A Multi-Objective Approach to Simultaneous Strategic and Operational Planning in Supply Chain Design

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ABSTRACT

In this research, an integrated multi-objective supply chain (SC) model is developed for use in simultaneous strategic and operational SC planning. Multi-objective decision analysis is adopted to allow use of a performance measurement system that includes cost, customer service levels (fill rates), and flexibility (volume or delivery). This measurement system provides more comprehensive measurement of supply chain system performance than do traditional, single-measure approaches. Moreover, This model incorporates production, delivery, and demand uncertainty, and provides a multi-objective performance vector for the entire SC network. The model developed here will aid in the: 1) design of efficient, effective, and flexible supply chain systems and 2) evaluation of competing SC networks.

Keywords: mathematical programming, supply chain, multi-objective decision analysis.

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1 Introduction

A supply chain is a set of facilities, supplies, customers, products and methods of controlling inventory, purchasing, and distribution. The chain links suppliers and customers, beginning with the production of raw material by a supplier, and ending with the consumption of a product by the customer. In a supply chain, the flow of goods between a supplier and a customer passes through several echelons, and each echelon may consist of many facilities.

The problems of supply-production, production-distribution, and inventory-distribution systems have been studied for many years. Most of these studies focus only on a single component of the overall supply-production-distribution system, such as procurement, production, transportation, or scheduling, although limited progress has been made towards integrating these components in a single supply chain.

Supply chain management (SCM) is a subject of increasing interest to academics, and practitioners. SCM can be divided into two levels: strategic and operational. Models have been developed for optimizing supply chain operations at these two levels. The primary objective of strategic optimization models is to determine the most cost-effective location of facilities (plants and distribution centers), flow of goods throughout the supply chain (SC), and assignment of customers to distribution centers (DCs). These types of models do not seek to determine required inventory levels, and customer service levels. The main purpose of the optimization at the operational level is to determine the safety stock for each product at each location, the size and frequency of the product batches that are replenished or assembled, the replenishment transport and production lead times, and the customer service levels.

Uncertainty is one of the most challenging but important problems in SC management. Indeed, it is a primary difficulty in the practical analysis of SC performance. In the absence of randomness, the problems of material and product supply are eliminated; all demands, production, and transportation behavior would be completely fixed, and therefore, exactly predictable. This work seeks to integrate strategic and operational analysis of a SC subject to uncertainty, using a performance vector designed to describe the efficiency and effectiveness of the chain.

2 Literature Review

The supply chain (SC) has been viewed as a network of facilities that performs the procurement of raw material, transformation of raw material to intermediate and end products, and distribution of finished products to retailers or directly to customers. These facilities, which usually belong to different companies, consist of production plants, distribution centers, and end-product stockpiles. They are integrated in such a way that a change in any one of them affects the performance of others. Substantial work has been done in the field of optimal SC control. Various SC strategies and different aspects of SC management have been investigated in the literature. However, most of the developed models study only isolated parts of the SC.

2.1 Deterministic Supply Chain Models

The production/distribution model (PILOT) of [1] is a global, deterministic, periodic (annual), cost function, mixed integer mathematical program with a nonlinear objective function. This model extends the classic, multi-commodity distribution system model of [2]. PILOT is concerned with the global supply strategy for manufacturing, thus it seeks to determine the number and locations of plants and distribution centers, material (raw material, intermediate, and finished products) flows, plant production volumes, and the allocation of customers to distribution centers.

PILOT to investigate the effects of certain variables (unit transport costs and plant fixed cost) on the optimal supply chain structure in [3] and [4]. The objective function minimizes total cost subject to constraints on demand, raw material supply, production and distribution center (DC) capacities, production-distribution network structure, and customer location.

A deterministic, “international”, non-linear model that uses a cost objective that considers before- and after-tax profitability is developed in [5]. The authors also add a trade balance constraint to the model because in some countries, it is a must to carry out a minimum level of manufacturing inside these countries, in order to gain entry into their markets. The major contribution of this model is the inclusion of fixed vendor costs and trade balance constraints.

An “international”, dynamic, non-linear supply chain model is developed in [6] and [7]. The authors differentiate this model from a single-country (domestic) model by including local content constraints, which require a minimum expenditure on production and raw material acquisition within a particular country based on the level of sales in that country. After solving for the optimal financial variable values, an iterative procedure is used to obtain an optimal solution to the overall problem.

A mixed-integer programming, cost function model for a two-echelon uncapacitatied distribution location problem is developed in [8]. The authors provide sensitivity, cost-service trade-offs, and what-if analyses to clarify all major costs and service trade-offs. A fixed-charge network programming technique is used to determine the best shipment routings and shipment size through the distribution system.

A deterministic mathematical program is proposed in [9], with the objective of minimizing the cost of a sub-supplier/supplier/manufacturer supply chain. Three different operational scenarios of optimizing the supply chain are assumed and examined. In the first one, manufacturing optimizes its operational costs without considering the suppliers. In the second scenario, overall cooperation exists among the three levels of the supply chain. In the last scenario, each level of the SC is optimized separately with a partial cooperation among the SC. A numerical example is provided to illustrate the above scenarios, and the simulation technique is used to verify the results. In this example, the differences in the total costs are very small, and the computational times are almost identical for the three scenarios.

The strategic aspects of the supply chain are also considered in [10]. They develop an integrated supply chain decision model for a large global international company by optimizing cost and cycle time. The major contribution of this work is considering trade balance, local content, and duty constraints in an international supply chain model. The latter constraints are valid whenever a country or a nation charges an import tax or duty to an imported product.

An interactive tool for reengineering P&G's North American product sourcing and distribution system is developed in [11]. The authors use a decomposition approach to divide the overall SC problem into two easily-solved sub-models: an ordinary uncapacitated distribution location mix integer model and transportation linear model. Near-optimal solutions are generated to help in coupling the two sub-models.

A mixed integer linear programming model is developed in [12] to streamline operations and improve the scheduling process, while avoiding material stockout or resource violation for a formulation and packaging chemical plant. The objective function is formulated to maximize flexibility, which is represented by capacity slacks, to absorb unexpected demand, and subjected to five types of constraints.

Each of the preceding supply chain models is deterministic, but in reality, SCs operate in an uncertain environment. Uncertainty is associated with customer demand, and internal and external supply deliveries throughout the SC. The following models seek to capture the uncertainty of the supply chain environment by considering stochastic supply and/or demand.

2.2 Stochastic Supply Chain Models

A non-linear, stochastic, multi-echelon inventory model is presented in [13] to determine the optimal stocking policy for a spare parts stocking system, based on achieving an optimal trade-off between holding costs and transportation costs, subject to response time constraints. This service system has unique characteristics, such as low demand rates, a complex echelon structure, and the existence of emergency shipments to meet unforeseen demand. Solution to this complex model is found using a branch and bound procedure.

A stochastic optimization supply chain model is presented in [5], which uses raw material, production, inventory, and distribution sub-models. All locations use (s, S) or (Q, R) control policies. A decomposition approach is used to optimize each sub-model individually, and then link them together by target fill rates; the sub-models are not optimized simultaneously. In this work, the network is limited in this research to a single manufacturing site.

Five different approaches to reduce SC demand amplification are proposed in [14]. These are:

- Fine tune the existing ordering policy parameters
- Reduce time-delays using primarily JIT techniques
- Integrate information flow throughout the supply chain
- Improve the individual echelon decision rules
- Remove the distributor echelon under the assumption that the length of the supply pipeline between the factory and the retailer is nearly equivalent to that between the distributor and the retailer.

Simulation modeling, using the Forrester production distribution system as a reference, is used to evaluate and compare these approaches. Improved information flow proved to be the most effective strategy in reducing demand amplification. In closely-related work, [15] compares the above strategies on the bases of cost and demand amplification reduction. The authors conclude that removing the distribution echelon is the best strategy for improving the performance of the supply chain.

