

Traverse Adjustment Using Microsoft Excel Solver

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Abstract

Simple two-dimensional traverses can easily be adjusted using Compass Rule or Transit Rule. However, when a traverse becomes more complicated with multiple junction points yielding redundancies greater than three, Compass Rule or Transit Rule adjustments will not be able to do an effective job, particularly when observations (distances and angles) have different weights. Although several least squares adjustment packages are available, Microsoft Excel provides another option for performing a rigorous least squares traverse adjustment. This paper provides: a brief overview of Excel's **Add-in** Solver; basic theory of least squares as implemented within the Solver; advantages and disadvantages of the Excel Solver in least squares, and two numerical examples outlining the steps involved in carrying out a traverse adjustment. The reasons the use of Excel for traverse adjustment can be considered a viable option are: a. Excel is readily available in any Windows platform without any additional cost. b. Excel is easy to use. c. The data transfer to and from Excel is very flexible. The paper also provides some useful Visual Basic functions used in traverse adjustment.

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1. INTRODUCTION

Solver is an Add-in tool in Microsoft Excel designed to perform optimization solutions for modeling systems. This paper describes various techniques for modeling survey traverse data to be adjusted by least squares. The models include a single traverse loop and interconnected traverse loops where observations are independently weighted.

1.1 About Microsoft Excel Solver

The Excel Solver Engine, developed by Frontline Systems, Inc., is a standard feature of any Microsoft Office product. Since the solution of least squares problems requires minimization, Excel with its strong Graphic User Interface (GUI) provides a convenient and readily available tool, the Solver, to adjust small to medium size traverses encountered in day to day surveying practices. Furthermore, the user does not need to know (1) how to form the condition equations (2) how to differentiate complex equations (3) how to solve the normal equations used in a conventional least squares approach.

Solving an optimization model using Excel Solver requires:

- Identifying model resource data
- Specifying the constraints for the model
- Specifying the model objective

The above three steps are carried out through the Solver's GUI dialog box.

In addition to the standard Solver, Frontline Systems provides Premium Solver Platforms to handle very large optimization models including a DLL Platform which can interface with Visual Basic and C/C++ programming languages. For additional information the reader is encouraged to visit the Frontline web site at <http://www.frontsys.com>. The site contains links to the most current information on the Solver platforms, examples, tutorials on different optimization models, although not much on least squares application to surveying problems. It should be pointed out that other spreadsheets such as Lotus 1-2-3 and Quattro Pro also provide a similar optimization model also developed by Frontline Systems, Inc.

1.1.1 Microsoft Excel Solver Installation

As stated earlier, Solver is an Excel Add-in tool available from **TOOLS/SOLVER...** If the Solver is not available, then it could be due to: (1) Solver is installed but not selected or (2) Solver is not installed.

1.1.1.1 Solver is installed but not selected

If Solver is installed, then from the **TOOLS** menu select **ADD-INS...** This will display all the available **ADD-INS** as shown in Figure 1.

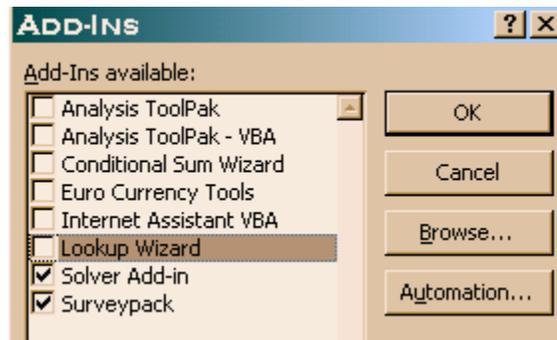


Figure 1

Check **Solver Add-in** box and click **OK**.

1.1.1.2 Solver is not installed

If Solver is not installed, then the **Solver Add-in** option in Figure 1 will not be available. In this case, use Microsoft Office CD and install the Excel Add-in. The Microsoft Excel online help and the [Microsoft Office Web site](#) provide excellent sources of reference to review when needed.

1.2 Adaptation of Traverse to Excel Solver

While it is not necessary for the user to understand the theory of traverse adjustment by least squares, it is however, important to understand the basic geometry of positioning. The geometric definition of the functional model described by Mikhail (1976) represents the mathematical relationships between points and lines. To aid in this analysis the following variables are defined:

- n_0 is the minimum number of observations needed to fix the model uniquely. Obviously, there must be some redundant observations in the model in order for any adjustment to take place.
- n is the total number of observations in the model.
- r is the total number of redundancies in the model, total number of observations minus the minimum number of observations needed to fix the model uniquely ($r = n - n_0$). Typically, in a close traverse where all the distances and angles are measured, there are three redundancies.
- w is the observation weight, and it is calculated as the inverse of the standard deviation squared. For angular observations, the standard deviation must be expressed in radians.
- v is the residual which is equal to the adjusted observation minus the observation ($v = \text{Adjusted} - \text{observed}$).

1.2.1 Least Squares Principle

The optimized solution in least squares is to minimize the weighted sum of the squares of the residuals ($\sum W_i \cdot v_i^2 \rightarrow \text{Minimum}$). In a two-dimensional traverse, the observations consist of distances, and horizontal angles. The adjusted distances and adjusted angles are computed as functions of latitudes

(distance X Cos(observed azimuth) and departures (distance X Sin(observed azimuth)). The observed azimuth is:

$$Az.2-1 = Az.1-2 \pm \pi + Angle \ 1 \ 2 \ 3 \quad (1)$$

Where Az.1 - 2, and Az. 2 - 3 are the respective azimuths from 1 to 2 and 2 to 3 derived from the observations.

The adjusted distances and adjusted angles are calculated as follows:

$$Adj. \ Dist = \sqrt{Latitude^2 + Departure^2} \quad (2)$$

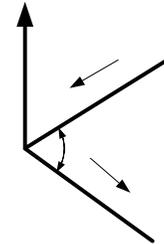


Figure 2

$$Adj. \ Angle = Arc \ tan\left(\frac{Dep_{2-3}}{Lat_{2-3}}\right) + \pi - Arc \ tan\left(\frac{Dep_{2-1}}{Lat_{2-1}}\right) \quad (3)$$

Note that equation (3) is specific to Figure 1 where line 1 2 is in quadrant 1, and line 2 3 is in quadrant 2. Similar provisions must be made for lines in other quadrants.

