Bond Implied CDS Spread and CDS-Bond Basis

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Abstract

We derive a simple formula for calculating the CDS spread implied by the bond market price. Using no-arbitrage argument, the formula expresses the bond implied CDS spread as the sum of bond price, bond coupon and Libor zero curve weighted by risky annuities. We show that the bond implied CDS spread is consistent with the standard CDS pricing model if the survival probabilities and recovery are consistent with the bond price.

1. Introduction

A CDS contract is an OTC transaction between two parties in which the protection buyer pays a stream of coupon payment to the protection seller until the earlier of maturity or entity default in exchange for a default contingent payment. The common default settlement is the physical settlement where the protection buyer delivers a bond from a pool of eligible bonds to the protection seller in exchange for par. CDS contracts can also be cash settled where the protection buyer receives from the protection seller the cash amount of par less recovery.

With the physical settlement, the CDS protection buyer holds a delivery option where he can choose any bond from a pool of bonds to deliver into the CDS contract. Empirical evidence shows that bonds of the same entity do not necessarily have the same market value following default [1]. As a result, the standard CDS pricing with a flat recovery rate cannot properly price in the value of the delivery option embedded in the CDS contracts. Given the issuer default probability, the bond price is determined by the recovery and other fundamental and market technical factors such as supply and demand, and liquidity. From modeling perspective, it is difficult to separate recovery, default probability, and other market fundamental and technical factors since they are intertwined. The recovery swap prices can be used as the expected recovery rate but the market has not yet fully developed. Even if the recovery rate can be determined independently, the default probabilities calibrated to the market CDS spreads or bond prices are still contaminated by other factors such as supply and demand, funding cost, bond trading away from par (see [7] for a detailed exposition on factors impacting CDS and cash basis).

Empirical studies show that the markets apears to price CDS based on Libor curve rather than the treasury curve [2]. CDS discounting should be based on Libor. Since the Libor

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are the borrowing rates between banks of AA rating, the Libor curve is implicitly an AA rated yield curve. As a result, we use Libor as the risk free interest rate.

An asset swap (ASW) is a package transaction between two parties in which the ASW buyer purchases a bond from the other party and simultaneously enters into an interest rate swap transaction, usually with the same counterparty, to exchange the coupon on the bond for Libor plus a spread. The spread is called the asset swap spread. A common asset swap is the par asset swap where the buyer pays par at the inception of the deal. Unlike CDS, ASW continues following bond default.

CDS-Bond basis is the difference between the CDS spread and the ASW spread on the same bond. It is a general indicator of relative value of CDS versus the cash bond. For example, when the CDS spread is higher than the ASW spread, i.e. the basis is positive, the CDS is generally considered to be more attractive than the bond. The reverse is true if the basis is negative.

Bond implied CDS spreads have been previously investigated. Davies and Pugachevsky proposed an approximation method for calculating the bond implied CDS spread based on the Z spread adjusted by the bond's market price, duration, convexity and recovery rate [3]. In a series of paper, Berd *et al* proposed to use the survival-based modeling as a consistent measure for credit-risky bond pricing and risk management [4]. In their model, the survival probability term structure of an issuer is estimated by regressing model prices against market prices across all bonds of that issuer under a constant recovery rate. The resulting survival probability term structure is not necessarily the same as that implied by the CDS market. The idiosyncratic fitting error is accounted for by the OAS-to-fit which is essentially a measure in spread of aggregate effect of market factors on the bond.

In this paper, we describe a simple model for estimating the bond implied CDS spread. The model concept is not new as it is based on survival probability. But it is cast in such an explicit way that it is easy to analyze the effects on the CDS spread of bond coupon, recovery rate, bond price, and interest rate and credit curves. These effects have been discussed in literature, but an explicit formula that combines all these factors seems needed and this paper provide a such formula.

The paper is organized as follows. Section 2 describes the underlying theory and main formulas. Section 3 gives some numerical examples, and section 4 concludes the paper. The detailed model derivation is given in Appendices A and B.

2. CDS and Asset Swap Spreads

In this section, we describe a model for calculating the bond implied CDS spread and the CDS-Bond basis. We define the bond implied CDS spread as the spread that equates the bond market price with the bond fair theoretical price. We demonstrate the effects of Libor curve, bond coupon and the price deviation from par on the credit spread.