A stochastic heuristic model is introduced in [16] for managing material flows in decentralized supply chains by either determining stock levels subject to a target service level (fill rate) or determining the service level performance given stock levels. The authors assume a pull-type, periodic base stock inventory system and a normally distributed demand pattern. The demand generated downstream becomes the required demand at the upstream site, while the supply delivery uncertainty at the upstream site influences material availability at the downstream site.

A two-phase design model is presented in [17] to aid in the evaluation of different production/inventory location strategies. The first phase uses mathematical programming and heuristic techniques to minimize the number of product types. The second phase uses a spreadsheet inventory model to estimate the minimum safety stock based on the service level, demand level, lead time, demand variability, lead time variability, and product size flexibility. Finally, capital investment and competitors' strategies are also considered before determining a final recommendation for the best strategy to be implemented.

The work in [18] presents a stochastic model of an integrated production-distribution system comprised of one factory, one stockpile of finished goods, and one retailer. It extends the work of [19] by considering a more complicated production network. Multiple products, independent demand, and expedited batches are considered in the model. The objective is to minimize the total costs subject to constraints on service levels for all products across the supply chain. Replenishment batch size, stockpile inventory, and retailer inventory trade-offs are the decision variables used. It is concluded that expediting capability reduces the otherwise dramatic effects of the sub-optimal selection of these decision variables.

Simulation models are developed by [20] to compare the dynamic performance improvements among different SC redesign strategies. More specifically, a case study is provided to illustrate four SC redesign process phases. These are:

- Phase 1: "Just in time"
- Phase 2: Interplant, and logistics integration
- Phase 3: Vendor integration
- Phase 4: Time based management

The effects of postponement strategy on SC cost are investigated by [21]. They provide a simplified analytical model to determine the optimal decoupling point (P), which refers to the point of product differentiation, by minimizing the cost function. The problem considers a supply chain with one factory serving multiple DCs. The authors determine, from the case example, that the inventory level is the main factor in determining the product configuration (decoupling) point, and that the fixed costs of improving DC postponement capabilities is negligible compared to this factor.

2.3 Literature Summary

The existing SC literature identifies a gap in the development of comprehensive supply chain models. Models that assume that demand is stochastic either consider only two echelons or consider the operational level of the supply chain exclusively. Other models that deal with larger networks at the strategic level do not consider supply chain uncertainty.

Other important observations that can be obtained from the existing body of literature are:

- Only one author [12] considers SC flexibility as a performance measure, which is represented by capacity slacks of operational resources, although these slacks are the only performance measure used.

- All strategic-level models are deterministic. All deterministic models, except for [10], have been established either for optimizing SC cost alone or maximizing profitability. Other performance measures are not considered.

- Strategic and operational considerations have not been integrated into a single approach.

The objective of this research is to develop a supply chain model that facilitates simultaneous strategic and operational planning. This model incorporates production, delivery, and demand uncertainty, and provides an appropriate performance measure by using a multi-objective analysis for the entire SC network. The model developed here will aid in the design of efficient, effective, and flexible supply chain and in the evaluation of competing SC networks.

3 Model Structure and Formulation

In this research, the supply chain structure consists of four echelons: (1) suppliers, (2) plants, (3) distribution centers (DCs), and (4) customer zones (CZs). Each SC echelon has a set of control parameters that affects the performance of other components. The performance of each echelon will be optimized simultaneously at two planning levels using the strategic and operational sub-models.

3.1 Model Overview

The objective of the strategic sub-model is to optimize the SC configuration and material flow. Since there are numerous sources of uncertainty in a typical supply chain, applying deterministic supply chain models is unrealistic. A stochastic operational level sub-model is integrated into the solution approach in order to accommodate uncertainty and to give insight into the trade-offs among cost, customer service level, and flexibility. Within the operational level sub-model, various sources of uncertainty are considered, such as customer demand, production lead-time, and supply lead times throughout the SC. In this operational sub-model, the work of [5] is extended to allow for simultaneous optimization of the entire SC system, and to estimate the actual production, distribution and transportation costs.

Fill rate is a common measure of service level performance [22]. Fill rate measures the percentage of orders filled immediately. Flexibility can be defined as the ability to respond to customer requirements [23]. Two types of flexibility, which depend on SC configuration, are considered here: (1) volume flexibility, and (2) delivery flexibility. Volume and delivery flexibility are defined in [23] as the ability to change the level of produced products and planned delivery dates, respectively. Volume flexibility, which is measured by capacity slack, is commonly used in industry. However, delivery flexibility, which is measured by lead-time slack, is not used often in industry or in literature. This is because the majority of inventory and SC models in literature assume fixed lead times. However, in many practical situations, lead time, either probabilistic or deterministic, may be controllable and thus must be considered as a decision variable as in this research. Lead-time is defined as the length of time between the time when an order for an item is placed and when it is actually available for satisfying customer demands [24].

A multi-objective problem arises in this model, due to consideration of multiple performance measures at each sub-model. The e-constraint method [25] is selected in this research over other solution methodologies for the following reasons: 1) the e-constraint method can solve non-linear models, 2) no specific conditions are required to achieve the solutions, and 3) the e-constraint method is simple, since it transforms the multi-objective problem into a single-objective optimization problem. This method allows the analyst the ability to specify bounds on the objectives in a sequential manner. The magnitude of e reflects the relative importance of the various objectives to decision-makers.

3.2 The Strategic-Level Sub-Model

The strategic-level sub-model considers an integrated, multi-product, multi-echelon, and procurement-production-distribution system design problem in a flexible facility network configuration. It optimizes material flows throughout the supply chain, gives the optimal number and locations for plants, and distribution centers, and provides the best assignment of distribution centers to customer zones. A multi-objective function is formulated to minimize cost, while ensuring a sufficient amount of volume flexibility, subject to supplier, plant and distribution capacities, production and DC throughput limits, and customer demand requirements. Total costs include production and distribution fixed costs (facility overhead, duties, manufacturing technology, labor, and maintenance), and production, distribution, and transportation variable costs. This sub-model is integrated with the operational sub-model to incorporate the uncertainty and non-linearity of variable production, distribution, and transportation costs.

Four echelons are considered in this sub-model: (1) suppliers (vendors), (2) plants (production), (3) distribution centers, and (4) customer zones.
Before presenting the problem formulation, the notation is introduced in Table 1.

Variables	Definitions
i	Product type index, $i = 1,....,I$
v	Vendor (echelon 1) index, $v = 1,....,V$
j	Plant (echelon 2) index, $j = 1,....,J$
k	Distribution center (echelon 3) index, $k = 1,....,K$
m	Customer zone (echelon 4) index, $m = 1,....,M$
r	Raw material type index, $r = 1,....,R$
Inputs	(Fixed values)
e	Volume flexibility performance index [0,1]
w_2, w_3	Weight factors for capacity utilization [0,1]
f_{2j}	Fixed charges for plant j ($/period)
f_{3k}	Fixed charges for DC k ($/period)
Ψ_{rv}	Production capacity of vendor v for raw material r (units/period)
d_{2ij}	Standard (equivalent) units at plant j per unit of product i
d_{3ik}	Standard (equivalent) units at DC k per unit of product i
Φ_j	Production capacity (standard units) for each plant (units/period)
t_{ri}	Utilization rate for each raw material per unit of product
x_{ij}	Minimum production volume for product i at plant j (units/period)
z_{ij}	Maximum production volume for i at j (units/period)
a_k	Minimum throughput(handling and inventory) at DC k (units/period)
b_k	Maximum throughput at DC k (units/period)
a_{rvj}	Unit transportation cost from v to j for raw material r ($/unit)
c_{ijk}	Unit transportation cost from j to k for product i ($/unit), from operational sub-model
d_{ikm}	Unit transportation cost from k to m for product i ($/unit)
l_{rv}	Unit cost of raw material r for vendor v ($/unit)
U_{2ij}	Unit production cost for product i at plant j ($/unit), from operational sub-model
U_{3ik}	Unit cost of throughput (handling and inventory) for product i at DC k ($/unit), from operational sub-model
D_{im}	Average demand for product i at CZ m (units/period)
Outputs	(Decision variables)
X_{ij}	Quantity of product i produced at plant j (units/period)
C_{ijk}	Quantity of product i shipped from j to k (units/period)
A_{rvj}	Quantity of raw material r shipped from v to j (units/period)
Z	Total cost ($/period)

