

# A Fast Triangle-Triangle Intersection Test

Tomas Möller

## Abstract

This paper presents a method, along with some optimizations, for computing whether or not two triangles intersect. The code, which is shown to be fast, can be used in, for example, collision detection algorithms.

## 1 Introduction

Most collision detection algorithms, such as OBBTree [Gottschalk96], sphere hierarchies [Hubbard96] and BV-trees [Klosowski97], try to minimize the number of primitive-primitive intersections that have to be computed. Still, a fast and reliable method for computing the primitive-primitive intersection is desired. Since rendering hardware is often targeted for triangles, the primitives in collision detection algorithms are often triangles as well. This paper describes a method for determining if two triangles intersect.

## 2 Intersection Test Method

Let us denote the two triangles  $T_1$  and  $T_2$ ; the vertices of  $T_1$  and  $T_2$  by  $V_0^1, V_1^1, V_2^1$ , and  $V_0^2, V_1^2, V_2^2$  respectively; and the planes in which the triangles lie  $\pi_1$  and  $\pi_2$ .

First, the plane equation  $\pi_2 : N_2 \cdot X + d_2 = 0$  (where  $X$  is any point on the plane) is computed:

$$\begin{aligned} N_2 &= (V_1^2 - V_0^2) \times (V_2^2 - V_0^2) \\ d_2 &= -N_2 \cdot V_0^2. \end{aligned} \quad (1)$$

Then the signed distances from the vertices of  $T_1$  to  $\pi_2$  (multiplied by a constant  $N_2 \cdot N_2$ ) are computed by simply inserting the vertices into the plane equation:

$$d_{v_i^1} = N_2 \cdot V_i^1 + d_2, \quad i = 0, 1, 2. \quad (2)$$

Now, if all  $d_{v_i^1} \neq 0$ ,  $i = 0, 1$  and  $2$  (that is, no point is on the plane) and all have the same sign, then  $T_1$  lies on one side of  $\pi_2$  and the overlap is rejected. The same is done for  $T_2$  and  $\pi_1$ . These two early rejection tests avoid a lot of computations for some triangle pairs. Indeed, for a pair to pass this test there must be some line of direction  $N_1 \times N_2$  that meets both.

If all  $d_{v_i^1} = 0$ ,  $i = 0, 1$  and  $2$ , then the triangles are co-planar, and this case is handled separately and discussed later. If not, the intersection of  $\pi_1$  and  $\pi_2$  is a line,  $L = O + tD$ , where  $D = N_1 \times N_2$  is the direction of the line and  $O$  is some point on it. Note that due to our previous calculations and rejections, both triangles are guaranteed to intersect  $L$ . These intersections form intervals on  $L$ , and if these intervals overlap, the triangles overlap as well. A similar interval test is used in a different context by Laidlaw et al. [Laidlaw86]. Two situations that can occur are depicted in figure 1.

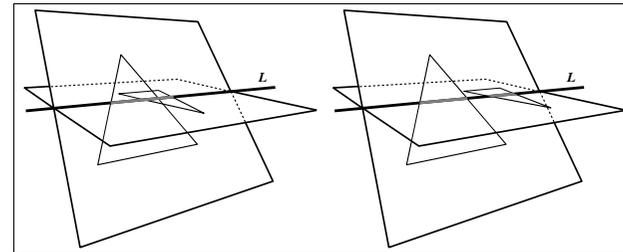


Figure 1: Triangles and the planes in which they lie. Intersection intervals are marked gray in both figures. Left: the intervals along  $L$  overlap as well as the triangles. Right: no intersection, the intervals do not overlap.

Now, assume that we want to compute a scalar interval (on  $L$ ) that represents the intersection between  $T_1$  and  $L$ , and that, for example,  $V_0^1$  and  $V_2^1$  lie on the same side of  $\pi_2$  and that  $V_1^1$  lies on the other side (if not, you have already rejected it). To find scalar values that represent the intersection between the edges  $\overline{V_0^1 V_1^1}$  and  $\overline{V_1^1 V_2^1}$  and  $L$ , the vertices are first projected onto  $L$ :

$$p_{v_i^1} = D \cdot (V_i^1 - O). \quad (3)$$

The geometrical situation is shown in figure 2. Then we want to compute a line parameter value,  $t_1$ , for  $B = \overline{V_0^1 V_1^1} \cap L = O + t_1 D$ . Letting  $K_1^1$  denote the projection of  $V_1^1$  onto  $\pi_2$ , we see that  $\triangle V_0^1 B K_1^1$  and  $\triangle V_1^1 B K_1^1$  are similar, so

$$t_1 = p_{v_0^1} + (p_{v_1^1} - p_{v_0^1}) \frac{d_{v_0^1}}{d_{v_0^1} - d_{v_1^1}}. \quad (4)$$