2.1 Asset Swap Spread

In a par asset swap transaction, the investor buys a package consisting of a cash bond and a payer interest swap where the investor swaps the bond coupon for Libor plus a (ASW) spread. The price of the asset swap package is par (hence the term par asset swap) which the investor pays at the deal inception. This means that the bond dirty price and the initial swap value must sum to par. For a discount bond, the initial swap value is positive to the investor. For a premium bond, it is negative. The market practice is asset swap does not knock out when the underlying bond defaults. As a result, the investor bears some interest rate risk in that he needs to seek new funding to pay the fixed rate should the bond default.

Let T_0 be today, and the bond coupon payment dates be $T_k, k = 1,...,N$. Furthermore, we assume that the ASW payment dates coincide with the bond coupon payment dates.

The spread of a par asset swap is given by

$$S_{ASW} = \frac{M - (1 - D)}{A} = C - \overline{L} + \frac{D}{A} \tag{1}$$

where M is the bond risk-free price, 1-D is the bond's dirty price, C is the bond coupon, L_{k-1} is the $T_{k-1}-$ maturity forward Libor rate paid at T_k, and A is the risk-free annuity given by

$$A = \sum_{k=1}^{N} \Delta T_{k-1} DF(T_k) \tag{2}$$

where $DF(T_k)$ is the discount factor for maturity T_k, $\Delta T_{k-1} = T_k - T_{k-1}$, and the risk-free average Libor weighted by the discount factors is

$$\overline{L} = \frac{1}{A} \sum_{k=1}^{N} L_{k-1} \Delta T_{k-1} DF(T_k) \tag{3}$$

The first form of ASW spread in formula (1) is well known. It states that the ASW spread is the difference between the risk-less price and the market price amortized over the swap life. Since the bond risk-free price is always greater than the market price, ASW spread is always positive. Note that the spread would have to be negative if the bond market price is greater than the bond risk-free price.

The second form of formula (1) is less familiar, but more intuitive. It shows that the ASW spread is composed of three components: the bond coupon, the average Libor over the swap life and the difference of bond dirty price and par scaled by the riskless annuity. The last component can also be interpreted as the bond discount amount amortized over the swap life. This discount amount is paid upfront by the investor and recouped over the life of the swap as part of the swap spread.

Remarks:

1) If the bond is trading at par (D = 0), the ASW spread is the difference between the bond coupon and the average Libor rate over the ASW term.
2) The ASW spread increases as the bond price decreases (D increases). Hence, the more deeply discounted is the bond, the higher is the ASW spread.
3) We will show in the next section that the bond implied spreads – given by formula (7) – has characteristics similar to the ASW spread. The bond implied spread contains three terms with interpretations similar to those of terms in formula (1). And the bond implied spread increases with decreasing bond price (increasing D) but at a faster rate than ASW spread does, resulting in an increasing basis.

2.2 Bond Implied CDS Spread

Suppose an investor executes a so-called negative basis trade in which he buys the cash bond on the ASW basis and buys CDS protection. The market price of the bond is 1-D where D represents bond's discount relative to par. Assuming a recovery rate R, the investor needs to buy 1-D/(1-R) notional CDS protection in order to make the combined position default neutral. Given a non-zero recovery, this amount is less (more) than 1-D for a discount (premium) bond.

Let us assume that the investor borrows 1-D to purchase a bond with fixed coupon C. He then buys 1-D/(1-R) notional CDS protection on the bond. Denoting the recovery rate by R, the bond implied CDS spread is given by (See Appendix A for derivation and definitions of the terms in equations (4), (5) and (6)).

$$S_{CDS} = \frac{1}{W}\left\{ C\frac{PV01}{\overline{\overline{PV01}}} - \overline{L}^{Risky} + \frac{D}{\overline{\overline{PV01}}}\left[1 - \frac{R \times Loss}{(1-D)}\right]\right\} \tag{4}$$

where $W = \left(1 - \frac{D}{1-R}\right)/(1-D) = 1 - \frac{R}{1-R}\frac{D}{1-D}$ under the constraint

$$1 - D = C \times PV01 + R \times Loss + P(\tau > T_N) \times DF(T_N) \tag{5}$$

Equation (4) shows that the bond implied CDS spread is the sum of contributions from the bond coupon, the Libor curve, the difference between the bond (dirty) price and par, and the recovery rate augmented by the risky annuities and W which is the ratio of the CDS notional amount to the bond price.