Variables	Definitions
W	Volume flexibility
Binary	(Decision variables)
q_{2j}	1, if plant j is open; 0 otherwise
q_{3k}	1, if DC k is open; 0 otherwise
y_{km}	1, if DC k serves CZ m; 0 otherwise

Table 1. Notation for Strategic-Level Sub-Model

The objective functions are given by:

$$Z=\left[\sum_{rvj}(a_{rvj}+l_{rv})A_{rvj}\right]+\left[\sum_{j} f_{2j}q_{2j}+\sum_{ij}U_{2ij}X_{ij}\right]+ \left[\sum_{k} f_{3k}q_{3k}+\sum_{ikm}U_{3ik}D_{im}y_{km}+\sum_{ijk}c_{ijk}C_{ijk}\right]+\left[\sum_{ikm}d_{ikm}D_{im}y_{km}\right] \quad (2)$$

Thus, the strategic level sub-model is formulated as follows:

$$W=\left[\sum_{j}\left(q_{2j}\Phi_{j}-\sum_{i}d_{2ij}X_{ij}\right)\right]w_2+\left[\sum_{k}\left(q_{3k}b_{k}-\sum_{im}d_{3ik}D_{im}y_{km}\right)\right]w_3 \quad (3)$$

$$\text{Minimize } Z \quad (4)$$

Subject to:

$$W \geq e \quad (5)$$

Also:

$$\sum_{j} A_{rvj} \leq \varnothing_{rv} \quad \forall r,v \quad (6)$$

$$\sum_{i} t_{ri}X_{ij} \leq \sum_{v} A_{rvj} \quad \forall r,j \quad (7)$$

$$\sum_{i} d_{2ij}X_{ij} \leq \ddot{O}_{j}q_{2j} \quad \forall j \quad (8)$$

$$x_{ij}q_{2j} \leq X_{ij} \leq z_{ij}q_{2j} \quad \forall i,j \quad (9)$$

$$a_k q_{3k} \leq \sum_{im} d_{3ik} D_{im} y_{km} \leq b_k q_{3k} \quad \forall k \tag{10}$$

$$\sum_k y_{km} = 1 \quad \forall m \tag{11}$$

$$X_{ij} = \sum_k C_{ijk} \quad \forall i, j \tag{12}$$

$$\sum_{jk} C_{ijk} = \sum_m D_{im} \quad \forall i \tag{13}$$

$$\sum_j C_{ijk} = \sum_m y_{km} D_{im} \quad \forall i, k \tag{14}$$

$$X_{ij}, C_{ijk}, A_{rvj} \geq 0 \quad \forall r, i, v, j, k \tag{15}$$

$$q_{2j}, q_{3k}, y_{km} = 0 \; or \; 1 \quad \forall j, k, m \tag{16}$$

The first mixed-integer linear objective function (Z) minimizes the total fixed and variable costs, and is divided into four components: (1) the raw material purchase price and transportation cost from vendors to plants, (2) the fixed and variable costs associated with plant operations, (3) the variable cost of handling and inventory of products at DCs, and transportation of products from plant to DCs, and (4) the transportation cost of products from DCs to CZs. The second linear objective function (W) represents the volume flexibility, which is the sum of the following flexibility performance measures:

- Plant volume flexibility, which is measured as the difference between plant capacity and plant capacity utilization, and thus represents the available plant capacity.
- Distribution volume flexibility, which is calculated as the difference between the available throughput and demand requirements, and thus represents the available distribution capacity.

Equations (6) through (16) of the strategic level sub-model represent, respectively:

- Equation (6) ensures that the required quantities of raw materials are within the supplier's capabilities (raw material supply constraints).
- Equation (7) matches raw materials to the production requirements.
- Equation (8) specifies that the total production quantities do not exceed plant capacity. Determining d_{2ij} is illustrated in Example 1 below (plant production capacity constraints).

- Equation (9) enforces the minimum and maximum production capacities for plants.
- Equation (10) enforces the minimum and maximum throughput capacities for DCs, and ensures that customer zone assignments can be made only to open DCs. Determining d_{3ik} follows the same procedure of calculating d_{2ij} as given in Example 1 below.
- Equation (11) specifies that each customer zone must be assigned to exactly one single distribution center.
- Equation (12) ensures that the amount shipped from a plant is equal to what is available at that plant.
- Equation (13) ensures satisfaction of all demand requirements, i.e., that the total shipments to customer zones are exactly equal to the forecasted demands there.
- Equation (14) ensures that the demand requirements at each DC be satisfied.
- Equation (15) ensures non-negativity for all variables.
- Equation (16) restricts the binary variables.

3.2.1 Estimating Subjective Parameters for the Strategic Sub-model

Decision-Maker (DM) specification of the desired minimum flexibility level (e) results in a preferred solution. However, a DM may need to examine many of non-dominated solutions (with the associated tradeoffs) prior to making a selection, especially in the absence of a specific target. This analysis may be accomplished by varying e to generate several non-dominated solutions. Although the formulation of the e -constraint method is straightforward, it is not clear how to determine the variation bounds on e . One way to accomplish this is as follows:

Step 1. Set the e bound at 0. Examine the available total flexibility (W).

Step 2. Decide either to continue in generating more non-dominated solutions (if the value of W is not sufficient), or to stop if the total available flexibility is satisfactory.

Step 3. Choose an appropriate e step size and add it to the previous lower bound value.

Step 4. Go to step 3, unless the best solution has been reached. The best solution is reached when adding additional flexibility results in reduced profit.

The weights w_2 and w_3 are specified by the DM for each flexibility performance measure. A weight can be interpreted as “the relative weight or worth” of that objective when compared to the other objectives. This is determined according to DM’s preferences. Therefore, the solution to the strategic-level sub-model is equivalent to the best-compromise solution; that is, the optimal solution is relative to a DM’s particular preference structure.

3.3 The Operational-Level Sub-Model

Given the output (decision variables) of the strategic-level sub-model, customer demand requirements, minimum required service and flexibility levels, cost and lead time data, and bill of material data, variable costs are estimated under uncertainty. Also, various operational variables are determined by optimizing inventory variables such as lot sizes, reorder points, and safety stock. A multi-objective function is developed to incorporate all cost, customer service level (fill rate), and flexibility (delivery) trade-offs.

Three echelons are considered in this sub-model: (1) suppliers, (2) plants (production and finished products stockpiles), and (3) distribution centers. The supplier and distribution echelon models are solved using analytical techniques, while the production and stockpile models are simultaneously optimized using non-linear programming. A single solution for the three-echelon supply chain at the operational level is determined using a heuristic approach as described in the following subsections.
The notation used in the operational sub-model is as follows:

Variables	Definitions
i	Product index, $i = 1,....,I$
v	Vendor index, $v = 1,....,V$
j	Plant index, $j = 1,....,J$
k	Distribution center index, $k = 1,....,K$
m	Customer zone index, $m = 1,....,M$
r	Raw material index, $r = 1,....,R$
Inputs	(Fixed values)
h	Customer service performance index [0,1]
g	Delivery flexibility performance index [0,1]
Y_i	Set of required raw materials for product i (bill of material)
F_{rj}	Set of products which consumes r in the production at j
t_{ri}	Utilization rate for each raw material per unit of product
m_{rvj}	Expected lead time (review periods) for r from v to j (period)
P_{1rv}	Material availability (fill rate) for r at v [0,1]
J_{rv}	Expected material delay time of r at v (period)
$\Im_{rvj}$	Variance of lead time for r from v to j (period^2)
$\Re_{rv}$	Variance of material delay time of r at v (period^2)
q_{1rj}	Order setup cost of replenishing r at j ($)
H_{1rj}	Unit holding cost for r at j ($/period/unit)
p_{1rj}	Unit backorder penalty cost for shortage of r at j ($/unit)
Δ_{rvj}	Unit holding cost for r en route from v to j ($/period/unit)
q_{2ij}	Production setup cost of i at j ($)
X_{ij}	Quantity of product i produced at plant j (units/period)
Γ_{ij}	Unit processing cost of i at j ($/unit)
Ω_{ij}	Unit work-in-process holding cost for i at j ($/period/unit)
P'_{2ij}	Minimum required service level of i at j [0,1]
H_{2ij}	Unit holding cost for i at j ($/period/unit)
e_{ij}	Cost of initiating an expedited production order for i at j ($)
P'_{3ik}	Minimum required service level of i at k [0,1]
N_{ijk}	Normal transportation lead time for i from j to k (period)
E_{ijk}	Expedited transportation lead time for i from j to k (period)
q_{3ik}	Order setup cost of i at k ($)
C_{ijk}	Quantity of product i shipped from j to k (units/period)
H_{3ik}	Unit holding cost of i at j ($/period/unit)