Similar calculations are done to compute  $t_2$ , and an interval for  $T_2$  is computed as well. If these intervals overlap, the triangles intersect.

If the triangles are co-planar, they are projected onto the axis-aligned plane where the areas of the triangles are maximized. Then a simple two-dimensional triangle-triangle overlap test is performed. First, test all closed edges of  $T_1$  for intersection with the edges of  $T_2$ . If any intersection is found, then the triangles intersect.

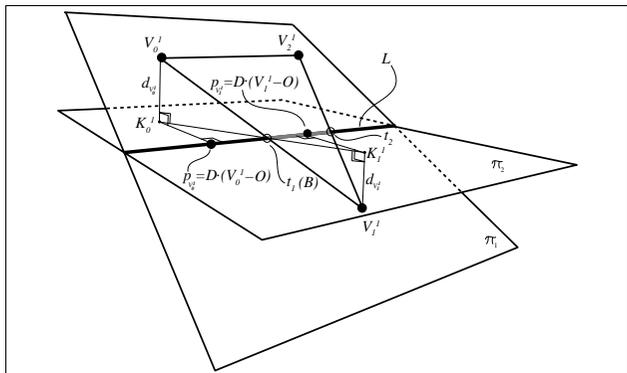


Figure 2: The geometrical situation:  $V_i^1$  are the vertices of  $T_1$ ,  $\pi_1$  &  $\pi_2$  are the planes in which  $T_1$  and  $T_2$  lie;  $d_{v_i^1}$  are the signed distances from  $V_i^1$  to  $\pi_2$ ;  $K_i^1$  are the projections of  $V_i^1$  onto  $\pi_2$ ; and  $p_{v_i^1}$  are the projections of  $V_i^1$  onto  $L$ , which is the line of intersection.

Otherwise, we must test if  $T_1$  is totally contained in  $T_2$  or vice versa. This can be done by performing a point-in-triangle test [Haines94] for one vertex of  $T_1$  against  $T_2$  and vice versa.

## 2.1 Optimizations

Since the intervals can be translated without altering the result of the interval overlap test, equation (3) can be simplified into:

$$p_{v_i^1} = D \cdot V_i^1, \quad i = 0, 1, 2. \quad (5)$$

Therefore  $O$  does not need to be computed.

Also, the result of the overlap test does not change if we project  $L$  onto the coordinate axis with which it is most closely aligned. Therefore equation (5) can be simplified further:

$$p_{v_i^1} = \begin{cases} V_{ix}^1, & \text{if } |D_x| = \max(|D_x|, |D_y|, |D_z|) \\ V_{iy}^1, & \text{if } |D_y| = \max(|D_x|, |D_y|, |D_z|) \\ V_{iz}^1, & \text{if } |D_z| = \max(|D_x|, |D_y|, |D_z|) \end{cases}, \quad i = 0, 1, 2. \quad (6)$$

Here,  $V_{ix}^1$  means the  $x$ -component of  $V_0^1$  and so on. The same principle was used by Mirtich [Mirtich96] in order to get a numerically stable simplification of an integral over a polygon's area.

## 3 Implementation & Performance

To summarize, the steps of the algorithm are as follows (complete C code is available at <http://www.acm.org/jgt/papers/Moller97/>):

1. Compute plane equation of triangle 2.
2. Reject as trivial if all points of triangle 1 are on same side.
3. Compute plane equation of triangle 1.
4. Reject as trivial if all points of triangle 2 are on same side.
5. Compute intersection line and project onto largest axis.
6. Compute the intervals for each triangle.
7. Intersect the intervals.

