

Non-Boolean Descriptions for Mind-Matter Problems

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Abstract

A framework for the mind-matter problem in a holistic universe which has no parts is outlined. The conceptual structure of modern quantum theory suggests to use complementary Boolean descriptions as elements for a more comprehensive non-Boolean description of a world without an *a priori* given mind-matter distinction. Such a description in terms of a locally Boolean but *globally non-Boolean* structure makes allowance for the fact that Boolean descriptions play a privileged role in science. If we accept the insight that there are no ultimate building blocks, the existence of holistic correlations between contextually chosen parts is a natural consequence.

The main problem of a genuinely non-Boolean description is to find an appropriate partition of the universe of discourse. If we adopt the idea that all fundamental laws of physics are invariant under time translations, then we can consider a partition of the world into a *tenseless* and a *tensed* domain. In the sense of a regulative principle, the material domain is defined as the tenseless domain with its homogeneous time. The tensed domain contains the mental domain with a tensed time characterized by a privileged position, the Now. Since this partition refers to two *complementary descriptions* which are not given *a priori*, we have to expect correlations between these two domains. In physics it corresponds to Newton's separation of *universal laws of nature* and *contingent initial conditions*. Both descriptions have a non-Boolean structure and can be encompassed into a single non-Boolean description. Tensed and tenseless time can be synchronized by holistic correlations.

1. Introduction

The study of the relationship between mind and matter is often based on tacit preconceptions of natural science. The starting point of historical classical science is atomism: the idea that there exist theory-independent objects. Of course, today nobody defends Newton's atomistic ontology (Newton 1730; in the Dover edition of 1952 on p. 400):

God in the Beginning form'd Matter in solid,
massy, hard, impenetrable moveable Particles,

but this idea is in modified forms still virulent in much of modern science. Reductionism tries to explain phenomena in terms of fundamental entities. When a system can be decomposed into parts, it makes sense to understand the system in terms of these parts. If no fundamental parts exist, reductionism fails. Classical science relies on the seemingly obvious notion of a system, namely “a complex unity formed of many parts”. However, this historical approach cannot serve as a conceptual starting point since it presupposes that the decomposition of the universe into subsystems is given *a priori*.

Modern quantum mechanics put an end to atomism and hence to reductionism: The so-called “elementary particles” (such as electrons, quarks, or gluons) are *patterns of reality*, not building blocks of reality. They are not primary, but arise as secondary manifestations, for example as field excitations, in the same sense as solitons are localized excitations of water, and not building blocks of water.

In the material world, patterns like molecules, atoms, electrons, or photons arise by an appropriate decomposition of the fundamentally holistic universe of discourse.¹ In this sense, quantum mechanics is the paradigmatic example of a theory which allows the description of a whole which does not consist of parts. It will form the *inspirational* background for my analysis. What we can learn from quantum mechanics is that Boolean descriptions are indispensable, but that it is nevertheless not possible to encompass the whole world into a single Boolean description. We have to give up the idea that we can describe the world (including the material and the mental domain) in terms of ontologically existing primary elements. To cover the full range of our capacities of insight, non-Boolean descriptions are compulsory. Since experimental science requires *Boolean frames of reference*, Boolean descriptions play a privileged role. For this reason we look for non-Boolean descriptions with a locally Boolean structure so that aspects of reality can be perceived by projections onto empirically accessible Boolean reference frames.

This suggests the idea to *use mutually incompatible Boolean descriptions as elements for a more comprehensive non-Boolean description*. But it would be inappropriate to rely on typical mathematical and conceptual tools of quantum theory (e.g., “measurement”, “observer”, “observable”, “probability”, “linear spaces”, “lattices”) or to assume a spatiotemporal structure for the description of non-physical phenomena. Moreover, the conceptual foundation of quantum mechanics is often poorly understood, and it makes no sense to transfer these misunderstanding to other sciences.

¹The often used decomposition in terms of group-theoretically defined elementary “bare” systems is far from optimal. The definition of less entangled “dressed” systems which include self-fields is extremely complicated. In spite of the fact that quantum mechanics put an end to atomism, modern science is still to a large extent based on an atomistic ontology.

To reach this aim I have first to explain what Boolean descriptions are and why they have (at least in present-day science) a privileged status. Next I discuss the necessity of non-Boolean descriptions. Finally I address the difficult problem of how to divide a universe which is ontologically undivided.

2. Boolean Descriptions

2.1 Boolean Logics and Boolean Algebras

Science, as we know it, requires a language based on the unambiguous framework of classical two-valued logic. It accepts the doctrine that every proposition is either true or false. George Boole discovered that the well-known rules which govern the way by which we infer a statement from other statements can be expressed algebraically (Boole 1847, 1854). He connected logic with an algebra representing, as he supposed, the “laws of thought”:

- *The law of contradiction*,
saying that nothing can be both A and $not-A$.
- *The law of excluded middle*,
saying that anything must either be A or $not-A$.
- *The law of identity*,
saying that if anything is A , then it is A .

The propositional calculus of logic defines a proposition as an *unambiguous sentence that is either true or false*. All false propositions are considered equal and are identified with falsehood, denoted by the absurd proposition $\mathbf{0}$. The universal truth is denoted by the trivial proposition $\mathbf{1}$. In classical logic one examines how sentences are combined by means of sentential connectives. Identifying equivalent propositions in the set of propositions of a context, the closure \mathcal{B} of this set under the operations of *join* \vee (the supremum), *meet* \wedge (the infimum), and the *complementation* $^\perp$ becomes a *Boolean algebra*.² The basic operations in a Boolean algebra are the conjunction $A \wedge B$ and the disjunction $A \vee B$. The comparison relation $A \leq B$ is defined by $A = A \wedge B$ or, equivalently by $B = A \vee B$. In Boolean classical logic the following relations hold:

²Discussing axioms for determining this class of logical structures introduced by Boole, Sheffer (1913) referred to them as *Boolean algebras*. But it was not Boole but Schröder (1890-1895), who introduced the now familiar representation of these algebras of two-value logic by the operations of union, intersection, and complement applied to the collection of subsets of a given set. According to the representation theorem by Stone (1936) every finite Boolean algebra is isomorphic to a Boolean algebra of subsets of some finite set.

Proposition	A	designates a statement
Negation	A^\perp	the proposition <i>not-A</i>
Conjunction	$A \wedge B$	$A \wedge B$ is true iff both A and B are true
Disjunction	$A \vee B$	$A \vee B$ is true iff at least one of the two statements is true
Equality	$A = B$	A is true iff B is true
Implication	$A \leq B$	is defined by $A \wedge B = A$ and means that A entails B
The trivial proposition	$\mathbf{1}$	is defined by $\mathbf{1} := A \vee A^\perp$ for every proposition A
The absurd proposition	$\mathbf{0}$	is defined by $\mathbf{0} := A \wedge A^\perp$ for every proposition A
Double negation		$(A^\perp)^\perp = A$
Contradiction		$A \wedge A^\perp = \mathbf{0}$
Excluded middle		$A \vee A^\perp = \mathbf{1}$

Contrariwise, by a *Boolean logic* we mean a Boolean algebra of propositions in which the Boolean operations of join, meet, and complementation correspond to the logical operations of disjunction, conjunction, and negation, respectively.

Definition: Boolean algebra

A *Boolean algebra* $\mathfrak{B} = \{\mathcal{B}, \vee, \wedge, \perp, \mathbf{0}, \mathbf{1}\}$ is a set \mathcal{B} with two distinguished elements $\mathbf{0}$ and $\mathbf{1}$, and with an algebraic operation \perp of rank one, called *negation*, as well as with two operations of rank two, the *conjunction* \wedge and the *disjunction* \vee . The following axioms are postulated:

- (i) $(A^\perp)^\perp = A$ for all $A \in \mathcal{B}$,
- (ii) \wedge and \vee are each associative and commutative,
- (iii) \wedge and \vee are distributive with respect to one another,
- (iv) $(A \vee B)^\perp = A^\perp \wedge B^\perp$, $(A \wedge B)^\perp = A^\perp \vee B^\perp$,
- (v) $A \vee A = A \wedge A = A$,
- (vi) $\mathbf{1} \wedge A = A$, $\mathbf{0} \vee A = A$.

This set of axioms can be simplified. Boolean algebras have an embarrassingly rich structure and there are many equivalent sets of axioms characterizing them. The usual definitions are highly redundant, but the elimination of this redundancy is remarkably involved.³

If in a Boolean algebra $A \wedge B = A$ holds, we write $A \leq B$ and say that A logically entails B . An element $A \neq \mathbf{0}$ in \mathcal{B} such that $B \leq A$ implies either $B = \mathbf{0}$ or $B = A$ is called an *atom*. A Boolean algebra is called *atomic* if every nonzero element has an atom under it.

In classical logic, every proposition is either true or false. A truth evaluation can be formalized by a homomorphism τ of the Boolean algebra \mathfrak{B} of propositions onto the Boolean algebra \mathfrak{B}_2 , consisting of the two elements $\mathbf{1}$ and $\mathbf{0}$, together with the convention that a proposition is true if and only if it is mapped on the element $\mathbf{1}$, that is

$$\begin{aligned} \tau(A) &= \mathbf{1} && \text{if and only if } A \text{ is true ,} \\ \tau(A) &= \mathbf{0} && \text{if and only if } A \text{ is false .} \end{aligned}$$

The value $\tau(A)$ of a proposition $A \in \mathcal{B}$ is called the *truth value* of A . Since a homomorphism is a structure-preserving mapping, the truth functional τ respects Boolean operations.