We will show in Appendix B that the bond implied CDS spread calculated using equation (4) automatically satisfies the standard CDS pricing equation

$$S_{CDS} = \frac{1-R}{\overline{PV01}} \times Loss \tag{6}$$

This implies that, if and only if the recovery and default probability are consistent with the bond price, is the model consistent with the standard CDS pricing model (6). The reverse does not hold. Given a CDS spread and a recovery rate R, the default probability implied by equation (6) is generally inconsistent with equation (5).
Substituting equation (6) into equation (4) and solve for S_{CDS}, we find a simplified form

$$S_{CDS} = C \times \frac{PV01}{\overline{PV01}} - \overline{L}^{Risky} + \frac{D}{\overline{PV01}} \tag{7}$$

It is clear from formula (7) that for a par bond (D = 0) and a flat Libor curve, we have $S_{CDS} = C\frac{PV01}{\overline{PV01}} - L$ which is slightly less than the C-L implied by the risky par floater replication model. The difference is due to that in our model, the bond accrued coupon is not paid but the accrued CDS premium and Libor interest are paid upon default, while there is no such default payment discrepancy in the par floater replication.

Given the bond's market price 1-D, the model framework of (5-7) can be used in several ways:

1) Single bond: In this situation, we assume a recovery rate R, and calculate the constant bond implied hazard rate using equation (5). The bond implied CDS spread is then calculated using equation (7). The bond implied hazard rate is not necessarily consistent with the hazard rate implied by the market CDS quote of the same issuer. The bond implied hazard rates are determined by the fundamental and technical factors in the cash market while the CDS hazard rates are determined by the CDS market fundamental and technical factors. However, if we are concerned only with the CDS spread, equation (7) is all that matters. Therefore, equation (7) can be used to compare the bond implied CDS spread to the market CDS quote.
2) Multiple bonds of differing maturities: Assuming a constant recovery, we bootstrap to obtain a term structure of the bond implied hazard rate consistent with the given bond prices. The bond implied CDS spread for a maturity can then be calculated from (7).
3) Term structures of market CDS spread and bond price: We bootstrap to calculate the hazard rate term structure and the bond implied recovery rate by simultaneously solving equations (5) and (6). The resulting hazard rate and recovery rate term structures are consistent with the bond market prices and the market CDS spreads of the issuer. However, the recovery rates are influenced by the cash market factors and are not necessarily the expected percentage recovery amount of par.

Remarks:

1) Interestingly, the CDS spread formula (7) can also be directly obtained if the CDS notional is 1-D. 1-D/(1-R) CDS notional is default neutral. 1-D notional would result in a small default payoff of DR which is offset by the larger carry (1-D)*S.
2) The form of equation (7) is useful as it explicitly expresses the CDS spread in terms of bond coupon, interest rate curve, and bond price augmented by the risky annuities, allowing easy analysis of individual factors.
3) It is important to note that in equations (4-7) we have not imposed any restriction on the shape of credit and interest rate curves.

2.3 CDS–Bond Basis

The CDS–Bond basis is defined as the difference between the bond implied CDS spread (7) and the par asset swap spread (1)

$$Basis = \left(\overline{L} - \overline{L}^{Risky}\right) - C\left(1 - \frac{PV01}{\overline{\overline{PV01}}}\right) + \frac{D}{\overline{\overline{PV01}}}\left(1 - \frac{\overline{PV01}}{A}\right) \tag{8}$$

The CDS-Bond basis consists of three terms:

1) The first term is the difference between the average risk-free Libor rate and the average risky Libor rate. This term increases with increasing bond discount D, or increasing default risk. It is positive for an upward sloping interest rate curve, and negative for a downward sloping curve. It is zero when the forward curve is flat. Therefore, the first term can be interpreted as the impact of the yield curve slope to the basis. Since the normal yield curve shape is upward sloping, the interest rate curve effect on the basis is generally positive (see Table 1).
2) The second term is due to the payment mismatch between the bond coupon and CDS premium and borrowing cost upon default. While the investor does not receive the bond's accrued interest in the event of default, he still needs to pay the accrued CDS premium and interest on the loan. This term always contributes negatively to the CDS-Bond basis.
3) The third term represents the effect of bond's market price on the CDS-Bond basis. It is positive for discount bond and negative for premium bond. It explains why, rough speaking, CDS-Bond basis is positive for discount bond and negative for premium bond. However, Tables 1 and 2 show that this is not strictly correct. They show that the CDS-Bond basis can be either positive or negative for par bond depending on the interest rate curve shape.