Variables	Definitions
p_{3ik}	Unit backorder penalty cost for shortage of i at k ($/unit)
c_{ijk}	Unit holding cost for i en route from j to k ($/period/unit)
T'_{ijk}	Standard delivery time at j when i is out of stock at k (period)
g_{ij}	Production setup time for i at j (period)
p_{ij}	Production processing time for i at j (period)
l_{ij}	Waiting time at the workstations for i at j (period)
D_{im}	Expected demand for i at m (units/period)
<u>Outputs</u>	
Q^*_{1rj}	Optimal batch size of r at j (units)
P_{1rj}	Customer service level (fill rate) of r at j [0,1]
s_{1rj}	Reorder point for r at j (units)
Θ_{1rj}	Expected total replenishment lead time of r at j
L_1	Expected demand of r over a replenishment lead time at j
s_1	Standard deviation of lead time demand of r
Q_{2ij}	Production batch size for i at j (units)
t_{2ij}	Total production lead time for i at j (period)
Θ_{2ij}	Material delay time of the production of i at j
L_2	Expected demand of i over production lead time at j
s_2	Standard deviation of production lead time demand of i
t_{3ik}	Expected transportation lead time for i to k (period)
s_{2ij}	Reorder point for i at j (units)
P_{2ij}	Customer service level of i at j [0,1]
u_{1ij}	Unit cost involved in controlling all r required for i ($/unit)
u_{2ij}	Unit cost of the production of i at j ($/unit)
u_{3ij}	Unit cost related to the finished goods stockpile for i at j ($/unit)
L_3	Expected demand of i over a replenishment lead time at k
s_3	Standard deviation of lead time demand of i at k
P_{3ik}	Customer service level of i at k [0,1]
T_{ijk}	Expected replenishment lead time for i from j to k (period)
s_{3ik}	Reorder point for i at k (units)
S_{3ik}	Order-up-to level for i at k (units)
Q_{3ik}	Order batch size of i at k (units)

Variables	Definitions
<u>Outputs</u>	(decision variables)
U_{2ij}	Unit total production cost for i at j (\$/unit)
U_{3ik}	Unit cost of throughput for i at k (\$/unit)
c_{ijk}	Unit transportation cost from j to k for i (\$/unit)
TC_{ij}	Total expected costs of production and stockpile of i at j
PS_{ij}	Customer service (fill rate) availability of product i at j
PD_{ijk}	Delivery flexibility availability of product i from j to k

Table 2. Notation for Operational-Level Sub-Model

3.3.1 Supplier Control Echelon Model

This model assumes continuous review of the inventory position for each raw material r involved in producing the finished production set F_{rj} at plant j, using an (s, Q) inventory control policy. A fixed quantity, Q_{rj}, of raw material r is ordered at plant j whenever the inventory position drops exactly (the undershoots are assumed to be negligible) to the reorder point s. The demand requirement for raw material r is determined from the production requirement of product i at plant j (X_{ij}), which is determined at the strategic level, and the unit usage rate of r in i (t_{ri}) is specified in the bill of material data.

The raw material shortages are assumed to be backordered. To simplify the computations, a normal lead-time demand distribution is also assumed. The total cost of controlling raw material inventory involves setup, holding, and backorder (delay) costs. Using standard terms, as in [26], the total cost involved in controlling raw material r at plant j per period can be specified as

$$\mathrm{TC}^{\mathrm{s}}_{\mathrm{1rj}} = q_j \left[\left(\sum_{i \in F_{rj}} \frac{t_{ri} X_{ij}}{Q_{1rj}} \right) a_{1rj} + H_{1rj} I_{1rj} + p_{1rj} s_1 G_b(n) \right] \tag{17}$$

where the on-hand inventory level (average inventory level plus safety stock) is given by:

$$I_{1rj} = \frac{Q_{1rj}}{2} + n_1 s_1 \tag{18}$$

The safety factor n_1 is selected to control the safety stock associated with a specified customer service level, and represents the number of standard deviations of lead time demand covered by the on-hand inventory. Suppose the decision rule of a specified probability, P_{1rj}, of no stockout per replenishment cycle (period) is selected in determining n_1. Then the safety factor n_1 should satisfy the following:

$$\Pr(B \geq n_1) = 1 - P_{1rj} \quad (19)$$

Where B is a standard normal variable. The value n_1 can be read directly from a standard normal table.

A simplified expression to determine the safety factor n_1 has been proposed for periodic review, to eliminate the need to use standard normal distribution tables [27] [16]. This expression is extended to be used in the continuous review case by making the required transformations given in [26]. The approximate expression for n_1 is given as follows (see Appendix A for derivation):

$$n_1 \approx \left(\frac{1}{2} \sqrt{\frac{p}{2}} \right) \ln \left(\frac{P_{1rj}}{1 - P_{1rj}} \right) \quad (20)$$

The required reorder point s can be determined directly using the following relationship:

$$s_{1rj} = L_1 + n_1 s_1 \quad (21)$$

Where L_1 is the expected demand over a replenishment lead-time, which can be determined as follows:

$$L_1 = \left[\sum_i t_{ri} X_{ij} \right] \Theta_{1rj} \quad (22)$$

Where Θ_{1rj} is the average total replenishment lead-time of r at j, which is calculated as the sum of the raw material lead time and delay time, considering all suppliers:

$$\Theta_{1rj} = \frac{\sum_v \left(m_{rvj} + J_{rv}(1 - P_{1rv}) \right)}{V} \quad (23)$$

the variance of Θ_{1rj} can be calculated as follows:

$$\mathrm{var}\left(\Theta_{1rj}\right) = \mathrm{var}\left(\frac{\sum_{v} m_{rvj}}{V}\right) + \mathrm{var}\left(\frac{\sum_{v} J_{rv}\left(1 - P_{1rv}\right)}{V}\right) \tag{24}$$

or

$$\mathrm{var}(\Theta_{1rj}) = Max_{v}\left(\Im_{rvj}\right) + Max_{v}(\Re_{rv}(1 - P_{1rv})) + Max_{v}\left(J_{rv}^{2}\left(1 - P_{1rv}\right)P_{1rv}\right) \tag{25}$$

Then,

$$\mathrm{var}\left(L_{1}\right) = \left[\sum_{i} t_{ri} X_{ij}\right]^{2} \mathrm{var}\left(\Theta_{1rj}\right) + \mathrm{var}\left[\sum_{i} t_{ri} X_{ij}\right]\Theta_{1rj} \tag{26}$$

For a derivation of the variance of lead-time demand, see [28].
Since production demand X_{ij} is fixed, and thus has variance zero, the lead time demand variance is given by:

$$\mathrm{var}\left(L_{1}\right) = \left[\sum_{i} t_{ri} X_{ij}\right]^{2} \mathrm{var}\left(\Theta_{1rj}\right) \tag{27}$$

and

$$s_{1} = \sqrt{\mathrm{var}\left(L_{1}\right)} \tag{28}$$

Now, the optimal lot size (Q_{1rj}^{*}) may be determined by minimizing equation (17), which is accomplished by finding the first derivative of the total cost with respect to Q_{rj}, and setting it equal to zero.
This yields,

$$Q_{1rj}^{*} = \sqrt{\frac{2q_{1rj}\sum_{i} t_{ri} X_{ij}}{H_{1rj}}} \tag{29}$$

Also, the optimal service level for raw material r at plant j may be determined by differentiating the total cost equation (17) with respect to P_{1rj} and setting it equal to zero.

