

To gain some insight into the solution of the problem, let us consider the system shown in Figure 1(a). The state estimation problem for this system is not always solvable, because if we keep observing events $bababababa\ldots$, then we cannot determine if the system is in states 1 or 2 (and states 3 or 4); but if the system execute $a\,l$, for example, then we can determine that the system is in state 4 On the other hand, if we remove the b on the left as shown in Figure 1(b), then no matter what strings the system execute, we can determine the current state after a finite number of observations.



(a) (b)

Figure 1. (a) System is (weakly) detectable but not strongly detectable

(b) System is strongly detectable

Formally, let us define the following two types of detectabilities.

Strong Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o = \Sigma$ and $Y = \text{emptyset}$ is strongly detectable if we can determine the current state and the subsequent states of the system after a finite number of event observations for all trajectories of the system.

The requirement detectability for all trajectories may be too strong. We can relax this requirement for some applications. This leads to the following definition.

(Weak) Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o = \Sigma$ and $Y = \text{emptyset}$ is (weakly) detectable if we can determine the current state and the subsequent states of the system after a finite number of event observation for some trajectories of the system.

To check detectability and strong detectability, we proceed as follows. Firstly, we add an initial state q_o to G and add e-transitions from q_o to all states in G. This addition models the fact that initially we do not know which state the system is in. Formally, let

$$G_{nd} = (Q \cup \{q_o\}, \Sigma \cup \{e\}, d_{nd}, q_o)$$

where $d_{nd} = d \cup \{(q_o, e, q) : q \in Q\}$, that is, d_{nd} consists of all transitions in d and e-transitions from q_o to all states $q \in Q$. Note that G_{nd} is now nondeterministic.

Secondly, we convert the nondeterministic automaton G_{nd} into a deterministic automaton G_{obs} using the standard method [2].

$$G_{obs} = Ac(X, \Sigma, x, x_o)$$

where $X = 2^{Q \cup \{q_o\}}$, $x_o = Q \cup \{q_o\}$, $x(x, s) = \{q \in Q \cup \{q_o\} : (\exists q' \in x) d_{nd}(q', s) = q\}$,

and Ac denotes the accessible part. The automaton G_{obs} tells us which subset of states the system could be in after observing a sequence of events.

Thirdly, let us mark the states in G_{obs} that contain singleton state of Q.

$$X_m = \{x \in X : |x| = 1\}$$

where $|x|$ denotes the number of elements in *x*. X_m has the following property: if the system reaches a state in X_m, then we know exactly the current state and the subsequent states of the system. Since the original G is deterministic, if the system reaches X_m, it will stay inside X_m forever.

Fourthly, we check if there are loops among states outside X_m. If there are not such loops, then the system can always reach X_m after a finite number of observations (transitions). This is because of the assumption that the system is deadlock free. Therefore, the system is strongly detectable. If there are such loops, then the system is not strongly detectable. However, if X_m is not empty, then the system will enter X_m for some trajectories. Hence the system is detectable. Formally, let us define the automaton remained after removing the states in X_m as

$$G_{obs}^{rem} = Ac(X - X_m, \Sigma, x\,|_{X-X_m}, x_o),$$

where $x\,|_{X-X_m}$ is the restriction of x in $X - X_m$.

We have the following criterions for checking strong detectability and detectability.

Criterion for Checking Strong Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o = \Sigma$ and $Y = \text{emptyset}$ is strongly detectable if and only if there are no loops in G_{obs}^{rem}.

Proof: If there are no loops in G_{obs}^{rem}, then the system will enter X_m and stay within X_m after a finite number of observations for all possible trajectories of the system. Therefore, the system is strongly detectable. If there are loops in G_{obs}^{rem}, then the system can stay in the loops and is not strongly detectable.

Q.E.D.

Criterion for Checking (Weak) Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o = \Sigma$ and $Y = \text{emptyset}$ is detectable if and only if X_m is not empty.

Proof: If X_m is not empty, then X_m is accessible from the initial state x_o. Therefore, there exist some trajectories of the system, for which the system will enter X_m and stay within X_m

after a finite number of observations, that is, the system is detectable. On the other hand, if X_m is empty, then the system is not detectable.

Q.E.D.

For the system in Figure 1(a), G_{obs} is shown in Figure 2. The system is not strongly detectable because there are loops of ab outside X_m, but the system is detectable because X_m is not empty.

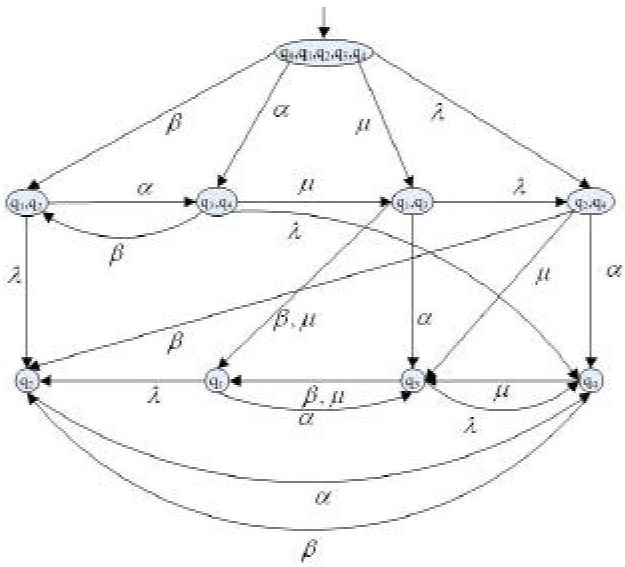


Figure 2. G_{obs} for the system in Figure 1(a)

Let us now consider the system in Figure 1(b). G_{obs} is shown in Figure 3. The system is strongly detectable because there are no loops outside X_m.

Obviously, if we assume that all events are observable and no states are observable, then the estimation problem is not difficult to solve.