Equations (1), (2) and (3) are the principle equations the user needs to know. Furthermore, the Appendix contains a collection of user-defined Visual Basic (VB) functions that can be used to further simplify the process, for example, the function *azrad(x1, y1, x2, y2)* can be used to calculate the azimuth in radians from point 1 to point 2.

1.2.2 Design of a Traverse Model

Identifying model resource data: The traverse resource data are observations (angles, distances), fixed coordinates and fixed azimuths. Observed azimuths are calculated from angle observations using equation (1). A traverse must have a minimum of one point fixed (known coordinates) and one fixed azimuth in order to carryout any adjustment. Latitudes and departures are calculated using the observed distances and the observed azimuths. Using the computed latitudes and departures, adjusted distances and angles are calculated using equations (2) and (3). The observation standard deviations are used to compute the associated weights as $w = 1/\sigma^2$, where σ is the standard deviation. The standard deviation of angles must be expressed in radians.

Specifying the constraints for the model: In traverse adjustment, the constraints are:

- Sum of the latitudes = 0
- Sum of the departures = 0
- Sum of the interior angles = $(n-2)*180^\circ$ for a geometrically closed traverse, where n is the number of sides in the traverse. If the traverse originates from a fixed azimuth and closes to

another fixed azimuth, then the azimuth of closing line derived from the observations must equal to the fixed azimuth of the closing line.

Interconnecting traverses must be broken down into segments where the above three constraints for each closed traverse sub-segment is applied.

Specifying the model objective: The model objective in traverse adjustment, as stated earlier, is to minimize the weighted sum of the squares of the residuals (minimize $\sum_{i=1}^n W_i \cdot v_i^2$), where n is the number of observations.

2. SIMPLE TRAVERSE EXAMPLE

Figure 2 shows a simple traverse with point 10 being the fixed point and an external azimuth from 10 to Azimuth mark is held fixed. All the angles including the tie from the Azimuth mark to points 1, and 5 are measured including all the distances.

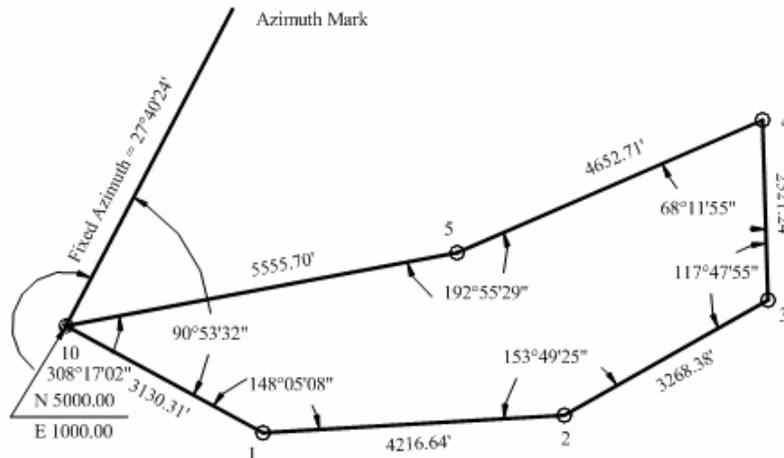


Figure 3

2.1 Traverse Setup in Excel

Figure 4 shows the observations (B6:B11, E6:12) their standard deviations (C6:C11, F6:F12) and weights (D6:D11, G6:G12), the azimuth of the Azimuth mark (H5 and I5) and the fixed coordinates of point 10 (J5, K5). The distance weights are calculated as $\frac{1}{0.05^2} = 400$, and the

angle weights are calculated as $\left(\frac{206264.8}{5}\right)^2 = 1701806709$.

	A	B	C	D	E	F	G	H	I	J	K
1	Traverse Adjustment - Example 1										
2		Observed	Standard	Distance	Observed	Standard	Angle	Fixed Az.	Fixed Az.	Coordinates	
3	Points	Distances	deviations	Weights	Angles	deviations	Weights	(dms)	(radians)	N 10	E 10
4					(dms)	(sec. arc)					
5	10 AzM							27.4024	0.482991	5000.00	1000.00
6	10 1	3130.310	0.05	400	90.5332	5	1701806709				
7	1 2	4216.640	0.05	400	148.0508	5	1701806709				
8	2 3	3268.380	0.05	400	153.4923	5	1701806709				
9	3 4	2521.240	0.05	400	117.4755	5	1701806709				
10	4 5	4652.710	0.05	400	68.1157	5	1701806709				
11	5 10	5555.700	0.05	400	192.5529	5	1701806709				
12	10 AzM				308.1702	5	1701806709				
13											

Figure 4

Figure 5 contains the setup for Solver and the cell entries are described as follows:

- 1 B23 contains a formula referencing the distance in cell B6. The formula in B23 ($=B6$) is copied through B28. This actually is not necessary as the formulae referencing the distances could have been reference directly to cells B6:B11. If B6:B11 contained slope distances then B23:B28 could include the reduction formula for horizontal distances.

	A	B	C	D	E	F	G	H	I	J	K	L	M	
18														
19		Observed	Computed	Observed	Observed	Adjusted	Adjusted	Residuals in:		Weights Times				
20	Points	distances	distances	angles	azimuths	azimuths	angles	angles	distances	Residuals Squared	angles	distances	Latitudes	Departures
21				(radians)	(radians)	(radians)	(radians)	(radians)						
22	10 AzM				0.482991	0.482991			0.482991					
23	10 1	3130.310	3130.310	1.586369	2.069359	2.069359	1.586369	0.000E+00	0.000	0.000E+00	0.000	-1496.801	2749.259	
24	1 2	4216.640	4216.640	2.584581	1.512347	1.512347	2.584581	0.000E+00	0.000	0.000E+00	0.000	246.319	4209.439	
25	2 3	3268.380	3268.380	2.684719	1.055473	1.055473	2.684719	0.000E+00	0.000	0.000E+00	0.000	1610.710	2843.927	
26	3 4	2521.240	2521.240	2.055974	6.253040	6.253040	2.055974	0.000E+00	0.000	0.000E+00	0.000	2520.094	-75.993	
27	4 5	4652.710	4652.710	1.190300	4.301747	4.301747	1.190300	0.000E+00	0.000	0.000E+00	0.000	-1857.353	-4265.906	
28	5 10	5555.700	5555.700	3.367172	4.527326	4.527326	3.367172	0.000E+00	0.000	0.000E+00	0.000	-1022.296	-5460.834	
29	10 AzM			5.380569	0.483117	0.482991	5.380443	-1.261E-04		2.704E+01				
30										27.040	0.000	0.672	-0.107	
31								Target						
32								27.040						
33														