This yields,

$$P_{1rj} = -\frac{H_{1rj}}{p_{1rj}} + 1 \tag{30}$$

Now, the unit cost associated with material control for material r at plant j is given by:

$$u_{1rj} = \frac{TC_{1rj}^{s}}{\sum_{i} t_{ri} X_{ij}} \tag{31}$$

The unit costs associated with controlling all raw material required for product i at plant j is:

$$u_{1ij} = \sum_{r \in Y_i} t_{ri} u_{1rj} \tag{32}$$

3.3.2 Plant Echelon Model

This sub-section describes the sub-models associated with the manufacturing (plant) echelon.

3.3.2.1 Production Model

The cost function to be minimized in this system consists of setup costs, processing costs, and work-in-process carrying costs. More specifically, the total costs for the production of product i at plant j per period can be specified as follows:

$$TC_{2ij}^{P} = q_{2ij}\left(\frac{X_{ij}}{Q_{2ij}}\right) + \Gamma_{ij} X_{ij} + \Omega_{ij} X_{ij} t_{2ij} \tag{33}$$

Also, the unit cost of producing product i at plant j can be given by:

$$u_{2ij}^{P} = \frac{TC_{2ij}^{P}}{X_{ij}} \tag{34}$$

The total production lead-time (t_{2ij}) is given as the sum of setup time (g_{ij}), waiting time at the workstations (l_{ij}), processing time (p_{ij}), and material delay times (Θ_{1rj}). The processing time of a batch of product i at plant j can be calculated as:

$$p_{ij} = \frac{Q_{2ij}}{h_{ij}} \tag{35}$$

Where h_{ij} denotes the average work rate for the processing of product i at plant j .

As long as $r > 1$, i.e., there is more than one raw material type in Y_i for a certain finished product i, and if the manufacturer cannot begin production until all raw materials in Y_i have been received, then the lead or delay time in the model is represented by the maximum average realized lead or delay time from suppliers for raw materials in Y_i .
Then, the material delay time can be determined from the following relationship:

$$\Theta_{2ij} = Max_{r \in Y_i} \left(\Theta_{1rj} \left(1 - P_{1rj} \right) \right) \tag{36}$$

So,

$$t_{2ij} = g_{ij} + p_{ij} + l_{ij} + \Theta_{2ij} \tag{37}$$

Assuming deterministic setup and processing times, the variance of the production lead-time is given by:

$$\operatorname{var}(t_{2ij}) = \operatorname{var}(l_{ij}) + \operatorname{var}(\Theta_{2ij}) \tag{38}$$

Where,

$$\operatorname{var}(\Theta_{2ij}) = Max_{r \in Y_i} \left(\operatorname{var}(\Theta_{1rj})(1 - P_{1rj}) + \Theta_{1rj}^2 (1 - P_{1rj}) P_{1rj} \right) \tag{39}$$

3.3.2.2 The Finished Products Stockpile Model

An (s, Q) inventory control policy is used for the operation of the finished products stockpile. It is assumed that distribution demand shortages are met by expedited shipment. The total costs related to the finished products stockpile for product i at plant j per period are given by [5]:

$$TC_{2ij}^{F} = q_{2j}\left[H_{2ij}\left(\frac{Q_{2ij}}{2} + n_2 s_2\right) + \sum_k c_{ijk} C_{ijk}\left(N_{ijk} P_{2ij} + E_{ijk}\left(1 - P_{2ij}\right)\right) + e_{ij}\frac{X_{ij}}{Q_{2ij}}\left(1 - P_{2ij}\right)\right] \quad (40)$$

where the total cost consists of stockpile holding cost, transportation holding cost from plant j to the DCs, and the expedited order setup cost, respectively.

The following parameters for the finished products stockpile are calculated similarly to those in the supplier echelon model. These parameters are given by:

$$\text{var}(L_2) = X_{ij}^{2}\ \text{var}(t_{2ij}) \quad (41)$$

$$s_{2ij} = L_2 + n_2 s_2 = X_{ij} t_{2ij} + \left(\frac{1}{2}\sqrt{\frac{2}{p}}\right)\left(\frac{P_{2ij}}{1 - P_{2ij}}\right)\sqrt{\text{var}(L_2)} \quad (42)$$

$$u_{2ij}^{F} = \frac{TC_{2ij}^{F}}{X_{ij}} \quad (43)$$

Now, the unit variable production cost of product i at plant j is:

$$U_{2ij} = u_{1ij} + u_{2ij}^{P} + u_{2ij}^{F} \quad (44)$$

Assuming a Bernoulli random lead-time offered by plant j to each distribution k [5], the expected replenishment lead time for product i from plant j to DC k is:

$$T_{ijk} = N_{ijk} P_{2ij} + \left(t_{2ij} + E_{ijk}\right)\left(1 - P_{2ij}\right) \quad (45)$$

and

$$\text{var}(T_{ijk}) = P_{2ij}\left(1 - P_{2ij}\right)\left[T_{ijk} - \left(t_{2ij} + E_{ijk}\right)\right]^2 \quad (46)$$

3.3.2.3 Finding the Optimal $Q_{2ij}, P_{2ij}, T_{ijk}$

A multiple objective function is developed to take into account cost, customer service level (fill rate), and delivery flexibility tradeoffs. The first objective function considers cost as a performance measure, and is given by:

$$TC_{ij} = TC_{2ij}^{P} + TC_{3ij}^{F} \qquad \forall i, j \tag{47}$$

The second objective function represents service levels (fill rates) for replenishing the distribution centers from the finished product stockpile at plant j, and is given by:

$$PS_{ij} = P_{2ij} - P_{2ij}^{'} \qquad \forall i, j \tag{48}$$

Finally, the delivery flexibility objective function is given by:

$$PD_{ijk} = T_{ijk}^{'} - T_{ijk} \qquad \forall i, j, k \tag{49}$$

Using the e-constraint method, the multi-objective is formulated as follows:

[Operational performance index] Minimize TC_{ij} (50)

Subject to

$$PS_{ij} \geq h \qquad \forall i, j \tag{51}$$

$$PD_{ijk} \geq g \qquad \forall i, j, k \tag{52}$$

The h, g values are specified to ensure the desired minimum levels of fill rate and delivery flexibility. The procedure for determining e discussed in the previous section may be used here to determine h and g.

3.3.3 The Distribution Echelon

For the distribution echelon, a continuous-review (s, S) inventory control policy is assumed, in which a replenishment quantity is made whenever the inventory position drops exactly to the reorder point s. The size of the replenishment quantity is sufficient to raise the inventory position to the order-up-to level S. The simple sequential determination method (neglecting undershoots), is used to determine the order-up-to level S. Demand is periodic, stochastic, and independently distributed among customer zones and over time. Additionally, the lead-time demand at each DC is assumed to be normally distributed. If retailer demand is normally distributed, and if the lead-times between DCs and CZs are fixed, then the DC lead-time demand will also be normally distributed. It is further assumed that customer demand shortages are backordered.