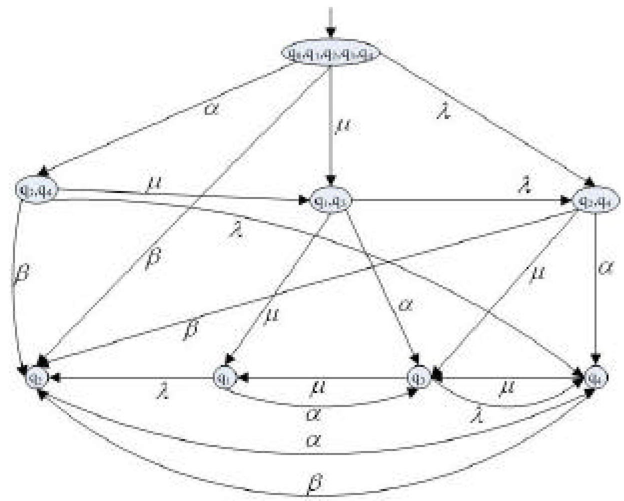


Figure 3. G_{obs} for the system in Figure 1(b)

4. Complete Event Observation and Some State Observation

We now consider the state estimation problem with complete event observation and some state observation, that is, we consider the case $\Sigma_o = \Sigma$ and $Y \neq \text{emptyset}$. The problem can be stated as follows.

State Estimation Problem 2

Given a discrete event system

$$(G, P, h, \Sigma_o, Y)$$

we do not know the initial state of $G = (Q, \Sigma, d)$. We have complete event observation and some state observation, that is, $\Sigma_o = \Sigma$ and $Y \neq \text{emptyset}$. Can we determine the current state of the system after a finite number of event observations?

The additional observation of states will certainly help us in determine the current state of the system. Of course, such state observation is not complete, that is, the state output mapping is not one-to-one; otherwise, the state estimation problem is trivial.

The definition of strong detectability and (weak) detectability is similar to the case without state observation.

Strong Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o = \Sigma$ and $Y \neq$ the empty set is strongly detectable if we can determine the current state and the subsequent states of the system after a finite number of events observation for all trajectories of the system.

(Weak) Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o = \Sigma$ and $Y \neq$ emptyset is (weakly) detectable if we can determine the current state and the subsequent states of the system after a finite number of event observations for some trajectories of the system.

To check detectability and strong detectability with partial state observation, we proceed as follows.

Step 1, we add an initial state q_o to G. We extend the event set from Σ to $(\Sigma \cup \{f\}) \times Y$ (f means no event transition is observed). We also add and modify transitions in G as follows. For all states $q \in Q$, we add transitions from q_o to q with label (f, y) where $y = h(q)$; that is, initially by observing state output, we know which subset of states the system is in (this introduces nondeterminism into the automaton). We then modify the original transitions (q', s, q) in G by re-labeling them with labels (s, y) where $y = h(q)$ (each transition is now labeled by both the event observation and state observation). Formally,

$$G_{s,nd} = (Q \cup \{q_o\}, (\Sigma \cup \{f\}) \times Y, d_{s,nd}, q_o)$$

where $d_{s,nd} = \{(q_o, (f, h(q)), q) : q \in Q\} \cup \{(q', (s, h(q)), q) : (q', s, q) \in d\}$.

Meanings of these transitions are as follows. Initially, the system is in the added new state q_o. When state output $y = h(q)$ is observed before any event occurs, then the system is in state $d_{s,nd}(q_o, (f, h(q)))$ (there may be more than one such states). If the system is in state q' and event s occurs that takes the system to $d(q', s) = q$, then event s and state output $y = h(q)$ are observed. Therefore the system is in state $d_{s,nd}(q', (s, h(q))) = q$.

Step 2, we convert the nondeterministic automaton $G_{s,nd}$ into a deterministic automaton $G_{s,obs}$:

$$G_{s,obs} = Ac(X, (\Sigma \cup \{f\}) \times Y, x_s, x_{s,o})$$

where $X = 2^{Q \cup \{q_o\}}$, $x_s(x, (s, y)) = \{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (s, y), q) \in d_{s,nd}\}$, and $x_{s,o} = Q \cup \{q_o\}$.

The reason for constructing $G_{s,obs}$ is that it describes the estimate of possible states of the system as stated in the following two lemmas.

Lemma 1

If the current estimate of possible states of G is $x \in X$ (that is, $x \subseteq Q \cup \{q_o\}$) and an event $s \in \Sigma$ occurs and $y \in Y$ is observed, then the next estimate of possible states of G is $x' = \{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (s, y), q) \in d_{s,nd}\}$.

Proof: By the definition of $d_{s,nd}$

$$\begin{aligned} x' &= \{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (s, y), q) \in d_{s,nd}\} \\ &= \{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (s, h(q)), q) \in d_{s,nd} \wedge h(q) = y\} \\ &= \{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', s, q) \in d \wedge h(q) = y\}. \end{aligned}$$

That is, x' consists of all states that can be reached from a state in x if event $s \in \Sigma$ occurs and $y \in Y$ is observed.

Q.E.D.

Lemma 2

1. The initial estimate of possible states of G after state output $y \in Y$ is observed and before any event occurs is $x_s(x_{s,o},(f,y)) = \{q \in Q \cup \{q_o\}: (q_o,(f,y),q) \in d_{s,nd}\}$.

2. The estimate of possible states of G after events $s_1 s_2 ... s_k$ occur and state outputs $y_0 y_1 y_2 ... y_k$ are observed is

$$x_s(x_{s,o},(f,y_0)(s_1,y_1)(s_2,y_2)...(s_k,y_k)).$$

Proof:

1. By the definition of $G_{s,nd}$, $x_s(x_{s,o},(f,y)) = \{q \in Q \cup \{q_o\}: (q_o,(f,y),q) \in d_{s,nd}\}$ is the set of all state possible from q_o when state output $y = h(q)$ is observed before any event occurs. Hence it is the initial estimate.

2. By the definition of $G_{s,nd}$ and repeated applications of Lemma 1.

Q.E.D.

Step 3, we mark the states in $G_{s,obs}$ that contain singleton state and denote the set by $X_m = \{x \in X : |x| = 1\}$.

Step 4, we remove states in X_m from $G_{s,obs}$ and denote the automaton remained after removing the states in X_m as $G_{s,obs}^{rem}$:

$$G_{s,obs}^{rem} = Ac(X - X_m, \Sigma, x_s|_{X-X_m}, x_{s,o}).$$

Step 5, we check strong detectability and detectability with partial state observation using the following criterions.

Criterion for Checking Strong Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o = \Sigma$ and $Y \neq \text{emptyset}$ is strongly detectable if and only if there are no loops in $G_{s,obs}^{rem}$.