2.2 On the Origin of Two-Valued Logic

The search for regularities is the principle concern of all scientific inquiry. Regularities can be found if and only if we suppress or ignore irrelevant features. It goes without saying that *every empirical statement is conditioned by what is ignored*. What makes science successful is the view that ignorance is admissible. We intentionally disregard the oneness of the undivided world and proceed by particularization. One can push this idea so far that “empirical” statements are always taken to be either true or false, so that the logic of science becomes Boolean. By adopting this dichotomy we destroy the natural interconnection of all phenomena, but we have to accept this sacrifice as the inevitable price for unambiguous empirical statements in terms of two-valued logic.

What is relevant and what is irrelevant is not determined by some natural law but by some convention, or by our own interest, or by our cognitive apparatus, or by the evolutionary history, or by pattern recognition devices. If we isolate a phenomenon and assign individuality to

³For a short history compare Grätzer (1998, p. 477). All operations of a Boolean algebra $\{\mathcal{B}, \mathbf{1}, \mathbf{0}, \wedge, \vee, \perp\}$ can be expressed in terms of the *binary rejection* $A^\perp \vee B^\perp$, called “Sheffer stroke”: $A|B := A^\perp \vee B^\perp$, $A \wedge B = (A|A)|(B|B)$, $A \vee B = (A|B)|(B|A)$, $A^\perp = (A|A)$, $A|(A|A) = \mathbf{0}$. In digital circuitry the Sheffer stroke is realized by a *NAND logical gate*. By means of an automated reasoning computer program, McCune *et al.* (2002) have shown that the single postulate $\{A|[[(B|A)|A]]|[B|(C|A)]] = B$ is the shortest axiom for a Boolean algebra in terms of Sheffer strokes.

it, we create an entity which we call a *pattern*. All concepts of empirical science refer to observations obtained by some pattern recognition methods which distinguish between *relevant and irrelevant features* and which therewith create so-called “empirical facts.” Recognition of patterns is a crucial activity both in living and in technical systems. In living systems relevant features are determined by biological evolution, in technical systems by design principles.

The binary choice yes/no or true/false has become a dominating principle of scientific reasoning. Neither its outstanding simplicity nor its practical success proves that it is given *a priori*. But *we have an innate preference for two-valued logic*, presumably developed in the struggle for survival. As Szent-Györgi (1962) explains:

Primarily the human brain is an organ of survival. It was built by nature to search for food, shelter, and the like, to gain advantage – before addressing itself to the pursuit of truth.

Individual neurons operate basically with two-valued threshold logic, so that it is tempting to assume that Boole’s *Laws of Thought* are not *a priori*, but may have an evolutionary basis.⁴ As a consequence, *some* of our beliefs and logical concepts may also depend on this neurological structure of the brain.

Threshold gates do not only have evolutionary advantages, but they are also important in modern technology as “tertium non datur”-devices. In the age of digital devices, practically all modern measuring instruments create well-defined numbers on the readout display with the aid of built-in threshold devices. But this exact number is generated by the measuring instrument and cannot be attributed to a property of the object system. The requirement that experiments have to be reproducible does not refer to individual experiments, but to equivalence classes of experimental data.⁵

2.3 Boolean Classification

In the simplest case classification refers to the process of grouping *individual objects* into mutually disjoint classes according to certain attributes they have in common. This characterization *presupposes* that there are individual objects which can be characterized by well-defined attributes. For provisional guidance we assume that the universe of discourse can be represented by a set $\mathfrak{U} = \{u_1, u_2, \dots\}$ of elements. The elements u_1, u_2, \dots represent *individual* objects which may be material

⁴See Platt (1956); compare also Russell (1948), part VI, chapt 1.

⁵In quantum mechanics von Neumann’s (1927, p. 271) repeatability axiom refers to the Boolean classification of final statistical decision methods necessary to arrive at “experimental facts” and not, as often wrongly claimed, to the Boolean structure of the projection-valued measures associated with the observables of the object system.

objects, electrical signals, mathematical objects, documents, pictures, or even ideas.

The elimination of irrelevant features is mandatory for any classification, *there are no unprejudiced classifications*. Every classification is generated by a particular equivalence relation \sim on the universe \mathfrak{U} of discourse (Ore, 1942). Recall that a binary relation \sim on the set \mathfrak{U} is called an *equivalence relation* if it is reflexive ($u_j \sim u_j$ for all $u_j \in \mathfrak{U}$), symmetric (whenever $u_j \sim u_k$, then $u_k \sim u_j$), and transitive (if $u_j \sim u_k$ and $u_k \sim u_\ell$, then $u_j \sim u_\ell$). Equivalence relations allow us (for a well-specified purpose) to ignore some distinguishing characteristics, and to merge together similar objects into equivalence classes.

Mathematically, a *classification* in the universe of discourse \mathfrak{U} with respect to an equivalence relation \sim is a homomorphism φ of the partition (\mathfrak{U}, \sim) into a partition $(\mathfrak{P}, =)$, where the classification set \mathfrak{P} consists of disjoint pattern classes $\mathfrak{P}_j \subset \mathfrak{U}$, ($j = 1, 2, \dots$) such that

$$u_k, u_j \in \mathfrak{U}, u_k \sim u_j \quad \text{implies} \quad \varphi(u_k) = \varphi(u_j) \in \mathfrak{P}_j .$$

This classification is *exhaustive*, $\mathfrak{U} = \cup_j \mathfrak{P}_j$, and the pattern classes are *mutually exclusive*, $\mathfrak{P}_j \cap_{j \neq k} \mathfrak{P}_k = \emptyset$.

2.4 Classifications as Filters

For the following it is convenient to represent classifiers by *filters*. We use this concept in the engineering sense, meaning any device which accepts or passes certain elements in a set and rejects others (Hammer, 1969, p.107). With respect to a given partition (\mathfrak{U}, \sim) of the universe of discourse, we represent the homomorphism φ by filters $\mathbb{F}_1, \mathbb{F}_2, \dots$, which accept the elements from \mathfrak{U} and passes them to one of the disjoint pattern classes $\mathfrak{P}_1, \mathfrak{P}_2, \dots$. The *selective filter* \mathbb{F}_j can be represented by a selection operator $F_j : \mathfrak{U} \rightarrow \mathfrak{P}_j$, defined by

$$F_j(\mathfrak{U}) := \mathfrak{P}_j, \quad \mathbf{1}(\mathfrak{U}) := \mathfrak{U}, \quad \mathbf{0}(\mathfrak{U}) := \emptyset,$$

where $\mathbf{1}$ is the identity operator and $\mathbf{0}$ the zero operator.

The series and parallel connections of two arbitrary filters \mathbb{F} and \mathbb{G} can be represented algebraically by the corresponding selection operators F and G , respectively: The *series combination* of two filters \mathbb{F} and \mathbb{G} is by convention written multiplicatively and represented by the selection operator GF . The *parallel combination* of filters \mathbb{F} and \mathbb{G} is by convention written additively and represented by the operator $F + G$ (see Fig. 1).

The classification $\varphi : (\mathfrak{U}, \sim) \rightarrow (\mathfrak{P}, =)$ can be represented as a parallel connection of selective filters $\mathbb{F}_1, \mathbb{F}_2, \dots$ with the idempotent selection operators F_1, F_2, \dots . It follows that

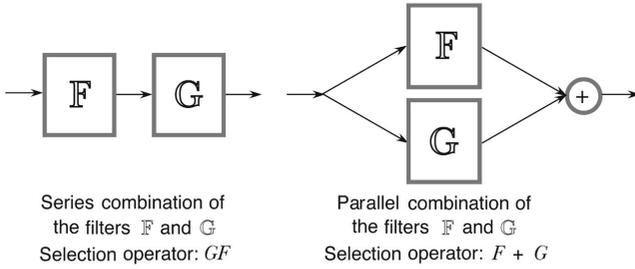


Figure 1: Combinations of filters.

- (i) the classification is *exhaustive*, $\sum_j F_j = \mathbf{1}$,
- (ii) the classification is *reproducible*, $F_j F_j = F_j$,
- (iii) the patterns are *mutually exclusive*, $F_j F_k = \mathbf{0}$ for $j \neq k$.

2.5 Classifications Generate Boolean Algebras

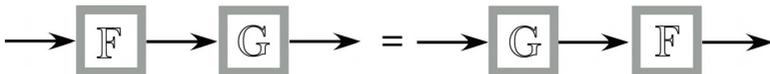
Consider a classification generated by selective filters $\mathbb{F}_1, \mathbb{F}_2, \dots$ with the idempotent selection operators F_1, F_2, \dots . With the definitions

$$F_j \wedge F_k := F_j F_k, \quad F_j \vee F_k := F_j + F_k - F_j F_k, \quad (F_j)^\perp := \mathbf{1} - F_j,$$

the set \mathcal{B} generated by the closure of the selection operators F_1, F_2, \dots of a classification under the operations meet \wedge , join \vee , and complementation $^\perp$ is a *Boolean algebra* $\mathfrak{B} = \{\mathcal{B}, \vee, \wedge, ^\perp, \mathbf{0}, \mathbf{1}\}$. If the Boolean algebra \mathfrak{B} is atomic and all selection operators F_1, F_2, \dots are atoms, we call the classification φ *atomic*.

All classifications can be formalized in terms of Boolean algebras. Since there is no universal Boolean frame of reference, there exist even in classical science incompatible classifications which cannot be embedded into a single Boolean description. We say that two Boolean classifications are *compatible* if they can be embedded in a common Boolean classification. If not, they are called *incompatible*.