Figure 5

- 2 C23 contains a formula for the computed distance from the Latitude and Departure ($=SQRT(L23^2+M23^2)$), see equation (2). This formula is copied through C28.
- 3 D23 references the observed angle in E6 and uses a user-defined function, dms2r, converting the angle observation to radians ($=dms2r(E6)$). This formula is copied through D29.
- 4 E22 contains a formula referencing the cell I5 containing the fixed azimuth in radian ($=I5$).
- 5 E23 contains a formula for computing the azimuth from 1 to 2 using the previously computed azimuth in E22 and the observed angle in D23 ($=E22+D23$).
- 6 L23 contains a formula for computing the latitude of line 10 to 1 ($=B23 * COS(E23)$). This formula is copied through L28.

- 7 M23 contains a formula for computing the departure of line 10 to 1 ($=B23 * SIN(E23)$). This formula is copied through M28.
- 8 E24 contains a formula for computing the azimuth from 1 to 2 using the previously computed azimuth from 10 to 1 in cell E23 and the observed angle in cell D24. See equation (1) in section 1.2.1 ($=IF(IF(E23 > PI(), E23 - PI() + D24, E23 + PI() + D24) > 2 * PI(), IF(E23 > PI(), E23 - PI() + D24, E23 + PI() + D24) - 2 * PI(), IF(E23 > PI(), E23 - PI() + D24, E23 + PI() + D24))$)

This formula seems intimidating; however, a close examination will reveal that it is simply the implementation of equation (1) in section 1.2.1, as further demonstrated with the flowchart in Figure 6.
- 9 F22 contains a formula referencing the cell I5 containing the fixed azimuth in radian ($=I5$).
- 10 F23 contains a formula that calculates the azimuth from 10 to 1 using latitude in cell (L23) and departure in cell (M23). The formula calls a user-defined function, azrad (see Appendix) to calculate the azimuth between two points given the Easting and Northing coordinates of the ends of the line. The formula is $=azrad(0,0,M23,L23)$
- 11 G23 contains a formula for computing the angle using the fixed azimuth in cell F22, and the adjusted azimuth from 10 to 1 in cell F23 ($=F23 - F22$).
- 12 24 contains a formula for computing the adjusted angle at 1 using the adjusted azimuth from 10 to 1 cell F23 and the adjusted azimuth from 1 to 2. A user-defined function called Angle (see Appendix) is used to compute the angle at a point given the azimuths of the two lines making the angle. The formula is $=Angle(F23, F24)$. This formula is copied through G29.
- 13 H23 contains a formula for computing the angle residuals in radians ($=G23 - D23$). This formula is copied through H29. Note that initially all the residuals will be zeroes.
- 14 I23 contains a formula for computing the distance residuals ($=C23 - B23$). This formula is copied through I28. Note that initially all the residuals will be zeroes.
- 15 J23 contains a formula for computing the weights in cell G6 times the residual in cell H23 squared ($=G6 * H23^2$). This formula is copied through J29.
- 16 K23 contains a formula for computing the weights in cell D6 times the residual in cell I23 squared ($=D6 * I23^2$). This formula is copied through I28.
- 17 J30 contains a formula for computing the weighted sum of the squares of the residuals in angles ($=SUM(J23:J29)$).
- 18 K30 contains a formula for computing the weighted sum of the squares of the residuals in distances ($=SUM(K23:K28)$).
- 19 L30 contains a formula for computing the algebraic sum of the latitudes which should equal to zero ($=SUM(L23:L28)$).

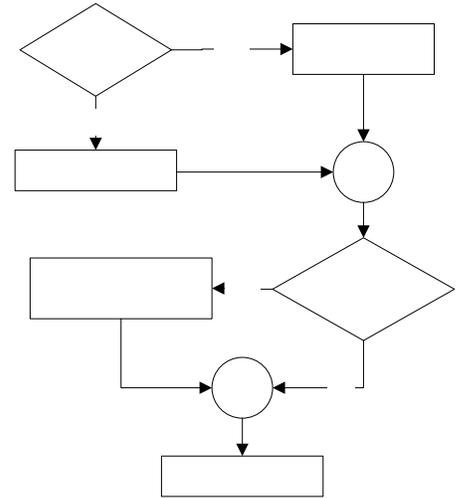


Figure 6

- 20 M30 contains a formula for computing the algebraic sum of the departures which should equal to zero ($=SUM(M23:M28)$).
- 21 H32 is the target cell. This in Excel Solver is a single cell whose value will be minimized, maximized, or set equal to a certain value when the Solver finds a solution. In a least squares application it the cell that contains the weighted sums of the squares of the residuals that must be minimized. The formula in the target cell is $=SUM(J30,K30)$.

2.2 Invoking the Excel Solver

Select **TOOLS/SOLVER...** from Excel's main menu the **SOLVER PARAMETER** dialog box as shown in Figure 7 below will be displayed.



Figure 7

2.2.1 Set Target Cell: The target cell is \$H\$32 and it contains the weighted sums of the squares of the residuals in angles and distances to be minimized.

2.2.2 Equal To: The **Min** radio button should be selected.

2.2.3 By Changing Cells: \$L\$23 to \$M\$28 are the latitudes and departures that are calculated from the observed distances and the azimuth of the lines derived from the observed angles. It also contains \$E\$29, the azimuth from 10 to Azimuth mark which ensures that the angle in cell D29 is included in the adjustment.