3. Numerical Examples

We now show some pricing examples. The interest rate curve is the swap zero curve for July 16, 2008 taken from Bloomberg. The payment frequency is semiannual. We linearly interpolate the swap zero curve to obtain the zero rates for all payment dates. For our purpose, linear interpolation is deemed adequate because we only need a forward Libor curve. However, different interpolation scheme may and will result in slightly different bond implied CDS spread. The impact of interpolation scheme on ASW spread seems to be smaller than on CDS spread.

Let Z_k be the zero rate for maturity T_k, the forward Libor rate for period (T_{k-1}, T_k) is

$$L_{k-1} = L(0, T_{k-1}) = \frac{T_k Z_k - T_{k-1} Z_{k-1}}{T_k - T_{k-1}} \tag{9}$$

Table 1: Bond implied CDS spread and ASW spread as function of bond price 1-D using method I. The bond has 10 year to maturity with 7% semiannual coupon, 40% recovery. The day count convention is 30/360. Note that all values are in percentage except for W.

D	-10	-5	0	5	10	15	20
W	1.06	1.03	1.0	0.96	0.93	0.88	0.83
CDS	0.96	1.60	2.31	3.09	3.97	4.97	6.13
ASW	1.05	1.67	2.29	2.92	3.54	4.16	4.79
Basis	-0.09	-0.07	0.01	0.17	0.43	0.81	1.34
λ	1.58	2.64	3.80	5.09	6.65	8.19	10.11
$\overline{L} - \overline{L}^{Risky}$	0.03	0.05	0.08	0.11	0.14	0.17	0.21
$C\left(1 - PV01/\overline{PV01}\right)$	0.03	0.05	0.07	0.09	0.11	0.14	0.18
$D/\overline{PV01} - D/A$	-0.09	-0.07	0	0.15	0.40	0.78	1.31

Table 1 shows the bond implied CDS spread (formula (7)), ASW spread (formula (1)) and CDS basis (formula (8) as a function of the bond price discount D. The bond pays 7% coupon semiannually and has 10 years to maturity. We assume 40% recovery rate and 30/360 day count convention. The CDS spread, ASW spread and bond implied hazard rate all increase with decreasing bond price (increasing D). As expected, the CDS-Bond basis decreases with increasing bond price.

Table 1 also shows the three terms in the CDS-Bond basis formula (7). The individual contribution of the three terms in formula (7) can be easily inferred from Table 1. In this case, the first term slightly dominates the second term, and the net effect is small due to offsetting. But this is not always the case. For example, the first term in (7) is zero if the Libor curve is flat. For bond trading substantially away from par, the 3rd term in formula (7) dominates the basis.

To demonstrate the curve effect, Table 2 shows the results for the same bond as in Table 1 but with a flat Libor curve of 4.7% which is the average Libor in Table 1. We can see that the shape of interest rate curve has a small effect on the spreads and basis.

Table 2: Flat Libor curve. The same bond as in Table 1. All values are in percentage.

D	-10	-5	0	5	10	15	20
CDS	0.92	1.55	2.24	3.00	3.86	4.83	5.95
ASW	1.04	1.67	2.30	2.93	3.56	4.20	4.83
Basis	-0.12	-0.12	-0.06	0.07	0.29	0.63	1.12
λ	1.51	2.55	3.68	4.94	6.35	7.96	9.81
$\overline{L} - \overline{L}^{Risky}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$C\left(1 - PV01/\overline{PV01}\right)$	0.03	0.04	0.06	0.09	0.11	0.14	0.17
$D/\overline{PV01} - D/A$	-0.09	-0.08	0.00	0.15	0.4	0.77	1.30

4. Conclusions

We have presented a simple explicit formula to calculate the CDS spread implied by the bond market price. The value of the model is that it can be used either for issuers having a single bond outstanding or issuers having multiple bond issues. The formula explicitly expresses the bond implied CDS spread as the weighted sum of three factors: bond coupon, bond discount percentage and the Libor curve.