The total cost of distribution, which consists of holding, reorder, and backorder cost for product i at DC k per period is:

$$TC_{3ik} = q_{3k}\left[H_{3ik}\left(\frac{Q_{3ik}}{2} + n_3 s_3\right) + q_{3ik}\frac{\sum_m y_{km} D_{im}}{Q_{3ik}} + p_{3ik} s_3 G(n_3)\right] \qquad (53)$$

The following parameters are also calculated similarly to those in the supplier echelon model, and are given by:

$$t_{3ik} = \frac{\sum_j q_{2j} T_{ijk}}{\sum_j q_{2j}} \qquad (54)$$

$$\operatorname{var}(t_{3ik}) = Max_j\left(q_{2j} \operatorname{var}(T_{ijk})\right) \qquad (55)$$

$$\operatorname{var}(L_3) = \left[\sum_m y_{km} D_{im}\right]^2 \operatorname{var}(t_{3ik}) + t_{3ik}\sum_m y_{km} \operatorname{var}(D_{im}) \qquad (56)$$

Also,

$$s_{3ik} = L_3 + n_3 s_3 = L_3 + \left(\frac{1}{2}\sqrt{\frac{2}{p}}\right)\left(\frac{P_{3ik}}{1 - P_{3ik}}\right)\sqrt{\operatorname{var}(L_3)} \qquad (57)$$

where

$$L_3 = t_{3ik}\sum_m y_{km} D_{im} \quad . \qquad (58)$$

Then

$$S_{3ik} = s_{3ik} + Q_{3ik} \tag{59}$$

and

$$U_{3ik} = \frac{TC_{3ik}}{I_{3ik}} \tag{60}$$

where

$$I_{3ik} = \frac{Q_{3ik}}{2} + n_3 s_3 \quad . \tag{61}$$

To obtain the optimal batch size for product i at DC k, we differentiate the total cost equation (53) with respect to Q and set it equal to zero. Which yields,

$$Q_{3ik}^{*} = \sqrt{\frac{2q_{3ik} \sum_{m} y_{km} D_{im}}{H_{3ik}}} \quad . \tag{62}$$

Also, the optimal service level for product i at DC k is determined by setting the derivative (with respect to P_{3ik}) of the total cost equation (53) equal to zero. Then,

$$P_{3ik} = \frac{-H_{3ik}}{p_{3ik}} + 1 \tag{63}$$

Now, we can summarize the actual unit variable costs relationships, which will be used as input to the strategic sub-model, as follows:

$$U_{2ij} = u_{1ij} + u_{2ij}^{P} + u_{2ij}^{F} \tag{64}$$

$$U_{3ik} = \frac{TC_{3ik}}{I_{3ik}} \tag{65}$$

$$c_{ijk} = c_{ijk}\left(N_{ijk} P_{2ij} + E_{ijk}\left(1 - P_{2ij}\right)\right) \tag{66}$$

4 Solution Methodology

This section presents an iterative procedure in which the strategic-level optimization sub-model is combined with the operational-level optimization sub-model to determine the optimal SC performance vector. The steps of the algorithm are given below and illustrated in Figure 1.

The Strategic-Operational Optimization Solution Algorithm:

Step 1. Optimize the strategic-level sub-model for an existing or proposed SC network to obtain the initial optimal configuration, using mixed integer linear programming by considering the base-case (initial) values for production, distribution, and transportation unit variable costs.

Step 2. Use the decision variable outputs of the strategic-level sub-model as input data to the operational-level sub-model, after dividing by the review period factor (number of operational review periods per strategic review period).

Step 3. Optimize the operational-level sub-model based on the configuration obtained in Step 2 above.

Step 4. Optimize the strategic-level sub-model with the new actual unit variable costs determined in Step 3, after multiplying them by the review period factor.

Step 5. Check if the new unit costs have significant effect on the optimal configuration, (i.e., check the binary decision variables for convergence). If all binary variables are equal, go to Step 6; otherwise, go to Step 2.

Step 6. Determine the values of the SC performance vector.

Step 7. Stop.

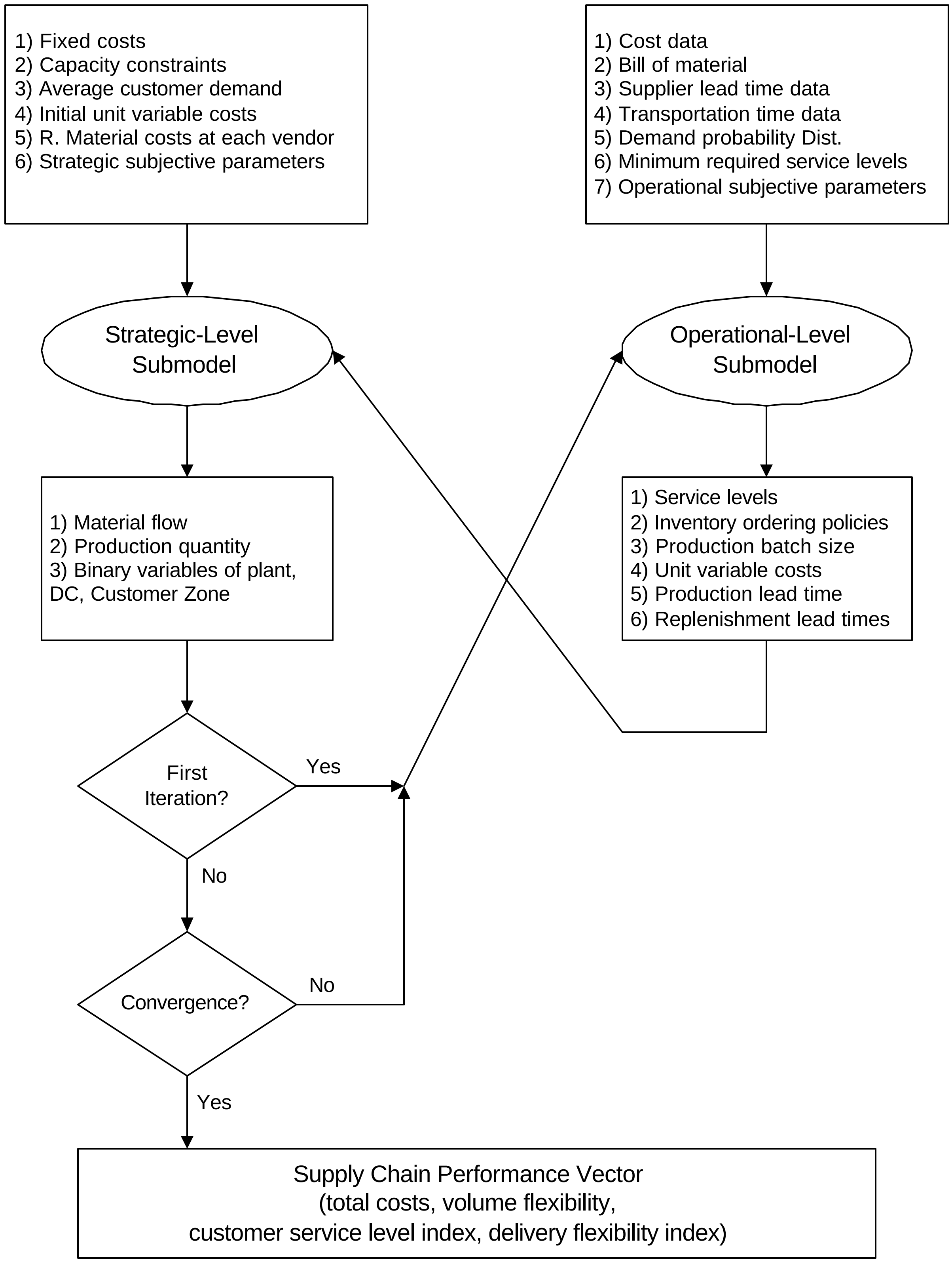

Figure 1: Model Structure

5 Numerical Example and Model Performance

The example developed here illustrates the algorithm proposed in Section 4, as well as the applicability and effectiveness of the model. The example system consists of three raw materials, two finished products, five vendors, three plants, four distribution centers, and five customer zones. This example consists of two sub-models: 1) strategic (27 binary variables and 123 continuous variables), and 2) operational (0 binary variables and 91 continuous variables, 18 of which are non-linear). For this example system, five different scenarios were examined, and the performance vectors and final SC configurations were determined (see Table 3).