For two classifications represented by filters \mathbb{F} and \mathbb{G} , the classifications are compatible if and only if the result of a series connection of these two filters does not depend on the order:



If the two classifications are incompatible, the result depends on the order of the filters. The series combination “first \mathbb{F} , then \mathbb{G} ” is usually represented algebraically by the product GF of the corresponding selection operators F and G . But this is an arbitrary convention, the opposite convention would be equally valid. This fact is not trivial, but implies the existence of an involutory automorphism of the underlying mathematical structure.⁶ If we represent the series combination “first \mathbb{F} , then \mathbb{G} ”, by the product F^*G^* with the corresponding selection operators F^* and G^* , we get an involutory automorphism $F \mapsto F^*$ with

$$(F^*)^* = F, \quad (G + F)^* = F^* + G^*, \quad (GF)^* = F^*G^*,$$

which is related to a new operator J with $J^2 = -1$ which is compatible with all selection operators. While any Boolean classification can be represented by an algebraic structure over the real numbers, *non-Boolean classifications with the operator J require an algebraic structure over the complex numbers.*⁷

3. Complementarity and Partially Boolean Descriptions

3.1 Duality Versus Complementarity

Many popular and philosophical books still use the misleading notion of the wave-particle *duality*, holding that light and matter exhibit properties of both waves and of particles. This historical notion goes back to the analysis of the photoelectric effect by Albert Einstein in 1905 and to the hypothesis by Louis de Broglie (in his doctoral dissertation in 1924) that all matter has a wave-like nature. Nowadays we know that photons or electrons are neither waves nor particles. They can be in particle-like and in wave-like states, but they can also be in infinitely many other states which are neither particle-like nor wave-like. In terms of modern quantum mechanics, we should not any more speak of *duality* but of *complementarity*. Dualistic statements refer to elements in different categories, while complementary pertain to holistic situations where Boolean fragmentation into parts is not possible.

There is no general agreement among historians, philosophers and physicists how to understand complementarity. From an operational point

⁶This fact is often overlooked, but clearly stated by Schwinger (1959), p. 1548.

⁷Quantum mechanics is the first physical theory which necessitates the use of the imaginary number i with the property $i^2 = -1$. Equivalently, one can introduce an operator J which satisfies $J^2 = -1$ and commutes with all operators, and use an algebraic structure over the real numbers. Compare also Stueckelberg (1959, 1960) and Stueckelberg and Guenin (1960).

of view, complementarity refers to the empirical fact that, what is in principle knowable is not necessarily knowable simultaneously. According to Bohr (1949, p. 210),

evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as *complementary* in the sense that only the totality of the phenomena exhausts the possible information about the objects.

That is, a comprehensive description of nature requires an expansion of the frame of discourse to include complementary descriptions based on incompatible mutually exclusive aspects which cannot be combined into a single description in terms of a two-valued logic. *A complementary description refers always to a contextually chosen decomposition of the universe of discourse.* Note that these formulations do not make any reference to physics.

3.2 Boolean Frames of Reference

Bohr's concept of complementarity is founded on the privileged role of Boolean descriptions (Bohr 1949, p. 209):

However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. The argument is simply that by the word 'experiment' we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.

This requirement reflects the actual scientific practice: every experiment ever performed in physics, chemistry, biology or psychology can be described in a language based on classical Boolean logics.⁸ We call such a domain in which all what is in principle knowable is also simultaneously knowable a *Boolean frame of reference* or a *Boolean context*.

The fact that every *single* experiment allows a description in terms of classical Boolean logic does not imply that the family of all feasible experiments can be combined into a single Boolean context. Bohr's complementarity provided a basis for describing physical reality in the presence of classically incompatible concepts. For this reason Bohr (1948, p. 317) proposed to use

⁸Bohr's emphasis on the special role of classical physics is somewhat misleading. According to Bohr (1948, p. 317) "... all well-defined experimental evidence, even if it cannot be analyzed in terms of classical physics, must be expressed in ordinary language making use of common logic." Important for Bohr's arguments is therefore only that facts have to be described in a Boolean language, but not necessarily in terms of classical physics.

the word phenomenon to refer exclusively to observations obtained under specified circumstances, including an account of the whole experiment.

No formal definition of complementarity can be found in Bohr’s papers. Since there is no *a priori* fixed decomposition of the universe of discourse but many mutually incompatible Boolean contexts, there are always countless different complementary descriptions of one and the same universe of discourse. This situation suggests the following definition:

Two Boolean descriptions are said to be complementary if they cannot be embedded into a single Boolean description.

3.3 Incompatible Features in Classical Science

The view that the existence of incompatible properties or complementary descriptions is a peculiarity of quantum theory seems to be an ineradicable preconception. Already in classical engineering science there are many examples of genuine incompatibility and complementarity. For example, the important “time-bandwidth uncertainty relation” for linear electrical circuits found its precise formulation in 1924,⁹ and is mathematically exactly equivalent to Heisenberg’s uncertainty relations of 1927. Nevertheless, it is still claimed that Heisenberg’s “uncertainty principle is certainly one of the most famous and important aspects of quantum mechanics” (Hilgevoord and Uffink 2001).

Explanation: Time-limited and band-limited signals

In communication theory signals are represented by functions of time $t \in \mathbb{R}$. We consider real-valued deterministic signals $s : \mathbb{R} \rightarrow \mathbb{R}$ for which the Fourier transform $\hat{s} : \mathbb{R} \rightarrow \mathbb{C}$ exists,

$$\hat{s}(\lambda) := \int_{\mathbb{R}} e^{-2\pi i \lambda t} s(t) dt, \quad \lambda \in \mathbb{R}.$$

Because of the frequency limiting nature of transfer channels, communication engineers introduced the class $\hat{\mathcal{C}}_W$ of band-limited signals. Each member of $\hat{\mathcal{C}}_W$ can be written as

$$s_W(t) = \int_{-W}^W e^{2\pi i \lambda t} \hat{s}(\lambda) d\lambda, \quad s_W \in \hat{\mathcal{C}}_W, \quad t \in \mathbb{R}.$$

A signal of $\hat{\mathcal{C}}_W$ is said to be of bandwidth W . Since every signal is observed only in a finite interval of time, say $(-T/2, T/2)$ (after a

⁹See Küpfmüller (1924). For a discussion of the widespread application of the complementarity relation between bandwidth and duration of acoustic or electric signals in engineering compare Slepian (1983).

suitable time translation), we also introduce the class \mathcal{C}_T of time-limited signals

$$s_T(t) := \begin{cases} s(t) & \text{if } |t| \leq T, \\ 0 & \text{if } |t| \geq T, \end{cases} \quad s_T \in \mathcal{C}_T, \quad t \in \mathbb{R}.$$

It is well-known that a band-limited function cannot be time-limited. In fact, the only signal that is both band-limited and time-limited is the trivial zero signal.¹⁰

We can classify signals according to their bandwidth W , or according to their duration T , but not both simultaneously. To get a precise formulation we define two filters, a band-limiting filter \mathbb{F} and a time-limiting filter \mathbb{G} . With respect to two arbitrary Borel sets \mathcal{F} and \mathcal{G} on the real line, we characterize the band-limiting filter \mathbb{F} by the selection operator $F(\mathcal{F})$, and the time-limiting filter \mathbb{G} by the selection operator $G(\mathcal{G})$:

$$\begin{aligned} \{F(\mathcal{F})s\}(t) &= \begin{cases} s(t) & \text{if } t \in \mathcal{F}, \\ 0 & \text{if } t \notin \mathcal{F}, \end{cases} & \{F(\mathcal{F})\}^2 &= F(\mathcal{F}), \\ \{G(\mathcal{G})s\}(t) &= \int_{\mathcal{G}} d\lambda \int_{-\infty}^{\infty} dt' e^{-2\pi i\lambda(t-t')} s(t'), & \{G(\mathcal{G})\}^2 &= G(\mathcal{G}). \end{aligned}$$

With the definitions

$$\begin{aligned} F(\mathcal{F}_1) \wedge F(\mathcal{F}_2) &:= F(\mathcal{F}_1 \cap \mathcal{F}_2), & G(\mathcal{G}_1) \wedge G(\mathcal{G}_2) &:= G(\mathcal{G}_1 \cap \mathcal{G}_2), \\ F(\mathcal{F}_1) \vee F(\mathcal{F}_2) &:= F(\mathcal{F}_1 \cup \mathcal{F}_2), & G(\mathcal{G}_1) \vee G(\mathcal{G}_2) &:= G(\mathcal{G}_1 \cup \mathcal{G}_2), \\ F(\emptyset) &= 0, & F(\mathbb{R}) &= 1, & G(\emptyset) &= 0, & G(\mathbb{R}) &= 1, \end{aligned}$$

these selection operators generate two Boolean algebras \mathfrak{F} and \mathfrak{G} which cannot be embedded into a single Boolean description. The spectral measures associated with the families $\{F(\mathcal{F})\}$ and $\{G(\mathcal{G})\}$ generate on the Hilbert space of Lebesgue square-integrable complex-valued functions on the real axis the selfadjoint operators T and Λ .

$$T := \int_{-\infty}^{\infty} t dF(t), \quad \Lambda := \int_{-\infty}^{\infty} \lambda dG(\lambda),$$

which fulfill the commutation relation $T\Lambda - \Lambda T = (i/2\pi)\mathbf{1}$, hence the inequality $\Delta T \Delta \Lambda \geq 1/(4\pi)$.

Besides these well-known cases, non-Boolean propositional systems has been used in classical engineering science in situations where a gain of knowledge involves a projection from a non-Boolean structure onto a Boolean context. Here we can only mention a few examples, say non-Boolean pattern recognition methods (Kulikowski 1970, Schadach 1973,

¹⁰Proof: If $t \mapsto s(t)$ has a band-limited Fourier transform, then $z \mapsto s(z)$ is an entire function of the complex variable z whose vanishing on a finite segment implies $s \equiv 0$. For mathematical details compare Wiener (1933).