A potential use of the spread formula (7) is to explore the difference between the market CDS spread quote and the fair bond implied CDS spread.

5. References

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Appendix A

We derive the pricing formula (4) for the bond implied CDS spread. As stated previously, given the bond discount D, the negative trade investor buys the bond and hedge with buying 1-D/(1-R) notional CDS protection. This CDS notional amount makes the combined CDS-bond position default neutral.

Suppose the investor funds the purchase at Libor flat which is a reasonable assumption because CDS spread is based on Libor [2]. We assume that the cash flow terminates upon default. Furthermore, we assume the investor will pay the accrued CDS premium and loan interest but not receive bond accrued coupon when the bond defaults. Based on these assumptions, the cash flow to the investor is described in the following table.

	CDS	Loan	Bond	Total
Initial	0	1-D	-(1-D)	0
Payment Date	-(1-D/(1-R))S	-(1-D)L	C	C-(1-D)L - (1-D/(1-R))S
Default	(1-D-R)-Accrued premium	-(1-D)-accrued loan interest	R	-Accrued CDS and loan interest
Maturity	0	-(1-D)	1	D

The no-arbitrage condition means that the expected present value of all cash flow, initial and future, to the investor must be zero. We arrive at

$$E_0\left\{\sum_{k=1}^{N}\left[\frac{\left(C-(1-D)L(T_{k-1},T_{k-1})-\left(1-\frac{D}{1-R}\right)S_{CDS}\right)\Delta T_{k-1}1(\tau>T_k)}{B(T_k)}\right]\right.$$
$$-\sum_{k=1}^{N}\left[(1-D)L(T_{k-1},T_{k-1})+\left(1-\frac{D}{1-R}\right)S_{CDS}\right]\times\frac{\tau-T_{k-1}}{B(\tau)}\times 1(T_{k-1}<\tau\le T_k) \qquad \text{(A.1)}$$
$$\left.+D\frac{1(\tau>T_N)}{B(T_N)}\right\}=0$$

In the above equation, $L(t,T)$ is the *T*-maturity forward Libor seen at time *t*, R is the expected recovery rate, τ is the default time, *B(t)* is the money market account and 1(A) is the indicator function. The first term in equation (A.1) is the net coupon payment to the investor on scheduled payment dates. The second term is the payment upon default of accrued loan interest and CDS spread since the last scheduled coupon payment date. We assume the accrued bond coupon is not paid upon default. The third term is the payoff to the investor at maturity if the bond has not defaulted.

Adopting the usual assumption of independence between credit spread and interest rate, using the Lando formula (see [5]), and approximating the integral using trapezoidal rule, we get

$$E_0\left\{\frac{1(T_{k-1} < \tau \le T_k)}{B(\tau)}\right\} = -\int_{T_{k-1}}^{T_k} E^I\left(\frac{1}{B(t)}\right) dP(\tau > t) = -\int_{T_{k-1}}^{T_k} DF(t) dP(\tau > t)$$

$$= \frac{DF(T_{k-1}) + DF(T_k)}{2}\left[P(\tau > T_{k-1}) - P(\tau > T_k)\right] \tag{A.2}$$

$$E_0\left\{\frac{(\tau - T_{k-1})1(T_{k-1} < \tau \le T_k)}{B(\tau)}\right\} = -\int_{T_{k-1}}^{T_k} E^I\left(\frac{t - T_{k-1}}{B(t)}\right) dP(\tau > t)$$

$$= -\int_{T_{k-1}}^{T_k} (t - T_{k-1}) DF(t) dP(\tau > t) = \frac{1}{2}\Delta T_{k-1} DF(T_k)\left[P(\tau > T_{k-1}) - P(\tau > T_k)\right] \tag{A.3}$$

Using the fact that the $L(t, T_{k-1})$ is a martingale under the $T_k -$forward measure, we have

$$E_0\left\{L(T_{k-1}, T_{k-1})\frac{(\tau - T_{k-1})1(T_{k-1} < \tau \le T_k)}{B(\tau)}\right\} = -\frac{1}{2}E^I\left(L(T_{k-1}, T_{k-1})\frac{\Delta T_{k-1}}{B(T_k)}\right)\int_{T_{k-1}}^{T_k} dP(\tau > t)$$

$$= \frac{1}{2}\Delta T_{k-1} DF(T_k) E^{T_k}\left(L(T_{k-1}, T_{k-1})\right)\left[P(\tau > T_{k-1}) - P(\tau > T_k)\right] \tag{A.4}$$

$$= \frac{1}{2}\Delta T_{k-1} L_{k-1} DF(T_k)\left[P(\tau > T_{k-1}) - P(\tau > T_k)\right]$$

where $L_{k-1} = L(0, T_{k-1})$ is the $T_{k-1} -$forward rate.