5.1 Sensitivity Analysis

For the base case (scenario 1), no constraints on flexibility and customer service levels were included. The performance vector (Figure 1) and final SC configuration were determined, resulting in one plant and two DCs. Next, a number of sensitivity analysis runs were performed. The volume flexibility was kept fixed, while the customer service level and delivery flexibility were increased simultaneously to explore the sensitivity to these performance parameters (scenario 2), resulting in an increase in the total cost (as expected) and a change in the customer zone-DC assignments. Increasing the customer service level and delivery flexibility was accomplished by choosing suitable values for the customer service index ($h = 0.15$), and delivery flexibility index ($g = 0.015$). This resulted in customer service levels greater than or equal to 0.95 (the minimum required service level was 0.8 for this example), and expected lead times from plants to DCs less or equal to 0.025 periods (the standard delivery time was assumed to be 0.04 periods). Scenario 3 examined the sensitivity to volume flexibility. In this scenario, the volume flexibility requirement was increased and no service level or delivery flexibility improvements were needed. However, an additional DC was required, resulting in additional cost to accommodate the increase in volume flexibility. In scenario 4, by increasing the flexibility (volume and delivery) and improving the customer service level (stockpile fill rates) to test the joint effect of these performance parameters, the total cost reached its maximum value among the first four scenarios. Equal weight was given to plant and distribution flexibility in each of the first four scenarios. In scenario 5, more weight was given to plant volume flexibility (Table 3) which resulted in the addition of another plant. It is interesting to note here that in scenario 3, an additional DC was opened (instead of a plant) when volume flexibility was increased. The total cost in scenario 5 was more than the total cost in scenario 3 because the fixed cost associated with the additional plant exceeds that for an additional DC. The scenarios used for the sensitivity analysis are summarized below in Table 3.

Scenario	e	w_2, w_3	h, g	Performance vector (Z, W, h, g)	SC Configuration q_{2j}, q_{3k}, y_{km}
1	0	.5, .5	0, 0	13337, 35, 0, 0	[0,0,1], [0,1,1,0], $\begin{bmatrix} 0,0,0,0,0 \\ 1,0,1,0,1 \\ 0,1,0,1,0 \\ 0,0,0,0,0 \end{bmatrix}$
2	0	.5, .5	.15, .015	13685, 35, .15, .015	[0,0,1], [0,1,1,0], $\begin{bmatrix} 0,0,0,0,0 \\ 1,0,0,1,1 \\ 0,1,1,0,0 \\ 0,0,0,0,0 \end{bmatrix}$
3	100	.5, .5	0, 0	13575, 135, 0, 0	[0,0,1], [1,1,1,0], $\begin{bmatrix} 0,1,0,0,0 \\ 1,0,1,0,1 \\ 0,0,0,1,0 \\ 0,0,0,0,0 \end{bmatrix}$
4	100	.5, .5	.15, .015	13823, 135, 0, 0	[0,0,1], [1,1,1,0], $\begin{bmatrix} 0,1,0,0,0 \\ 1,0,0,1,1 \\ 0,0,1,1,0 \\ 0,0,0,0,0 \end{bmatrix}$
5	100	.9, .1	0, 0	14649, 127, 0, 0	[0,1,1], [0,1,1,0], $\begin{bmatrix} 0,0,0,0,0 \\ 1,0,0,1,1 \\ 0,1,1,0,0 \\ 0,0,0,0,0 \end{bmatrix}$

Table 3. The Performance Vector and SC Configuration for the Example Scenarios

The proposed solution algorithm was able to find optimal solutions in a few iterations for all five scenarios. It took an average of 2.2 iterations to reach to the convergence limit (all the binary variables were equal), with an average of 1.5 seconds of CPU time on a Pentium 233 MHz using the LINGO solver by LINDO Systems, Inc.

Although the main objective of the example was to test the performance of the solution algorithm, this example was used also to test the effectiveness of the model formulation by evaluating cost, customer service, and flexibility tradeoffs among the various scenarios. A summary of the results for the five scenarios of this example system are given in Table 4 below.

Scenario #	1	2	3	4	5
# plants	1	1	1	1	2
# DCs	2	2	3	3	2
Volume Flexibility	35	35	135	135	127
Avg. Customer Service	0.93	0.983	0.93	0.983	0.93
Avg. Delivery Flexibility	0	0.015	0	0.015	0
TOTAL COST	**13,337**	**13,685**	**13,575**	**13,823**	**14,649**

Table 4. Numerical Example Summary Results (Sensitivity Analysis)

In the next phase of the study, the effect of eight different levels of volume flexibility on the total cost is evaluated. The volume flexibility index was varied from $e = 0$ to 200 units. Figure 2 shows the results of this analysis. This graph illustrates the cost-volume flexibility index tradeoffs, and provides evidence to support decisions affecting SC performance. For example, total cost increased slightly from 13685 to 13689 (0.03 %) when e was increased from 0 to 75 units. This may support management's preference for a e = 75 because large increases in flexibility for values between zero and 175 results in a small cost penalty.

Similarly, Figures 3 and 4 show the relationship between the total cost and customer service performance index and delivery flexibility performance index, respectively. Moreover, from these figures, it appears that the total cost is more dramatically affected by changes in the customer service index and delivery flexibility index for smaller ranges and then levels out. These flexibility measures have large cost increases for increases in smaller index values and small cost increases beyond given (larger) index values; in contrast, for volume flexibility, there appears to be small cost increases for small volume flexibility values, and then large cost increases beyond a given (larger) flexibility value.