Watanabe 1969), non-commutative probability and information theory (Watanabe 1969, Chap. 9, Niestegge 2001), theory reduction in terms of a non-Boolean covering theory (Primas 1977), interactive control systems whose “interactive logic” is non-Boolean (Finkelstein and Finkelstein 1983), computational complementarity of finite automata (Moore 1956, Svozil 1993, 1998, 2005, 2006), partition logic formed by pasting together Boolean algebras generated by partitions of a set (Svozil 1993, 2006), classical dynamics with incompatible partitions of the phase space (Graben 2004, Graben and Atmanspacher 2006).

3.4 Complementarity Beyond Physics

This limitation of Boolean descriptions, which is clearly recognized in engineering science and in quantum physics, is also relevant in many other fields. In fact, William James (1890, p. 206) introduced the notion of complementarity more than hundred years ago to describe split modes of consciousness “which coexist but mutually ignore each other.” Henri Bergson (1911, pp. 343–344) distinguished two types of knowledge of time: the one alludes to physical time, the other to our intuition of flowing time. He referred to this situation as *two opposed although complementary ways of knowing*:

Not only may we thus complete the intellect and its knowledge of matter by accustoming it to install itself within the moving, but by developing also another faculty, complementary to the intellect, we may open a perspective on the other half of the real. ... To intellect, in short, there will be added intuition.

Satosi Watanabe (1961) suggested that physical and mental languages are complementary and that a synthesis of physical and mental aspects of the mind-body relation requires a non-Boolean logical framework.

The complementarity between the intuition of discreteness (in the sense of constructivism) and the intuition of continuity (via topological notions) has been suggested by Willem Kuyk (1977) as a basis for a philosophy of mathematics. Similarly, Paul Bernays (1946, p. 79) pointed out that an understanding of mathematical existence in the discussion of existential versus constructive aspects of mathematics requires *complementary perspectives*. According to Bernays (1935) intuitionistic logic is founded on the relation of the reflecting and acting subject, while the customary manner of doing mathematics consists in establishing theories detached as much as possible from the thinking subject. Mark Balaguer (1998) convincingly defended the view that both mathematical Platonism (i.e. the view that non-spatiotemporal mathematical objects such as numbers exist independently of us) and anti-Platonism (understood as mathematical fictionalism, i.e. the view that abstract objects such as numbers do not exist) are perfectly workable philosophies of mathematics. In spite of the

fact that there is no way of deciding which view is right, both views are necessary for a deeper understanding of mathematics. In this situation it seems to be natural to conclude that *both* platonism and fictionalism are complementary views (an option not considered by Balaguer).

3.5 Partially Boolean Descriptions

3.5.1 Families of Complementary Descriptions

If an universe of discourse admits a description in terms of two complementary Boolean descriptions, then usually there exists a whole family of different mutually incompatible pairs of complementary Boolean descriptions.¹¹ By “pasting” together mutually complementary Boolean contexts one can arrive at a non-Boolean description in terms of partial Boolean algebras.¹² The resulting non-Boolean structure is determined by the underlying partially overlapping Boolean algebras and the way they are pasted together.¹³ There are many ways to do so. The most important example to get a globally non-Boolean descriptions is to patch local Boolean descriptions together *smoothly*, in the same sense as geometric manifolds can be constructed out of Euclidean spaces.

3.5.2 A Simple Geometric Analogy

The locally Euclidean geometrical structure of the globally non-Euclidean theory of general relativity is an apt analogy for the locally Boolean behavior of globally non-Boolean descriptions. The proper tool for a mathematical formulation of this analogy are *Boolean manifolds*. Boolean contexts play an analogous role Euclidean spaces play for *geometric manifolds*.

Recall that a *geometric n-manifold* is a topological space that is locally like a n -dimensional Euclidean space (that is, every point has a neighborhood homeomorphic to an open set in \mathbb{R}^n) such that the locally Euclidean patches are pasted together continuously. In spite of the fact that manifolds look *locally* like Euclidean spaces, their *global* structure can be unimaginably complex. Examples for geometric 2-manifolds are the plane, the sphere, the torus, the Möbius strip, the Klein bottle.

¹¹The simplest examples are given by canonically conjugated physical quantities. As a particular case we may consider the time operator T and the frequency operator Λ . A unitary fractional Fourier transform brings about a rotation in the time-frequency plane such that $T_\varphi := T \cos \varphi + \Lambda \sin \varphi$ and $\Lambda_\varphi := \Lambda \cos \varphi - T \sin \varphi$. The operators T_φ and Λ_φ generate the complementary Boolean algebras \mathfrak{F}_φ and \mathfrak{L}_φ , respectively. The family $\{(\mathfrak{F}_\varphi, \mathfrak{L}_\varphi) | 0 \leq \varphi < 2\pi\}$ then represent infinitely many different mutually incompatible pairs of complementary Boolean descriptions.

¹²By a partial algebraic structure we mean a set \mathcal{P} with certain partial operations which are defined only for certain elements of \mathcal{P} .

¹³An appropriate technique for the unification of mutually incompatible Boolean descriptions has been introduced by Greechie (1968) as “pasting”. Compare also Finch (1969).

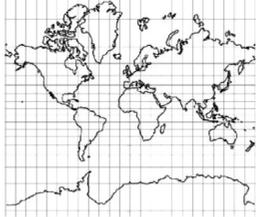
<p><i>Mercator projection</i> preserves angles and circles useful for aeronautical charts</p> $x = \lambda$ $y = \arctan\{\sin(\phi)\}$	
<p><i>Stereographic projection</i> preserves shapes and directions used in the polar regions</p> $x = k \cos(\phi) \sin(\lambda)$ $y = k \sin(\phi)$	
<p><i>Lambert azimuthal projection</i> preserves areas used for mapping large regions in middle latitude</p> $x = \frac{\sqrt{2} \cos(\phi) \sin(\lambda)}{\sqrt{1 + \cos(\phi) \cos(\lambda)}}$ $y = \frac{\sin(\phi)}{\sqrt{1 + \cos(\phi) \cos(\lambda)}}$	

Figure 2: Examples of cartographic maps. Map projections taken from <http://mathworld.wolfram.com/topics/MapProjections.html>.

As a simple geometric analogy we may consider *cartographic map projections*. Map projections are methods of constructing latitudes and longitudes on maps of the earth. The surface of the earth – here for simplicity idealized as a sphere – is a manifold. Locally the earth seems to be flat, but viewed as a whole it is spherical. The surface of a sphere is a two-dimensional manifold, denoted by \mathcal{S}^2 . Any function defined on the sphere \mathcal{S}^2 with values on the plane \mathbb{R}^2 is called a *projection*.

Every projection map $\mathcal{S}^2 \rightarrow \mathbb{R}^2$ which transfers a feature (like area, shape, direction, distance, scale) of the surface of the sphere onto the plane \mathbb{R}^2 produces some distortions. Some projections preserve relative distances in all directions from the center of the map (equidistant projections), some preserve areas (equal-area projections), while others preserve angles (conformal projections). Therefore, various complementary types of map projections are inescapable. The particular projection chosen for a given map depends on the purpose for which the map is intended. There are literally hundreds of cartographic projections used in cartography and

in national grid systems.¹⁴ The fact that all cartographic maps are valid only “in the small”, i.e. locally, is a typical property of the description of geometric manifolds.

3.5.3 Boolean Manifolds

The basic idea to discuss a globally non-Boolean description is to introduce a *Boolean atlas* as a structured family of Boolean algebras, called *Boolean charts*. These Boolean charts are patched together by smooth maps such that overlapping Boolean charts are compatible. A Boolean atlas carries all relevant information of a non-Boolean description.¹⁵

A Boolean manifold is a structured family of partially overlapping Boolean algebras that are pasted together such that the Boolean operations agree on their overlaps so that the manifold looks locally like a Boolean algebra. It is natural to require that the various Boolean algebras agree concerning their unit and zero elements. Therefore we consider a family $\{\mathcal{B}_\gamma \mid \gamma \in \Gamma\}$ of Boolean algebras $\mathfrak{B}_\gamma = \{\mathcal{B}_\gamma, \vee_\gamma, \wedge_\gamma, \perp_\gamma, \mathbf{0}, \mathbf{1}\}$ with the same smallest element $\mathbf{0}$ and the same greatest element $\mathbf{1}$. With this we arrive at the following definition:

Definition: Boolean manifolds¹⁶

A family of Boolean algebras $\{\mathcal{B}_\gamma, \vee_\gamma, \wedge_\gamma, \perp_\gamma, \mathbf{0}_\gamma, \mathbf{1}_\gamma\}$ is called a *Boolean manifold* if it satisfies the following conditions:

$$(i) \quad \mathbf{0}_\alpha = \mathbf{0}_\beta \text{ and } \mathbf{1}_\alpha = \mathbf{1}_\beta \text{ for all } \alpha, \beta \in \Gamma ; \quad (1)$$

$$(ii) \text{ for all } \alpha, \beta \in \Gamma \text{ there is a } \gamma \in \Gamma \\ \text{such that } \mathcal{B}_\alpha \cap \mathcal{B}_\beta = \mathcal{B}_\gamma ; \quad (2)$$

$$(iii) \text{ if } F, G \in \mathcal{B}_\alpha \cap \mathcal{B}_\beta, \text{ then the Boolean operations} \\ \text{in } \mathcal{B}_\alpha \text{ and in } \mathcal{B}_\beta \text{ coincide:}$$

$$F \wedge_\alpha G = F \wedge_\beta G, \quad F \vee_\alpha G = F \vee_\beta G, \quad F^{\perp_\alpha} = F^{\perp_\beta}. \quad (3)$$

Two elements F, G in a Boolean manifold are *compatible*, if there is a Boolean algebra $\mathcal{B}_\alpha \subset \cup_\gamma \mathcal{B}_\gamma$, such that $F, G \in \mathcal{B}_\alpha$.