2.2.4 Subject to the Constraints: Clicking the **Add** button will display the **ADD CONSTRAINT** dialog box. Figure 8 shows how the first constraint is added. For this example, there are three constraints to be introduced, and the remaining two are added in a similar fashion.

1. Sum of the latitudes in cell L30 = 0. The constraint formula is $LS30=0$.

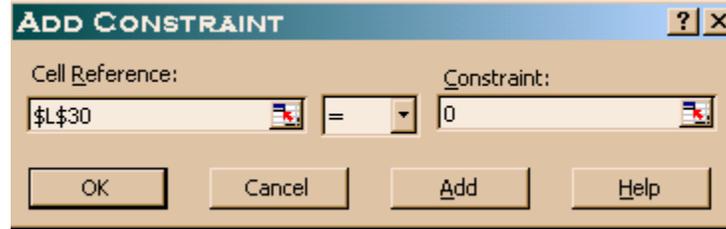


Figure 8

2. Sum of the departures in cell M30 = 0. The constraint formula is $SM\$30=0$.
3. The computed azimuth from 10 to Azimuth mark in cell E29 should equal to the fixed azimuth from 10 to Azimuth mark in cell F29. Therefore, the constraint formula is $SE\$29=FE\29

The \$ sign before the column designation and before the row designation implies an absolute address cell in Excel. Within the **SOLVER PARAMETER** dialog box cells referenced can be selected by clicking the cell or windowing around a group of contiguous cells and hence Excel will insert the absolute address.

2.2.5 SOLVER OPTIONS: Clicking the **Options** button in the **SOLVER PARAMETER** dialog box will display the **SOLVER OPTIONS** dialog box as shown in Figure 9. For a traverse adjustment the options can be selected as shown in Figure 9.

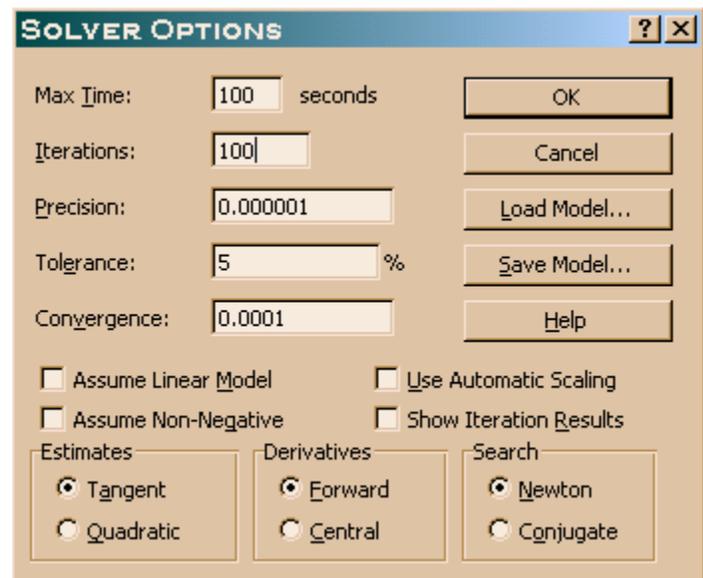


Figure 9

2.2.6 Performing the Adjustment: At this stage the adjustment can be performed by clicking the **Solve** button in the **SOLVER PARAMETER** dialog box. If the adjustment is successful the Solver Results dialog box will be displayed as shown in Figure 10. Click **OK** to save the solution results.

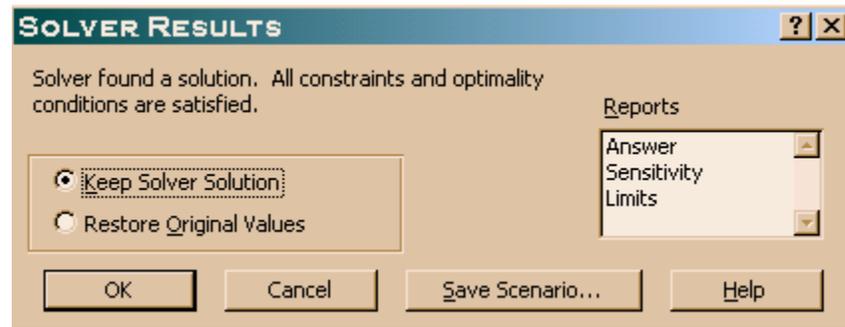


Figure 10

TRAVADJ.XLS														
	A	B	C	D	E	F	G	H	I	J	K	L	M	
18														
19		Observed	Computed	Observed	Observed	Adjusted	Adjusted	Residuals in:		Weights Times				
20	Points	distances	distances	angles	azimuths	azimuths	angles	angles	distances	Residuals	Squared	Latitudes	Departures	
21				(radians)	(radians)	(radians)	(radians)	(radians)		angles	distances			
22	10	AzM			0.482991	0.482991								
23	10	1	3130.310	3130.320	1.586369	2.069359	2.069342	1.586352	-1.694E-05	0.010	4.883E-01	0.037	-1496.759	2749.293
24	1	2	4216.640	4216.651	2.584581	1.512347	1.512317	2.584567	-1.349E-05	0.011	3.095E-01	0.044	246.447	4209.442
25	2	3	3268.380	3268.389	2.684719	1.055473	1.055428	2.684704	-1.455E-05	0.009	3.605E-01	0.036	1610.842	2843.863
26	3	4	2521.240	2521.239	2.055974	6.253040	6.252976	2.055955	-1.887E-05	-0.001	6.062E-01	0.000	2520.089	-76.154
27	4	5	4652.710	4652.700	1.190300	4.301747	4.301658	1.190275	-2.517E-05	-0.010	1.078E+00	0.038	-1857.729	-4265.731
28	5	10	5555.700	5555.690	3.367172	4.527326	4.527217	3.367152	-2.007E-05	-0.010	6.858E-01	0.040	-1022.890	-5460.713
29	10	AzM			5.380569	0.482991	0.482991	5.380552	-1.695E-05		4.891E-01			
30											4.018	0.195	0.000	0.000
31								Target						
32								4.213						
33														
34			Residuals in:	Adjusted	Adjusted									
35			angles	distances	azimuths	distances								
36			(sec)		(dms)									
37	10	1	-3.5	0.010	118.3353	3130.320								
38	1	2	-2.8	0.011	86.3858	4216.651								
39	2	3	-3.0	0.009	60.2818	3268.389								
40	3	4	-3.9	-0.001	358.1609	2521.239								
41	4	5	-5.2	-0.010	246.2801	4652.700								
42	5	10	-4.1	-0.010	259.2325	5555.690								
43	10	AzM	-3.5											
44														