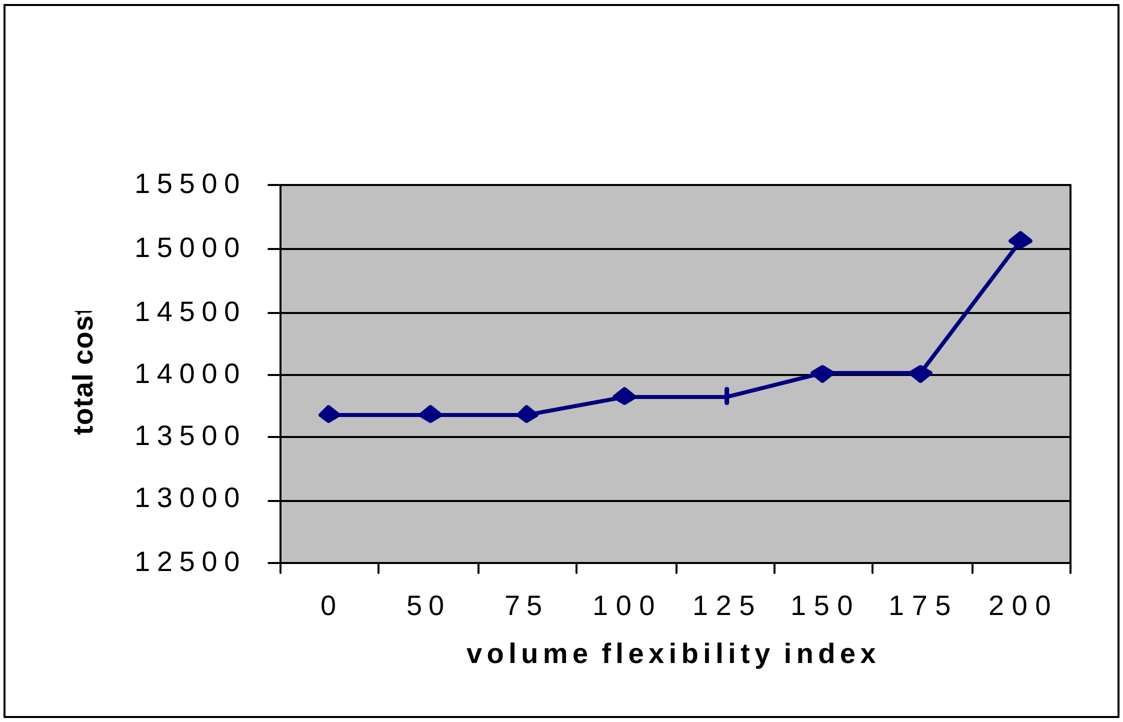

Figure 2. The Total Cost - Volume Flexibility Index Curve

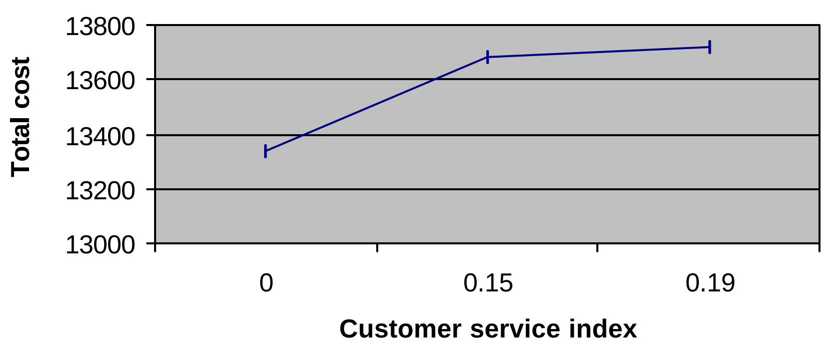

Figure 3. The Total Cost - Customer Service Index Curve

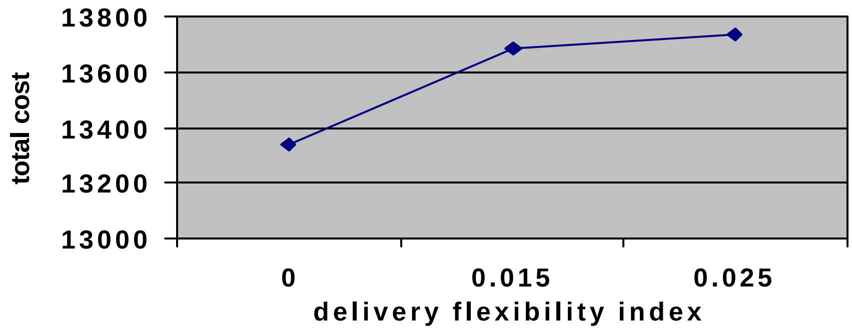

Figure 4. The Total Cost - Delivery Flexibility Index Curve

This example illustrates the effects of operational variables (such as fill rates, and lead times) on the strategic variables, which demonstrates the importance of simultaneously specifying the operational and strategic solutions. Increasing the stockpile fill rate or decreasing distribution replenishment lead times (increasing delivery flexibility), increases production unit cost, which then increases the total cost. In this example, although the number of plants and DCs is insensitive, to changes in customer service or delivery flexibility parameters, these parameters do affect the SC configuration (Table 3).

Applying this model to large-scale problems (recall that in the example considered, the SC structure considers three raw material and two finished products), is not expected to present major problems since there are only two major issues of concern in a large-scale application of the model:

- *The performance (speed and memory) of optimizing strategic-level (mixed integer linear programming) and operational-level (nonlinear programming) sub-models.* The extended version of LINGO 5.0 no longer places limits of the number of constraints and variables. Also, LINGO 5.0's nonlinear solver handles large model more efficiently.
- *Reasonable convergence time of the iterative procedure on binary variables between the strategic-level and operational-level sub-models.* By setting the convergence limit in the algorithm to equal binary variables instead of equal unit costs hastens convergence, and is designed to result in reasonably small convergence times for even large scale applications.

5.2 Regression Analysis

A regression analysis was carried out to analyze the relationships among the total cost parameter, the volume flexibility, customer service, delivery flexibility, and the weight factor of the volume flexibility. Additional runs were executed to develop these relationships. These results appear in Table 5.

Model Total cost =	Volume flexibility	Customer service	Delivery flexibility Weight factor
R^2 = 0.991,	Model F = 164.97,	P-value = 0.006	
Parameter	Estimate (C_i)	T	P-value
Intercept	7976.5	5.907	0.0275
Volume flexibility (VF)	1.787	4.983	0.0380
Customer service (CS)	4.223.5	2.955	0.0980
Delivery flexibility (DF)	5698.7	1.355	0.3082
Weight factor (WF)	2797.4	22.427	0.002

Table 5. Linear Regression Analysis

The first column of Table 5 represents the independent variables, while the cost is the dependent variable. Thus, the relationship is represented in the following linear functional form:

$$Cost = C_o + C_1 (VF) + C_2 (CS) + C_3 (DF) + C_4 (WF) \quad (67)$$

The coefficient values (C_i) can be obtained from the second column of Table 5. Each coefficient has been tested using a t-test (column 3). It is found that all the estimated coefficients were significant (at $a = 10\%$) except for delivery flexibility (see the fourth column of Table 5). This is due to the high correlation between customer service (stockpile fill rate) and delivery flexibility (lead-time), which means that only one of these factors could be used in this scenario (example). It is also interesting to note that the volume flexibility and its weight factor are dominant parameters for determining the cost.

The linear regression model in Table 5 indicates that the independent variables (volume flexibility, customer service, delivery flexibility, and weight factor) explain variation in the dependent variable (total cost) with a high R^2 value of 0.991. These results give the justification to accept this linear regression model to explain the relationship between the cost and other performance measures. This model quantifies the impact of changes in the customer service and flexibility performance on the SC total cost.

6 Summary and Conclusions

This research developed a supply chain model that facilitates simultaneous strategic and operational planning using an iterative method. This model incorporates production, delivery, and demand uncertainty, and reduces complexity via reasonable simplifications. The model also provides an appropriate performance measure by using multi-objective analysis for the entire SC network. The model developed here aids in the design of efficient, effective, and flexible supply chains, and in the evaluation of competing SC networks.

Although it may appear that this model requires the determination of a large number of input parameters, considering that this model is designed to solve a wide range of problems from the strategic-level down to the operational-level of a multi-echelon SC, the number of required parameters is relatively small. It is also important to note that in real-world applications, most of these inputs readily obtained from organizational databases.

The developed model (which consists of the conceptual framework, mathematical formulation, and solution algorithm) gives valuable insights into the modeling and analysis of complex SC configurations, and facilitates coordinated decision-maker interaction to solve specific problems. The model formulations described in this research represent a departure from the standard and optimization methods currently used to analyze SC. The key innovation lies in the integration of strategic and operational levels, and the associated linkages of decisions and performance measures.

This model is general at the strategic level and can accommodate a wide variety of different supply chain strategies. These strategies can be investigated and compared by determining the performance vector for each strategy. Additionally, this model is flexible for modifications and changes at the operational level. Various operational policies can be studied to determine the best policy for a given SC configuration. Such policies could, for example, involve the choice between 'make to order' vs. 'make to stock' or 'periodic' vs. 'continuous' review period. An example system, in which "make to stock" and "continuous review period" policies are considered, is described and solved to illustrate the applicability of the model.

In the example considered, the solution algorithm was very successful in generating solutions in short computation times. The results obtained confirmed the significance of cost, customer service, and flexibility and also demonstrated the need to integrate operational and strategic decisions in SC design.

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Appendix A: Derivation of k (continuous case)

The following approximate expression was presented in [27] in order to avoid the cumulative standard normal distribution:

$$P = 1 - \left[\frac{1}{1 + \exp\left(\frac{2\left(S - (L+R)m\right)\sqrt{2/p}}{s_{L+R}} \right)} \right]$$

An (R, S) system is equivalent to an (s, Q) system if one makes the following transformations [26]:

Periodic (R, S)	Continuous (s, Q)
S	s
DR	Q
$R + L$	L

Therefore,

$$P = 1 - \left[\frac{1}{1 + \exp\left(\frac{2\left(s - Lm\right)\sqrt{2/p}}{s_{L}} \right)} \right] \quad \text{(for continuous case)}$$

where,

$$s = X_L + n s_L = Lm + n s_L ,$$

$$n = \frac{s - Lm}{s_L}$$

So,

$$P = 1 - \frac{1}{1 + \exp\left(2\sqrt{\frac{2}{p}}\, n \right)}$$

Then $$n = \left(\frac{1}{2}\sqrt{\frac{p}{2}}\right)\ln\left(\frac{P}{1-P}\right).$$