Definition: Coherent Boolean manifolds

A Boolean manifold is said to be *coherent* if it fulfils the condition:

$$(iv) \text{ If } F, G \in \mathcal{B}_\alpha, \quad G, H \in \mathcal{B}_\beta, \quad H, F \in \mathcal{B}_\gamma \\ \text{then there is a } \delta \in \Gamma \text{ such that } F, G, H \in \mathcal{B}_\delta. \quad (4)$$

¹⁴The *Map Projection Catalog* at www.ilstu.edu/microcam/map_projections/ contains 317 educational-quality *Map Projection Graphics* to be downloaded.

¹⁵Domotor (1974), Hardegree and Frazer (1981, p. 60), Lock and Hardegree (1984).

¹⁶Hardegree and Frazer (1981, p. 63), Lock and Hardegree (1984), Hughes (1985a).

3.5.4 Partial Boolean Algebras

Elements in a Boolean manifold $\{\mathcal{B}_\gamma \mid \gamma \in \Gamma\}$ which belong to a common Boolean subalgebra are said to be *compatible*.

Definition: Compatibility relation

We write $\mathcal{P} = \cup_{\gamma \in \Gamma} \mathcal{B}_\gamma$ and define a compatibility relation \leftrightarrow on \mathcal{P} for two elements $F, G \in \mathcal{P}$ by

$$F \leftrightarrow G \text{ if and only if } F, G \in \mathcal{B}_\gamma \text{ for some } \gamma \in \Gamma .$$

Due to Eq. (3) we can drop the subscripts associated with the join, the meet, and the complementation and introduce operations \vee , \wedge , and $^\perp$:

$$\vee|_{\mathcal{B}_\gamma} = \vee_\gamma , \quad \wedge|_{\mathcal{B}_\gamma} = \wedge_\gamma , \quad ^\perp|_{\mathcal{B}_\gamma} = ^\perp_\gamma .$$

The unary complementation $^\perp$ is defined over all \mathcal{P} , while the binary operations \vee and \wedge are partial operations on \mathcal{P} with the domain \leftrightarrow :

$$\text{domain}(\vee) = \text{domain}(\wedge) = \leftrightarrow .$$

According to Eqs. (2,3) we can also drop the subscripts associated with the minimum and maximum element. If $\{\mathcal{B}_\gamma \mid \gamma \in \Gamma\}$ is a *coherent* Boolean manifold, the resulting structure $\{\mathcal{P}, \leftrightarrow, \vee, \wedge, ^\perp, \mathbf{0}, \mathbf{1}\}$ is a *partial Boolean algebra* (Hughes 1985a,b).

Definition: Partial Boolean algebra

If $\{\mathcal{B}_\gamma \mid \gamma \in \Gamma\}$ is a *coherent* Boolean manifold, then the algebra $\{\mathcal{P}, \leftrightarrow, \vee, \wedge, ^\perp, \mathbf{0}, \mathbf{1}\}$ with $\mathcal{P} = \cup_{\gamma \in \Gamma} \mathcal{B}_\gamma$ is a *partial Boolean algebra*.

The notion of a *partial Boolean algebra* is a generalization of the concept of a Boolean algebra. Roughly speaking, a partial Boolean algebra is a collection of overlapping Boolean algebras in which not every pair of elements may belong to a common Boolean subalgebra. In contrast to a Boolean algebra, the binary operations \vee and \wedge are *partial* operations which are defined only for certain elements of a partial Boolean algebra.¹⁷ Nevertheless, all elements of a partial Boolean algebra are connected:

¹⁷Strauss (1936) was the first to propose a predicate logic with a partial Boolean algebra as a logical codification of Bohr’s ideas on complementarity (compare also Strauss 1938, 1967, 1970, 1973). His proposal differs from that of Birkhoff and von Neumann (1936) who require that the structure of propositions is a non-distributive lattice and, thus, that the conjunction of two meaningful proposition should again be a meaningful proposition. In contrast, Strauss admits the distributive law but denies that the logical connectives $F \vee G$ and $F \wedge G$ are meaningful whenever F and G are meaningful. His important contribution remained hidden to most scientists for a long time. Nowadays Strauss’ work is easily accessible in a volume collecting his most important papers, all in English translation, some with new postscripts (Strauss 1972).

when two or more Boolean algebras overlap, their operations agree with each other.

According to the *axiomatic characterization* by Kochen and Specker (1965b) a *partial Boolean algebra* $\{\mathcal{P}, \leftrightarrow, \vee, \wedge, \perp, \mathbf{0}, \mathbf{1}\}$ is an algebraic structure where \mathcal{P} is a set with two distinguished elements $\mathbf{0}$ and $\mathbf{1}$. The binary partial relation \leftrightarrow is symmetric and reflexive (but not transitive) and is called *compatibility*. The *disjunction* \vee (the “Boolean sum”) and *conjunction* \wedge (the “Boolean product”) are two *partial* binary relations defined only for pairs of compatible elements in \mathcal{P} . They have domain \leftrightarrow and range \mathcal{P} . The *negation* \perp is a unary relation defined for every element in \mathcal{P} . Apart from the partial definability of the logical relations, a partial Boolean algebra satisfies all the relations of a Boolean algebra. In particular, in any partial Boolean algebra the following relations hold for every $F, G, H \in \mathcal{P}$:

- $F \leftrightarrow \mathbf{0}, F \leftrightarrow \mathbf{1}, F \leftrightarrow F,$
- if $F \leftrightarrow G$, then $G \leftrightarrow F,$
- if $F \leftrightarrow G$, then $F \leftrightarrow G^\perp,$
- if $F \leftrightarrow G$, then $G \vee F$ and $G \wedge F$ are defined,
- if $F \leftrightarrow G \leftrightarrow H \leftrightarrow F$, then F, G, H together with $\mathbf{0}, \mathbf{1}$ generate a Boolean algebra relative to operations \vee, \wedge and $\perp,$
- if $F \leftrightarrow G \leftrightarrow H \leftrightarrow F$, then $F \vee G \leftrightarrow H.$

Every Boolean algebra $\{\mathcal{B}, \vee, \wedge, \perp, \mathbf{0}, \mathbf{1}\}$ is a partial Boolean algebra with $\leftrightarrow = \mathcal{B} \times \mathcal{B}$

3.5.5 Transitive Partial Boolean Algebras

In a partial Boolean algebra $\{\mathcal{P}, \leftrightarrow, \vee, \wedge, \perp, \mathbf{0}, \mathbf{1}\}$ one can define for all elements of \mathcal{P} a binary relation \leq by

$$F \leq G \text{ if and only if } F \vee G = G, \quad F, G \in \mathcal{P}.$$

Note that $F \leq G$ only if $F \leftrightarrow G$. The relation \leq is reflexive and anti-symmetric, but it need not be transitive so that a partial Boolean algebra is in general not a partially ordered set. A Boolean algebra is said to be transitive if the relation \leq is transitive:

Later, partial Boolean algebras were first systematically developed by Kamber (1964) and in a series of papers by Kochen and Specker (1965a,b, 1967). The interrelations of the lattice approach and the partial Boolean algebra approach to quantum logics have been investigated by Czelakowski (1974, 1975). As discussed by Specker (1960), partial Boolean algebras arise naturally in situations with propositions which are not simultaneously decidable.

Definition: Transitive partial Boolean algebra¹⁸

A partial Boolean algebra $\{\mathcal{P}, \leftrightarrow, \vee, \wedge, \perp, \mathbf{0}, \mathbf{1}\}$ is *transitive* if for all F, G, H the relations $F \leq G$ and $G \leq H$ imply $F \leq H$.

In transitive partial Boolean algebras the relation \leq is a partial order, with the greatest element $\mathbf{1}$ and the smallest element $\mathbf{0} = \mathbf{1}^\perp$.

3.6 Classical Modes in Quantum Mechanics

Quantum physics describes material reality in terms of a transitive partial Boolean algebra.¹⁹ That this structural property of quantum mechanics provides the proper relationship between classical and quantum mechanics has been acknowledged only more than half a century after the creation of quantum mechanics. The crucial point characterizing this relation is the *partial Boolean structure of this theory, not any limiting procedures*.

The idea that the formalism of quantum theory should contain classical mechanics as a limiting case was expressed by Max Planck (1906, p. 143):

The classical theory can simply be characterized by the fact that the quantum of action becomes infinitesimally small.

Planck's correspondence principle and the later amplifications by Niels Bohr played an outstanding role in the historical development of quantum mechanics but do not get to the core of the matter. Popular expansions in powers of Planck's constant make conceptually no sense – Planck's constant is not a dimensionless parameter but a fundamental physical constant which cannot “go to zero”.²⁰

From the modern point of view, quantum mechanics is the basis of all physics, the microscopic and the macroscopic. That is, all so-called “classical systems” are quantum systems in a very special context. Thereby Planck's constant \hbar retains its physical value. Such systems can be described in terms of Kähler manifolds which then play the role of a classical symplectic phase space. For example, a symplectic manifold can arise as the manifold of appropriate coherent states which via Klauder's

¹⁸Compare Kochen and Specker (1965a).

¹⁹The still more popular description of the propositional structure of quantum mechanics as a non-distributive complete, irreducible, atomic, orthomodular and semi-modular lattice is mathematically (but not conceptually) equivalent to the description by a transitive partial Boolean algebra.