Figure 10

2.2.7 Adjusted Results: Cells B37 through B43 contain the residuals (corrections) for the angle observations. They are simply the value of cells H23 through H29 converted from radians to seconds using the formula ($=H23*206264.8$) in cell B37. This formula in cell H23 is copied through cells B43. Similarly, cells C37 through C42 are the residuals for the distances. The values of these cells are identical to those in cells I23 through I28. The formula in cell C37 is ($=I23$) and copied through cells C42. The adjusted azimuths in DDD.MMSS, cells D37 through D42, are calculated from the latitudes and departures, cells L23 through M28. The formula in

D37 is(=*azdms(0,0,M23,L23)*), and copied through H42. It uses the user-defined *azdms* function provided in the Appendix of this paper. Adjusted distances, cells E37 through E42 are identical to cells C23 through C29.

3. INTERCONNECTING TRAVERSE EXAMPLE

Figure 11 shows an interconnecting traverse. The observations, their respective standard deviations and weights are entered in an Excel worksheet, similar to the example in section 2.1, as shown in Figure 12.

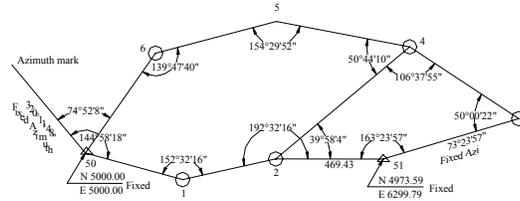


Figure 11

	A	B	C	D	E	F	G	H	I	J	K
1	Traverse Adjustment - Example 2										
2					Observed						
3					Standard						
4	Points	Distances	Standard deviations	Weights	Angles (dms)	deviations (sec. arc)	Weights	Fixed Az. (dms)	Fixed Az. (radians)	Fixed Coordinates	
5	50 AzM							320.1148	5.588486	5000.00	5000.00
6	50 1	439.06	0.05	400	144.5818	5	1701806709				
7	1 2	416.08	0.05	400	152.3216	5	1701806709				
8	2 4	763.16	0.05	400	152.1940	5	1701806709				
9	4 5	592.75	0.05	400	50.4410	5	1701806709				
10	5 6	544.62	0.05	400	154.2952	5	1701806709				
11	6 50	532.92	0.05	400	139.4740	5	1701806709				
12	50 AzM				285.0752	5	1701806709				
13	Second Loop									N 51	E 51
14	2 51	469.43	0.05	400	39.5804	5	1701806709			4973.59	6299.79
15	51 3	617.13	0.05	400	163.2357	5	1701806709	73.2357	1.281057		
16	3 4	570.07	0.05	400	50.0022	5	1701806709				
17	4 2				106.3755	5	1701806709				
18											

Figure 12

3.1 Traverse Setup

The Excel formulae used in Figure 13 are very similar to those described in section 2.1. For the sake of completeness, the formulae for the first cells in each column are listed in the table below. In the remaining cells within each row the formula is simply copied:

TRAVADJ.XLS													
	A	B	C	D	E	F	G	H	I	J	K	L	M
19	Observed	Computed	Observed	Observed	Adjusted	Adjusted	Residuals in:		Weights Times				
20	Points	distances	distances	angles	azimuths	azimuths	angles	angles	distances	Residuals Squared	Residuals Squared	Latitudes	Departures
21				(radians)	(radians)	(radians)	(radians)	(radians)		angles	distances		
22	50 AzM				5.588486	5.588486							
23	50 1	439.06	439.06	2.530233	1.835534	1.835534	2.530233	0.000E+00	0.000	0.000E+00	0.000	-114.883	423.7637
24	1 2	416.08	416.08	2.662286	1.356227	1.356227	2.662286	0.000E+00	0.000	0.000E+00	0.000	88.59432	406.5386
25	2 4	763.16	763.16	2.658621	0.873256	0.873256	2.658621	0.000E+00	0.000	0.000E+00	0.000	490.2039	584.9045
26	4 5	592.75	592.75	0.885512	4.900361	4.900361	0.885512	0.000E+00	0.000	0.000E+00	0.000	110.7654	-582.3088
27	5 6	544.62	544.62	2.696495	4.455263	4.455263	2.696495	0.000E+00	0.000	0.000E+00	0.000	-138.498	-526.7156
28	6 50	532.92	532.92	2.439873	3.753544	3.753544	2.439873	0.000E+00	0.000	0.000E+00	0.000	-436.21	-306.1443
29	50 AzM			4.976477	5.588428	5.588486	4.976535	5.818E-05		5.760E+00			
30										5.760	0.000	-0.0271	0.0381
31													
32	Second Loop												
33	2 51	469.43	469.43	0.697569	1.570825	1.570825	0.697569	0.000E+00	0.000	0.000E+00	0.000	-0.01366	469.43
34	51 3	617.13	617.13	2.851853	1.281086	1.281086	2.851853	0.000E+00	0.000	0.000E+00	0.000	176.2983	591.4122
35	3 4	570.07	570.07	0.872771	5.295450	5.295450	0.872771	0.000E+00	0.000	0.000E+00	0.000	313.8702	-475.8837
36	4 2			1.861079	4.014936	4.014849	1.860991	-8.727E-05		12.960			
37													
38										12.960	0.000	-0.0491	0.0540
39												0.1081	-0.0577
40									Target				
									18.720				