Proof: Since G is a deterministic automaton, X_m in $G_{s,obs}$ has the following property: if $G_{s,obs}$ reaches X_m, it will stay inside X_m forever. The condition that there are no loops in $G_{s,obs}^{rem}$ is a necessary and sufficient condition for $G_{s,obs}$ to reach X_m after a finite number of event observations. The result then follows from Lemma 2.

Q.E.D.

Criterion for Checking (Weak) Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o = \Sigma$ and $Y \neq \text{emptyset}$ is detectable if and only if X_m is not empty.

Proof: Again, X_m in $G_{s,obs}$ has the following property: if $G_{s,obs}$ reaches X_m, it will stay inside X_m forever. Therefore X_m is not empty is a necessary and sufficient condition for the existence of some trajectories under which $G_{s,obs}$ will reach X_m after a finite number of event observations. The result then follows from Lemma 2.

Q.E.D.

Consider, for example, the system in Figure 1(a). Let us add a state observation as follows. There are two outputs for states: $Y = \{1, 2\}$. The output map is defined as $h(q_1) = 1$, $h(q_2) = 2$, $h(q_3) = 1$, $h(q_4) = 2$. Using the above procedure, $G_{s,nd}$ and $G_{s,obs}$ are shown in Figures 4 and 5 respectively. It can be verified that the system is strongly detectable with partial state observation. Note that without state observation, the system is not strongly detectable.

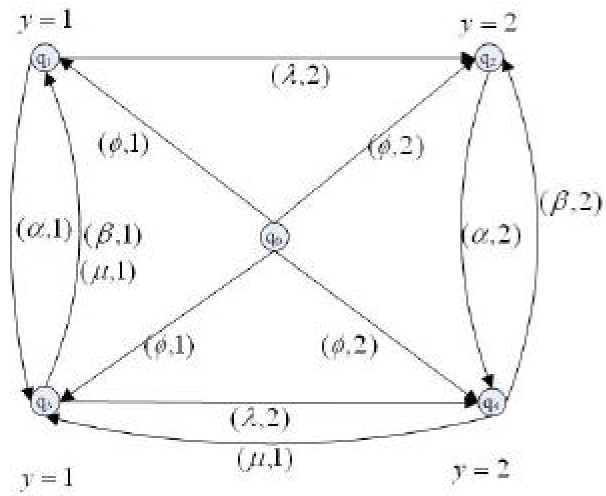


Figure 4. $G_{s,nd}$ for the system in Figure 1(a) with $h(q_1)=1$, $h(q_2)=2$, $h(q_3)=1$, $h(q_4)=2$

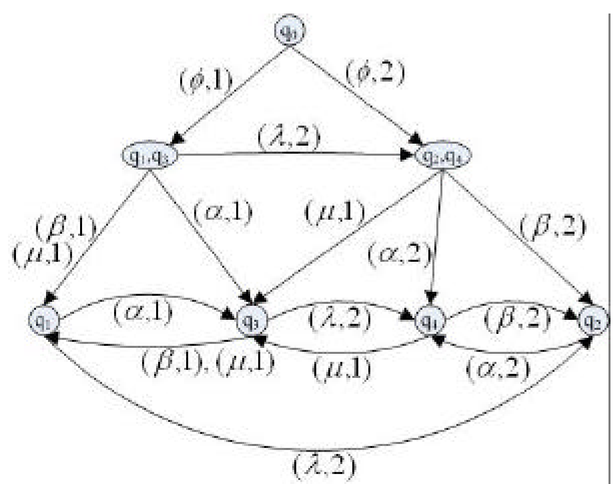


Figure 5. $G_{s,obs}$ for the system in Figure 4

On the other hand, for the same system in Figure 1(a), if we change the state observation and let the output map be $h(q_1)=1$, $h(q_2)=1$, $h(q_3)=2$, $h(q_4)=2$, then $G_{s,nd}$ and $G_{s,obs}$ are shown in Figures 6 and 7 respectively. It can be seen that the system is detectable but not strongly detectable with state observation.

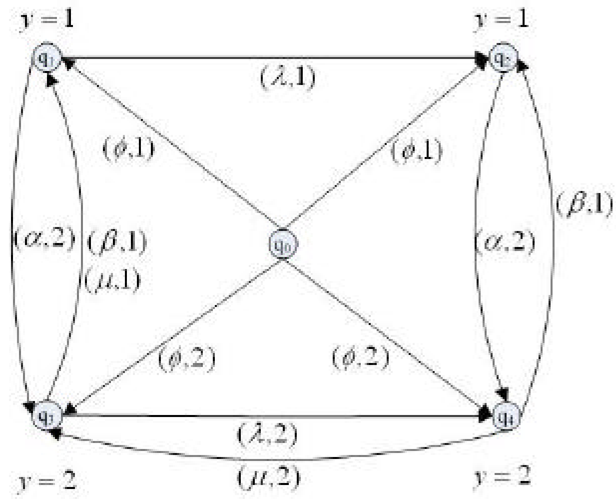


Figure 6. $G_{s,nd}$ for the system in Figure 1(a) with $h(q_1) = 1$, $h(q_2) = 1$, $h(q_3) = 2$, $h(q_4) = 2$

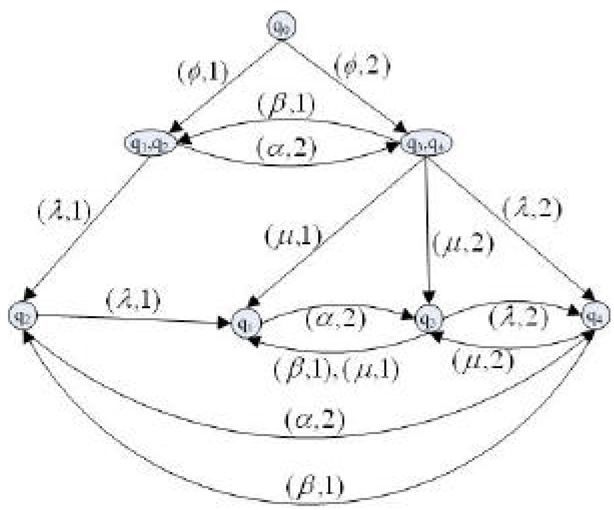


Figure 7. $G_{s,obs}$ for the system in Figure 6

5. Partial Event Observation and No State Observation

We now consider case 3, that is, some events are observable and no state is observable. We need to make an assumption tha t no infinite strings exist whose events are all unobservable. In other words, no loops in G contain only unobservable events. This assumption ensures that the system will not look like in deadlock. We will discuss how to relax this condition at the end of the paper.