²⁰Many conceptually very different limiting procedures have been discussed extensively in the last sixty years, for example: the limit of high quantum numbers, the limit of vanishing Planck's constant, the macroscopic limit of very many degrees of freedom, the high temperature limit. Most textbook treatments of such limits are incomplete, often misleading and not too seldom plainly erroneous. Since the fictitious limit $\hbar \rightarrow 0$ does not exist in the norm topology, there is no universal classical limit of quantum mechanics.

variational and correspondence principle gives rise to an effective classical Hamiltonian description with classical nonlinear equations of motion (Klauder 1962, 1967, 1981, 1986).

The fact that quantum mechanics allows without any limiting procedures a contextual, locally classical description of some features of quantum systems is not at all trivial. It is due to the partially Boolean character of quantum mechanics. Without such a structure there would be no link between the fundamental quantum description and classical engineering science and experimental investigations.

Example: The astronomical Kepler problem

A fully quantum-theoretical description of the astronomical Kepler problem (e. g. the motion of the earth around the sun in a gravitational field) can be given in terms of the $O(4)$ -coherent states (as defined by Barut and Xu, 1993). Provided that proper initial conditions are chosen, these Keplerian coherent quantum states do not spread under the time evolution and mimic with great accuracy all Kepler orbits familiar from celestial mechanics. This description depends on Planck's constant \hbar , but nevertheless it is empirically indistinguishable from the motion of the Kepler problem of classical mechanics.

4. Parts and Wholes

4.1 A Remark on Different Concepts of Holism

There are at least two conceptually different concepts of holism. To avoid a verbal confusion between them I will call them *Boolean holism* and *non-Boolean holism*, respectively. Boolean holism is most succinctly expressed by Hegel's doctrine (Hegel 1969, p. 167):

The whole consists of parts.

This type of holism is usually considered as characterizing the central issue of systems science:²¹

General system theory is a general science of "wholeness", [in which] the whole is more than the sum of its parts.

But in spite of the claims of their advocates,²² general system theory cannot describe holistic situations as we know them for example in the

²¹Compare Bertalanffy (1968, pp. 37, 55). This phrase is usually considered as characterizing systems science and is commonly but improperly attributed to Aristotle (Metaphysica, book VIII, 1045a). More relevant would be a reference to Plato: "The all is not the whole" (Theatetus, 204B), where the context shows that "the all" means the sum of its parts.

²²Compare the boastful claim that "general system theory deals with the most fundamental concepts and aspects of systems" (Mesarovic and Takahara 1975, p. 1).

quantum world. The reason is the tacit presupposition that it makes sense to describe a “system” as a collection of “things”.

In contrast, *non-Boolean holism refers to a whole which has no parts*. Since there are no pre-existent parts, the main problem of a genuinely non-Boolean description is the choice of an appropriate partition of the universe of discourse. First of all, we have to give up the deeply seated presupposition that the universe of discourse is composed of individuals. *Decompositions of the world are neither given a priori nor determined by first principles*. As Suzanne Langer (1978, p. 273) observed:

Our world divides into facts because we so divide it.

Every description of a holistic universe of discourse requires a partition adapted to a particular context. This partition breaks the holistic unity of the world and requires the introduction of *correlations between such contextually chosen subsystems*. In a non-Boolean holistic world there are infinitely many possible, mutually incompatible decompositions of the whole world into parts.

4.2 Holistic Correlations

From our everyday experience we believe to know that certain things are quite independent of others, notably those distant in time or space. In physics, this observation has been elevated to the so-called *spatio-temporal separability principle*.²³ According to Howard (1989, pp. 225–226), it

asserts that the contents of any two regions of space-time separated by a nonvanishing spatio-temporal interval constitute separable physical systems, in the sense that (1) each possesses its own distinct physical state, and (2) the joint state of the two systems is wholly determined by these separate states.

From an operational viewpoint, two physical objects are considered as separated if and only if an experiment performed on one of the systems does not change the state of the other one. Einstein argued forcefully for the general validity of this principle of separation (“Trennungsprinzip”) but there is no convincing reason for its approval. We are so accustomed to accept the separability of nature as something self-evident that we easily forget how artificial this doctrine really is. The fact that quantum

²³This principle is independent of both the “principle of local action” (“Nahwirkungsprinzip”, due to which physical interactions propagate only from points to points in an infinitesimal neighborhood) and “Einstein causality” (“no physical effect can propagate faster than light”). Unfortunately, the three entirely different principles are often referred to as “locality principles”. Compare the reviews by Howard (1985, 1990).

mechanics violates Einstein's separability principle²⁴ appears as counter-intuitive only if one adopts an atomistic world view.

The nonseparability predicted by quantum theory is usually described by *entanglement*, a term introduced by Erwin Schrödinger.²⁵ His historical characterization still adopts an atomistic ontology, assuming that the quantum world consists of parts. From the modern viewpoint it is therefore somewhat misleading to speak of "an entanglement of quantum systems", since subsystems have no independent existence. Moreover, genuine holistic correlations are not restricted to physical systems. They are independent of Planck's constant of action, they are independent of spatial separations, and do not arise from known physical forces. They cannot be reproduced by any system with a Boolean logical structure. In systems which allow a Boolean description there are no entanglements.

Entanglement is not *The Greatest Mystery in Physics*,²⁶ but one of the most often misunderstood and misrepresented concepts – even in physics, where the related experiments form already an important part of modern technology. It is an inevitable and generic consequence of partitioning a non-Boolean whole which has no parts. If we accept the insight that there are no ultimate building blocks, there is nothing spectacular or controversial about the existence of holistic correlations. In particular, holistic correlations require no explanations in terms of "forces".

Since there is no *a priori* given decomposition of the world, *entanglement is a contextual concept*. It always refers to a particular, contextually chosen decomposition of a whole. The degree of entanglement can be changed by changing the partition.

5. Complementarity of Mind and Matter

5.1 On the Distinction Between Mind and Matter

Even if the world does not consist of parts, we can consider *notional distinctions* and therewith create patterns, thus providing a *partial description* of the world. For a scientific description of the external world some kind of a mind-matter distinction is inevitable, but is not "given",

²⁴The nonseparability of the material world as predicted by quantum mechanics has been verified beyond any reasonable doubt in a series of beautiful experiments in recent years.

²⁵"When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." (Schrödinger 1935, p. 555)

²⁶This is the title of a popular book by Aczel (2001).

or “natural”. The traditional characterization of the mental and physical domains does not allow us to construct a workable theory for the mind-matter problem.

Even the answer to the apparently easy question “what is matter?” has changed dramatically several times since 1641, when in his *Meditationes de Prima Philosophia* Descartes characterized matter by the essential attribute of extension (*res extensa*). In terms of modern physics, Pauli (1954) spoke of “matter as an aspect of the nature of things”:

Matter has always been and will always be one of the main objects of physics. . . . even light has become matter now, due to Einstein’s discoveries. It has mass and also weight; it is not different from ordinary matter, it too having both energy and momentum. . . . Taking the existence of all these transmutations into account, what remains of the old ideas of matter and substance? The answer is *energy*. This is the true substance, that which is conserved; only the form in which it appears is changing.

A fundamental theorem by Emmy Noether (1918) says that for every continuous symmetry there exists a conservation law, and for every conservation law there exists a continuous symmetry. By definition, the conserved quantity related to the time-translation symmetry is called *energy*. That is, *the conservation law of energy holds if and only if the corresponding equations of motion are invariant under time translations*.

Time has two aspects which refer to two mutually exclusive, *complementary* ways of looking at a fundamental ordering principle. For a conceptually clean specification of the initial conditions of physical experiments, the homogeneous parameter time of physics has to be complemented by a time with nowness. The pertinent time concepts were introduced by McTaggart as A-time and B-time.²⁷ The tensed A-time assigns a privileged position to the Now, the present moment. It involves the relationship between past (“before Now”), present (“concurrent with Now”), and future (“after Now”). In contrast, the time used to formulate the fundamental laws of physics is purely sequential, described by a tenseless *relational parameter* (McTaggart’s B-time), and characterized by the relations “earlier than”, “later than”, and “simultaneous with”.

All known fundamental principles of physics refer to laws which are invariant under time translations. That is, *the fundamental laws of physics do not contain any tensed notions*. In contrast, one of the most distinguishing qualities of consciousness is the *Now*.²⁸ According to Bertrand Russell the present, past and future arise only from the fact that we apply

²⁷See McTaggart (1908). For a survey of the many philosophical controversies about theories of tensed and tenseless time compare Oaklander and Smith (1994).

²⁸“An entity is said to be now if it is simultaneous with what is present to me, i.e., with this, where ‘this’ is the proper name of an object of sensation of which I am aware.” (Russell 1915, p. 213)

a time relation between subject and object. He suggested to distinguish *mental time* and *physical time* (Russell 1915, p. 212):

It will be seen that past, present, and future arise from time-relations of subject and object, while earlier and later arise from time-relations of object and object. In a world in which there was no experience there would be no past, present, or future, but there might well be earlier and later. Let us give the name of mental time to the time which arises through relations of subject and object, and the name physical time to the time which arises through relations of object and object.

Since a description of the relations between subject and object requires tensed time, *tenseless physics with a homogeneous time cannot give a complete description of the world.*

The division of the world into a tensed domain and a tenseless domain refers to a *description*, not to a state of affairs. This distinction is motivated by our desire to get context-independent, non-indexical universal “first principles”.²⁹ It is therefore not given *a priori*, but generated by an *epistemic* breaking of the holistic symmetry of the universe of discourse. The tensed domain is supposed to contain the mental domain, while the tenseless domain refers to *first principles* describing matter and energy. However, the tenseless domain is not identical with physics, it more resembles Plato’s non-temporal *world of immutable ideas*. What is missing is the concept of the Now, among others necessary for the specification of initial conditions of physical experiments.