Figure 13

Cell Number	Formula
B23:B28 and B33:B35	Distance observations
C24	=SQRT(L23^2+M23^2)
C33	=SQRT(L33^2+M33^2)
D23	=dms2r(E6)
D33	=dms2r(E14)
E22	=dms2r(H5)
E23	=E22+D23-2*PI()
E24	=IF(IF(E23>PI(),E23-PI()+D24,E23+PI()+D24)>2*PI(),IF(E23>PI(),E23-PI()+D24,E23+PI()+D24)-2*PI(),IF(E23>PI(),E23-PI()+D24,E23+PI()+D24))
E33	=E25+D33
E34	=IF(IF(E33>PI(),E33-PI()+D34,E33+PI()+D34)>2*PI(),IF(E33>PI(),E33-PI()+D34,E33+PI()+D34)-2*PI(),IF(E33>PI(),E33-PI()+D34,E33+PI()+D34))
F22	=I5
F23	=azrad(0,0,M23,L23)
F29	=I5
F33	=azrad(0,0,M33,L33)
G23	=F23-F22+2*PI()
G24	=Angle(F23,F24)
G33	=F33-F25
G34	=Angle(F33,F34)
H23	=G23-D23

H33	=G33-D33
I23	=C23-B23
I33	=C33-B33
J23	=G6*H23^2
J30	=SUM(J23:J29)
J33	=G14*H33^2
J38	=SUM(J33:J36)
K23	=D6*I23^2
K30	=SUM(K23:K28)
K33	=D14*I33^2
K38	=SUM(K33:K35)
L23	=B23*COS(E23)
L30	=SUM(L23:L28)
L33	=B33*COS(E33)
L38	=SUM(L33:L35)-L25
L40	=J5+SUM(L23:L24,L33)-J14
M23	=B23*SIN(E23)
M30	=SUM(M23:M28)
M33	=B33*SIN(E33)
M38	=SUM(M33:M35)-M25
M40	=K5+SUM(M23:M24,M33)-K14
H40	=SUM(J30,K30,J38,K38)

3.2 Invoking the Excel Solver

Select **TOOLS/SOLVER...** from Excel's main menu the **SOLVER PARAMETER** dialog box as shown in Figure 14 below will be displayed.

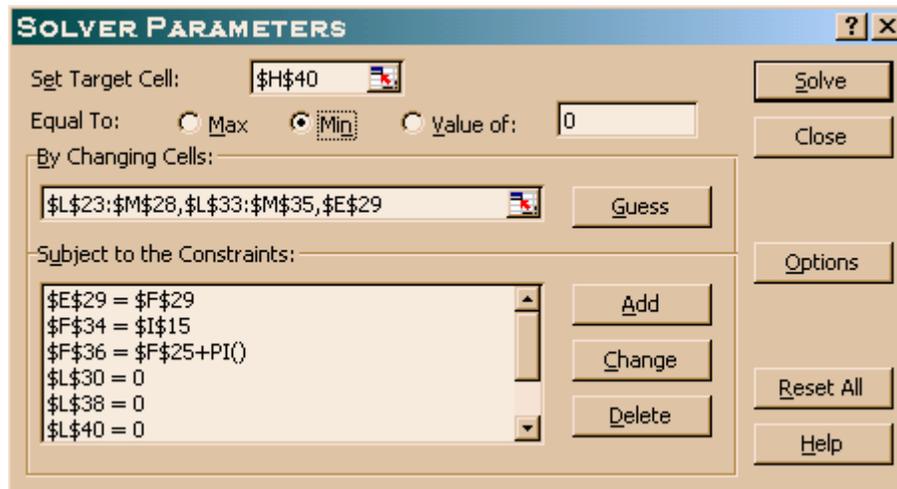


Figure 14

3.2.1 Set Target Cell: The target cell is \$H\$40 and it contains the weighted sums of the squares of the residuals in angles and distances that need to be minimized.

3.2.2 Equal To: The **Min** radio button should be selected.

3.2.3 By Changing Cells: \$L\$23 to \$M\$28, \$L\$33 to \$M\$35 are the latitudes and departures that are calculated from the observed distances and the azimuths of the lines derived from the observed angles. It also contains cell \$E\$29 which is the azimuth from point 50 to Azimuth mark. This ensures that the angle in cell D29 is also included in the adjustment.

3.2.4 Subject to the Constraints: Identifying the constraints warrant some discussions. In a simple traverse where all angles and distances are observed, the number of redundancies (r) which is also equal to the number of constraint entries is equal three. They include: (1. sum of the latitudes = 0, 2. sum of the departures = 0, 3. sum of the interior angles in a closed figure = to $(n-2)*180^\circ$, where n is the number of sides). This example, in reality, consists of three simple traverses. The first one being points 50 1 2 4 5 6, the second one being points 2 51 3 4, and the third one being points 50 1 2 51. Knowing that each simple traverse contributes three redundancies, three simple traverses create three times three = nine redundancies. Alternatively, the number of redundancies can be determined by total number of observations ($n = 20$, nine distances and eleven angles) minus the minimum number of observations (n_0) needed to fix the positions of six unknown points where there is a known direction to one unknown point (fixed azimuth from 51 to 3). Therefore, $n_0 = 11$, and $r = n - n_0 = 9$ redundancies. As stated earlier, the number of constraints to be introduced is equal to number of redundancies, nine in this case.

Clicking the  button provides the opportunity to include the constraints to be used. The nine constraints to be introduced in this example are:

1. The algebraic sum of the latitudes in cell L30 = 0. The constraint formula is **$SL\$30=0$** .
2. The algebraic sum of the departures in cell M30 = 0. The constraint formula is **$SM\$30=0$** .
3. The computed azimuth from 50 to Azimuth mark in cell E29, should equal to the fixed azimuth from 50 to Azimuth mark, cell F29. Therefore the constraint formula is **$SE29=FE29$**
4. The algebraic sum of the latitudes in the second loop in cell L38 = 0. The constraint formula is **$SL\$38=0$** .
5. The algebraic sum of the departures in the second loop in cell M38 = 0. The constraint formula is **$SM\$38=0$** .
6. The computed azimuth from 4 to 2 in cell F26 should equal to the computed azimuth from 2 to 4 plus Pi in cell F25. Therefore, the constraint formula is **$FF26=FF25+PI()$** . Note that PI() is an Excel built-in function for Pi.
7. The Northing coordinate of 50 plus the algebraic sum of the latitudes (50 to1, 1 to 2, 2 to 51) should equal to the Northing coordinate of point 51. The constraint formula is **SL40=0$** .
8. The Easting coordinate of 50 plus the algebraic sum of the departures (50 to1, 1 to 2, 2 to 51) should equal to the Easting coordinate of point 51. The constraint formula is **SM40=0$** .
9. The computed azimuth from 51 to 3 in cell F34 should equal to the fixed azimuth from 51 to 3 in cell I15. The constraint formula is **$FF34=IF15$** .