State Estimation Problem 3

Given a discrete event system

$$(G, P, h, \Sigma_o, Y)$$

We do not know the initial state of $G = (Q, \Sigma, d)$. We have partial event observation and no state observation, that is, $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$. Can we determine the current state of the system after a finite number of event observations?

For this problem, the definitions of strong detectability and (weak) detectability are as follows.

Strong Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is strongly detectable if we can determine the current state and the subsequent states of the system after finite number of event observations for all trajectories of the system.

(Weak) Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is detectable if we can determine the current state and the subsequent states of the system after finite number of event observations for some trajectories of the system.

The procedure to check detectability and strong detectability with partial event observation is more difficult than that for complete events observation. We need to handle unobservable events properly.

Step 1, we add an initial state q_o to G. We extend the event set Σ to $(\Sigma \cup \{e\})$. For all states $q \in Q$, we add e-transition from q_o to q. For observable transitions $s \in \Sigma_o$, we write the transition as (q', s, q). For unobservable transitions $s \notin \Sigma_o$, we re-label the transition as (q', e, q). Formally,

$$G_{p,nd} = (Q \cup \{q_o\}, \Sigma_o \cup \{e\}, d_{p,nd}, q_o)$$

where $d_{p,nd} = \{(q_o, e, q) : q \in Q\} \cup \{(q', e, q) : (q', s, q) \in d \wedge s \notin \Sigma_o\}$

$\cup \{(q', s, q) : (q', s, q) \in d \wedge s \in \Sigma_o\}$.

In other words, $d_{p,nd}$ is a mapping $d_{p,nd}$: $(Q \cup \{q_o\}) \times (\Sigma_o \cup \{e\}) \rightarrow 2^{Q \cup \{q_o\}}$, which can be easily extended to

$$d_{p,nd} : (Q \cup \{q_o\}) \times (\Sigma_o \cup \{e\})^* \rightarrow 2^{Q \cup \{q_o\}}.$$

Step 2, we convert the nondeterministic automaton $G_{p,nd}$ into a deterministic automaton $G_{p,obs}$. Since $G_{p,nd}$ has unobservable transition (q', e, q), we need first define the unobservable reach from a subset of states $x \subseteq Q \cup \{q_o\}$ as follows:

$$UR(x) = x \cup \{q \in Q \cup \{q_o\} : (\exists q' \in x) q \in d_{p,nd}(q', e)\}.$$

Then we can define

$$G_{p,obs} = Ac(X, \Sigma_o, x_p, x_{p,o})$$

where $X = 2^{Q \cup \{q_o\}}$, $x_p(x, s) = UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', s, q) \in d_{p,nd}\})$, and $x_{p,o} = Q \cup \{q_o\}$. The reason for constructing $G_{p,obs}$ is that it describes the estimation of possible states of the system as follows.

Lemma 3

If the current estimate of possible states of G is $x \in X$ (that is, $x \subseteq Q \cup \{q_o\}$) and an event $s \in \Sigma$ occurs, then the next estimate of possible states of G is $x' = UR(\{q \in Q \cup \{q_o\}:$ $(\exists q' \in x)(q', s, q) \in d_{p,nd}\})$.

Proof: Similar to Lemma 1.

Q.E.D.

Lemma 4

1. The initial estimate of possible states of G is $x_{p,o} = UR(\{q_o, q_1, \cdots, q_n\})$.
2. The estimate of possible states of G after transitions $s_1 s_2 \ldots s_k$ are observed is $x_p(x_{p,o}, s_1 s_2 \ldots s_k)$.

Proof: By Lemma 3 and similar to Lemma 2.

Q.E.D.

Step 3, we mark the states in $G_{p,obs}$ that contain singleton state and denote the set by $X_m = \{x \in X : |x| = 1\}$.

Step 4, we remove states in X_m from $G_{p,obs}$ and denote the automaton remained after removing the states in X_m as $G_{p,obs}^{rem}$:

$$G_{p,obs}^{rem} = Ac(X - X_m, \Sigma, x_p |_{X - X_m}, x_{p,o}).$$

Step 5, under the assumption that no loops in G contain only unobservable events, there exists no deadlock state in the observer $G_{p,obs}$. We check the following criterions on strong detectability and detectability with partial event observation and no state observation.

Criterion for Checking Strong Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is strongly detectable if and only if all loops in $G_{p,obs}$ are entirely within X_m.

Proof: By Lemma 4, the observer $G_{p,obs}$ describes the estimate of states the system may be in. When $G_{p,obs}$ enters states in X_m, we know exactly which state the system is in. By our assumptions, the observer $G_{p,obs}$ is deadlock free. Since $G_{p,obs}$ is finite, after some finite observations, $G_{p,obs}$ must enter some loops. If all loops in $G_{p,obs}$ are entirely within X_m, then the current state and the subsequent states of the system are known no matter which trajectory the system follows, that is, the system is strongly detectable. On the other hand, if there exists some loops in $G_{p,obs}$ that are not entirely within X_m, then the system can follow those loops and hence not strongly detectable.

Q.E.D.

Criterion for Checking Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is detectable if and only if X_m is not empty and there are loops in X_m.

Proof: If X_m is not empty then it is accessible from the initial state in $G_{p,obs}$. So for any state in X_m, there must exist some trajectories for which the system can enter that state. Furthermore, if there are loops in X_m, then any such loop can produce at least one infinite string by which the system can always stay within X_m. Hence the system is detectable by Lemma 4. On the other hand, if there are no loops in X_m, the system will eventually leave X_m along any trajectory of the system. Hence the system is not detectable.

Q.E.D.

Let us examine Figure 1(b), we assume that event b is not observable, that is, $\Sigma_o = \{a, m, l\}$. We obtain $G_{p,nd}$ and $G_{p,obs}$ as shown in Figures 8 and 9 respectively.