5.2 The Role of Tensed Time in Experimental Physics

In his early writings on time Russell (1915) argued that the notion of temporal becoming has no objective counterpart and, consequently, that it is psychological or illusory. Yet, to dismiss the Now as a subjective illusion is a mistake: *science is also concerned with tensed time.*

Every laboratory experiment presupposes a distinction between contingent facts and natural laws. This fact is presumably the most consequential conceptual insight by Isaac Newton, implying a necessary division of the world into a tensed and a tenseless domain:³⁰

Any description of the world has to be separated into two categories:
contingent initial conditions and *universal laws of nature.*

²⁹Note that there are no context-independent time-irreversible processes – all such processes contain non-fundamental parameters (like relaxation times, half-lives, etc.). Among other things, this fact implies that it is not possible to introduce an “arrow of time” at a fundamental level.

³⁰Compare Houtappel *et al.* (1965, p. 596), Wigner (1964). The distinction between initial conditions and laws of nature can be found in Newton’s *Philosophia Naturalis Principia Mathematica*, even though only in an implicit fashion.

The laws of nature determine that, if a physical system is in state Σ_1 at time t_1 , it is in state Σ_2 at time t_2 . Yet, there are no natural laws which determine the initial state Σ_1 of the system.³¹ We will call the separation between contingent initial conditions and natural laws the *Newtonian cut*. This distinction has been a paradigm of physics for centuries. According to Einstein and Wigner “it would mean an enormous change of our concepts if the laws of nature and initial conditions could not be separated.”³² The Newtonian cut is always taken for granted in experimental physics, but is not without problems.

In Cartesian physics it was presumed that God created the universe as a system of particles in motion, thereby fixing its initial conditions, so that the cosmos runs like a clockwork with no divine intervention after creation. While the Cartesian world view denies the possibility of free actions, Newton was prone to admit intentional agents.³³ He introduced some supramechanical “active principles” which were produced by the intentional activity of spirits. That is, Newton never envisaged a “block universe”, complete and whole for its entire history. What we carelessly call “Newtonian physics” is an illegitimate extrapolation of Newton’s work. Modern classical mechanics of conservative systems is not Newton’s physics. It refers to strictly isolated systems, not to the actual universe containing intentional agents.

Experimental physics demands the distinction of past and future, the concept of the Now, the freedom of the experimenter to manipulate (within appropriate limits) initial conditions,³⁴ and the feasibility to repeat experiments at any particular instant. Without this freedom it would not be possible to test, verify or falsify theories. Every stimulus-response experiment necessitates a Newtonian cut which breaks the time-translation symmetry of the physical laws assumed to hold in the tenseless domain. On the basis of the time-translation invariant first principles of physics it is impossible to introduce the concept of a Now. The thesis that the physical realm is causally closed is, thus, untenable if experimental physics is included. However, the fact that any description of the behavior of matter based on first principles alone is fundamentally incomplete does not imply that these principles are inconsistent or invalid. It only implies that they cannot account for deliberately chosen initial conditions.

³¹The Laplacian assumption that every event is causally determined by events taking place at some antecedent times (a view still defended by McAllister 1999) is without justification since there are unending sequences of events with a limit (for details compare Jordan 1963, p. 23).

³²Discussion remark by Wigner, in Doncel *et al.* (1987), p. 631.

³³Compare Newton (1729), pp. 543–544, Newton (1730), pp. 397 ff. For more details compare Kubrin (1967), Force (2004).

³⁴It is important to distinguish between nondisturbing *observations* and *creations* of initial conditions. For example, the initial conditions used by astronomers are not freely selectable, but are relative to an antecedent system determined by universal laws.

The initial conditions generated by the experimenter introduces the Now into *experimental* physics, bringing about quantifiable physical effects into the material world. Therefore, the Now does not exclusively refer to the subjective present of the experimenter, but to a *universal Now of the tensed domain*. According to Heidegger (2001, p. 48),

the ‘now’ is neither something first found in the subject, nor is it an object which can be found among other objects, as for instance this table and this glass. Nevertheless, at any given time the spoken ‘now’ is immediately received-perceived jointly by everyone present. We call this accessibility of ‘now’ the publicness [Öffentlichkeit] of ‘now.’

Subjective time is a special case of mental time. In the same sense as the brain is an enormously complex structured piece of matter, subjective time is mental time with a complex additional structure.

In experimental physics the Now originates from an intentional action of an experimenter, and not from an observation. However, according to the so-called “Copenhagen interpretation”, a measurement allegedly implies a state reduction (a “reduction of the wave packet”), which is said to occur whenever a scientist makes an observation of the quantum system, or even, when there is an interaction between the physical measuring apparatus and the psyche of some observer.³⁵ Such claims are in flagrant contradiction with experimental practice. In most modern experiments the data of measurements are not directly observed by a human observer but collected automatically and stored in the memory of a computer. Before anybody takes note of measured results, these data can be automatically duplicated arbitrarily often and sent around. Up to this point, no human observer is involved.

Moreover, the measurement process is not instantaneous. The idea of “quantum jumps” goes back to an old idea of Niels Bohr (1913), suggesting that light and matter interact in such a way that an atom jumps instantaneously between stationary states and thereby absorbs or emits a light quantum. The question whether these transitions, later called state reductions, really take place or are just a useful metaphor to describe some phenomena, has been a contentious subject of debate since the early days of Bohr’s model of the hydrogen atom. As shown by many authors, *all experiments can be explained without ever using the fiction of*

³⁵This popular misinterpretation should be contrasted with the irreproachable formulation by Pauli (1933, Ziff. 9, p. 148): “Eine solche Setzung einer physikalischen Tatsache . . . ist ein besonderer, naturgesetzlich nicht im voraus determinierter Akt, dem nachträglich durch Reduktion der Wellenpakete . . . Rechnung zu tragen ist”. Here Pauli refers to a *posit of a fact* which *subsequently* has to be taken into account by a reduction of the wave packets. Unfortunately the English translation (Pauli 1980, Sect. 9, p. 71) of Pauli’s penetrative German is misconceived and erroneous.

*state reductions.*³⁶ Modern quantum mechanics can rigorously describe the measurement process as a *continuous* natural physical process which transforms a pure state asymptotically into a classical mixture of disjoint pure quantum states, independently of any attention of an observing scientist.³⁷ *Measurements with asymptotically disjoint final states do not require an infinite measuring time.* In special cases, the effective measuring time can be very short, but there are no quantum jumps. The so-called “state reduction” is simply a short-cut notion for an inherently continuous measurement. We conclude, that *the Now in experimental physics is not related to a “state reduction”, but appears exclusively in the context of the preparation of the experiment.*

5.3 Complementarity of Mental and Physical Time

The contention that physical laws are deterministic applies only to the tenseless domain with its time-translation invariant laws. The associated time-evolution group is given by one-parameter group of automorphisms which only change the point of view, so that in the tenseless domain *nothing happens.*³⁸ For that reason the first principles of physics do not describe “time” as something that “flows”. The Now in experimental physics refers to the initial conditions chosen by the experimenter, that is, to the tensed domain. The so-called “flow of time” is a relational concept, *it refers to the relation between tensed and tenseless time.* Neither the mental nor the physical time does “flow” or “pass”. The view that the Now is passing in a block universe is equivalent with the view that the Now is resting and the material states are flowing. *Becoming is a relative notion.*

Since tensed and tenseless time refer to different domains, the still prevailing discussion whether the tensed or the tenseless theory of time is “true” makes no sense. Physical time is a crucial element in theoretical physics, but the experienced time cannot be dismissed as irrelevant for the understanding of physics. These two concepts of time are not contradictory but *complementary*. None of them is sufficient, none can replace the other, both are necessary.

³⁶A large number of publications to this controversial topic are referred to in a paper by Beige *et al.* (1997); compare also Beige and Hegerfeldt (1996). As reported by Margenau (1962), von Neumann himself regarded the projection postulate “as pictorially useful and consistent with the axioms of quantum mechanics . . . but also as avoidable.”

³⁷See Lockhart and Misra (1986). Further comments and references can be found in Primas (1987, 1997, 2000).

³⁸Every description in terms of a well-defined mathematical structure has to use irrelevant elements. Two descriptions which differ only by irrelevant elements but leave the relevant structure invariant are called isomorphic. An isomorphism of the mathematical structure to itself is called an automorphism. That is, an automorphism is nothing but a logical symmetry.

According to quantum mechanics, the material domain does not allow a comprehensive Boolean description. For the mental domain we have no satisfactory theory. If the tensed domain would allow a comprehensive Boolean description, then the mind-matter distinction would be given *a priori*. If we reject this possibility as unnatural, then it follows that both the tensed and the tenseless domain require a non-Boolean description so that we have to expect that these two domains are holistically correlated. In this case there are mind-matter correlations which are not due to interactions. Yet, mental time cannot be reduced to physical time (or the other way round). However, it is possible that mental time and physical time are *synchronized* by holistic correlations.³⁹ If we synchronize the physical situation in the tenseless domain with the Now of the tensed domain, the time describing matter behaves as a stochastic time which *never has a sharp value*.

Since breaking the time-inversion symmetry in the tenseless domain implies the *emergence* of a Now, we may conjecture that physical time and experienced time are in some way related. An obvious idea is that the time-inversion group mediates between the tenseless and the tensed domains. To obtain a full account of the relations between physical and the mental time a proper mathematical formulation of the two aspects of time is inevitable. Although challenging, this issue remains open.