Note that after step 2, there is enough information to immediately test whether the triangles are co-planar, but because this is a rare occurrence, the test is deferred until after several more frequently occurring rejection tests have been performed.

Robustness problems may arise when the triangles are nearly co-planar or when an edge is near co-planar to the other triangle (especially when the edge is close to an edge of the other triangle). To handle these cases in a reasonable way the source code provides a constant `EPSILON` ( $\epsilon$ ) which the user defines. As a result, if any  $|d_{v_k}| < \epsilon$ , they are reset so that  $d_{v_k} = 0$ . Geometrically, this means that if a point is "close enough" to the other triangle's plane, it is considered as being on the plane. The source code does not handle degenerate triangles (i.e., lines and points). If it did, then those cases would have to be detected first and then handled as special cases.

Performance was measured for several different scenarios in a collision detection program [RAPID97]. Both our method and the method from ERIT [Held97] were found to be faster than the brute-force method,<sup>1</sup> which took between 1.3 and 1.6 times longer to execute. Our method was found to be slightly faster (in RAPID) in most cases when compared to the author's implementation of ERIT's method. We also tested the performance exactly like ERIT and found that our method was approximately 7 percent faster on an SGI Impact. However, on the SGI O2, we found that our method had a slight tendency to be faster for lower collision ratios (i.e. the number of triangle-triangle collisions divided by the number of tests), and the break even point was around 45 percent. Therefore, having worse bounding volumes in a collision detection program implies that our method should perform better than ERIT and vice versa.

Our method is also used in an in-house collision detection package for a commercial VR-platform [Oxygen97].

<sup>1</sup>Here, each closed edge of each triangle is tested for intersection with the other triangle and if, at any time, an intersection occurs, then the triangles intersect. However, this method does not (easily) handle cases where one or three edges are parallel to the plane of the other triangle.

**Acknowledgements.** A special thanks to Stefan Gottschalk (UNC) for many valuable discussions on intersection tests. Constructive criticism and testing on an early version of this work came from Jacob Ström, Per Carleberg, Pär Oscarsson, Patric Fredin and Ola Liljedahl. Finally, thanks to both John F. Hughes and Ronen Barzel for helping me improve the contents this paper.

## References

- [Gottschalk96] S. Gottschalk, M.C. Lin, D. Manocha, "OBBTree: A Hierarchical Structure for Rapid Interference Detection," *Computer Graphics (SIGGRAPH'96 Proceedings)*, pp. 171–180, August, 1996.
- [Haines94] Eric Haines, "Point in Polygon Strategies," in *Graphics Gems IV*, edited by Paul S. Heckbert, Academic Press Professional, Cambridge, MA, pp. 24–46, 1994.
- [Held97] Martin Held, "ERIT – A Collection of Efficient and Reliable Intersection Tests," submitted to *journal of graphics tools*, 1997.
- [Hubbard96] Philip M. Hubbard, "Approximating Polyhedra with Spheres for Time-Critical Collision Detection," *ACM Transactions on Graphics*, vol. 15, no. 3, pp. 179–210, 1996.
- [Klosowski97] James T. Klosowski, Martin Held, Joseph S.B. Mitchell, Henry Sowizral, Karel Zikan, "Efficient Collision Detection Using Bounding Volume Hierarchies of k-DOPs," submitted for publication, *IEEE Transactions on Visualization and Computer Graphics*, 1997.
- [Laidlaw86] David H. Laidlaw, W. Benjamin Trumbore, John F. Hughes, "Constructive Solid Geometry for Polyhedral Objects," *Computer Graphics (SIGGRAPH'86 Proceedings)*, vol. 20, no. 4, pp. 161–168, 1986.
- [Mirtich96] Brian Mirtich, "Fast and Accurate Computation of Polyhedral Mass Properties," *journal of graphics tools*, vol. 1, no. 2, pp. 31–50, 1996.
- [Oxygen97] "Oxygen Base User's Guide 1.0", Gothenburg, Sweden, Prosolvia Clarus AB, 1997.
- [RAPID97] Source code for collision detection by the Research Group on Modeling, Physically-Based Simulation and Applications at University of North Chapel Hill, "RAPID – Robust and Accurate Polygon Interference Detection;" available at <http://www.cs.unc.edu/~geom/OBB/OBBT.html>, 1997.