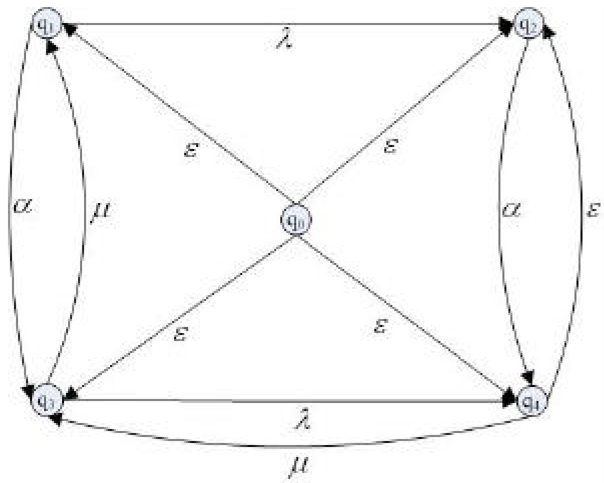


Figure 8. $G_{p,nd}$ of a discrete event system with partial events observation

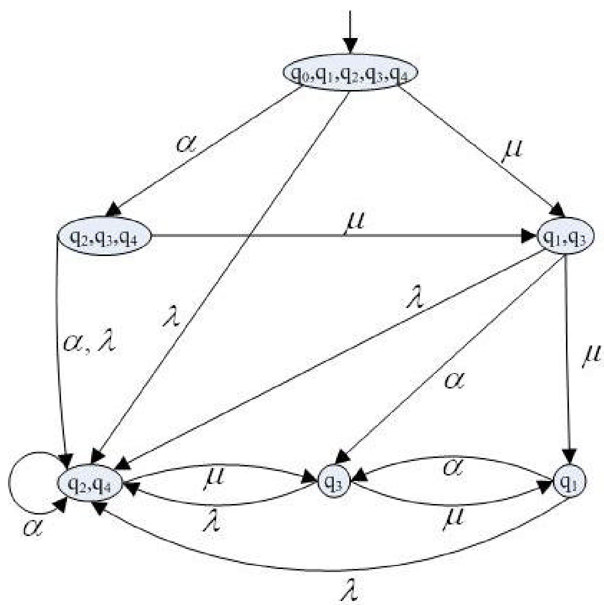


Figure 9. $G_{p,obs}$ for the system in Figure 8

From Figure 9, we can see that the system is not strongly detectable. But it is detectable. If we compare partial event observation with complete event observation in Figure 3, we will see a major difference between the two figures. Under complete event observation, if the observer enters X_m, it will remain in X_m forever for a deterministic discrete event system. However, this is not true for system under partial events observation. In other words, even if the current state of the system is known, it may become unknown since next event may not be observable. If this is the case, then our hope is that we can determine the current state periodically, which leads to the following two definitions.

Strong Periodic Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is strongly periodically detectable if we can periodically determine the current state of the system for all trajectories of the system.

(Weak) Periodic Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is periodically detectable if we can periodically determine the current state of the system for some trajectories of the system.

The procedure for checking strongly periodic detectability and (weak) periodic detectability is same as that for checking strong detectability and (weak) detectability, except that the criterions are different.

Criterion for Checking Strong Periodic Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o \subset \Sigma$ and $Y = \text{emptyset}$ is strongly periodically detectable if and only if there are no loops in $G_{p,obs}^{rem}$.

Proof: The condition of no loops in $G_{p,obs}^{rem}$ ensures that the system cannot always stay in $X - X_m$. Therefore, the system must visit X_m periodically. This implies that we can periodically determine the current state of the system for all trajectories of the system by Lemma 4. On the other hand, if there are loops in $G_{p,obs}^{rem}$, then the system may stay in the loop forever. Hence the system is not strongly periodically detectable.

Q.E.D.

Criterion for Checking Periodic Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o \subset \Sigma$ and $Y =$ emptyset is periodically detectable if and only if there are loops in $G_{p,obs}$ which include at least one state belonging to X_m.

Proof: If there are loops in $G_{p,obs}$ which include at least one state belonging to X_m, then the system can stay in this loop, and we can periodically determine the current state of the system for some trajectories of the system by Lemma 4. If no such loops exist, then the system is not periodically detectable.

Q.E.D.

For the system in Figure 1(b), let us remove the a on the right and assume that b is unobservable ($\Sigma_o = \{a, m, l\}$). We have $G_{p,nd}$ and $G_{p,obs}$ shown in Figures 10 and 11 respectively.

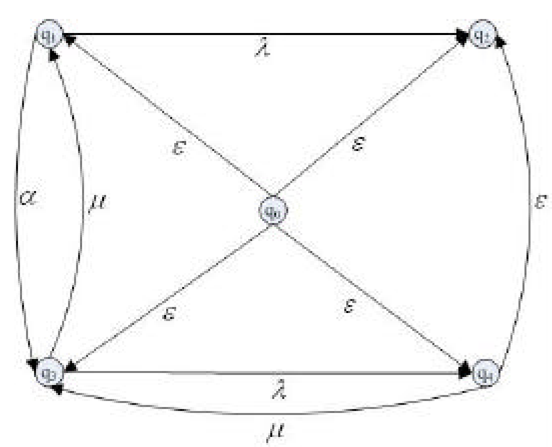


Figure 10. $G_{p,nd}$ of a discrete event system with partial event observation

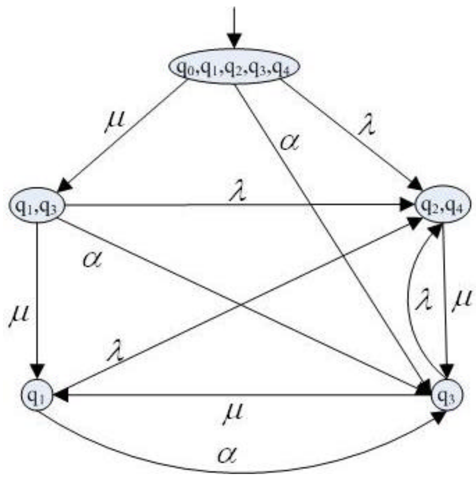


Figure 11. $G_{p,obs}$ for the system in Figure 10

From Figure 11, we can see that though the system is not strongly detectable, but it is strongly periodically detectable.

6. Partial Event Observation and Some State Observation

We now consider the state estimation problem for systems with partial event observations and some state observations. The problem can be stated as follows.