The order of how the above constraints are entered is not significant, however, Excel will sort them by columns and rows as shown in Figure 14.

3.2.5 Performing the Adjustment: After the constraints have been entered as defined in 3.2.4, the adjustment can be performed by clicking the **Solve** button in the **SOLVER PARAMETER** dialog box. If the adjustment is successful the Solver Results dialog box will be displayed as shown in Figure 9. Click **OK** to save the solution results.

3.2.6 Adjusted Results: Figure 15 shows the adjusted results. Note that all the nine constraints listed in 3.2.4 are satisfied. For example, the sums of latitudes and departures in cells L30, L38, L40, M30, M38 and M40 are all equal to zeroes. Furthermore, the observed azimuth in E29 is equal to F29, the computed azimuth from 51 to 3 in cell F34 is equal to cell I15. It is important to point out that after the solver is invoked to carryout the adjustment calculations by clicking the **Solve** button, all the formulae in the cells referenced in the **By Changing Cells**, see sections 2.2.3, and 3.2.3 will be changed to numerical values, therefore, invoking the Solver again will not work properly. This can be remedied by copying the entire worksheet into a new one before carrying out the adjustment. Then, when it is necessary to rerun the Solver, the original worksheet can be copied over the worksheet that has gone through the Solver calculations.

	A	B	C	D	E	F	G	H	I	J	K	L	M
19		Observed	Computed	Observed	Observed	Adjusted	Adjusted	Residuals in:		Weights Times			
20	Points	distances	distances	angles	azimuths	azimuths	angles	angles	distances	Residuals Squared	Residuals Squared	Latitudes	Departures
21				(radians)	(radians)	(radians)	(radians)	(radians)		angles	distances		
22	50 AzM				5.588486	5.588486							
23	50 1	439.06	439.15447	2.530233	1.835534	1.835587	2.530286	5.336E-05	0.094	4.846E+00	3.570	-114.93	423.8487
24	1 2	416.08	416.01303	2.662286	1.356227	1.356300	2.662306	1.958E-05	-0.067	6.524E-01	1.794	88.55041	406.4796
25	2 4	763.16	763.15936	2.658621	0.873256	0.873316	2.658608	-1.306E-05	-0.001	2.901E-01	0.000	490.1685	584.9334
26	4 5	592.75	592.812	0.885512	4.900361	4.900437	0.885529	1.656E-05	0.062	4.668E-01	1.538	110.8215	-582.3613
27	5 6	544.62	544.64296	2.696495	4.455263	4.455345	2.696501	5.812E-06	0.023	5.749E-02	0.211	-138.46	-526.7492
28	6 50	532.92	532.87476	2.439873	3.753544	3.753620	2.439867	-6.516E-06	-0.045	7.227E-02	0.819	-436.15	-306.1513
29	50 AzM			4.976477	5.588486	5.588486	4.976459	-1.757E-05		5.252E-01			
30										6.911	7.931	0.0000	0.0000
31													
32		Second Loop											
33	2 51	469.43	469.46165	0.697569	1.570825	1.570861	0.697545	-2.403E-05	0.032	9.831E-01	0.401	-0.03049	469.4617
34	51 3	617.13	617.12655	2.851853	1.281086	1.281057	2.851789	-6.494E-05	-0.003	7.177E+00	0.005	176.3145	591.4037
35	3 4	570.07	570.11818	0.872771	5.295450	5.295424	0.872775	3.305E-06	0.048	1.859E-02	0.929	313.8845	-475.932
36	4 2			1.861079	4.014936	4.014909	1.861077	-1.595E-06		0.004			
37													
38										8.183	1.334	0.0000	0.0000
39													
40								Target				0.0000	0.0000
41								24.359					
42		Adjusted Results											
43		Residuals		Adj. Az.	Adj. Dist.								
44		Angle	Dist.										
45	50 1	(sec)		(dms)									
46	1 2	11.0	0.094	105.1017	439.1545								
47	2 4	4.0	-0.067	77.423705	416.013								
48	4 5	-2.7	-0.001	50.021435	763.1594								
49	5 6	3.4	0.062	280.46278	592.812								
50	6 50	1.2	0.023	255.1621	544.643								
51	50 AzM	-1.3	-0.045	215.03596	532.8748								
52		-3.6											
53													
54	2 51	-5.0	0.032	90.00134	469.4617								
55	51 3	-13.4	-0.003	73.2357	617.1265								
56	3 4	0.7	0.048	303.24197	570.1182								
57	4 2	-0.3											

Figure 15

4. CONCLUSION

The Excel Solver is a tool that can easily be used for least squares adjustment projects, from level network, two-D and three-D traverses or any other adjustment projects encountered in surveying and mapping. Its advantages are: (1) Excel's familiar Graphics User Interface (GUI). (2) Relatively little or no understanding of least squares theory and problem solution methodologies. (3) Excel or similar spreadsheet products such as Lotus 1-2-3, Quattro Pro, etc. are available with no additional cost. (4) Provides some flexibility to do problems that a standard stand alone least squares adjustment package may not provide such as introducing constraints within observations and parameters (quantities derived from observations). The disadvantages, however, are: (1) the solution does not provide statistical information about the adjustment, i.e., variances and covariances of the adjusted observations and parameters for analysis. (2) The standard version of Solver will not handle large adjustment projects. It may not be prudent to purchase the Premium Solver only to be used for adjustment purposes.