5.4 Physics Refutes the Idea of a Causally Closed World

A popular but scientifically unfounded metaphysical doctrine is the idea of the *causal closure* of the physical (i.e. non-mental) world.⁴⁰ To say that the physical world is “causally closed” is to claim that the only causes of physical events are other physical events. Causal closure is often defended on *a priori* grounds, claiming that it is indispensable for a scientific world view. In addition, many philosophers argue that we have good empirical evidence that the domain of physical phenomena is causally closed, or that this is at least very plausible.

Such claims are without any support from physics. No serious physical theory makes the overarching claim that the physical laws uniquely

³⁹A quantum-theoretical model that proves the feasibility of a non-causal synchronization of tensed and tenseless time by a *maximal entanglement* with respect to the tensorization into a tensed and a tenseless domain has been discussed by Primas (2003). In this model all material observables commute with the mathematically well-defined time operator. Hence all observables of the material domain correspond to *substance observables* as defined by Römer (2006), while the observables of the mental domain correspond to *process observables*.

⁴⁰Chalmers (1996, p.150) exploits the following rather vacuous characterization: “The physical world is more or less causally closed, in that for any given physical event it seems there is a physical explanation (modulo a small amount of quantum indeterminacy).” Without giving any arguments, Heil (1998, p. 23) claims that “modern science is premised on the assumption that the material world is a causally closed system. . . . A natural law is exceptionless.”

determine the behavior of the whole material world. As Hermann Weyl (1932, p. 45) emphasized:

Physics has never given support to that truly consistent determinism which maintains the unconditioned necessity of everything which happens.

The first principles of physics are methodological regulative principles which refer to *strictly isolated systems*, not to the real world.⁴¹ No part of the world can be faithfully represented by an isolated physical system. But under appropriate circumstances the behavior of a part of the world can be modelled mathematically by a physical system. Whether the initial state of such a model system uniquely determines the motion depends on the choice of state space of the mathematical model. Under rather general conditions one can construct a state space such that the temporal behavior is given by a well-posed dynamical system whose present state determines uniquely the state at any future time.⁴² In such a dynamical system the state space can be extended in such a way that the dynamics is given by a one-parameter group of automorphisms, describing a *fictitious hidden determinism* (Primas, 2002).

However, there is no reason to assume that such a locally possible notional deterministic description can be extended to a global dynamical system for the whole material world. We conclude that *determinism is a property of local theoretical models, and not a feature of the world*. Therefore the prevailing opinion that the physical state of the material universe at any one time determines the state of the universe at all other times is both theoretically and empirically unfounded. *Both “Hadamard’s principle of scientific determinism” and the doctrine of the “causal closure” are not demonstrable theses but regulative principles to guide us in theory construction*. A regulative principle does not say anything about reality, it is accepted “as a guiding maxim, a regulative idea that gives [the scientist’s] work its logical coherence and its systematic unity” (Cassirer 1944, p. 219).

The idea of strict determinism is related to the principle of logical bivalence (according to which every statement is either true or false). Lukasiewicz has shown that a bivalent system of logic is adequate for the language of the deterministic world view, but is no longer suitable for the investigation of a non-deterministic world view (for details see Jordan

⁴¹This and the following considerations apply to both classical and quantum physics, as well as to individual and to statistical determinism.

⁴²This so-called Nerode state can be considered as a kind of memory, representing the minimal amount of information about the past history of the system which suffices to predict the effect of the past upon the future. It is defined as the equivalence class of all histories for $t < 0$ of the system which give rise to the same behavior for all conceivable future. This state representation fulfills *Hadamard’s principle of scientific determinism*.

1963). Therefore we cannot expect the global validity of determinism for a system which requires a non-Boolean description.

5.5 Physical and Mental Objectivity

According to Moritz Schlick (1985, p. 303) the doctrine of causal closure “rests on the the fact that the natural sciences must banish the sense qualities from their completed conceptualization”. Similarly, Niels Bohr claims⁴³

that what we have really learned in physics is how to eliminate subjective elements in the account of experience, and it is rather this recognition which in turn offers guidance as regards objective description in other fields of science. To my mind, this situation is well described by the phrase ‘detached observer’ . . .

But Pauli emphatically rejected descriptions which do “not contain any explicit reference to the actions or the knowledge of the observer”⁴⁴ and conjectured “that the observer in present-day physics is still too strongly detached and that physics will depart still further from the classical paradigm.”⁴⁵

Thomas Nagel (1986, p. 18) distinguishes “physical objectivity” (a maximally detached point of view, aspiring to conceive the world *sub specie aeternitatis*) from “mental objectivity” (a first person perspective, referring to “things which can only be understood from the inside”). He asserts that reality is more than just objective reality (Nagel 1979, p. 211) and challenges the view that the objective perspective is the correct one. Since the first-person perspective cannot be stated in terms of external truth conditions, mental objectivity is distinct from and irreducible to physical objectivity. Nagel (1979, p. 211) proposes “to resist the voracity of the objective appetite”:

Perhaps the best or truest view is not obtained by transcending oneself as far as possible. Perhaps reality should not be identified with objective reality.

For a complete understanding of the world, both the objective and the subjective descriptions are required; neither of these perspectives is clearly superior. But Nagel (1979, p. 212) believes

that a single world cannot contain both irreducible points of view and irreducible objective reality - that one of them must be what there really is and the other somehow reducible to or dependent on it. This is a very powerful idea. To deny it is in a sense to deny that there is a single world.

⁴³Letter by Bohr to Pauli of March 2, 1955 (letter 2035 in von Meyenn 2001, p. 136).

⁴⁴Letters by Pauli to Bohr of February 15, 1955 (letter 2015 in von Meyenn 2001, p. 104) and of March 11, 1955 (letter 2041 in von Meyenn 2001, p. 148).

⁴⁵Translated from Pauli (1957, p. 42).

For that reason Nagel (1986, p. 6) points out that

the two [objective and subjective] standpoints cannot be satisfactorily integrated, and in these cases [Nagel believes] the correct course is not to assign victory to either standpoint but to hold the opposition clearly in one's mind without suppressing either element.

Nagel's *view from nowhere* conception of objectivity is problematic since he adopts tacitly the idealized framework of classical physics with its Boolean logical structure. In contrast to Nagel's presuppositions, the material world is not reducible to a viewer-free description. Every experimental result depends on a particular Boolean context. One of the deepest insights of quantum theory is the result that the totality of all experimental facts can only be represented in a *globally non-Boolean* (but locally Boolean) theory. That is, experimental facts always refer to a *view from somewhere*. Since every view presupposes a subject, it is natural to call a conception subjective if it is represented from a specific point of view. In this sense we can understand objectivity as "a virtuous way of negotiating perspectival subjective views" (Chrisley 2001).

Nagel's conclusion that mental and physical objectivity cannot be integrated into a coherent common conception is tacitly based on a *tertium non datur* rule for these two kinds of understanding. If we "require no more of objectivity than what we require of physical theories concerning physical facts" (Chrisley 2001), then a unified non-Boolean description of mind and matter becomes feasible. In such a framework of non-Boolean descriptions subjectivity and objectivity are complementary Boolean structures so that first-person and third-person accounts are *complementary* rather than contradictory. Moreover, the fact that each subject's perspective is "just one manifestation of the mental" (Nagel 1986, p. 18), that particular aspects of the subjective can be intersubjectively available (e.g. the Now), and that "the distinction between more subjective and more objective views is really a matter of degree" (Nagel 1986, p. 5) are then natural consequences.

6. Final Remarks

A deeper understanding of the mind-matter problem requires locally Boolean but globally non-Boolean descriptions with their naturally associated holistic correlations. For a holistic universe there is no *a priori* given partition into parts. If we accept the thesis that the most fundamental physical laws are invariant under time translations, then we arrive at a partition of the world into a tenseless and a tensed domain. These two domains and their associated tensed and tenseless time refer to two complementary descriptions. The tensed domain contains the mental domain

and the Now (which is also necessary for specifying initial conditions in experimental physics), while the tenseless domain refers to tenseless natural laws describing matter and energy. Both descriptions have a non-Boolean structure and can be encompassed into a single non-Boolean description. Furthermore, tensed and tenseless time can be synchronized by holistic correlations.

This partition of the universe of discourse into a tenseless and a tensed domain is responsible for the successes and the failures of the traditional scientific approach. In physics it corresponds to the Newtonian cut – the distinction between contingent initial conditions and universal laws of nature. It is based on the problematic assumption of the existence of a kind of Platonic domain with empirically inaccessible “first principles” – an assumption which seems to contradict our experience of the world as it actually is. Sure enough, in contemporary physics we cannot easily give up this idea. It is not an empirical generalization about the structure of the world but in effect a *regulative principle* which allows us to formulate non-indexical and context-independent universal laws.

Any division of the holistic world into two (or more) parts implies that there are global phenomena which cannot be understood in terms of local theories of the parts. The proposed partition of the universe of discourse into a tenseless and a tensed domain is adapted to modern science with its distinction between fundamental theories (with context-independent and non-indexical fundamental laws) and experiments (where the experimenter can choose initial conditions). If one admits that the difference between the tensed and the tenseless domain is not ontologically grounded, then distinctions complementary to the Newtonian tensed/tenseless separation are imaginable. Different distinctions would probably lead to new insights, and most probably create as many problems as they solve. Only the totality of all complementary distinctions can yield a comprehensive picture.

Acknowledgments

I am grateful to Harald Atmanspacher and Peter beim Graben for stimulating discussions and for suggestions how to improve earlier versions of this article.

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Received: 19 October 2006

Accepted: 13 November 2006

Reviewed by Günter Mahler and Karl Svozil