State Estimation Problem4

Given a discrete event system

$$(G, P, h, \Sigma_o, Y)$$

we do not know the initial state of $G = (Q, \Sigma, d)$. We have partial event observations and some state observations, that is, $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$. Can we determine the current state and the subsequent states of the system after a finite number of observations?

For this problem, the definitions of strong detectability, (weak) detectability, strong periodic detectability and (weak) periodic detectability are as follows.

Strong Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$ is strongly detectable if we can determine the current state and the subsequent states of the system after a finite number of observations for all trajectories of the system.

(Weak) Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$ is (weakly) detectable if we can determine the current state and the subsequent states of the system after a finite number of observations for some trajectories of the system.

Strong Periodic Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$ is strongly periodically detectable if we can periodically determine the current state of the system for all trajectories of the system.

(Weak) Periodic Detectability

A discrete event system

$$(G, P, h, \Sigma_o, Y)$$

with $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$ is periodically detectable if we can periodically determine the current state of the system for some trajectories of the system.

The procedure to check the above four types of detectabilities is as follows.

Step 1, we add an initial state q_o to G. We extend the event set from Σ to $(\Sigma \cup \{f\}) \times Y \cup \{e\}$. For all states $q \in Q$, we add transitions from q_o to q with label (f, y) where $y = h(q)$. We re-label the transitions (q', s, q) in G to take into account of observation. We need to consider two types of transitions: For observable transitions $s \in \Sigma_o$, we re-label (q', s, q) as $(q', (s, h(q)), q)$. For unobservable transitions $s \notin \Sigma_o$, we re-label (q', s, q) as $(q', (f, h(q)), q)$ if $h(q') \neq h(q)$ and as (q', e, q) if $h(q') = h(q)$. The reason for the above re-labeling is that if $h(q') \neq h(q)$, then we know that some event has occurred because the system has changed states, although we do not know which event has occurred; if $h(q') = h(q)$, then nothing will be observed. Formally,

$$G_{ps,nd} = (Q \cup \{q_o\}, (\Sigma_o \cup \{f\}) \times Y \cup \{e\}, d_{ps,nd}, q_o)$$

where $d_{ps,nd} = \{(q_o, (f, h(q)), q) : q \in Q\} \cup \{(q', (s, h(q)), q) : (q', s, q) \in d \wedge s \in \Sigma_o\}$

$$\cup \{(q', (f, h(q)), q) : (q', s, q) \in d \wedge s \notin \Sigma_o \wedge h(q') \neq h(q)\}$$

$$\cup \{(q', e, q) : (q', s, q) \in d \wedge s \notin \Sigma_o \wedge h(q') = h(q)\}.$$

In other words, $d_{ps,nd}$ is a mapping $d_{ps,nd}: (Q \cup \{q_o\}) \times ((\Sigma_o \cup \{f\}) \times Y \cup \{e\}) \to 2^{Q \cup \{q_o\}}$, which can be easily extended to

$$d_{ps,nd} : (Q \cup \{q_o\}) \times ((\Sigma_o \cup \{f\}) \times Y \cup \{e\})^* \to 2^{Q \cup \{q_o\}}.$$

Step 2, we convert the nondeterministic automaton $G_{ps,nd}$ into a deterministic automaton $G_{ps,obs}$.

$$G_{ps,obs} = Ac(X, (\Sigma_o \cup \{f\}) \times Y, x_{ps}, x_{ps,o}),$$

where $X = 2^{Q \cup \{q_o\}}$, $x_{ps}(x, (s, y)) = UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (s, y), q) \in d_{ps,nd}\})$, and $x_{ps,o} = \{q_o\}$.

As before, the reason for constructing $G_{ps,obs}$ is to describe the estimate of possible states of the system as shown in the following lemma.

Lemma 5

If the current estimate of possible states of G is $x \in X$ (that is, $x \subseteq Q \cup \{q_o\}$), an event $s \in \Sigma$ occurs, then either s or nothing (f) is observed and state output $y \in Y$ is also observed. The next estimate of possible states of G is $x' = UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x)$ $(q', (s, y), q) \in d_{ps,nd}\})$, where s could be f.

Proof: If $s \neq f$, then

$$\begin{aligned} x' &= UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (s, y), q) \in d_{ps,nd}\}) \\ &= UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', s, q) \in d \wedge s \in \Sigma_o \wedge h(q) = y\}) \end{aligned}$$

That is, x' consists of all states in the unobservable reach of the set of states that can be reached from a state in x if event $s \in \Sigma_o$ and $y \in Y$ is observed.

If $s = f$, then

$$\begin{aligned} x' &= UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x)(q', (f, y), q) \in d_{ps,nd}\}) \\ &= UR(\{q \in Q \cup \{q_o\} : (\exists q' \in x) \\ &\qquad ((q', s, q) \in d \wedge s \notin \Sigma_o \wedge h(q) = y \neq h(q')) \vee (q' = q_o \wedge h(q) = y)\}) \end{aligned}$$

That is, x' consists of all states in the unobservable reach of the set of states that can be reached from a state in x if no event is observed but a new state output $y \in Y$ is observed.

Q.E.D.

Lemma 6

1. The initial estimate of possible states of G after state output $y \in Y$ is observed and before any event occurs is $x_{ps}(x_{ps,o}, (f, y)) = UR\{q \in Q \cup \{q_o\} : (q_o, (f, y), q) \in d_{ps,nd}\}$.

2. The estimate of possible states of G after transitions $s_0 s_1 s_2 \ldots s_k$ (s_i could be f) and state outputs $y_0 y_1 y_2 \ldots y_k$ are observed is $x_{ps}(x_{ps,o},(s_0,y_0)(s_1,y_1)(s_2,y_2) \ldots (s_k,y_k))$.

Proof:

3. By the definition of $G_{ps,nd}$, $x_{ps}(x_{ps,o},(f,y)) = UR\{q \in Q \cup \{q_o\} : (q_o,(f,y),q) \in d_{ps,nd}\}$ is the set of all state possible from q_o when state output $y = h(q)$ is observed before any event occurs. Hence it is the initial estimate.

4. By the definition of $G_{s,nd}$ and repeated applications of Lemma 5.

Q.E.D.

Step 3, we mark the states in $G_{ps,obs}$ that contain singleton state and denote the set by $X_m = \{x \in X : |x| = 1\}$.