Appendix

The following is a listing of some useful Visual Basic functions that have been used in the examples for this paper. Not all the functions in the list are used for this project. To make this listing a part of Excel proceed as follows:

1. Type the listing using a text editor such as Notepad where word wrap is turned off and save the file as SurePack.bas. Alternatively, the listing can be obtained by contacting the author.
2. Start Excel – While no worksheet is open click **Tools/Macro/Visual Basic Editor**
3. Inside Microsoft Visual Basic click **File/Import File**
4. Select **SurePack.bas** and click **Open**
5. Close Visual Basic to return back to Excel.
6. From Excel click **File/Save as**, select **Microsoft Excel Add-In** by clicking the down arrow on **Save as type**.
7. Type **SurePack** as the **File name** and click **Save**.
8. **Exit** Excel.
9. Restart Excel: **Tools/Add-Ins**, and check the **SurePack** box and click **OK**.

```
' Converts DDD.MMSSs to radians
Public Function DMS2R(dms As Double) As Double
    Dim pi As Double
    pi = 4# * Atn(1#)
    DMS2R = dms2d(dms) * pi / 180#
End Function
' Converts radians to DDD.MMSSsss
Public Function rad2dms(rad As Double) As Double
    Dim v, m, S, pi As Double
    Dim D, M1 As Integer
    pi = 4# * Atn(1#)
    v = d2dms(Abs(rad * 180# / pi))
    If rad < 0 Then v = -v
```

```

    rad2dms = v
End Function

' Converts DDD.MMSSs to Degrees
Public Function dms2d(Deg As Double) As Double
    Dim E, v, m, S As Double
    Dim D, M1 As Integer
    E = 10 ^ -12
    v = Abs(Deg) + E
    D = Fix(v)
    m = (v - D) * 100#
    M1 = Fix(m)
    S = (m - M1) * 100#
    v = D + M1 / 60# + S / 3600#
    If Deg < 0 Then v = -v
    dms2d = v
End Function

' Converts degrees to DDD.MMSSss
Public Function d2dms(Deg As Double) As Double
    Dim v, m, S As Double
    Dim D, M1 As Integer
    v = Abs(Deg)
    D = Fix(v)
    m = (v - D) * 60#
    M1 = Fix(m)
    S = (m - M1) * 60#
    v = D + M1 / 100# + S / 10000#
    If Deg < 0 Then v = -v
    d2dms = v
End Function

' Converts DDD.MMSSss to degrees, minutes, and seconds (3 separate values)
Public Sub dms2Dms(ByVal dms As Double, D As Integer, m As Integer, _
    S As Double)
    Dim D1 As Double, M1 As Double, E As Double
    E = 0.0000000000001
    D1 = Abs(dms2d(dms)) + E
    D = Fix(D1)
    M1 = (D1 - D) * 60#
    m = Fix(M1 + E)
    S = (M1 - m) * 60# + E
    If dms < 0 Then D = -D
End Sub

' Computes the azimuth from 1 to 2
' in DDD.MMSSss, given x1, y1, x2, and y2
Public Function AzDms(x1 As Double, y1 As Double, x2 As Double, _
    y2 As Double) As Double
    Dim dx As Double, dy As Double, pi As Double
    dx = x2 - x1
    dy = y2 - y1
    If dy = 0# Then
        If dx > 0 Then

```

```

    AzDms = 90
Else
    AzDms = 180#
End If
Exit Function
End If
pi = 4# * Atn(1#)
AzDms = Atn(dx / dy)
If dy < 0# Then
    AzDms = AzDms + pi
Else
    If dx < 0# Then
        AzDms = 2# * pi + AzDms
    End If
End If
AzDms = rad2dms(AzDms)
End Function

' Computes the azimuth from 1 to 2
' in radians, given x1, y1, x2, and y2
Public Function AzRad(x1 As Double, y1 As Double, x2 As Double, _
    y2 As Double) As Double
Dim dx As Double, dy As Double, pi As Double
dx = x2 - x1
dy = y2 - y1
If dy = 0# Then
    If dx > 0 Then
        AzRad = 90
    Else
        AzRad = 180#
    End If
Exit Function
End If
pi = 4# * Atn(1#)
AzRad = Atn(dx / dy)
If dy < 0# Then
    AzRad = AzRad + pi
Else
    If dx < 0# Then
        AzRad = 2# * pi + AzRad
    End If
End If
End Function

' Adds two angles in DDD.MMSS
' The result is also in DDD.MMSS
Public Function adddms(a As Double, b As Double) As Double
    adddms = d2dms(dms2d(a) + dms2d(b))
End Function

' Subtracts the second angle from the first, where angles are
' in DDD.MMSS. The result is also in DDD.MMSS
Public Function subdms(a As Double, b As Double) As Double
    subdms = d2dms(dms2d(a) - dms2d(b))

```

End Function

```
` Computes distance between two points, given X1, Y1, X2, Y2
Public Function Dist(x1 As Double, y1 As Double, x2 As Double, y2 As Double) As Double
    Dist = Sqr((x2 - x1) ^ 2 + (y2 - y1) ^ 2)
End Function
```

```
` Computes angle 1 2 3, where az1 is azimuth 1 to 2, and az2 is azimuth 2 to 3
Public Function Angle(az1 As Double, az2 As Double) As Double
    Dim pi As Double
    pi = 4# * Atn(1#)
    az1 = az1 + pi
    If az2 < az1 Then
        az2 = az2 + 2 * pi
    End If
    Angle = az2 - az1
    If Angle < 0 Then
        Angle = Angle + 2 * pi
    End If
End Function
```

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BIOGRAPHICAL SKETCH

Sayed R. Hashimi is Professor and Department Chair of the Surveying Engineering Department at Ferris State University. He is also a professional surveyor in the State of Michigan. Professor Hashimi holds a masters degree in geodesy from Purdue University, a bachelor of technology degree in surveying from Oregon Institute of Technology formerly known as Oregon Technical Institute, and a bachelor of science degree in computer information systems from Ferris State University. He is the author of Ez-Adjust, a comprehensive least squares adjustment package for level network, two-D traverse adjustment including State Plane Coordinates option, three-D adjustment including GPS and terrestrial observations combined, four-parameter coordinate transformation, geoid modeling using geoid 03 and much more. Information on Ez-Adjust can be obtained by visiting <http://www.srh-leastsquares.com>.

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