Substituting equations (A.2), (A.3) and (A.4) into (A.1), and rearranging terms results in

$$C \times PV01 - \left[(1 - D)\overline{L}^{Risky} + \left(1 - \frac{D}{1 - R}\right)S_{CDS}\right]\overline{PV01} + D \times DF(T_N)P(\tau > T_N) = 0\,. \tag{A.5}$$

where

$$PV01 = \sum_{k=1}^{N} DF(T_k)\Delta T_{k-1} P(\tau > T_k),$$

$$\overline{PV01} = \frac{1}{2}\sum_{k=1}^{N}\Delta T_{k-1} DF(T_k)\left(P(\tau > T_{k-1}) + P(\tau > T_k)\right)$$

$$\overline{L}^{Risky} = \frac{1}{2}\sum_{k=1}^{N} L_{k-1}\Delta T_{k-1} DF(T_k)\left(P(\tau > T_{k-1}) + P(\tau > T_k)\right) / \overline{PV01} \tag{A.5}$$

$$Loss = \frac{1}{2}\sum_{k=1}^{N}\left(DF(T_{k-1}) + DF(T_k)\right)\left(P(\tau > T_{k-1}) - P(\tau > T_k)\right)$$

$$P(\tau > T_k) = Exp\left(-\int_0^{T_k}\lambda dt\right), \qquad DF(T_k) = \prod_{j=1}^{k}\frac{1}{1 + L_{j-1}\Delta T_{j-1}}$$

Appendix B

Now we prove the equivalency between equations (4) and (6) under the constraint of equation (5).

Proposition: If equation (5) is satisfied, the bond implied CDS spread calculated using equation (4) satisfies equation (6).

Proof: By virtue of equation (5), we can rewrite equation (4) as

$$(1 - DF_N P_N) - \overline{L}^{Risky}\,\overline{PV01} - \frac{R \times Loss}{1-D} - W \times S_{CDS}\,\overline{PV01} = 0 \qquad \text{(B.1)}$$

where $DF_N = DF(T_N)$, $P_N = P(\tau > T_N)$.

Furthermore, we rewrite the term *Loss* in equation (A.1) into

$$Loss = \frac{1}{2}\sum_{k=1}^{N}(DF_{k-1} + DF_k)(P_{k-1} - P_k) = \frac{1}{2}[1 - DF_N P_N] + \frac{1}{2}\sum_{k=1}^{N}[DF_k P_{k-1} - DF_{k-1} P_k] \quad \text{(B.2)}$$

Notice that $DF_k L_{k-1} \Delta T_{k-1} = DF_{k-1} - DF_k$ and using (B.2), we have

$$\begin{aligned}
&(1 - DF_N P_N) - \overline{L}^{Risky}\,\overline{PV01} = 2Loss - \overline{L}^{Risky}\,\overline{PV01} - \sum_{k=1}^{N}(DF_k P_{k-1} - DF_{k-1} P_k) \\
&= 2Loss - \frac{1}{2}\sum_{k=1}^{N}[(DF_{k-1} - DF_k)(P_{k-1} + P_k) + 2(DF_k P_{k-1} - DF_{k-1} P_k)] \qquad \text{(B.3)} \\
&= 2Loss - \frac{1}{2}\sum_{k=1}^{N}(DF_{k-1} + DF_k)(P_{k-1} - P_k) = 2Loss - Loss = Loss
\end{aligned}$$

Substitute (B.3) into (B.1) yields

$$\left(1 - \frac{R}{1-D}\right) \times \frac{Loss}{W} - S_{CDS}\,\overline{PV01} = 0 \qquad \text{(B.4)}$$

Since $W = \left(1 - \frac{D}{1-R}\right)/(1-D)$, we have $\left(1 - \frac{R}{1-D}\right)/W = 1 - R$. This completes the proof.