Step 4, we remove states in X_m from $G_{ps,obs}$ and denote the automaton remained after removing the states in X_m as $G_{ps,obs}^{rem}$:

$$G_{ps,obs}^{rem} = Ac(X - X_m, \Sigma_o, x_{ps}|_{X-X_m}, x_{ps,o}).$$

Step 5, since there is no deadlock state in the observer $G_{ps,obs}$, we can now check detectability with partial event observation and partial state observation using the following criterions.

Criterion for Checking Strong Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o \subset \Sigma$ and $Y \neq$ emptyset is strongly detectable if and only if all loops in $G_{ps,obs}$ are entirely within X_m.

Proof: By Lemma 6, the observer $G_{ps,obs}$ describes the estimate of states the system may be in. When $G_{ps,obs}$ enters states in X_m, we know exactly which state the system is in. By our assumptions, the observer $G_{ps,obs}$ is deadlock free. Since $G_{ps,obs}$ is finite, after some finite observations, $G_{ps,obs}$ must enter some loops. If all loops in $G_{p,obs}$ are entirely within X_m, then the current state and the subsequent states of the system are known no matter which trajectory the system follows, that is, the system is strongly detectable. On the other hand, if there exists some loops in $G_{ps,obs}$ that are not entirely within X_m, then the system can follow those loops and hence not strongly detectable.

Q.E.D.

Crite rion for Checking Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$ is detectable if and only if there are loops in X_m.

Proof: If X_m is not empty then it is accessible from the initial state in $G_{ps,obs}$. So for any state in X_m, there must exist some trajectories for which the system can enter that state. Furthermore, if there are loops in X_m, then any such loop can produce at least one infinite string by which the system can always stay within X_m. Hence the system is detectable by Lemma 6. On the other hand, if there are no loops in X_m, the system will eventually leave X_m along any trajectory of the system. Hence the system is not detectable.

Q.E.D.

Criterion for Checking Strongly Periodic Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o \subset \Sigma$ and $Y \neq \text{emptyset}$ is strongly periodically detectable if and only if there are no loops in $G_{ps,obs}^{rem}$.

Proof: The condition of no loops in $G_{ps,obs}^{rem}$ ensures that the system cannot always stay in $X - X_m$. Therefore, the system must visit X_m periodically. This implies that we can periodically determine the current state of the system for all trajectories of the system by Lemma 2. On the other hand, if there are loops in $G^{rem}_{p,obs}$, then the system may stay in the loop forever. Hence the system is not strongly periodically detectable.

Q.E.D.

Criterion for Checking Periodic Detectability

A discrete event system (G, P, h, Σ_o, Y) with $\Sigma_o \subset \Sigma$ and $Y \neq$ emptyset is periodically detectable if and only if there are loops in $G_{ps,obs}$ which include at least one state belonging to X_m.

Proof: If there are loops in $G_{ps,obs}$ which include at least one state belonging to X_m, then the system can stay in this loop, and we can periodically determine the current state of the system for some trajectories of the system by Lemma 4. If no such loops exist, then the system is not periodically detectable.

Q.E.D.

Let us now consider the system in Figure 1(a). We assume that event m is not observable: $\Sigma_o = \{a, b, l\}$. We have the following state observations. $Y = \{1, 2, 3\}$. The state output is $h(q_1) = 1$, $h(q_2) = 2$, $h(q_3) = 3$, $h(q_4) = 2$. $G_{ps,nd}$ and $G_{ps,obs}$ are calculated as shown in Figures 12 and 13 respectively. From Figure 13, we can see that the system is strongly detectable.

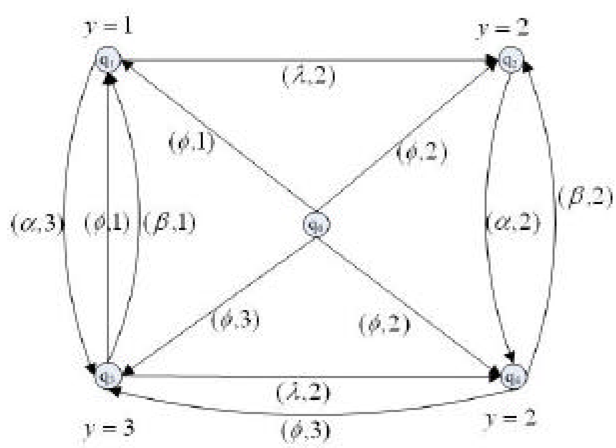


Figure 12. $G_{ps,nd}$ of a discrete event system with partial event observation

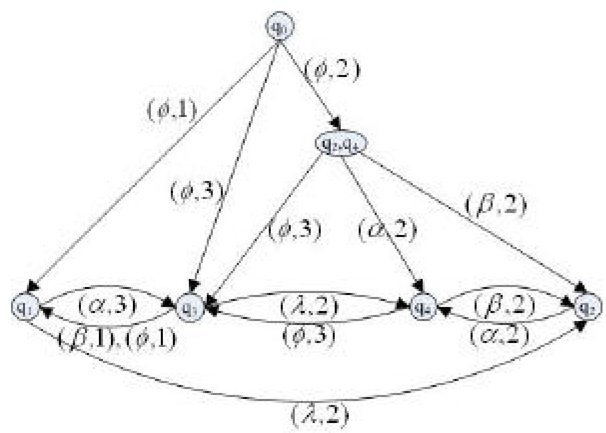


Figure 13. $G_{ps,obs}$ for the system in Figure 12

However, if we change the unobservable event from m to l, and state observation to $h(q_1)=1$, $h(q_2)=1$, $h(q_3)=3$, $h(q_4)=2$, then $G_{ps,nd}$ and $G_{ps,obs}$ are shown in Figures 14 and 15 respectively. It can be checked that the system is detectable but not strongly detectable.

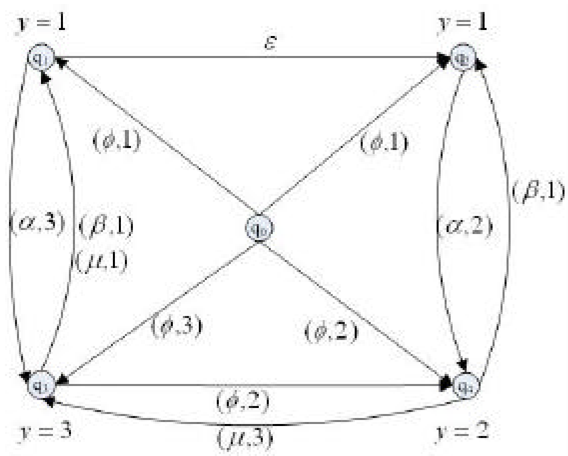


Figure 14. $G_{ps,nd}$ of a discrete event system with partial event observation

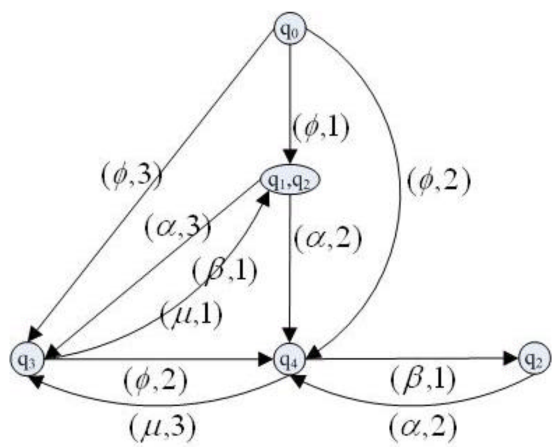


Figure 15. $G_{ps,obs}$ for the system in Figure 14

Let us consider again Figure 15, from it we can get $G_{ps,obs}^{rem}$ as in Figure 16:

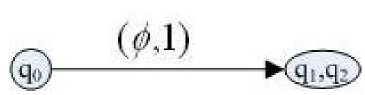


Figure 16. $G_{ps,obs}^{rem}$ for the system in Figure 14

From $G_{ps,obs}$ in Figure 15 and $G^{rem}_{ps,obs}$ in Figure 16 we can conclude the system is strongly periodic detectable.

Finally, let us look at another example for illustrating the difference between strongly periodic detectability and periodic detectability.

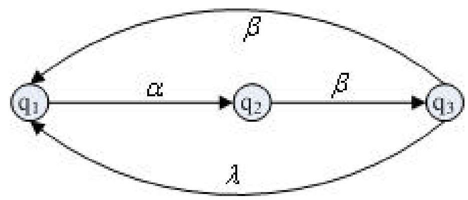


Figure 17. A discrete event system that is periodically detectable but not strongly periodically detectable

The system is showed in Figure 17. b is the unobservable event and the other events are all observable. There is no state output. The observer of the system is showed in Figure 18. From the figure, we know that the system is periodic detectable, but not strongly periodic detectable.

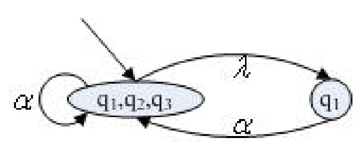


Figure 18. $G_{ps,obs}$ for the system in Figure 17

7. Conclusion

In this paper, we considered both event observation and state observation. We defined detectability and periodic detectability, both in a strong sense and in a weak sense. We constructed an observer, whose roles are to estimate the states of a system after a sequence of observation. We derived computable criterions for checking necessary and sufficient conditions for various types of detectability. These definitions and criterions extend the results of [9] and [10] significantly and covered most cases encountered in practice.

References

[1] L. Aguirre-Salas, O. Begovich and A. Ramirez-Trevino, "State estimation in DES modeled by a class of interpreted Petri nets," Proceedings of the 2002 IEEE International Symposium on Intelligent Control, pp. 574-579, 2002.

[2] C. Cao, F. Lin and Z.-H. Lin, "Why event observation: Observability Revisited," Discrete Event Dynamic Systems: Theory and Applications, 7(2), pp. 127-149, 1997.

[3] C.G. Cassandras, and S. Lafortune, "Introduction to Discrete Event Systems," Kluwer, 1999.

[4] R. Cieslak, C. Desclaux, A. Fawaz, and P. Varaiya, "Supervisory control of discrete-event processes with partial observations," IEEE Transactions on Automatic Control, Vol. 33, pp. 249-260, March, 1988.

[5] E. Fabre, A. Benveniste, C. Jard, L. Ricker and M. Smith, "Distributed state reconstruction for discrete event systems," Proceedings of the 39th IEEE Conference on Decision and Control, pp. 2252-2257, 2000.

[6] F. Lin, "Diagnosability of discrete event systems and its applications," Discrete Event Dynamic Systems: Theory and Applications, 4(1), pp. 197-212, 1994.

[7] F. Lin and W. M. Wonham, "On observability of discrete event systems," Information Sciences, 44(3):173-198, 1988.

[8] F. Lin and H. Ying, "Modeling and Control of Fuzzy Discrete Event Systems," IEEE Transactions on Man, Systems and Cybernetics - Part B, 32:408-415, 2002.

[9] C.M. Ozveren and A.S. Willsky, "Observability of Discrete Event Dynamic Systems," IEEE Transactions on Automatic Control, 35(7): 797-806, 1990.

[10] P.J. Ramadge. “Observability of discrete event systems,” In Proceedings of CDC, December, 1986.

[11] R.J. Ramadge and W. M. Wonham, “Supervisory control of a class of discrete event processes,” SIAM J. Control and Optimization, 25(1:206-230, 1987.

[12] Meera Sampath, Raja Sengupta, Stephane Lafortune, Kasim Sinnamohideen, Demosthenis Teneketzis, “Diagnosability of discrete event systems,” IEEE Transactions on Automatic Control, Vol. 40, No. 9, September, 1995.

[13] Meera Sampath, Raja Sengupta, Stephane Lafortune, Kasim Sinnamohideen, Demosthenis Teneketzis, “Failure diagnosis using descrete event models,” IEEE Transactions on Control Systems Technology, Vol. 4, No. 2, March, 1996,

[14] S. Hashtrudi Zad, R. H. Kwong and W. M. Wonham, "Fault diagnosis in discrete-event systems: Framework and model reduction," IEEE Transactions on Automatic Control, vol. 48, no. 7, pp. 1199-1212, July, 2003.

[15] W.M. Wonham, ‘Towards an abstract internal model principle,’ IEEE Trans. On Systems, Man, and Cybernetics, SMC-6 (11):735–740, 1976.